Mon. Not. R. astr. Soc. (1969) **142**, 213–227.

OBSERVATIONS IN SOME SIMPLE COSMOLOGICAL MODELS WITH SHEAR

P. T. Saunders

(Received 1968 July 11)

Summary

Homogeneous but anisotropic models of Bianchi type I with the two distinct forms of singularity, the 'pancake' and 'cigar', are considered in some detail. It is found that in some directions of observation neither the red shift nor the luminosity distance is a monotonic function of the emission time. Some numerical examples are given and possible consequences with regard to the selection of world models and the problem of quasars are discussed.

1. *Introduction.* There has recently been an increased interest in model universes which are spatially homogeneous but anisotropic. One of the simplest of these models is that in which the group of motions is of Bianchi type I and for which the metric can be written in the form

$$ds^2 = dt^2 - X^2(t)dx^2 - Y^2(t)dy^2 - Z^2(t)dz^2. \tag{1}$$

The solutions of the 'dust' field equations for this model have been found by Heckmann & Schücking (1962) (although they appear to have published only the solution for $\Lambda = 0$) and the special cases in which the model has an axis of symmetry have been further discussed by Kompaneets & Chernov (1964). Jacobs (1968) has solved the field equations with vanishing cosmical constant for several different equations of state and considered the problems of element formation, primordial magnetic fields and the black body background.

In this paper general expressions for the red shift (Kermack, McCrea & Whittaker 1932) and the luminosity distance, number counts and integrated background from sources (Saunders 1968, to be referred to as Paper I) are applied to the particular case of models of this type. These simpler expressions are studied both analytically and numerically and it is shown that a cosmological lens effect exists. Some of the results have been found independently (for the axi-symmetric case only) by Tomita (1968).

2. *Construction of the models.* The field equations for the metric (1) can readily be written down; they are

$$\begin{aligned}
\ddot{X}/X - (\dot{X}/X)^2 + (\dot{X}/X)(\dot{R}/R) &= \Lambda + \tfrac{1}{2}\kappa(\rho - p),\\
\ddot{Y}/Y - (\dot{Y}/Y)^2 + (\dot{Y}/Y)(\dot{R}/R) &= \Lambda + \tfrac{1}{2}\kappa(\rho - p),\\
\ddot{Z}/Z - (\dot{Z}/Z)^2 + (\dot{Z}/Z)(\dot{R}/R) &= \Lambda + \tfrac{1}{2}\kappa(\rho - p),\\
(\dot{X}\dot{Y})/(XY) + (\dot{Y}\dot{Z})/(YZ) + (\dot{Z}\dot{X})/(ZX) &= \Lambda + \kappa\rho,
\end{aligned} \tag{2}$$

where $R^3 = XYZ$ and a dot denotes differentiation with respect to t.

From the first three of these equations it follows that

$$\dot{X}/X = \dot{R}/R + AR^{-3}, \qquad \dot{Y}/Y = \dot{R}/R + BR^{-3},$$
$$\dot{Z}/Z = \dot{R}/R + CR^{-3}, \tag{3}$$

where A, B, C are three constants whose sum is zero. Equations (3) may then be substituted into the last of equations (2) yielding

$$(\dot{R}/R)^2 = \tfrac{1}{3}\Lambda + \tfrac{1}{3}\kappa\rho + \tfrac{1}{2}a^2R^{-6}, \tag{4}$$

where

$$3a^2 = A^2 + B^2 + C^2.$$

Writing the field equations in the form (3) and (4) has several advantages. First, it makes the integration easier, since one has only to solve equation (4) for R and then each of the other equations can be solved separately for X, Y, Z. Also since neither p nor ρ appears explicitly in equations (3) the consequences of a particular choice of equation of state are more easily studied. Finally the three constants A, B, C have a simple physical interpretation. Kristian & Sachs (1966) have shown that if an observer has velocity u^α and if $k^\alpha = dx^\alpha/dv$ (with v an affine parameter) is the tangent to the light ray with which he observes some distant object, then the relation between the red shift z and the distance by apparent size ξ in the given direction is

$$1 + z = u_{\alpha;\,\beta}e^\alpha e^\beta \xi + \tfrac{1}{2}u_{\alpha;\,\beta\gamma}e^\alpha e^\beta e^\gamma \xi^2 + \ \ldots$$
$$e^\alpha \equiv k^\alpha/(u^\alpha k_\alpha). \tag{5}$$

The coefficient of ξ may be interpreted as the Hubble parameter in the given direction. In the present case, if the observer is on a fundamental particle $u^\alpha = \delta_0{}^\alpha$, and it can readily be verified that the Hubble parameters in the three principal directions are given by

$$H_x = \dot{X}/X, \qquad H_y = \dot{Y}/Y, \qquad H_z = \dot{Z}/Z, \tag{6}$$

respectively. The mean value of the Hubble parameter is in this case $\dot{R}/R$. Hence A, B, C are proportional to the variations in the Hubble parameter in the three principal directions. Further, if an observer at the present epoch t_0 observes a root mean square fractional variation ϵ in the Hubble parameter then*

$$a = \epsilon H R_0{}^3, \tag{7}$$

where H is the present observed value of the mean Hubble parameter.

A reasonable equation of state is that of the 'dust-plus-radiation' universe, in which the density is written as the sum of a matter density ρ_m and a pressure density $\rho_p = 3p$. It then follows from the assumption of mass conservation that

$$\kappa\rho_m R^3 = \text{const.} = 6MG, \text{ say},$$
$$\kappa p R^4 = \text{const.} = \beta, \text{ say}, \tag{8}$$

so that finally equation (4) may be written

$$(\dot{R}/R)^2 = \tfrac{1}{3}\Lambda + 2MGR^{-3} + \beta R^{-4} + \tfrac{1}{2}a^2R^{-6}. \tag{9}$$

* A subscript o indicates that the variable is to be evaluated at the present epoch, $t = t_0$.

From this equation it is clear that in all models of this type R varies as $t^{1/3}$ as t tends to zero so that the temperature varies as $t^{-1/3}$ which, as was pointed out by Hawking & Tayler (1966), is a relation sufficient to provide the correct relative helium abundance. The only restriction on the models is that a must be sufficiently large to ensure that the last term dominates for temperatures as low as 10^8 °K. Taking the present temperature as 3° this implies the condition (neglecting the term $\frac{1}{3}\Lambda$ which is insignificant in the dense stage of any realistic model)

$$\epsilon^2 H^2 \geqslant 5 \cdot 5\kappa(\rho_0 + p_0 \times 10^8) \times 10^{-22}, \tag{10}$$

if the other terms are to contribute no more than 1 per cent of the total value of $\dot{R}/R$. (A more detailed description of the problem is given by Thorne (1967a) who discusses the nuclear reactions involved; see also Jacobs (1968).)

The exact solutions for the dust-plus-radiation universe have been obtained by Jacobs (1968) for the case $\Lambda = 0$. These involve elliptic integrals, but solutions in terms of elementary functions do exist in the case $\beta = 0$ (the ' dust ' universes) and it turns out that these are sufficient for the purposes of the present work. To see this one need only verify (again neglecting $\frac{1}{3}\Lambda$) that the condition that the term βR^{-4} never contributes more than 1 per cent of the value of $\dot{R}/R$ is

$$\epsilon^2 H^2 \geqslant \frac{9\kappa}{\rho_0{}^2}\,[66 p_0]^3. \tag{11}$$

Taking $H = 3 \times 10^{-18}\,\mathrm{s}^{-1}$ and $(\rho_p)_0 = 6 \cdot 8 \times 10^{-34}\,\mathrm{g\,cm}^{-3}$ (which is the energy density of the black body background) this condition becomes

$$\epsilon^2 \geqslant 5 \cdot 7 \times 10^{-66}\,\rho_0{}^{-2}. \tag{12}$$

If $\epsilon \leqslant 0 \cdot 01$ the only features which distinguish an anisotropic model from an isotropic one seem to be, at least within the limits of observation in the foreseeable future, phenomena associated with the very early and very dense stages of the universe; since this work is concerned with events at epochs much closer to our own it follows that it will suffice to consider dust models only. The field equations can now be integrated to yield

$\Lambda > 0$:

$$R_i = [(a/\omega)\sinh \omega t + (3MG/\Lambda)(\cosh \omega t - 1)]^{(1/3)-(2/3)\sin \alpha_i} \times [\cosh \omega t - 1]^{(2/3)\sin \alpha_i};$$

$\Lambda = 0$:

$$R_i = [\tfrac{9}{2}MGt^2 + at]^{(1/3)-(2/3)\sin \alpha_i}\,[t^2]^{(2/3)\sin \alpha_i}; \tag{13}$$

$\Lambda < 0$:

$$R_i = [(a/\omega)\sin \omega t + (3MG/\Lambda)(\cos \omega t - 1)]^{(1/3)-(2/3)\sin \alpha_i} \times [1 - \cos \omega t]^{(2/3)\sin \alpha},$$

where

$$R_1 = X, \quad R_2 = Y, \quad R_3 = Z, \quad \omega = \sqrt{(3|\Lambda|)},$$
$$\alpha_1 = \alpha, \quad \alpha_2 = \alpha + 2\pi/3, \quad \alpha_3 = \alpha + 4\pi/3,$$

with α an arbitrary real angle. If $a = 0$ each of these solutions reduces to a Robertson–Walker solution with $k = 0$ and cosmical constant greater than, equal to, or less than zero, respectively. If $a \neq 0$ there are two parameters to be fixed,

a itself and t_0, the present age of the Universe and the most natural way to do this is to require that at $t = t_0$

$$X = Y = Z = R,$$
$$\dot{X}/X + \dot{Y}/Y + \dot{Z}/Z = 3H. \tag{14}$$

One then finds

$\Lambda > 0$:

$$t_0 = \frac{1}{\omega}\cosh^{-1}\left\{\frac{P(P+Q) - 3\epsilon H^2 Q^2/(\Lambda\sqrt{2})}{(P+Q)^2 - 3H^2Q^2/\Lambda}\right\},$$
$$a = \frac{\omega(\cosh \omega t_0 - 1)}{\sinh \omega t_0}\left\{1 + \frac{P}{Q}(1 - \cosh \omega t_0)\right\}.$$

$\Lambda = 0$:

$$t_0 = \frac{H}{4\pi\rho_0 G}(1 - \epsilon/\sqrt{2}), \qquad a = \frac{3HQ\epsilon t_0}{9Gt_0/\sqrt{2} + 3HQ\epsilon}. \tag{15}$$

$\Lambda < 0$:

$$t_0 = \frac{1}{\omega}\cos^{-1}\left\{\frac{|P(P+Q)| - 3\epsilon H^2 Q^2/(\Lambda\sqrt{2})}{(P+Q)^2 - 3H^2Q^2/\Lambda}\right\},$$
$$a = \frac{\omega(1 - \cos \omega t_0)}{\sin \omega t_0}\left\{1 + \frac{P}{Q}(1 - \cos \omega t_0)\right\},$$
$$P \equiv 3G/\Lambda, \qquad Q \equiv 3/4\pi\rho_0.$$

3. *Calculation of the observations.* In this section the expressions for observations derived in Paper I will be applied to the models constructed in the previous section. Denoting differentiation with respect to an affine parameter v by a prime the null geodesics of the models are given by

$$x'X^2 = \text{const.}, \qquad y'Y^2 = \text{const.}, \qquad z'Z^2 = \text{const.} \tag{16}$$

so that for a light ray through the event whose co-ordinates are $(t_0, 0, 0, 0)$,

$$\begin{aligned} x &= \int_{t_0}^{t_e} \sin\theta\cos\phi X^{-2}(t')^{-1}\,dt, \\ y &= \int_{t_0}^{t_e} \sin\theta\sin\phi Y^{-2}(t')^{-1}\,dt, \\ z &= \int_{t_0}^{t_e} \cos\theta Z^{-2}(t')^{-1}\,dt, \\ t' &= [\sin^2\theta\cos^2\phi X^{-2} + \sin^2\theta\sin^2\phi Y^{-2} + \cos^2\theta Z^{-2}]^{1/2}, \end{aligned} \tag{17}$$

where θ, ϕ are the polar angles of the direction in which the observer, who is assumed to be on a fundamental particle, is making his observations. Then the Jacobian of the transformation from the cosmic co-ordinates (t_e, x, y, z) to the observer's polar co-ordinates (t_e, θ, ϕ, t_0) of any observed event is

$$J = -t_e' \sin\theta\Delta,$$

where

$$\Delta = \sin^2\theta\cos^2\phi I_2 I_3 + \sin^2\theta\sin^2\phi I_3 I_1 + \cos^2\theta I_1 I_2, \tag{18}$$

with

$$I_1 = \int_{t_0}^{t_e} Y^{-2}Z^{-2}(t')^{-3}\,dt,$$

$$I_2 = \int_{t_0}^{t_e} Z^{-2}X^{-2}(t')^{-3}\,dt,$$

$$I_3 = \int_{t_0}^{t_e} X^{-2}Y^{-2}(t')^{-3}\,dt.$$

Using the results of Paper I one can now write down the expressions for the red shift:

$$1+z = t_e'/t_0' = R_0[\sin^2\theta\cos^2\phi X_e^{-2}+\sin^2\theta\sin^2\phi Y_e^{-2}+\cos^2\theta Z_e^{-2}]^{1/2} \quad (19)$$

the luminosity distance:

$$D = (1+z)^2[R_e^3 t_e'\Delta]^{1/2}; \quad (20)$$

the rate of increase of number of sources:

$$dN/dt = \hat{n}_0 R_0^3 t_e'\Delta\delta\Omega \quad (21)$$

and the integrated background from radio sources:

$$I = \hat{n}_0 B_0 R_0^2 \int_{t_0}^{t_m} (1+z)^{x-2}R^{-3}(t')^{-1}\,dt.(\delta\Omega/4\pi), \quad (22)$$

where $\hat{n}_0$ is the present number density of sources, B_0 the rate of emission at the frequency of observation, x the spectral index, and t_m the cut-off time (see Paper I).

Thorne (1967a, b) has shown that on the assumption that the primordial black body radiation was kept isotropic until some time t_s and that it has since propagated freely, the black body spectrum is preserved and the temperature in any given direction is

$$T_0 = T_s(1+z)^{-1}, \quad (23)$$

where $(1+z)$ is the red shift which would be observed for a source in the given direction which emitted radiation at time t_s. The time t_s is determined by the relation

$$\int_{t_0}^{t_s} dt/\lambda(t) = 1, \quad (24)$$

where $\lambda(t)$ is the mean free path for Thomson scattering and is given by

$$\lambda(t) = 8\cdot 37\times 10^{-11}/\rho_m(t) \text{ light seconds.} \quad (25)$$

Thorne showed that in some homogeneous models of the type considered by Kantowski & Sachs (1966) the anisotropy in the black body background temperature between two principal directions varies exponentially as the anisotropy in the Hubble parameter. The same result holds in models of Bianchi type I; this can easily be seen by considering equations (3) and (6) from which it follows that

$$X_e/Y_e = (X_0/Y_0)\exp\left\{\epsilon_{xy}HR_0^3\int_{t_0}^{t_e} R^{-3}\,dt\right\}, \quad (26)$$

where ϵ_{xy} is the fractional variation in the Hubble parameter between the two directions at the present epoch. Since in a ' dust ' universe $\rho = \rho_m = \rho_0 R_0{}^3 R^{-3}$ the integral can be evaluated by using equations (24) and (25); it then follows from equation (23) that

$$T_x/T_y = \exp\{8 \cdot 37 \times 10^{11} H \epsilon_{xy}/\rho_0\}. \tag{27}$$

From this expression it is clear that a high matter density reduces the relative sensitivity of the background as a test for anisotropy; this is intuitively obvious since it corresponds to reducing the mean free path for scattering and consequently the time interval since the radiation may be considered to have become decoupled.

Before proceeding to numerical examples it is useful to consider the behaviour of the models as $t \to 0$. Whatever the value of Λ, for small t

$$R_i = O(t^{(1/3)+(2/3)\sin\alpha_i}). \tag{28}$$

In all cases $R(0) = 0$, so the models all originate from a singular state. In almost all cases two of the R_i vanish at $t = 0$ while the third becomes infinite, so the majority of these models develop from a ' cigar ' singularity. The exceptional case is that in which one of the α_i is equal to $\pi/2$; this model is axi-symmetric and one of the R_i vanishes at $t = 0$ while the other two remain finite (and equal) so the original singularity is a ' pancake '.*

It follows from equation (19) that for observations made in a principal direction

$$(1+z)_i = (R_i)_0/(R_i)_e. \tag{29}$$

Hence in the usual ' cigar ' case there will be one direction in which the red shift will increase to a maximum and then decrease, becoming a violet shift for sources sufficiently far away. In models with a pancake singularity there will be a plane of observation in which no red shifts greater than a certain limit will be observed.

To see what happens in the case of a source not lying in a principal direction, suppose without loss of generality that $X(0) = 0$. Then it is clear from equation (19) that as $t \to 0$ the red shift tends to infinity providing only that $\sin\theta\cos\phi \neq 0$. In this way it can easily be shown that it is only for sources lying precisely in the direction of the axis of the cigar or the plane of the pancake that the red shift fails to become infinite. A similar calculation shows that the luminosity distance exhibits analogous behaviour: tending to zero along the axis of a cigar, a finite limit in the plane of a pancake and infinity everywhere else.

Of course it would not be reasonable to conclude from the above analysis that if the universe is in fact Bianchi type I then in two antipodal directions there should be observed infinitely bright sources with infinite violet shifts! Quite apart from the fact that it is only precisely along the direction of the axis that the red shift and luminosity distance exhibit this unusual behaviour one would not expect that sources would exist at arbitrarily early epochs, and even if they did the radiation would almost certainly have been scattered on account of the high matter density. This scattering also ensures that the temperature of the black body background does not become infinite, since this radiation is supposed to

* See Kompaneets & Chernov (1964). It is interesting that in the case in which the model later collapses back to a singularity, models which originate as cigars collapse to pancakes and vice versa.

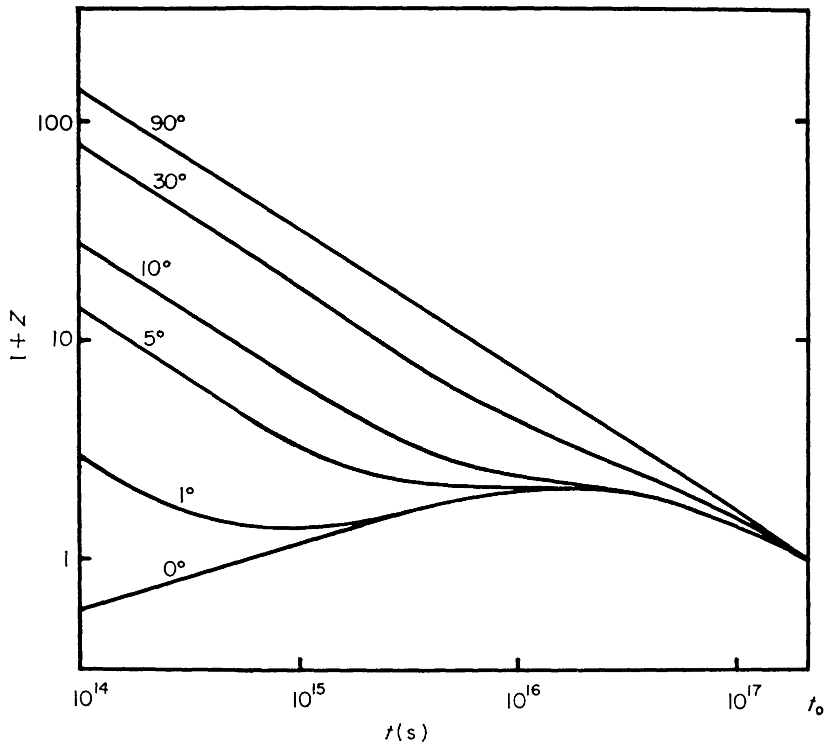

FIG. 1. *The relation between red shift and emission time for a model with a cigar singularity and $\epsilon = 0 \cdot 1$. Angles are measured from the axis of symmetry.*

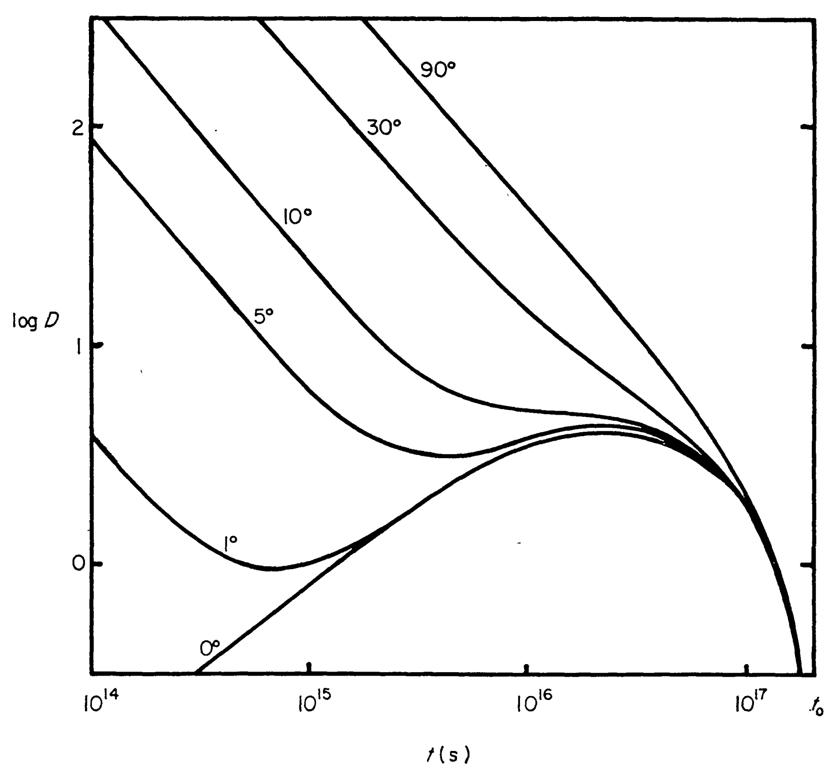

FIG. 2. *The relation between luminosity distance and emission time for a model with a cigar singularity and $\epsilon = 0 \cdot 1$. Angles are measured from the axis of symmetry.*

originate in the ' big bang ' itself. On the other hand, one might now expect that for directions of observation close to the axis of the cigar or the plane of the pancake the behaviour of the red shift and luminosity distance as functions of emission time might be somewhat peculiar; this will be illustrated in the next section.

4. *Numerical examples.* In this section some of the properties of anisotropic model universes are illustrated by means of numerical examples. For this purpose

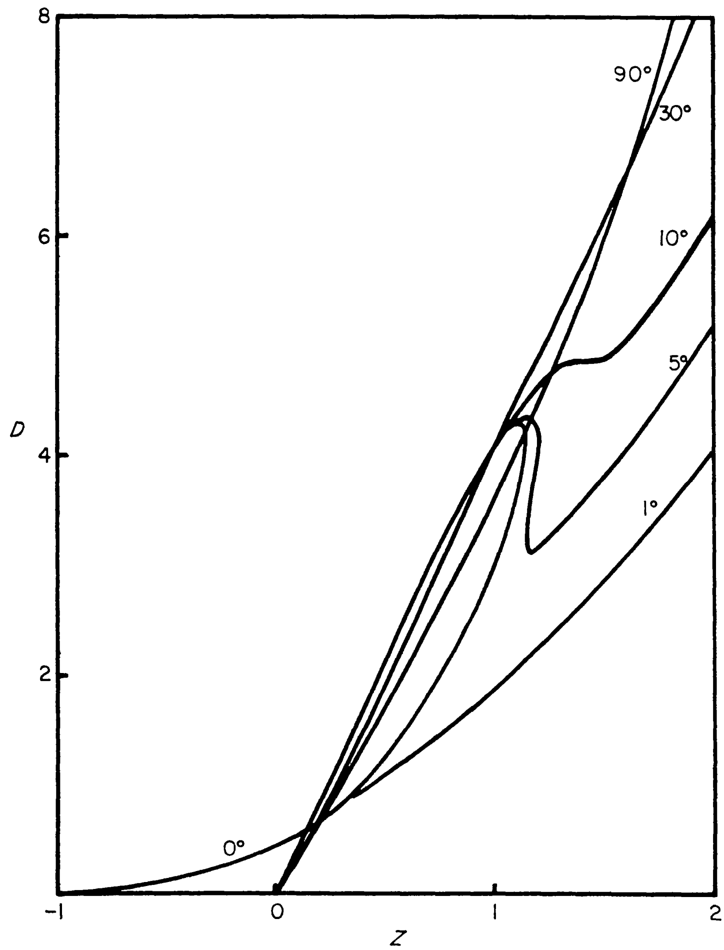

FIG. 3. *The relation between red shift and luminosity distance for a model with a cigar singularity and $\epsilon = 0 \cdot 1$. Angles are measured from the axis of symmetry.*

it will be sufficient to consider only those models which have an axis of symmetry and vanishing cosmical constant, since calculations on more general models have not revealed any essentially different properties.

Figs 1 and 2 illustrate the variation of red shift and luminosity distance with time of emission for the model with a cigar singularity and an anisotropy of $\epsilon = 0 \cdot 1$. Curves have been drawn for different directions of observation (at angles measured from the axis of symmetry). It can be seen that the form of the time dependence of both the red shift and the luminosity distance does vary considerably with the direction of observation. This fact is of great physical significance in any theory which admits evolution of sources, even if (as is not, in fact, the case) the relation between red shift and luminosity distance were to appear normal. To see how this comes about consider the simplest model of source evolution: suppose only that there were no discrete sources radiating before a certain epoch. Then, if, for example, the cut-off is chosen to be at $t_s = 4 \cdot 75 \times 10^{15}$ s (the age of the universe is about $2 \cdot 5 \times 10^{17}$ s in this model), within a solid angle of radius 10° about the axis of symmetry there would be no sources with red shifts greater than 2, while the upper limit for sources within 45° of the plane of symmetry could be as high as 10. Other assumptions about evolution would lead to different conclusions, but the arguments would be similar. It should also be noted that in this model, as in any model in which luminosity distance is not necessarily a monotonic

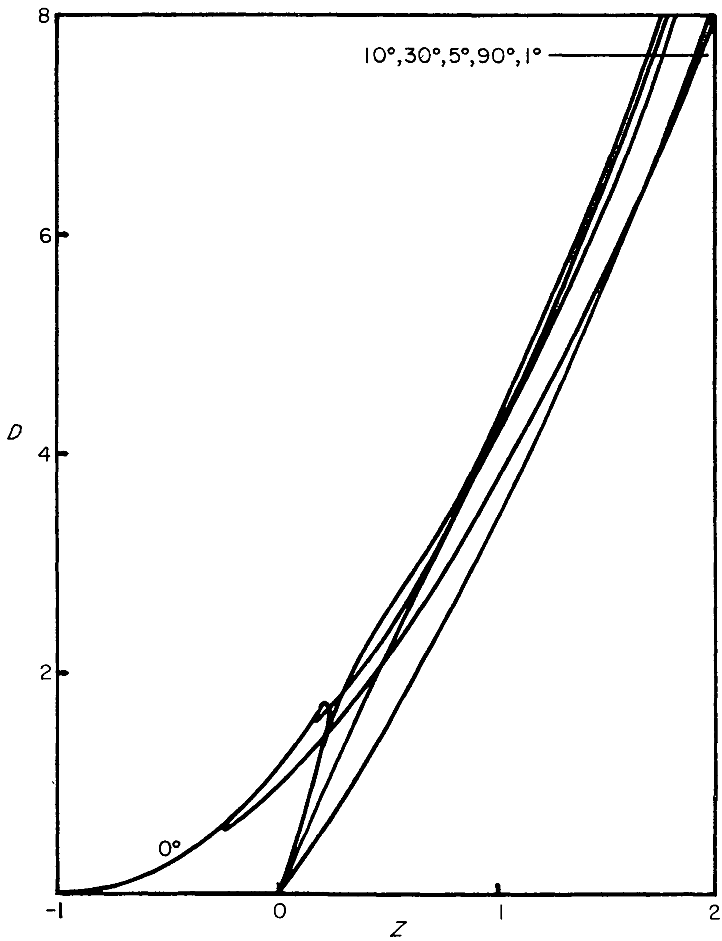

FIG. 4. *The relation between red shift and luminosity distance for a model with a cigar singularity and $\epsilon = 0 \cdot 3$. Angles are measured from the axis of symmetry.*

function of the time of emission, the whole of the number count–flux density (N–S) curve, not just the end corresponding to very distant sources, is sensitive to a change in the assumptions about evolution. A further difficulty encountered in anisotropic models is that the evolution of sources, which can play a very significant role, is itself more difficult to study than in isotropic universes. The reason for this is that in these models it is not possible to adopt the usual device of studying the evolution as a function not of the time of emission but of the red shift, which is directly observable.

Figs 3–6 illustrate the red shift–luminosity distance relation for four different models; these models illustrate the two types of singularity and have anisotropies $\epsilon = 0 \cdot 1$ and $0 \cdot 3$. The figures are for the most part self-explanatory, but a few points should be stressed. First, in almost all cases, for small red shifts the anisotropy is normally less than the nominal value (since the curves tend to converge) so that if an upper limit to the anisotropy can be fixed from observation, a model with ϵ less than or equal to this figure is automatically consistent with the observations in this respect. On the other hand, as can be seen from Fig. 3, the peculiar predicted behaviour of the red shift–luminosity distance relation in the intermediate range of red shift, say $z \doteqdot 1$, may enable us to reject as unrealistic models whose nominal anisotropy ϵ is well within the present upper bound of

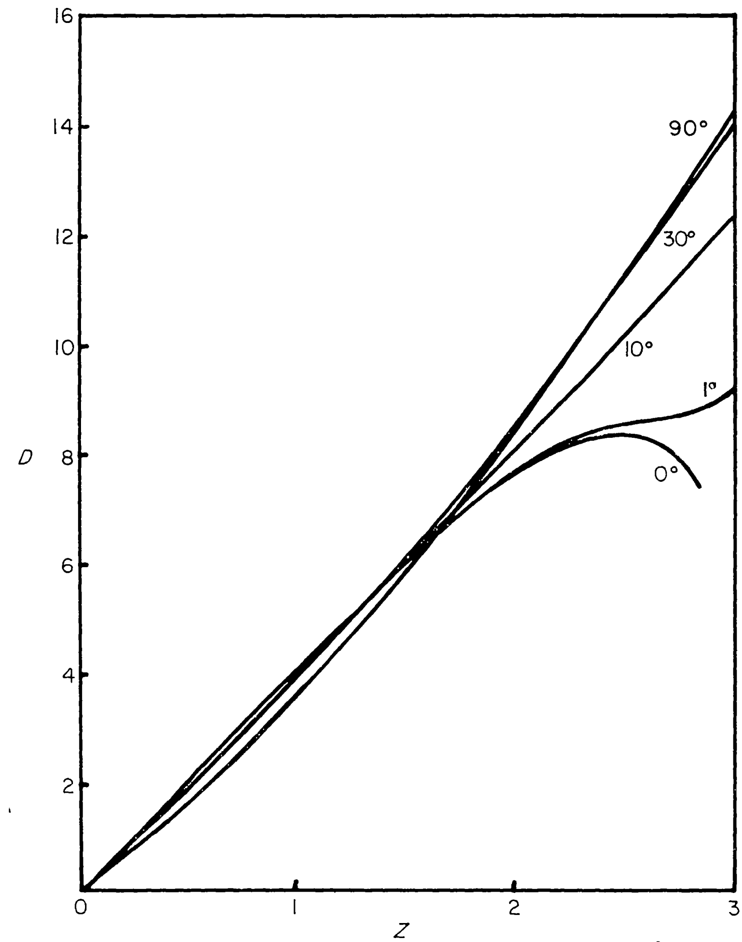

FIG. 5. *The relation between red shift and luminosity distance for a model with a pancake singularity and* $\epsilon = 0 \cdot 1$. *Angles are measured from the plane of symmetry. Note the reduced vertical scale.*

0·2 (Kristian & Sachs 1966). Finally, in the two cases with $\epsilon = 0 \cdot 3$ for $z > 1$ the luminosity distance is not a monotonic function of angle from the axis (for given red shift). This would clearly confuse any statistical study, the object of which was to seek systematic variations with angle in this relation, although an r.m.s. variation of 0·3 would probably be noticed from local observations.

In Fig. 7 the effect of evolution is illustrated by introducing the assumption that the absolute luminosity of a typical source is proportional to $t^{-0 \cdot 7}$. (This model of evolution, which is close to that of Partridge & Peebles (1967), was chosen for simplicity and because it implies a less marked variation than most other theories and hence is the most conservative assumption.) The effect is clearly much greater for sources near the axis of the cigar than for those seen in other directions; this comes about (as can be seen from Fig. 2) because in directions close to the axis it is possible to see sources which emitted at much earlier epochs than any sources viewed in other directions. In Fig. 8 is shown a typical *N–S* relation. Here no account has been taken of evolution except that a cut-off has been assumed at $t_s = 10^{15}$ s.

It is clear from Fig. 2 that in directions sufficiently close to the axis of the cigar there will be a ' cosmological lens ' effect and, as in another model universe

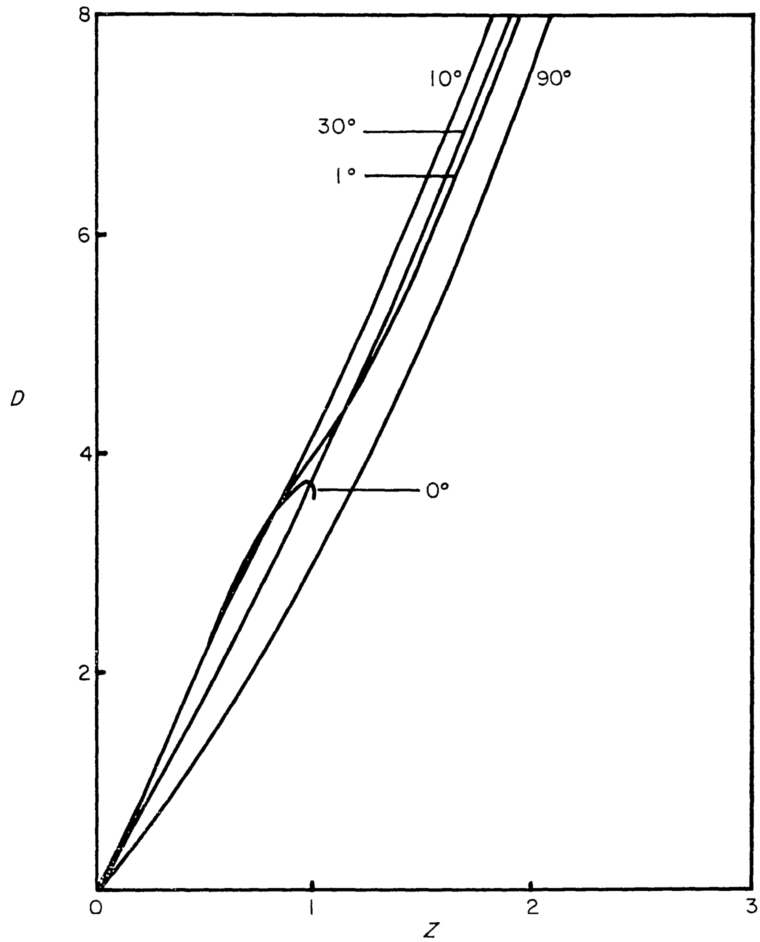

FIG. 6. *The relation between red shift and luminosity distance for a model with a pancake singularity and* $\epsilon = 0 \cdot 3$. *Angles are measured from the plane of symmetry.*

in which this is known to happen (the Lemaître model, discussed by Petrosian, Salpeter & Szekeres 1967), it is interesting to consider the possible consequences concerning the problem of quasars. Fig. 9, which is a plot of the red shift–luminosity distance relation for a model with a cigar singularity and $\epsilon = 0 \cdot 0625$ has been drawn to illustrate the situation. It is clear that in two antipodal regions of radius about seven degrees each there will be seen sources which have red shifts which are much greater than those which can be seen in other directions. A considerable number of these sources will have red shifts which are very close to $1 \cdot 95$ (the particular value of ϵ was of course chosen to make this happen) and there will be a very poor correlation between red shift and luminosity distance for the sources with large red shifts. The sources with large red shifts would be much older than sources of the same apparent magnitude but smaller red shifts, so they might very well have somewhat different physical properties; in particular this might explain why a much greater proportion of these sources have absorption lines. This could also be explained by the fact that since they are in the intuitive sense of the word 'further' from us (the light from them has been travelling for longer) there is in any case a much greater chance that the light from them would have encountered interfering matter on its journey; the matter need not have

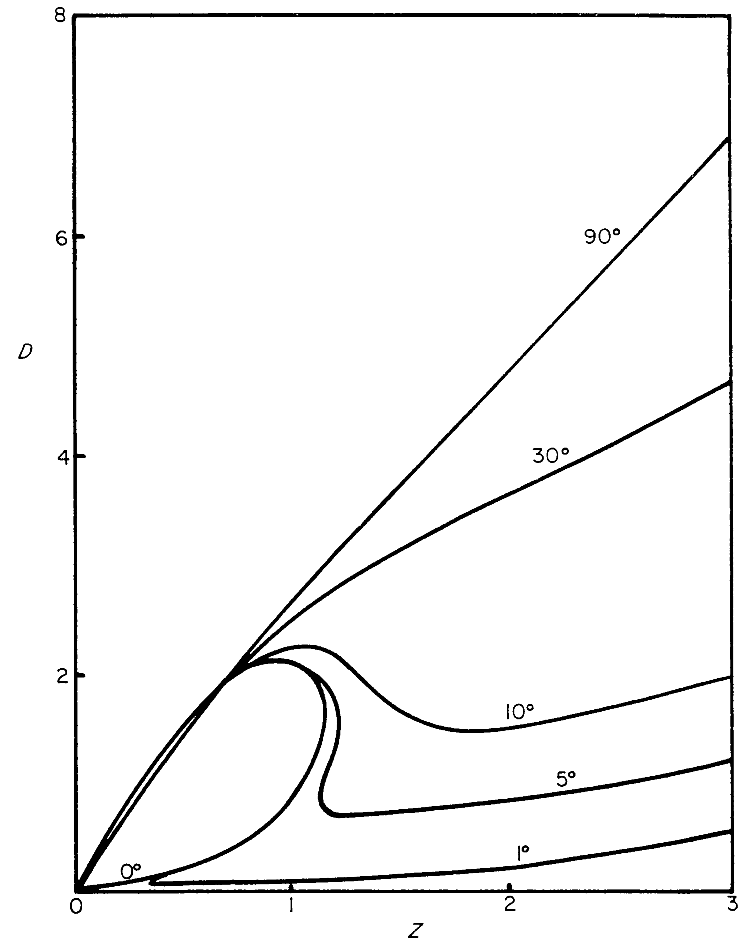

FIG. 7. *The relation between red shift and luminosity distance for a model with a cigar singularity and ϵ = 0·1. It is supposed that the absolute luminosity of a source is proportional to $t^{-0 \cdot 7}$. Angles are measured from the axis of symmetry.*

been associated with the object from which the radiation came since (cf. Fig. 1) $d(1+z)/dt_s$ is small in this region so the red shift of an absorption line caused by an object considerably removed from the emitter could well be about the same as the emission lines. This would also explain why the absorption lines are different from the emission lines and in fact could even explain how it is possible to have an absorption line red shift that is greater than the emission line red shift.

Unfortunately there are also difficulties involved in this explanation, one of the greatest of these being that while the quasars with large red shifts are in fact almost entirely contained within two small regions of the sky the regions are not antipodal, as they would be in the above model. No model has so far been found which provides a strikingly good fit to the actual distribution of quasars with small red shifts, as well as large ones. Also, while the present anisotropy required is consistent with local observation, it is much greater than that permitted by the observed isotropy of the black body background. Nevertheless it is interesting to speculate that the observed properties of quasars may lead to the supposition that the universe originated from a line, not a point, singularity, particularly since as has been shown, this hypothesis is also suggested by the problem of the relative helium abundance.

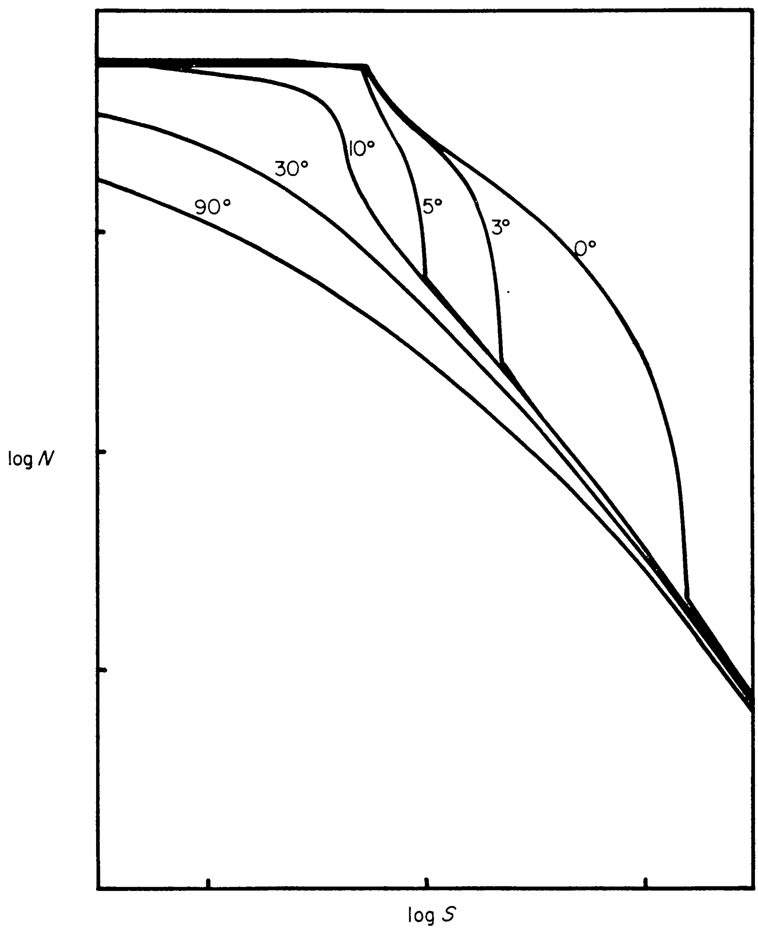

FIG. 8. *The N–Z relation for a model with a cigar singularity and $\epsilon = 0{\cdot}1$. A cut-off has been imposed at $t = 10^{15}$ s. Angles are measured from the axis of symmetry.*

5. *Conclusions.* From this analysis of observations in some models of Bianchi type I some insight can be gained into the behaviour of anisotropic models in general. The most striking phenomena are associated with the fact that the original singularity, where it exists, is typically a 'cigar' or a 'pancake'; this implies that the red shift will fail to become infinite in certain directions and that there can be a cosmological lens effect. The rate of expansion in the early stages is also quite different from that in the isotropic models and this has a significant effect on the primordial nuclear reactions and, consequently, the present relative abundances of the elements.

Of course the study of one of the simplest anisotropic models can only be a first step in the discovery of the effects of anisotropy. Some other models with

P

shear but no rotation are being studied at the present. It is interesting that so far not only have no essentially different characteristics been found, but the peculiar behaviour of the red shift–luminosity distance relation, while it still occurs, is less marked; this is caused by the characteristic tendency of geodesics to

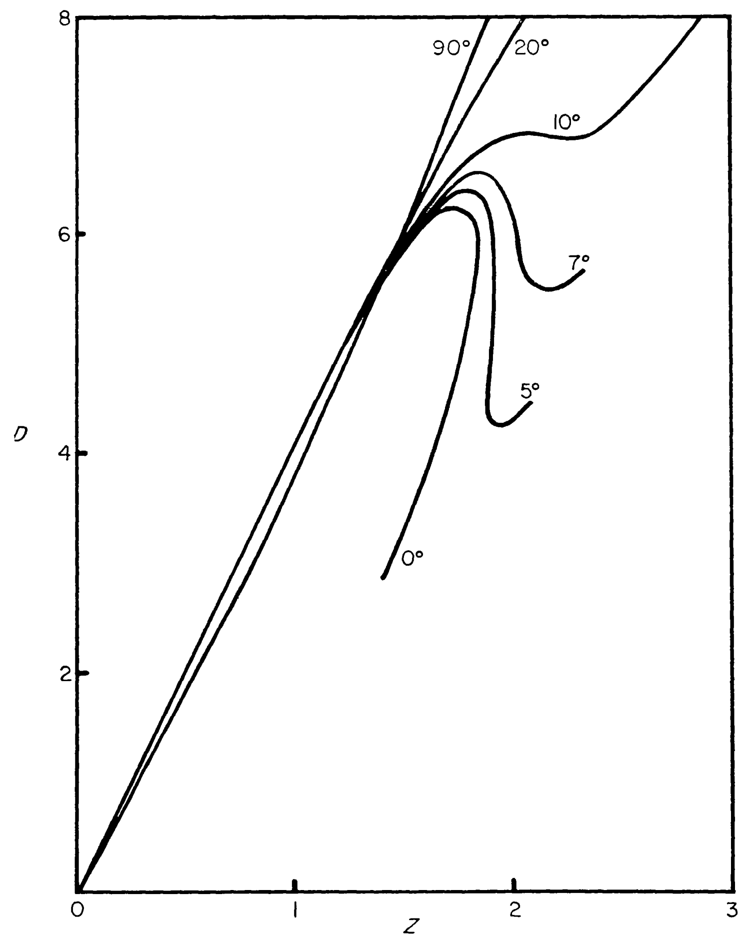

FIG. 9. *The relation between red shift and luminosity distance for a model with a cigar singularity and* $\epsilon = 0{\cdot}0625$. *Angles are measured from the axis of symmetry. A cut-off has been imposed at* $t = 2{\cdot}6\times10^{15}$ *s.*

'avoid' singularities. The expansion rate in the early stages appears, not surprisingly, to be the same.

Acknowledgment. I am grateful to Professor F. A. E. Pirani for his advice and encouragement.

Queen Elizabeth College,
*London, W.*8.
1968 *July*.

References

Hawking, S. W. & Tayler, R. J., 1966. *Nature, Lond.*, **209**, 1278.
Heckmann, O. & Schücking, E., 1962. In *Gravitation*, ed. by L. Witten, Wiley, New York.
Jacobs, K. C., 1968. Preprint.
Kantowski, R. & Sachs, R. K., 1966. *J. math. Phys.*, **7**, 443.
Kermack, W. O., McCrea, W. H. & Whittaker, J. M., 1932. *Proc. R. Soc. Edinb.*, **53**, 31.

Kompaneets, A. S. & Chernov, A. S., 1964. *Zk. eksp. teor. Fiz.*, **47**, 1939. English translation in 1965. *Soviet Phys. JETP*, **20**, 1303.
Kristian, J. & Sachs, R. K., 1966. *Astrophys. J.*, **143**, 379.
Partridge, R. B. & Peebles, P. J. E., 1967. *Astrophys. J.*, **148**, 377.
Petrosian, V., Salpeter, E. & Szekeres, P., 1967. *Astrophys. J.*, **147**, 1222.
Saunders, P. T., 1968. *Mon. Not. R. astr. Soc.*, **141**, 427.
Thorne, K. S., 1967a. *Astrophys. J.*, **148**, 51.
Thorne, K. S., 1967b. In *High Energy Astrophysics*, ed. by C. Dewitt, E. Schatzman and P. Veron, Gordon and Breach, New York.
Tomita, K., 1968. Preprint.