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# Observer and Command Filter-based Adaptive Fuzzy Output Feedback Control of Uncertain Nonlinear Systems

Jinpeng Yu, Peng Shi, *Fellow, IEEE*, Wenjie Dong, *Member, IEEE*, and Haisheng Yu

**Abstract**—In this paper, observer and command filter-based adaptive fuzzy output feedback control is proposed for a class of strict-feedback systems with parametric uncertainties and unmeasured states. First, fuzzy logic systems are used to approximate the unknown and nonlinear functions. Next, a fuzzy state observer is developed to estimate the immeasurable states. Then, command filtered backstepping control is designed to avoid the explosion of complexity in the backstepping design and compensating signals are introduced to remove the effect of the errors caused by command filters. The proposed method guarantees that all signals in the closed-loop systems are bounded. The main contributions of this paper are the proposed control method can overcome two problems of linear in the unknown system parameter and explosion of complexity in backstepping-design methods and it does not require that all the states of the system are measured directly. Finally, two examples are provided to illustrate its effectiveness.

**Index Terms**—Fuzzy control, Output feedback control, Backstepping, Observer, Command filter

## I. INTRODUCTION

Over the past few decades, backstepping method [1], [2] has been taken for one of the most popular and effective control approaches to deal with nonlinear systems with a strict-feedback form, particularly those systems which do not meet matching conditions. Many significant results have already been achieved, see for example [3]–[7], and the references therein. Nevertheless, the traditional backstepping requires that the exact knowledge of the model is available and parametric uncertainties are not considered [8]–[11]. This restriction limits the application scope of backstepping techniques. As an alternative, neural networks (NNs) [12]–[15] or fuzzy

logic system (FLS) [16]–[20] approximator-based adaptive-control approaches have been developed to deal with nonlinear systems with unstructured uncertainties. Adaptive NN or fuzzy control combined backstepping technique provides a systematic methodology to solve the problem of linear in the unknown system parameters contained in the backstepping design [21]–[25]. But the drawback of explosion of complexity caused by the repeated differentiations of virtual input [4], [5] for the adaptive backstepping is not resolved by approximation-based adaptive NN or fuzzy backstepping control [26].

Recently, several new techniques are proposed to solve this problem of explosion of complexity inherent in adaptive backstepping such as dynamic surface control (DSC) [27]–[29] and command filtered backstepping control [30]–[32]. A novel adaptive fuzzy control combined DSC technology proposed in [33] eliminates the explosion of complexity problem by introducing first-order filters for the backstepping approach. But it takes no account of the problem of compensating the errors caused by the filters, which will add the difficulty to gain a better control quality. The command filtered backstepping was proposed in [30] and then was extended to adaptive case for strict-feedback systems in [31]. By utilizing the output of a command filter to approximate the derivative of the virtual control at each step of backstepping approach, the problem of explosion of terms can be eliminated. And the errors caused by command filter can be reduced by introducing compensation signals. But the command filtered backstepping method has two major limitations for its applications. The first limitation is that those works only considered the case for systems without parametric uncertainty or for systems in which the unknown parameters are constants. Another limitation is that the proposed above works are all based on an assumption that the states of the controlled systems are measurable directly.

Motivated by the above investigations, observer and command filter-based adaptive fuzzy output feedback control is proposed for a class of strict-feedback systems with parametric uncertainties and unmeasured states. Compared with the existing results, the main advantages of the proposed controller can be summarized as follows: 1) By designing the state observer, the proposed control method does not require all the states of the controlled system are measurable; 2) the proposed method can overcome two problems of linear in the unknown system parameter and explosion of complexity in backstepping-design methods; 3) compensating signals are designed to remove the effect of the errors caused by command

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filters to gain a smaller tracking error; and 4) the proposed method in this paper only needs the information of the desired trajectory and its first derivative, which makes it more suitable for practical applications where higher order derivations of the desired trajectory cannot be obtained. It is shown that the proposed control approach can guarantee the semiglobal uniform ultimate boundedness for all the solutions of the closed-loop system. Two examples are given to verify the novelty of the new design method.

The rest of this paper is organized as follows. Section II formulates the control problem for the nonlinear system and introduces some related technical assumptions. Section III gives the fuzzy state observer design. In Section IV, the command filtered fuzzy adaptive backstepping control is designed to guarantee the boundedness of all signals in the closed-loop systems. Section V presents simulation studies. Finally, Section VI draws some conclusions.

## II. PRELIMINARIES

Consider the following SISO strict-feedback nonlinear system:

$$\begin{aligned}\dot{x}_1 &= x_2 + f_1(x_1) \\ \dot{x}_2 &= x_3 + f_2(x_1, x_2) \\ &\vdots \\ \dot{x}_n &= u + f_n(x_1, x_2, \dots, x_n) \\ y &= x_1\end{aligned}\quad (1)$$

where  $X = (x_1, x_2, \dots, x_n)^T \in R^n$  is the system state vector,  $u$  is the control input,  $y$  is the output of system,  $f_i(X_i)$  ( $i = 1, 2, \dots, n$ ) are unknown smooth functions of the system, and only the output variable  $y = x_1$  can be measured directly.

For a given reference input  $x_d$ , the control objective is to design the control law to realize the trajectory tracking. In this paper, unless otherwise stated, all norms are the standard 2-norm. For any  $a > 0$ , the symbols  $\Omega_a$  and  $\bar{\Omega}_a$  represent the standard open and closed level sets:  $\Omega_a = \{x \in R^n \mid \|x\| < a\}$  and  $\bar{\Omega}_a = \{x \in R^n \mid \|x\| \leq a\}$ . Superscripts in parentheses indicate derivatives with respect to the indicated argument. To facilitate the design, we need the following assumptions:

**Assumption 1:** The desired trajectory  $x_d$  and its first derivative  $\dot{x}_d$  are smooth, bounded and known.

**Remark 1:** The adaptive backstepping or approximation based adaptive backstepping requires the knowledge of  $x_d^{(i)}(t)$  for  $i = 0, \dots, n-1$ , whereas Assumption 1 only requires  $x_d(t)$  and  $\dot{x}_d(t)$ . It implies that Assumption 1 is less stringent, so that the command filtered backstepping control is more suitable for some applications in which the higher order derivatives can't be obtained such as land vehicle system.

**Assumption 2:** Let  $\Omega_d \subset R^n$  represent an open set that contains the origin, the initial condition  $x(0)$  and the trajectory  $x_d(t)$ . For the system (1), for  $i = 1, \dots, n$ :  $f_i^{(j)}(\cdot)$  are bounded on  $\bar{\Omega}_d$  for  $j = 1, \dots, n-i$ .

**Assumption 3:** There exist a set of constant  $h_i, i = 0, \dots, n$  for  $\forall X_1, X_2 \in R^i$  such that the following inequality holds:

$$|f_i(X_1) - f_i(X_2)| \leq h_i \|X_1 - X_2\|$$

where  $\|X_1 - X_2\|$  expresses the 2-norm of vector  $X_1 - X_2$ .

By Assumption 2, the function  $f_i$  is Lipschitz on  $\bar{\Omega}_d$ . And some useful Lemmas are introduced as follows.

**Lemma 1** [34]: Let  $f(x)$  be a continuous function defined on a compact set  $\Omega$ . Then for any scalar  $\varepsilon > 0$ , there exists a fuzzy logic system  $W^T S(x)$  such that

$$\sup_{x \in \Omega} |f(x) - W^T S(x)| \leq \varepsilon$$

where  $W = [W_1, \dots, W_N]^T$  is the ideal constant weight vector, and  $S(x) = [p_1(x), p_2(x), \dots, p_N(x)]^T / \sum_{i=1}^N p_i(x)$  is the basis function vector, with  $N > 1$  being the number of the fuzzy rules and  $p_i$  are chosen as Gaussian functions, i.e., for  $i = 1, 2, \dots, N$ ,  $p_i(x) = \exp[-\frac{(x-\mu_i)^T(x-\mu_i)}{\eta_i^2}]$  where  $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$  is the center vector, and  $\eta_i$  is the width of the Gaussian function.

**Lemma 2:** The command filter is defined as

$$\dot{\varphi}_1 = \omega_n \varphi_2 \quad (2)$$

$$\dot{\varphi}_2 = -2\zeta \omega_n \varphi_2 - \omega_n (\varphi_1 - \alpha_1). \quad (3)$$

If the input signal  $\alpha_1$  satisfies  $|\dot{\alpha}_1| \leq \rho_1$  and  $|\ddot{\alpha}_1| \leq \rho_2$  for all  $t \geq 0$ , where  $\rho_1$  and  $\rho_2$  are positive constants and  $\varphi_1(0) = \alpha_1(0)$ ,  $\varphi_2(0) = 0$ , then for any  $\mu > 0$ , there exist  $\omega_n > 0$  and  $\zeta \in (0, 1]$ , such that  $|\varphi_1 - \alpha_1| \leq \mu$ ,  $|\dot{\varphi}_1|$ ,  $|\ddot{\varphi}_1|$  and  $|\ddot{\varphi}_1|$  are bounded.

**Proof:** Define  $\delta\varphi_1 = \varphi_1 - \alpha_1$ ,  $\delta\varphi_2 = \dot{\varphi}_1 - \dot{\alpha}_1$ , then

$$\delta\dot{\varphi}_1 = \delta\varphi_2 \quad (4)$$

$$\delta\dot{\varphi}_2 = -2\zeta \omega_n \delta\varphi_2 - \omega_n^2 \delta\varphi_1 - (\ddot{\alpha}_1 + 2\zeta \omega_n \dot{\alpha}_1) \quad (5)$$

with  $\delta\varphi_1(0) = \varphi_1(0) - \alpha_1(0) = 0$  and  $\delta\varphi_2(0) = \varphi_2(0) - \dot{\alpha}_1(0) = -\dot{\alpha}_1(0)$ .

According to [31], we can obtain

$$\begin{aligned}\delta\varphi_1 &= \frac{1}{\omega_n} \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t) \delta\varphi_2(0) \\ &+ \int_0^t \frac{1}{\omega_n} \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n (t-\tau)} \times \\ &\sin[\omega_n \sqrt{1-\zeta^2} (t-\tau)] (2\zeta \omega_n \dot{\alpha}_1 + \ddot{\alpha}_1),\end{aligned}\quad (6)$$

which is followed by

$$\begin{aligned}|\delta\varphi_1(t)| &\leq \frac{1}{\omega_n} \frac{1}{\sqrt{1-\zeta^2}} |\delta\varphi_2(0)| + \frac{1}{\omega_n^2 \zeta \sqrt{1-\zeta^2}} \\ &\max_{0 \leq \tau \leq t} (2\zeta \omega_n |\dot{\alpha}_1(\tau)| + |\ddot{\alpha}_1(\tau)|) \\ &\leq \frac{1}{\omega_n} \frac{1}{\sqrt{1-\zeta^2}} \rho_1 \\ &+ \frac{1}{\omega_n \zeta \sqrt{1-\zeta^2}} \max_{0 \leq \tau \leq t} (2\zeta \rho_1 + \frac{\rho_2}{\omega_n}).\end{aligned}\quad (7)$$

It can be seen that by choosing the right  $\zeta$  and  $\omega_n$ , we can make  $|\delta\varphi_1(t)| \leq \mu$  and  $|\varphi_1(t) - \alpha_1| \leq \mu$ .

Then we can get

$$\begin{aligned}\dot{\varphi}_2(t) &= -2\zeta \omega_n \varphi_2(t) - \omega_n (\varphi_1(t) - \alpha_1) \\ &\leq -2\zeta \omega_n \varphi_2(t) + \omega_n \mu.\end{aligned}\quad (8)$$

From (8), we can get  $|\varphi_2(t)| \leq |\varphi_2(0) + \frac{\mu}{2\zeta}| \leq \frac{\mu}{2\zeta}$ , then it can be shown that  $|\varphi_2|$  and  $|\dot{\varphi}_2|$  are bounded. Notice that  $\varphi_1 = \omega_n \varphi_2$ , one can easily get  $|\dot{\varphi}_1|$ ,  $|\ddot{\varphi}_1|$  and  $|\ddot{\varphi}_1|$  are bounded.

**Remark 2:** From equation (7), it can be known that increasing  $\omega_n$  decreases the tracking error by decreasing  $\mu$ , but for a second-order command filter, increasing  $\omega_n$  also increases the magnitude of the command derivatives. Higher order filters have useful noise reduction properties, however, the computational load increases with the order of the command filter [31].

### III. FUZZY STATE OBSERVER DESIGN

In this section, since the state variables are not available, state observer should be designed to estimate the states. So system (1) can be rewritten in the following form:

$$\begin{aligned}\dot{X} &= AX + \sum_{i=1}^n B_i(f_i(\hat{X}) + \Delta f_i) + D_i u \\ y &= C^T X\end{aligned}\quad (9)$$

where  $\Delta f_i = f_i(X) - f_i(\hat{X})$ ,  $\hat{X} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]$  is the estimates of  $X$ ;  $A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & 1 & \dots \\ \vdots & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix}$ ,  $B_i = [0, \dots, 1, \dots, 0]$ ,  $C^T = [1 \dots 0 \dots 0]$ ,  $D_i = [0 \dots 0 \dots 1]$ .

Define  $W_i = \theta_i$ , then by Lemma 1, we can get  $\hat{f}_i(\hat{X}|\hat{\theta}_i) = \hat{\theta}_i^T S_i(\hat{X})$  where  $\hat{\theta}_i (i = 1, 2, \dots, n)$  are the estimation of the unknown optimal parameter vector  $\theta_i$  which are defined as

$$\theta_i = \arg \min_{\theta_i \in \Omega_i} \left[ \sup_{\hat{X} \in U_i} \left| \hat{f}_i(\hat{X}|\hat{\theta}_i) - f_i(\hat{X}) \right| \right]$$

where  $\Omega_i, U_i$  are compact regions for  $\hat{\theta}_i, \hat{X}$ , respectively. Also the fuzzy minimum approximation error  $\delta_i$  are defined by

$$\delta_i = f_i(\hat{X}) - \hat{f}_i(\hat{X}|\hat{\theta}_i)$$

where  $\delta_i$  satisfies  $|\delta_i| < \varepsilon_i$ , with  $\varepsilon_i$  being an unknown positive constant. Rewrite (9) as

$$\begin{aligned}\dot{X} &= A_0 X + \sum_{i=1}^n B_i(f_i(\hat{X}) + \Delta f_i) + D_i u + K y \\ y &= C^T X\end{aligned}\quad (10)$$

where  $K = [k_1, k_2, \dots, k_n]^T$  and  $A_0 = A - KC^T$ .

Choose vector  $K$  so that the matrix  $A_0$  is a strict Hurwitz matrix. Thus, given a  $Q^T = Q > 0$ , there exists a positive definite matrix  $P^T = P > 0$  which satisfies

$$A_0^T P + P A_0 = -Q. \quad (11)$$

According to Lemma 1, design fuzzy state observer as

$$\begin{aligned}\dot{\hat{X}} &= A_0 \hat{X} + \sum_{i=1}^n B_i \hat{f}_i(\hat{X}|\hat{\theta}_i) + D_i u + K y \\ \hat{y} &= C^T \hat{X}\end{aligned}\quad (12)$$

Let  $e = X - \hat{X}$  be the observer error, then from (10) and (12), we have the observer error equations as

$$\dot{e} = A_0 e + \sum_{i=1}^n B_i \left[ \tilde{\theta}_i^T S_i(\hat{X}) + \delta_i + \Delta f_i \right] \quad (13)$$

with  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ . Choose  $V_0 = e^T P e$ , then

$$\begin{aligned}\dot{V}_0 &= \dot{e}^T P e + e^T P \dot{e} = -e^T Q e \\ &\quad + 2e^T P \sum_{i=1}^n B_i \left[ \tilde{\theta}_i^T S_i(\hat{X}) + \delta_i + \Delta f_i \right].\end{aligned}\quad (14)$$

By using Young's inequality and Assumption 3, we obtain

$$\begin{aligned}2e^T P \sum_{i=1}^n B_i \left[ \tilde{\theta}_i^T S_i(\hat{X}) + \delta_i \right] &\leq 2ne^T e + \|P\|^2 \sum_{i=1}^n B_i (\tilde{\theta}_i^T \tilde{\theta}_i + \varepsilon_i^2), \\ 2e^T P \sum_{i=1}^n B_i \Delta f_i &\leq (n + \|P\|^2 \sum_{i=1}^n h_i^2) e^T e.\end{aligned}\quad (15)$$

Substituting (15) into (14), we can get

$$\begin{aligned}\dot{V}_0 &\leq -(\lambda_{\min}(Q) - 3n - \|P\|^2 \sum_{i=1}^n h_i^2) e^T e \\ &\quad + \|P\|^2 \sum_{i=1}^n B_i \left[ \tilde{\theta}_i^T \tilde{\theta}_i + \varepsilon_i^2 \right]\end{aligned}\quad (16)$$

where  $\lambda_{\min}(Q)$  is the minimum eigenvalue of  $Q$ .

**Remark 3:** Notice that if  $(\lambda_{\min}(Q) - 3n - \|P\|^2 \sum_{i=1}^n h_i^2) > 0$ , and the term  $\|P\|^2 \sum_{i=1}^n B_i \tilde{\theta}_i^T \tilde{\theta}_i$  is bounded, therefore, from (16), it will conclude that the designed nonlinear fuzzy state observer (12) is stable. However, the term  $\|P\|^2 \sum_{i=1}^n B_i \tilde{\theta}_i^T \tilde{\theta}_i$  may be unbounded, thus it is necessary to design a suitable controller to make the resulting closed-loop system be stable.

### IV. COMMAND FILTERED ADAPTIVE FUZZY BACKSTEPPING

This section presents a new adaptive backstepping implementation control approach. First of all, define the tracking error for the command filtered backstepping approach as

$$z_1 = x_1 - x_d, z_i = \hat{x}_i - x_{i,c}$$

for  $i = 2, \dots, n$ , where  $x_d$  is the desired trajectory and  $x_{i,c}$  are the output of the command filter with  $\alpha_{i-1}$  as the input. The design produce contains  $n$  steps. At each step, one command filter is needed to filter the virtual control. For  $i = 1, 2, \dots, n-1$ , the command filter is defined as:

$$\dot{\varphi}_{i,1} = \omega_n \varphi_{i,2} \quad (17)$$

$$\dot{\varphi}_{i,2} = -2\zeta \omega_n \varphi_{i,2} - \omega_n (\varphi_{i,1} - \alpha_i) \quad (18)$$

with  $x_{i,c}(t) = \varphi_{i,1}(t)$  as the output of each filter and  $\alpha_i$  is the input of the filter. The filter initial conditions are  $\varphi_{i,1}(0) = \alpha_i(0)$  and  $\varphi_{i,2}(0) = 0$ .

Define the virtual control functions  $\alpha_i (i=2, \dots, n)$  for the command filtered backstepping procedure as

$$\alpha_1 = -c_1 z_1 - z_1 + \dot{x}_d - \hat{\theta}_1^T S_1(\hat{X}) \quad (19)$$

$$\alpha_i = -c_i z_i - z_{i-1} + \dot{x}_{i,c} - k_i e_1 - \hat{\theta}_i^T S_i(\hat{X}) \quad (20)$$

$$u = \alpha_n \quad (21)$$

with  $c_i > 0$ . We choose the adaptive law as

$$\dot{\hat{\theta}}_i = r_i \nu_i S_i(\hat{X}) - m_i \hat{\theta}_i (i = 1, 2, \dots, n). \quad (22)$$

**Remark 4:** It should be pointed that the command filters may cause the filtering errors which will add the difficulty to get a small tracking error. In this paper we will design the compensating signals in order to remove the effect of the errors  $(x_{i+1,c} - \alpha_i)$  caused by the command filters.

The compensating signals  $\xi_i$  for  $i=2, \dots, n$  are defined as

$$\dot{\xi}_1 = -c_1\xi_1 - \xi_1 + \xi_2 + (x_{2,c} - \alpha_1) \quad (23)$$

$$\dot{\xi}_i = -c_i\xi_i - \xi_{i-1} + \xi_{i+1} + (x_{i+1,c} - \alpha_i) \quad (24)$$

$$\dot{\xi}_n = -c_n\xi_n - \xi_{n-1} \quad (25)$$

with  $\xi(0) = 0$ . According to Lemma 3 in [31], we can get  $\|\xi_i\|$  is bounded. If  $t$  extends to  $\infty$ , we have

$$\lim_{t \rightarrow \infty} \|\xi_i\| \leq \frac{\mu}{2k_0} \quad (26)$$

where  $k_0 = \frac{1}{2} \min_i(c_i)$ .

Then we define the compensated tracking error signals  $\nu_i$  as  $\nu_i = z_i - \xi_i$ , for  $i=1, 2, \dots, n$ .

*Step 1:* Consider the following Lyapunov function as  $V_1 = V_0 + \frac{1}{2}\nu_1^2 + \frac{1}{2r_1}\tilde{\theta}_1^T\tilde{\theta}_1$  where  $r_1 > 0$  is a constant. Then the time derivative of  $V_1$  is given as

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \nu_1(z_2 + x_{2,c} - \alpha_1 + \alpha_1 + e_2 + \theta_1^T S_1(\hat{X}) \\ &\quad + \delta_1 - \dot{x}_d - \dot{\xi}_1) - \frac{1}{r_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1. \end{aligned} \quad (27)$$

By using Young's inequality, we can get the following inequalities

$$\nu_1 e_2 \leq \frac{1}{2}\nu_1^2 + \frac{1}{2}e^T e, \nu_1 \delta_1 \leq \frac{1}{2}\nu_1^2 + \frac{1}{2}\varepsilon_1^2. \quad (28)$$

According to (19) and (28), we can obtain

$$\begin{aligned} \dot{V}_1 &\leq \dot{V}_0 + \nu_1[\nu_2 + \xi_2 + (x_{2,c} - \alpha_1) - c_1\nu_1 - c_1\xi_1 - \xi_1 \\ &\quad + \tilde{\theta}_1^T S_1(\hat{X}) - \dot{\xi}_1] + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 - \frac{1}{r_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1. \end{aligned} \quad (29)$$

Choosing the adaptive law  $\dot{\hat{\theta}}_1 = r_1\nu_1 S_1(\hat{X}) - m_1\hat{\theta}_1$ . and substituting (23) into the equation (29), we can get

$$\dot{V}_1 \leq \dot{V}_0 - c_1\nu_1^2 + \nu_1\nu_2 + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 + \frac{m_1}{r_1}\tilde{\theta}_1^T\hat{\theta}_1. \quad (30)$$

*Step 2:* Choose the Lyapunov function as  $V_2 = V_1 + \frac{1}{2}\nu_2^2 + \frac{1}{2r_2}\tilde{\theta}_2^T\tilde{\theta}_2$  with  $r_2 > 0$  being a constant. Obviously, the time derivative of  $V_2$  is computed by

$$\begin{aligned} \dot{V}_2 &\leq \dot{V}_0 - c_1\nu_1^2 + \nu_2(\nu_1 + z_2 - \dot{\xi}_2) + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 \\ &\leq \dot{V}_0 - c_1\nu_1^2 + \nu_2(\nu_1 + z_3 + x_{3,c} - \alpha_2 + \alpha_2 - \dot{x}_{2,c} \\ &\quad + \hat{\theta}_2^T S_2(\hat{X}) + \tilde{\theta}_2^T S_2(\hat{X}) - \tilde{\theta}_2^T S_2(\hat{X}) + k_2 e_1 \\ &\quad - \dot{\xi}_2) + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 + \frac{m_1}{r_1}\tilde{\theta}_1^T\hat{\theta}_1 - \frac{1}{r_2}\tilde{\theta}_2^T\dot{\hat{\theta}}_2. \end{aligned} \quad (31)$$

Utilizing Young's inequality, we have

$$-\nu_2\tilde{\theta}_2^T S_2(\hat{X}) \leq \nu_2^2 + \frac{1}{4}\tilde{\theta}_2^T\tilde{\theta}_2. \quad (32)$$

Substituting (20) and (32) into (31) results in

$$\begin{aligned} \dot{V}_2 &\leq \dot{V}_0 - c_1\nu_1^2 + \nu_2[-c_2\nu_2 - c_2\xi_2 - \xi_1 + \nu_2 \\ &\quad + \nu_3 + \xi_3 + (x_{3,c} - \alpha_2) - \dot{\xi}_2] + \frac{1}{2}e^T e \\ &\quad + \frac{1}{2}\varepsilon_1^2 + \frac{1}{4}\tilde{\theta}_2^T\tilde{\theta}_2 + \frac{m_1}{r_1}\tilde{\theta}_1^T\hat{\theta}_1 \\ &\quad + \frac{1}{r_2}\tilde{\theta}_2^T(r_2\nu_2 S_2(\hat{X}) - \dot{\hat{\theta}}_2). \end{aligned} \quad (33)$$

According to the compensating signal  $\xi_2$  and the adaptive law  $\hat{\theta}_2$ , (33) can be rewritten as

$$\begin{aligned} \dot{V}_2 &\leq \dot{V}_0 - c_1\nu_1^2 - c_2\nu_2^2 + \nu_2^2 + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \frac{m_1}{r_1}\tilde{\theta}_1^T\hat{\theta}_1 + \frac{m_2}{r_2}\tilde{\theta}_2^T\hat{\theta}_2 + \frac{1}{4}\tilde{\theta}_2^T\tilde{\theta}_2 + \nu_2\nu_3. \end{aligned} \quad (34)$$

*Step i:* ( $2 < i \leq n-1$ ) Consider the Lyapunov function as  $V_i = V_{i-1} + \frac{1}{2}\nu_i^2 + \frac{1}{2r_i}\tilde{\theta}_i^T\tilde{\theta}_i$  where  $r_i > 0$  is a constant. Then the time derivative of  $V_i$  can be expressed as follows:

$$\begin{aligned} \dot{V}_i &\leq \dot{V}_0 - c_1\nu_1^2 - \sum_{i=2}^{n-2} (c_i - 1)\nu_i^2 + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \sum_{i=1}^{n-2} \frac{m_i}{r_i}\tilde{\theta}_i^T\hat{\theta}_i + \frac{1}{4}\sum_{i=2}^{n-2} \tilde{\theta}_i^T\tilde{\theta}_i + \nu_i(z_{i-1} - \xi_{i-1} \\ &\quad + z_{i+1} + x_{i+1,c} - \alpha_i + \alpha_i + \hat{\theta}_i^T S_i(\hat{X}) + \tilde{\theta}_i^T S_i(\hat{X}) \\ &\quad - \tilde{\theta}_i^T S_i(\hat{X}) + k_i e_1 - \dot{x}_{i,c} - \dot{\xi}_i) - \frac{1}{r_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i. \end{aligned} \quad (35)$$

Similar to (32), we can get the following inequality

$$-\nu_i\tilde{\theta}_i^T S_i(\hat{X}) \leq \nu_i^2 + \frac{1}{4}\tilde{\theta}_i^T\tilde{\theta}_i. \quad (36)$$

By (20) and (36), we can gain

$$\begin{aligned} \dot{V}_i &\leq \dot{V}_0 - c_1\nu_1^2 - \sum_{i=2}^{n-2} (c_i - 1)\nu_i^2 + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \sum_{i=1}^{n-2} \frac{m_i}{r_i}\tilde{\theta}_i^T\hat{\theta}_i + \frac{1}{4}\sum_{i=2}^{n-1} \tilde{\theta}_i^T\tilde{\theta}_i + \nu_i[-c_i\nu_i \\ &\quad - c_i\xi_i - \xi_{i-1} + \nu_i + \nu_{i+1} + \xi_{i+1} + (x_{i+1,c} \\ &\quad - \alpha_i) - \dot{\xi}_i] + \frac{1}{r_i}\tilde{\theta}_i^T(r_i\nu_i S_i(\hat{X}) - \dot{\hat{\theta}}_i). \end{aligned} \quad (37)$$

From the compensating signal  $\xi_i$  and the adaptive law  $\hat{\theta}_i$ , (37) can be rewritten as

$$\begin{aligned} \dot{V}_i &\leq \dot{V}_0 - c_1\nu_1^2 - \sum_{i=2}^{n-1} (c_i - 1)\nu_i^2 + \frac{1}{2}e^T e + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \sum_{i=1}^{n-1} \frac{m_i}{r_i}\tilde{\theta}_i^T\hat{\theta}_i + \frac{1}{4}\sum_{i=2}^{n-1} \tilde{\theta}_i^T\tilde{\theta}_i + \nu_i\nu_{i+1}. \end{aligned} \quad (38)$$

*Step n:* Consider the following Lyapunov function as  $V_n = V_{n-1} + \frac{1}{2}\nu_n^2 + \frac{1}{2r_n}\tilde{\theta}_n^T\tilde{\theta}_n$  with  $r_n > 0$  being a con-



**Case 1.** Introduce the variables  $x_1 = q, x_2 = \dot{q}, x_3 = \tau$  and  $u = V$ , and the dynamics given by (47) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + \frac{1}{D}x_3 \\ \dot{x}_3 = f_3(x_1, x_2, x_3) + \frac{1}{M}u \\ y = x_1 \end{cases} \quad (48)$$

where  $f_2(x_1, x_2) = -\frac{N}{D}\sin(x_1) - \frac{B}{D}x_2$  and  $f_3(x_1, x_2, x_3) = -\frac{K_m}{M} - \frac{H}{M}x_3$  are unknown functions.

According to (48), the state observers are designed as:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + k_1(y - \hat{y}) \\ \dot{\hat{x}}_2 = \frac{1}{D}\hat{x}_3 + \hat{\theta}_2^T S_2 + k_2(y - \hat{y}) \\ \dot{\hat{x}}_3 = \frac{1}{M}u + \hat{\theta}_3^T S_3 + k_3(y - \hat{y}) \\ \dot{\hat{y}} = \hat{x}_1 \end{cases} \quad (49)$$

(a). First, the command filtered fuzzy controller is applied to system (48). Design the error and the virtual control as:

$$\begin{aligned} z_1 &= x_1 - x_d, z_2 = \hat{x}_2 - x_{2,c}, z_3 = \hat{x}_3 - x_{3,c} \\ \alpha_1 &= -c_1 z_1 - z_1 + \dot{x}_d, \\ \alpha_2 &= -c_2 z_2 - z_1 + \dot{x}_{2,c} - k_2 e_1 - \hat{\theta}_2^T S_2 \\ u &= -c_3 z_3 - z_2 + \dot{x}_{3,c} - k_3 e_1 - \hat{\theta}_3^T S_3 \end{aligned}$$

Choose the compensating signals as:

$$\begin{aligned} \dot{\xi}_1 &= -c_1 \xi_1 - \xi_1 + \xi_2 + (x_{2,c} - \alpha_1) \\ \dot{\xi}_2 &= -c_2 \xi_2 - \xi_1 + \xi_3 + (x_{3,c} - \alpha_2) \\ \dot{\xi}_3 &= -c_3 \xi_3 - \xi_2 \end{aligned}$$

Design  $\nu_i = z_i - \xi_i$  ( $i = 1, 2, 3$ ) and choose the adaptive laws as  $\dot{\hat{\theta}}_i = r_i \nu_i S_i - m_i \hat{\theta}_i$  ( $i = 2, 3$ ). The control parameters are

$$\begin{aligned} c_1 &= 16, c_2 = 75, c_3 = 50, r_1 = r_2 = 0.2, \\ m_2 &= 2, m_3 = 5, \zeta = 0.4, \omega_n = 2000. \end{aligned}$$

The fuzzy membership functions are:  $\mu_{F_i^j} = \exp\left[-\frac{(\hat{x}_i + l)^2}{2}\right]$  ( $i=2,3$ ) where  $j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$  and  $l = 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5$ , respectively. Choose observer gain vector  $K = [10, 300, 600]$  so that the matrix  $A_0$  is Hurwitz. Specify positive definite matrix  $Q = \text{diag}\{10, 10, 10\}$ , such that  $\lambda_{\min}(Q) - 2(n-1) - \frac{1}{4} > 0$ . The simulation is carried out under initial conditions are chosen to be zero and the reference signal is  $x_d = 0.5 \sin(t) + 0.5 \sin(0.5t)$ .

(b). Next, the DSC method in [35] is also utilized to control system (48) and the corresponding controller parameters  $c_1 = 300, c_2 = 30, c_3 = 100$ , and the other parameters are chosen as same as the proposed method in this paper.

The simulation results are shown in Figs. 2-7 in which Figs. 2(a)-7(a) are for the proposed method in this paper and Figs. 2(b)-7(b) are for the DSC method. From the simulation results, we can see that two kinds of methods all achieve satisfactory control performance and our proposed approach has smaller overshoots and achieves better tracking effect.

**Remark 6:** It should be noted that the fuzzy output-feedback DSC method proposed in [35] takes no account of the problem of compensating the errors caused by the filters, however, our proposed method introduces the filtering

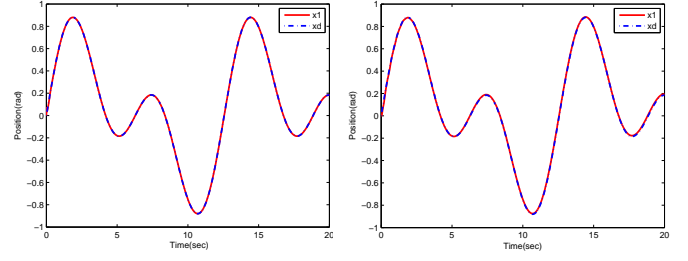


Fig. 2(a).  $x_1$  and  $x_d$ .

Fig. 2(b).  $x_1$  and  $x_d$ .

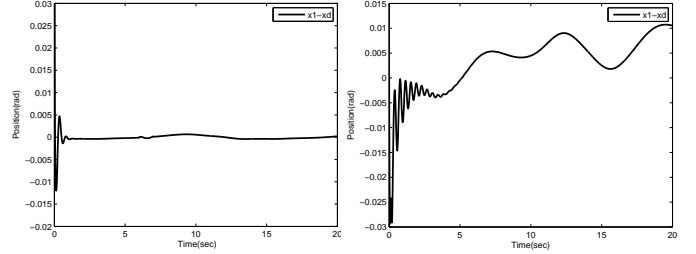


Fig. 3(a). Tracking error.

Fig. 3(b). Tracking error.

error compensation signals in this paper. Simulation results by comparing Figs. 2(a)-7(a) with Figs. 2(b)-7(b) shows the effectiveness of our proposed method.

**Case 2.** To further illustrate the robustness for the output measure noise of our proposed method, we give another set of simulation for the same electromechanical system and the measure noise  $0.003 \sin(5t)$  is added at  $t \in [6, 7]$ s for the input signal. From the simulation results in Figs. 8-9, it can be seen that the proposed command filtered-based fuzzy controller has a good robustness for the output measure noise.

**Example 2. (Numerical Example)** Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + x_1 \cos x_1^2 \\ \dot{x}_2 = x_3 + x_2 \sin x_1^2 \\ \dot{x}_3 = x_4 + a x_1 x_2 \\ \dot{x}_4 = \frac{u}{b} + \frac{x_2}{1+x_1 x_2} \\ y = x_1 \end{cases} \quad (50)$$

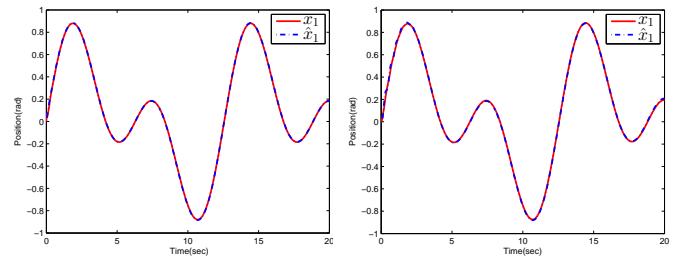


Fig. 4(a).  $x_1$  and  $\hat{x}_1$ .

Fig. 4(b).  $x_1$  and  $\hat{x}_1$ .

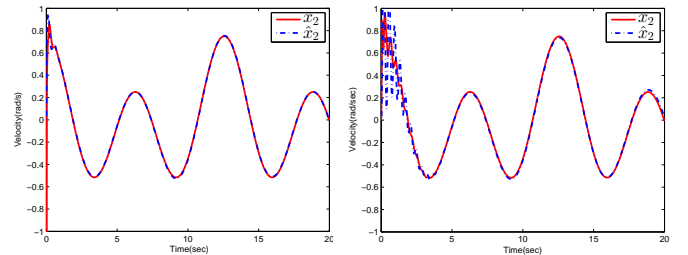
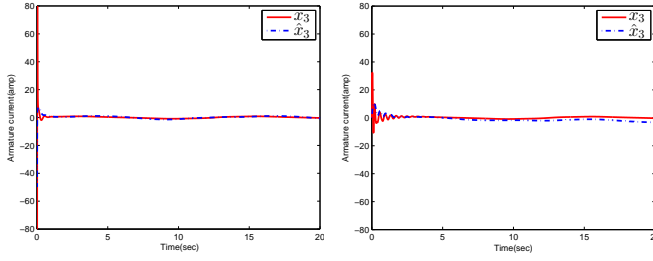
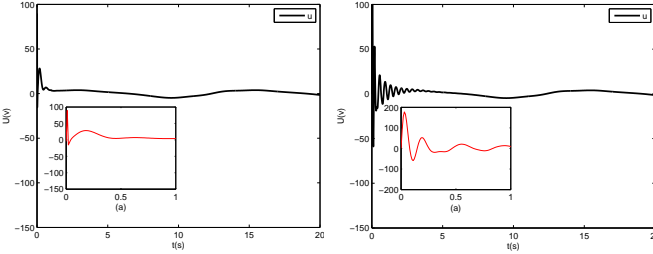


Fig. 5(a).  $x_2$  and  $\hat{x}_2$ .

Fig. 5(b).  $x_2$  and  $\hat{x}_2$ .

Fig. 6(a).  $x_3$  and  $\hat{x}_3$ .Fig. 6(b).  $x_3$  and  $\hat{x}_3$ .Fig. 7(a). Curve of  $u$ .Fig. 7(b). Curve of  $u$ .

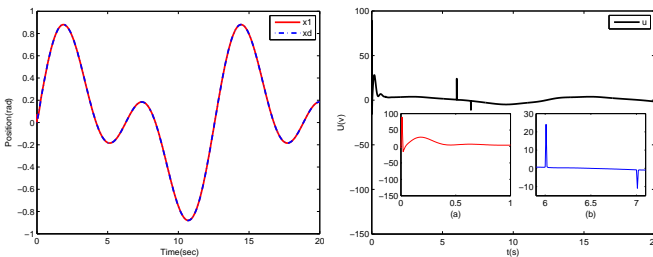
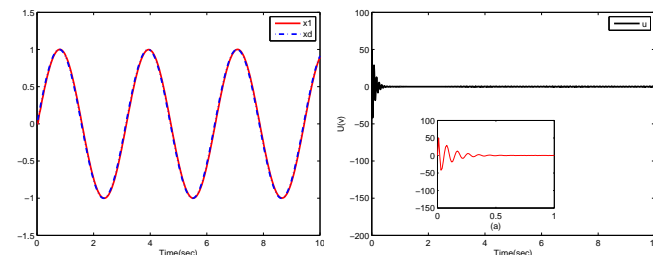
where  $a = 10$ ,  $b = 0.001$ ,  $f_1(x_1) = x_1 \cos x_1^2$ ,  $f_2(x_1, x_2) = x_2 \sin x_1^2$ ,  $f_3(x_1, x_2) = ax_1x_2$  and  $f_4(x_1, x_2) = \frac{x_2}{1+x_1x_2}$  are unknown functions. The design parameters are chosen as

$$\begin{aligned} k_1 &= 50, k_2 = 180, k_3 = 80, k_4 = 80, c_1 = 40, \\ c_2 &= 10, c_3 = 50, c_4 = 30, r_1 = r_2 = r_3 = r_4 = 0.18, \\ m_1 &= m_2 = m_3 = m_4 = 0.05, \zeta = 0.8, \omega_n = 270. \end{aligned}$$

The simulation is carried out under zero initial conditions and the reference signal is  $x_d = \sin(2t)$ . The main simulation results are shown in Figs. 10-11. From them, it can be seen that the proposed control method in this paper can track the reference signal well and achieve better control performances.

## VI. CONCLUSION

In this paper, a fuzzy command filtered adaptive output-feedback control has been proposed for a class of strict-feedback with parametric uncertainties and unmeasured states.

Fig. 8.  $x_1$  and  $x_d$  (Case 2).Fig. 9. Curve of  $u$  (Case 2).Fig. 10.  $x_1$  and  $x_d$ .Fig. 11. Curve of  $u$ .

Fuzzy logic systems are used to approximate the unknown nonlinear functions and a fuzzy state observer is designed to estimate the immeasurable states. Furthermore, command filtered backstepping control is constructed to avoid the explosion of complexity in the backstepping design and compensating signals are introduced to remove the effect of the errors caused by command filters. The proposed controller guarantees that all the signals in the closed-loop systems are bounded. Simulation results of two examples have shown the effectiveness of the proposed control scheme.

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