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Observer Based Coordinated Adaptive Robust Control of Robot Manipulators Driven by Single-Rod Hydraulic Actuators

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Abstract

This paper studies the coordinated motion control of a hydraulic arm driven by single-rod hydraulic actuators (a scaled down version of an industrial backhoe/excavator arm). Compared to conventional robot manipulators driven by electrical motors, hydraulic arms have a richer nonlinear dynamics and strong couplings among various joints (or hydraulic cylinders). This paper presents a physical model based adaptive robust controller (ARC) to explicitly take into account the strong coupling among various hydraulic cylinders (or joints). In addition, an observer is employed to avoid the need of acceleration feedback for ARC backstepping design. Theoretically, the resulting controller is able to take into account not only the effect of parametric uncertainties coming from the payload and various hydraulic parameters but also the effect of uncertain nonlinearities. Furthermore, the proposed ARC controller guarantees a prescribed output tracking transient performance and final tracking accuracy while achieving asymptotic output tracking in the presence of parametric uncertainties. Simulation and experimental results are presented to illustrate the proposed control algorithm.

Keywords
Robot Manipulators; Electro-Hydraulic System; Motion Control; Adaptive Control; Robust Control; Nonlinearity

1 Introduction

Compared with electrical motors, hydraulic actuators have a much smaller size-to-power ratio and the ability to apply very large force and torque. As such, they are widely used in industry in a wide number of applications, especially in earth moving equipment such as excavators and backhoe loader. In order to increase the productivity and performance of industrial hydraulic machines like excavators, it is essential to be able to control a robot-arm-like mechanical device driven by hydraulic actuators (i.e., a hydraulic robot arm) well.

The precision motion control of a hydraulic arm driven by hydraulic cylinders is difficult both theoretically and experimentally due to the following several reasons. Firstly, the coupling effects among various joints are strong. Unlike electric motors, hydraulic cylinders are linear actuators and can not be placed at the joints of the arm, which normally results in a complicated mechanical configuration. Secondly, in addition to the coupled multiple-input multiple-output (MIMO) nonlinear dynamics of the rigid hydraulic arm, the dynamics of the hydraulic actuators must be considered in the control of a hydraulic arm, which substantially increases the controller design difficulties. It is well known that a robot arm including actuator dynamics [1] has a "relative degree" more than three. Synthesizing controllers for such a system usually requires joint acceleration feedback for a complete state feedback, which may not be a practical solution. Furthermore, the single-rod hydraulic actuator studied here has a much more complicated dynamics than electrical motors. The dynamics of a hydraulic cylinder is highly nonlinear [2]. The dynamic equations describing the pressure changes in the two chambers of a single-rod hydraulic actuator cannot be combined into a single load pressure equation, which not only increases the dimension of the system to be dealt with but also brings in the stability issue of the added internal dynamics. Thirdly, a hydraulic arm normally experiences large extent of model uncertainties including the large changes in load seen by the system in industrial use, the large variations in the hydraulic parameters, leakages, the external disturbances, and frictions. Partly due to these difficulties, so far, the model-based robust control of a hydraulic arm has not been well studied and fewer results are available.

This paper is devoted to solving the theoretical and practical problems mentioned above. In [3, 4], the adaptive robust control (ARC) approach proposed by Yao and

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Tomizuka in [5, 6, 7, 8] was generalized to provide a rigorous theoretical framework for the high performance robust motion control of a one degrees-of-freedom (DOF) single-rod hydraulic actuator. The stability of zero output tracking error dynamics of single-rod hydraulic actuator was also addressed in [3, 4].

This paper will extend the nonlinear adaptive robust control strategies proposed in [3, 4] to the precision motion control of a MIMO 3 DOF robot arm driven by single-rod hydraulic actuators. The main theoretical issues to be addressed here are the strong couplings among various hydraulic cylinders due to the hydraulic arm dynamics, the need for avoiding acceleration feedback, and presence of parametric uncertainties and uncertain nonlinearities in both the robot dynamics and the hydraulic dynamics. A model-based adaptive robust observer is first constructed to obtain the estimates of the joint accelerations. The resulting observer based ARC controller is able to take into account not only the effect of the parametric uncertainties coming from the payload and various hydraulic parameters but also the effect of uncertain nonlinearities such as uncompensated friction forces, external disturbances. Theoretically, the ARC controller guarantees a prescribed output tracking transient performance and final tracking accuracy in general while achieving asymptotic output tracking in the presence of parametric uncertainties. Simulation and experimental results are presented to show the effectiveness of the proposed control algorithm.

The proposed adaptive robust observer design is motivated by the research done in [1], where an observer based adaptive controller is constructed for the robot manipulator with electrical motor dynamics. The main difference between the two observer designs is that the proposed adaptive robust observer design is able to take into account the effect of both parametric uncertainties and the uncertain nonlinearities and achieve a prescribed transient performance and final tracking accuracy in general while the observer design in [1] deals with parametric uncertainties only. It is well known that pure adaptive design may become unstable when the system is subjected to uncertain nonlinearities [9].

2 Problem Formulation and Dynamic Models

The system under consideration is depicted in Fig.1, which represents a 3 DOF robot arm driven by three single-rod hydraulic cylinders. The swing angle q_1, boom angle q_2 and stick angle q_3 are defined as shown in the figure. l_1, l_2 and l_3 are the pin-to-pin lengths for the swing, boom and stick arm respectively. $x = [x_s, x_b, x_{st}]^T$ is the displacement vector of the swing, boom, and stick cylinders, which is uniquely related to the angle q_1, q_2 and q_3, i.e., $x_s(q_1)$, $x_b(q_2)$ and $x_{st}(q_3)$. The goal is to have joint angles $q = [q_1, q_2, q_3]^T$ track any feasible desired motion

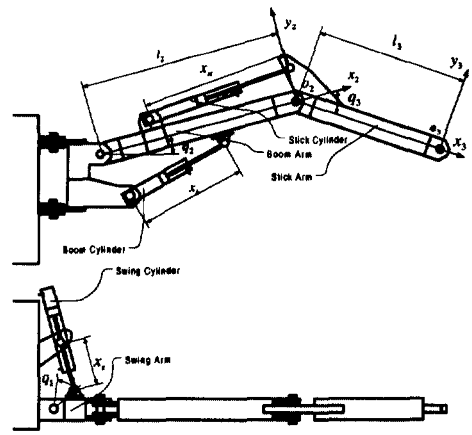

Figure 1: A Hydraulic Robot Arm

trajectories as closely as possible for precision maneuver of the inertia load of the hydraulic robot arm. The rigid-body dynamics of the hydraulic arm can be described by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \frac{\partial x}{\partial q}(A_1P_1 - A_2P_2) + T(t,q,\dot{q}) \quad (1)$$

where $P_1 = [P_{1s}, P_{1b}, P_{1st}]^T$,$P_2 = [P_{2s}, P_{2b}, P_{2st}]^T$ and P_{1s}, P_{1b}, P_{1st}, P_{2s}, P_{2b}, P_{2st} are the forward chamber pressures and return chamber pressures for the swing, boom and stick cylinder respectively, $A_1 = diag[A_{1s}, A_{1b}, A_{1st}]$ and $A_2 = diag[A_{2s}, A_{2b}, A_{2st}]$ are the ram areas of the two chambers of the swing, boom and stick cylinders respectively, and $T(t,q,\dot{q}) \in R^3$ represents the lumped disturbance torque including external disturbances and terms like the friction torque.

Let m_L be the unknown payload mounted at the end of the stick arm, which is treated as a point mass for simplicity. Then, the inertial matrix $M(q)$, coriolis terms $C(q,\dot{q})$ and gravity terms $G(q)$ in (1) can be linearly parametrized with respect to the unknown mass m_L as

$$\begin{array}{l} M(q) = M_c(q) + M_L(q)m_L, G(q) = G_c(q) + G_L(q)m_L \\ C(q,\dot{q}) = C_c(q,\dot{q}) + C_L(q,\dot{q})m_L \end{array} \quad (2)$$

where $M_c(q)$, $M_L(q)$, $C_c(q,\dot{q})$, $C_L(q,\dot{q})$, $G_c(q)$, $G_L(q)$ are known nonlinear functions of q and $\dot{q}$.

Assuming no cylinder leakages, the actuator (or the cylinder) dynamics can be written as [2],

$$\begin{array}{l} \frac{V_1(x)}{\beta_e}\dot{P}_1 = -A_1\dot{x} + Q_1 = -A_1\frac{\partial x}{\partial q}\dot{q} + Q_1 \\ \frac{V_2(x)}{\beta_e}\dot{P}_2 = A_2\dot{x} - Q_2 = A_2\frac{\partial x}{\partial q}\dot{q} - Q_2 \end{array} \quad (3)$$

where $V_1(x) = V_{h_1} + A_1 diag[x] \in R^{3\times3}$ and $V_2(x) = V_{h_2} - A_2 diag[x]$ are the diagonal total control volume matrices of the two chambers of hydraulic cylinders respectively, which includes the hose volume between the two chambers and the valves, $V_{h1} = diag[V_{h1s}, V_{h1b}, V_{h1st}]$ and $V_{h2} = diag[V_{h2s}, V_{h2b}, V_{h2st}]$ are the control volumes of the two chambers when $x = 0$, $diag[x] = diag[x_s, x_b, x_{st}]$, $\beta_e \in R$ is the effective bulk modulus, $Q_1 = [Q_{1s}, Q_{1b}, Q_{1st}]^T$ is the vector of the supplied flow rates to the forward chambers of the three cylinders, and

$Q_2 = [Q_{2s}, Q_{2b}, Q_{2st}]^T$ is the vector of the return flow rates from the return chambers of the cylinders.

Let x_{vs}, x_{vb}, and x_{vst} denote the spool displacements of the valves in the swing, boom, and stick loop respectively. Define the square roots of the pressure drops across the two ports of the swing valve g_{3s} and g_{4s} as:

$$g_{3s}(P_{1s}, sign(x_{vs})) = \begin{cases} \sqrt{P_s - P_{1s}} & \text{for} \quad x_{vs} \geq 0 \\ \sqrt{P_{1s} - P_r} & \quad x_{vs} < 0 \end{cases}$$
$$g_{4s}(P_{2s}, sign(x_{vs})) = \begin{cases} \sqrt{P_{2s} - P_r} & \text{for} \quad x_{vs} \geq 0 \\ \sqrt{P_s - P_{2s}} & \quad x_{vs} < 0 \end{cases} \tag{4}$$

where P_s is the supply pressure of the pump, and P_r is the tank reference pressure. Similarly, let g_{3b}, g_{4b}, g_{3st}, and g_{4st} be the square roots of the pressure drops for the boom and stick loops respectively. For simplicity of notation, define the diagonal square root matrices of the pressure drops as:

$$\begin{array}{l} g_3(P_1, sign(x_v)) = diag[g_{3s}(P_{1s}, sign(x_{vs})), \\ \qquad g_{3b}(P_{1b}, sign(x_{vb})), g_{3st}(P_{1st}, sign(x_{vst})] \\ g_4(P_2, sign(x_v)) = diag[g_{4s}(P_{2s}, sign(x_{vs})), \\ \qquad g_{4b}(P_{2b}, sign(x_{vb})), g_{4st}(P_{2st}, sign(x_{vst}))] \end{array} \tag{5}$$

Then, Q_1 and Q_2 in (3) are related to the spool displacements of the valves $x_v = [x_{vs}, x_{vb}, x_{vst}]^T$ by [2],

$$\begin{array}{l} Q_1 = k_{q1} g_3(P_1, sign(x_v)) x_v, \\ Q_2 = k_{q2} g_4(P_2, sign(x_v)) x_v \end{array} \tag{6}$$

where $k_{q1} = diag[k_{q1s}, k_{q1b}, k_{q1st}]$ and $k_{q2} = diag[k_{q2s}, k_{q2b}, k_{q2st}]$ are the *constant* flow gain coefficients matrices of the forward and return loops respectively.

Given the desired motion trajectory $q_{Ld}(t)$, the objective is to synthesize a control input $u = x_v$ such that the output $y = q$ tracks $q_{Ld}(t)$ as closely as possible in spite of various model uncertainties.

3 Adaptive Robust Controller Design

III.1 *Design Model and Issues to be Addressed*

In this paper, for simplicity, we consider the parametric uncertainties due to the unknown payload m_L, the bulk modulus β_e, the nominal value of the lumped disturbance T, T_n only. Other parametric ucnertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the system dynamics equation in terms of a set of unknown parameters. To achieve this, define the unknown parameter set as $\theta = [\theta_1, \theta_2, \theta_3]^T$ where $\theta_1 = m_L$, $\theta_2 = T_n$, and $\theta_3 = \beta_e$. The system dynamic equations can thus be linearly parametrized in terms of θ as

$$\begin{array}{l} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \frac{\partial x}{\partial q}(P_1 A_1 - P_2 A_2) \\ \quad + \theta_2 + \tilde{T}(t,q,\dot{q}), \qquad \tilde{T} = T(t,q,\dot{q}) - T_n \\ \dot{P}_1 = \theta_3 V_1^{-1}(q)\left[-A_1 \frac{\partial x}{\partial q}\dot{q} + Q_1(u, g_3(P_1, sign(u))\right] \\ \dot{P}_2 = \theta_3 V_2^{-1}(q)\left[A_2 \frac{\partial x}{\partial q}\dot{q} - Q_2(u, g_4(P_2, sign(u))\right] \end{array} \tag{7}$$

Since the extent of the parametric uncertainties and uncertain nonlinearities are normally known, it is assumed that

$$\begin{array}{rcl} \theta & \in & \Omega_\theta \triangleq \{\theta : \ \theta_{min} < \theta < \theta_{max}\} \\ |\tilde{T}(t,q,\dot{q})| & \leq & \delta_T(q,\dot{q},t) \end{array} \tag{8}$$

where $\theta_{min} = [\theta_{1min}, \ldots, \theta_{3min}]^T$, $\theta_{max} = [\theta_{1max}, \ldots, \theta_{3max}]^T$, and $\delta_T(t,q,\dot{q})$ are known.

At this stage, it can be seen that the main difficulties in controlling (7) are: (i) The system dynamics are highly nonlinear and coupled, due to either the nonlinear robot dynamics or the dependence of the effective driving torque on joint angle (terms like $\frac{\partial x(q)}{\partial q}$) and the nonlinearities in the hydraulic dynamics; (ii) the system has large extent of parametric uncertainties due to the large variations of inertial load m_L and the change of bulk modulus caused by the entrapped air or change of temperature, etc; (iii) The system may have large extent of lumped uncertain nonlinearities $\tilde{T}$ including external disturbances and unmodeled friction forces; (iv) The added nonlinear hydraulic dynamics are more complex than the electrical motor dynamics. (v) The model uncertainties are mismatched, i.e. both parametric uncertainties and uncertain nonlinearities appear in the dynamic equations which are not directly related to the control input $u = x_v$.

To address the challenges mentioned above, following general strategies will be adopted in the controller design. Firstly, the nonlinear physical model based analysis and synthesis will be employed to deal with the nonlinearities and coupling of the system dynamics. Secondly, the ARC approach [5, 8] will be used to handle the effect of both parametric uncertainties and uncertain nonlinearities; fast robust feedback will be used to attenuate the effect of various model uncertainties as much as possible while parameter adaptation will be introduced to reduce model uncertainties for high performance. Thirdly, backstepping design via ARC Lyapunov function will be used to overcome the design difficulties caused by the unmatched model uncertainties. Finally, an adaptive robust observer will be synthesized in order to avoid the need for acceleration feedback. The details are outlined below.

III.2 *ARC Controller Design*

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in [8, 3] as follows. Let $Proj_{\hat{\theta}}(\bullet)$ denote the discontinuous projection defined in [10, 11, 5], the adaptation law is given by

$$\dot{\hat{\theta}} = -Proj_{\hat{\theta}}(\Gamma\tau) \tag{9}$$

where $\Gamma > 0$ is a diagonal matrix and τ is an adaptation function to be synthesized later.

Step 1

From (7), define the load pressure as $P_L = A_1 P_1 - A_2 P_2$. If we treat P_L as the virtual control input to the first equation of (7), a virtual control law P_{Ld} for P_L will be synthesized such that $z_1 = q - q_d$ is as small as possible with a guaranteed transient performance, in which q_d is the reference trajectory. The ARC approach proposed in

[8] will be generalized to accomplish the objective. The control function P_{Ld} consists of two parts given by

$$P_{Ld}(q,\dot{q},\hat{\theta}_1,\hat{\theta}_2,t) = P_{Lda} + P_{Lds}$$
$$P_{Lda} = \left(\frac{\partial x}{\partial q}\right)^{-1}\left[\hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - \hat{\theta}_2\right]$$
$$P_{Lds} = P_{Lds1} + P_{Lds2}, \quad P_{Lds1} = -\left(\frac{\partial x}{\partial q}\right)^{-1} K_2 z_2$$
$$\tau_2 = \phi_2 z_2$$
$$\phi_2 = [M_L(q)\ddot{q}_r + C_L(q,\dot{q})\dot{q}_r + G_L(q), -I_{3\times3}, 0]^T \tag{10}$$

where $\hat{M}(q) = M_c(q) + M_L(q)\hat{\theta}_1$, $\hat{C}(q,\dot{q}) = C_c(q,\dot{q}) + C_L(q,\dot{q})\hat{\theta}_1$, $\hat{G}(q) = G_c(q) + G_L(q)\hat{\theta}_1$, $\dot{q}_r = \dot{q}_d - k_1 z_1$, $z_2 = \dot{q} - \dot{q}_r$, k_1 is a positive feedback gain, and K_2 is a symmetric positive definite(s.p.d) control gain matrix. The robust control function P_{Lds2} is synthesized to satisfy the following conditions

$$\begin{array}{ll} \text{condition i} & z_2^T[\frac{\partial x}{\partial q}P_{Lds2} + \phi_2^T\tilde{\theta} + \tilde{T}] \leq \varepsilon_2 \\ \text{condition ii} & z_2^T\frac{\partial x}{\partial q}P_{Lds2} \leq 0 \end{array} \tag{11}$$

where ε_2 is a positive design parameter which can be arbitrarily small. How to choose P_{Lds2} to satisfy constraints like (11) can be worked out in the same way as in [6, 7].

Let $z_3 = P_L - P_{Ld}$ denote the input discrepancy. Substituting (10) into the first equation of (7), one obtains

$$M(q)\dot{z}_2 + C(q,\dot{q})z_2 = -K_2 z_2 + \frac{\partial x}{\partial q}z_3 + \frac{\partial x}{\partial q}P_{Lds2} + \phi_2^T\tilde{\theta} + \tilde{T} \tag{12}$$

Define a positive semi-definite function (p.s.d.) as $V_r = \frac{1}{2}z_2^T M(q)z_2$. Notice that $\dot{M}(q) - 2C(q,\dot{q})$ is a skew-symmetric matrix [12], from (12), the time derivative of V_r can be expressed as:

$$\dot{V}_r = z_2^T\frac{\partial x}{\partial q}z_3 - z_2^T K_2 z_2 + z_2^T\left[\frac{\partial x}{\partial q}P_{Lds2} + \phi_2^T\tilde{\theta} + \tilde{T}\right] \tag{13}$$

Step 2

In this step, an actual control law will be synthesized so that z_3 converges to zero or a small value with a guaranteed transient performance and accuracy. If we were to use the backstepping design strategy via ARC Lyapunov function as in [3, 8], then, the resulting ARC law would require the feedback of the joint acceleration $\ddot{q}$ for adaptive model compensation since $\ddot{q}$ is needed in computing $\dot{\hat{P}}_{Ld}$, the calculable part of the derivative of the desired virtual control function P_{Ld}. In order to avoid the need for joint acceleration feedback, in the following, an adaptive robust observer is first constructed to obtain the estimates of the joint velocity and acceleration. Those estimates will be used to form $\hat{P}_{Ld}$ and $\dot{\hat{P}}_{Ld}$.

Observer Design

Define the observer errors as:

$$\begin{array}{ll} e_{o1} = q - y, & \dot{y}_r = \dot{y} - k_{o1}e_{o1} \\ e_{o2} = \dot{q} - \dot{y}_r, & \ddot{y}_r = \ddot{y} - k_{o1}(\dot{q} - \dot{y}) \end{array} \tag{14}$$

where y and $\dot{y}$ are the estimates of q and $\dot{q}$ respectively. The proposed nonlinear observer is :

$$\bar{M}(q)\ddot{y}_r + \bar{C}(q,\dot{q})\dot{y}_r + \bar{G}(q) = \frac{\partial x}{\partial q}P_L + (K_{o2} + K_{o2s})e_{o2} + T_{os} + \bar{\theta}_2 \tag{15}$$

where $\bar{M}(q) = M_c + M_L\bar{\theta}_1$, $\bar{C}(q,\dot{q}) = C_c + C_L\bar{\theta}_1$ and $\bar{G}(q) = G_c + G_L\bar{\theta}_1$, in which $\bar{\theta} = [\bar{\theta}_1, \bar{\theta}_2]^T$ is a new set of parameter estimate for θ and is used in the construction of the above observer only. K_{o2} is any positive definite gain matrix, and K_{o2s} is a nonlinear positive definite gain matrix to be specified later. T_{os} is a robust observer error feedback term which will be specified later for a guaranteed transient performance.

Define a p.s.d. function as $V_o = \frac{1}{2}e_{o2}^T M(q)e_{o2}$. Differentiating it and noting (1) and (15), it can be shown that

$$\dot{V}_o = -e_{o2}^T(K_{o2} + K_{o2s})e_{o2} + e_{o2}^T\left[\phi_o^T\tilde{\theta}_o + \tilde{T} - (I - \tilde{\theta}_{o1}M_L(q)\bar{M}^{-1}(q))T_{os}\right] \tag{16}$$

where $\tilde{\theta}_o = \theta - \bar{\theta}$ is the parameter estimation error in the observer, and $\phi_o = [M_L(q)\bar{M}^{-1}(\frac{\partial x}{\partial q}P_L + (K_{o2} + K_{o2s})e_{o2} + \bar{\theta}_2 - \bar{C}(q,\dot{q})\dot{y}_r - \bar{G}(q)) + C_L(q,\dot{q})\dot{y}_r + G_L(q), -I_{3\times3}, 0]^T$

If the parameter variation $\tilde{\theta}_{o1}$ is within certain limit such that $\|\tilde{\theta}_{o1}M_L(q)\bar{M}^{-1}(q)\| < 1$, then, a robust feedback function $T_{os}(q,\dot{q},\bar{\theta},y,\dot{y},t)$ can be determined in the same way as in (11) to satisfy following conditions:

$$\begin{array}{ll} \text{condition i} & e_{o2}^T[-(I - \tilde{\theta}_{o1}M_L\bar{M}^{-1})T_{os} + \phi_o^T\tilde{\theta}_o + \tilde{T}] \leq \varepsilon_o \\ \text{condition ii} & -e_{o2}^T(I - \tilde{\theta}_{o1}M_L\bar{M}^{-1})T_{os} \leq 0 \end{array} \tag{17}$$

where ε_o is a positive design parameter. The adaptation law for parameter estimates is given by

$$\dot{\bar{\theta}} = -Proj_{\bar{\theta}}(\Gamma_o\tau_o), \qquad \tau_o = \phi_o^T e_{o2} \tag{18}$$

From (15), $\ddot{y}_r$ can be computed by:

$$\ddot{y}_r = \bar{M}^{-1}(q)[\frac{\partial x}{\partial q}P_L + (K_{o2} + K_{o2s})e_{o2} + T_{os} + \bar{\theta}_2 - \bar{C}(q,\dot{q})\dot{y}_r - \bar{G}(q)] \tag{19}$$

and can be used in the design of the control law u. In general, the observer (19) needs the calculation of the inverse of the inertial matrix estimate $\bar{M}(q)$, which may become singular during parameter adaptation process [1]. Here, due to the use of projection mapping in (18), the parameter estimate is guaranteed to stay within the prescribed bound [5], and the singularity problem [1] may be avoided in our design.

Note that $\dot{y}_r$ and $\ddot{y}_r$ are the estimates of the joint velocity and acceleration respectively. By using the velocity estimate $\dot{y}_r$ to replace the joint velocity $\dot{q}$ in the virtual control law P_{Ld}, we obtain the estimate of the P_{Ld} as $\hat{P}_{Ld} = P_{Ld}(q,\dot{y}_r,\hat{\theta}_1,\hat{\theta}_2,t)$. Let $\hat{z}_3 = P_L - \hat{P}_{Ld}$ be the new input discrepancy. Then, (13) becomes:

$$\dot{V}_r = z_2^T\frac{\partial x}{\partial q}\hat{z}_3 - z_2^T K_2 z_2 + z_2^T\left[\frac{\partial x}{\partial q}P_{Lds2} + \phi_2^T\tilde{\theta} + \tilde{T}\right] + z_2^T\mu \tag{20}$$

where $\mu = \frac{\partial x}{\partial q}(\hat{P}_{Ld} - P_{Ld})$ represents the effect of the observer error, which has the property summarized by the following lemma.

Lemma 1 *μ is bounded by*

$$\|\mu(q,\dot{y}_r,\hat{\theta}_1,e_{o2},z_2,t)\| \leq \sigma_e(q,\dot{y}_r,\hat{\theta}_1,e_{o2},z_2,t)\|e_{o2}\| \tag{21}$$

where σ_e is a positive scalar function.

Proof. The proof can be obtained from the author. □.

The following is to synthesize a control input u such that $\hat{z}_3 = P_L - \hat{P}_{Ld}$ converges to zero or a small value with a guaranteed transient performance. From (7),

$$\dot{\hat{z}}_3 = \dot{P}_L - \dot{\hat{P}}_{Ld} = \theta_3[-(A_1^2V_1^{-1} + A_2^2V_2^{-1})\frac{\partial x}{\partial q}\dot{q} + (A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2)] - \dot{\hat{P}}_{Ld} \quad (22)$$

where $\dot{\hat{P}}_{Ld} = \frac{\partial \hat{P}_{Ld}}{\partial q}\dot{q} + \frac{\partial \hat{P}_{Ld}}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \hat{P}_{Ld}}{\partial t} + \frac{\partial \hat{P}_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}}$. Define the load flow Q_L as

$$Q_L = A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2 = [A_1V_1^{-1}k_{q1}g_3(P_1, sign(u)) + A_2V_2^{-1}k_{q2}g_4(P_2, sign(u))]u \quad (23)$$

then step 2 is to synthesize a control function Q_{Ld} for Q_L such that P_L tracks the desired control function $\hat{P}_{Ld}$ with a guaranteed transient performance. Consider the augmented p.s.d. function $V_m = V_r + \frac{1}{2}\hat{z}_3^T\hat{z}_3$. From (22)

$$\dot{V}_m = \dot{V}_r + \hat{z}_3^T\dot{\hat{z}}_3 = \dot{V}_r\mid_{\hat{z}_3=0} + \hat{z}_3^T\left[\theta_3 Q_L + Q_{Lde} + \phi_3^T\tilde{\theta}\right] \quad (24)$$

where $\dot{V}_r\mid_{\hat{z}_3=0}$ is a short-hand notation used to represent $\dot{V}_r$ when $\hat{z}_3 = 0$, $Q_{Lde} = \frac{\partial x}{\partial q}z_2 - \hat{\theta}_3(A_1^2V_1^{-1} + A_2^2V_2^{-1})\frac{\partial x}{\partial q}\dot{q} - \dot{\hat{P}}_{Ld}$, $\phi_3 = [0, 0, -Q_{La} + (A_1^2V_1^{-1} + A_2^2V_2^{-1})\frac{\partial x}{\partial q}\dot{q}]^T$.
The control function Q_{Ld} consists of two parts given by

$$\begin{aligned} &Q_{Ld}(q, \dot{q}, \dot{y}_r, \ddot{y}_r, P_1, P_2, \hat{\theta}, t) = Q_{Lda} + Q_{Lds} \\ &Q_{Lda} = -\frac{1}{\hat{\theta}_3}Q_{Lde} \\ &Q_{Lds} = Q_{Lds1} + Q_{Lds2}, \quad Q_{Lds1} = -\frac{1}{\theta_{3min}}K_3\hat{z}_3 \end{aligned} \quad (25)$$

where K_3 is a constant positive definite control gain matrix, Q_{Lds2} is a robust control function satisfying the following two conditions

$$\begin{aligned} &\text{condition i} \quad \hat{z}_3^T\left[\theta_3 Q_{Lds2} + \phi_3^T\tilde{\theta}\right] \le \varepsilon_3 \\ &\text{condition ii} \quad \hat{z}_3^T Q_{Lds2} \le 0 \end{aligned} \quad (26)$$

where ε_3 is a positive design parameter.
Once the control function Q_{Ld} for Q_L is synthesized as given by (25), the actual control input u can be backed out from the continuous one-to-one nonlinear load flow mapping (23) as follows. Noting that the elements of the diagonal matrices g_3, g_4, V_1, and V_2 are all positive functions, u_i should have the same sign as Q_{Ldi}. Thus

$$u = [A_1V_1^{-1}k_{q1}g_3(P_1, sign(Q_{Ld})) + A_2V_2^{-1}k_{q2}g_4(P_2, sign(Q_{Ld}))]^{-1}Q_{Ld} \quad (27)$$

III.3 *Main Theoretical Results*

Theorem 1 *Let the parameter estimates be updated by the adaptation law (9) in which* $\tau = \phi_2 z_2 + \phi_3\hat{z}_3$. *Let* k_2, k_3, *and* k_{o2} *be any positive gains and construct a p.s.d. function as* $V = V_m + V_o$. *If the control gain matrices* K_2, K_3 *and* K_{o2} *are chosen such that* $\lambda_{min}(K_2) \ge k_2 + \frac{1}{2}$, $\lambda_{min}(K_3) \ge k_3$, $\lambda_{min}(K_{o2}) \ge k_{o2}$, and $\lambda_{min}(K_{o2s}) \ge \frac{1}{2}\sigma_e^2$, *then, the following results hold if the control law (27) with the adaptation law (9) is applied:*

A. *In general, the tracking errors and the observer errors,* $z_1, z_2, \hat{z}_3, e_{o1}$, and e_{o2}, *are bounded. Furthermore,* V, *an index for the bound of the tracking errors and observer errors, is bound above by*

$$V(t) \le exp(-\lambda_V t)V(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \quad (28)$$

where $\lambda_V = \frac{2min\{k_2, k_3, k_{o2}\}}{max\{k_M, 1\}}$, $\varepsilon_V = \varepsilon_2 + \varepsilon_3 + \varepsilon_o$, *and* k_M *is the upper bound of the inertial matrix (i.e.,* $M(q) \le k_M I_{3\times3}$ *).*

B *If after a finite time* t_0, $\tilde{T} = 0$, *i.e., in the presence of parametric uncertainties only, in addition to results in A, asymptotic output tracking is also obtained.* △

Proof : The theorem can be proved in the same way as in [3, 4]. The details can be obtained from the authors. □.

4 Simulation and Experimental Results

To test the proposed nonlinear ARC strategy and study fundamental problems associated with the control of electro-hydraulic systems, a three-link robot arm (a scaled down version of industrial backhoe loader arm) driven by three single-rod hydraulic cylinders shown in Fig.1 has been set up. The three hydraulic cylinders are controlled by two proportional directional control valves and one servovalve manufactured by Parker Hannifan company.
The exact model of the hydraulic arm shown in Fig.1 is quite messy and can be obtained from the authors. Parameters of the actual arm are: m_1 =22.98kg, m_2 =24.94kg, m_3 =19.68kg, m_L = 20kg, l_1 =0.3683m, l_2= 0.9906m, and l_3 =0.8001m. Cylinder parameters are: $A_1 = diag[2.0268\times10^{-3}, 2.0268\times10^{-3}, 2.0268\times10^{-3}]m^2$, $A_2 = diag[1.0688\times10^{-3}, 1.0688\times10^{-3}, 1.0688\times10^{-3}]m^2$, $V_{h1} = diag[4.9953\times10^{-4}, 5.2125\times10^{-4}, 4.8505\times10^{-4}]m^3$, and $V_{h2} = diag[9.0676\times10^{-4}, 8.7237\times10^{-4}, 9.2667\times10^{-4}]m^3$. The valve parameters are: $k_{q1} = diag[3.5904\times10^{-8}, 7.4965\times10^{-8}, 7.4965\times10^{-8}]\frac{m^3}{sec\sqrt{Pa}V}$, $k_{q2} = diag[3.7206\times10^{-8}, 6.9047\times10^{-8}, 6.9047\times10^{-8}]\frac{m^3}{sec\sqrt{Pa}V}$, $P_s = 6.9\times10^6 Pa$, and $\beta_e = 2.7148\times10^8 Pa$. The desired joint position vector is $q_d = [0.9 + 0.25sin(\frac{\pi}{4}t), 0.5 + 0.25sin(\frac{\pi}{4}t), -0.6 + 0.25sin(\frac{\pi}{4}t)]rad$.
Firstly, simulation is conducted to validate the design of the controller. The control gain and gain matrices are chosen as $k_1 = 200$, $K_2 = K_3 = diag[310, 250, 240]$, $k_{o1} = 500$, and $K_{o2} = diag[500, 1000, 1000]$. Adaptation gain matrices are $\Gamma = diag[1, 100, 20, 1, 3]$ and $\Gamma_o = diag[100, 100, 100, 100, 0]$. As seen in Fig. 2, the system has very small tracking errors during both transient period and steady-state period. Parameter estimates for

θ_1 shown in Fig.3 reveals that both the parameter estimate $\hat{\theta}_1$ used in the controller and $\bar{\theta}_1$ used in the observer approach to their true value θ_1.

The controller is then implemented on the actual system to test its performance. Due to the severe measurement noise problem associated with the position and pressure sensors and the limited valve bandwidth (around 10Hz for the servovalve and around 7Hz for the proportional valves), much smaller controller gains than those in the simulation is used: $k_1 = 20$, $K_2 = K_3 = diag[20, 25, 30]$, $k_{o1} = 50$, and $K_{o2} = diag[50, 60, 65]$. Adaptation gain matrices are $\Gamma = diag[0.01, 0.3, 1, 1.6, 3]$ and $\Gamma_o = diag[0.3, 11, 16, 19, 0]$. The tracking errors of all joints are shown in Fig. 4. The results are not as good as those in simulation results, which is mainly due to the severe measurement noises and the neglected valve dynamics. Further efforts are needed to address these practical implementation issues, which is the focus of our current research.

5 Conclusion

In this paper, a physical model based adaptive robust controller (ARC) is constructed for the coordinated motion control of a hydraulic arm driven by single-rod hydraulic actuators. The ARC controller explicitly takes into account the strong coupling among various hydraulic cylinders (or joints). In addition, an observer is employed to avoid the need of acceleration feedback. Simulation and experimental results are presented to illustrate the proposed algorithm.

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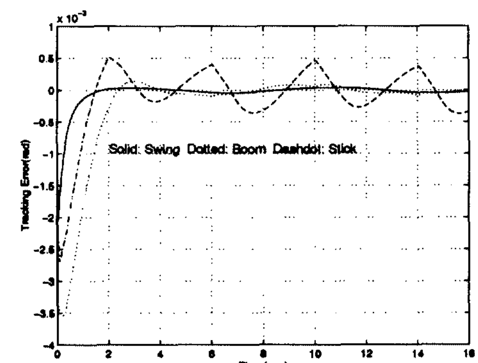

Figure 2: Tracking Errors for Each Joint

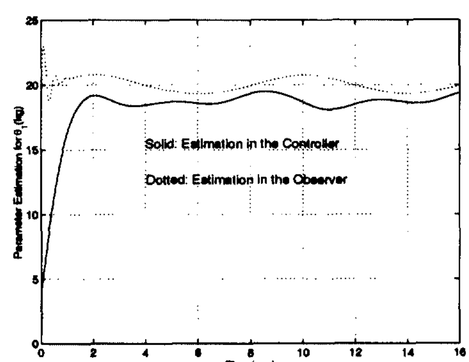

Figure 3: Parameter Estimations for θ_1

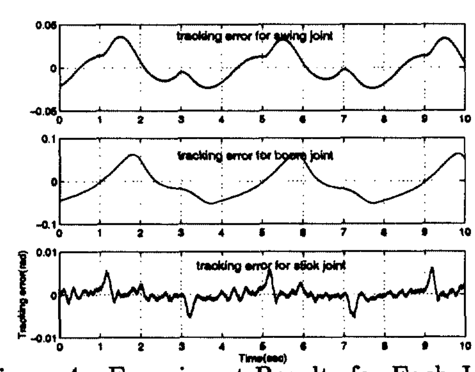

Figure 4: Experiment Results for Each Joint