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Abstract

The paper deals with an observer-based event-triggered control strategy for linear systems using only local (that is available) variables. Sufficient conditions based on linear matrix inequalities (LMI) associated to convex optimization problems are proposed to ensure the asymptotic stability of the closed loop and the output convergence to a constant reference in both emulation and co-design contexts. Indeed, the proposed approach allows either to design the event-triggering rules or co-design the event-triggering rule along with the controller gain.

Keywords: Event-triggered observer, stability, co-design, constant reference tracking, LMI.

1 Introduction

Nowadays the implementation of modern control systems is performed through digital communication networks using both wired and wireless technologies. In this context, aperiodic event- and self-triggering strategies have been proposed to deal with issues such as limited communication capabilities, energy and computation constraints. In particular, in many distributed applications the point of measurement is geographically separated from the location of the control processing. The sensor information is therefore sent through a wireless network, where the energy consumption can be a critical issue, since these devices are in general feed by batteries. Indeed, there are peaks of energy consumption for transmission/reception of data. Hence, to reduce the sampling activity, i.e., the instants where the measurement information is transmitted is of great importance. On the other hand, in classical wired networks, it can be of interest to reduce the number of messages sent through the network, alleviating in this way the traffic and problems regarding delays and package losses. For more details, the reader may refer, for instance, to [17, 19, 23, 28] and references therein.

Self-triggered strategies pre-define the sampling instants based on the available measurements and on predictions of the plant response. On the other hand, event-triggered controllers consider only the current measurements in order to define the next sampling instant. Self-triggering strategies for observer-based controllers have been proposed in [4] based on a cascade interconnection of a discrete-time observer and a

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controller designed for state feedback. This approach has also been extended to deal with interconnected systems [3].

For event-triggered strategies, [14] proposes the use of a state observer in the event generator node and an upper bound on the estimation error for designing an event-triggered mechanism to guarantee asymptotic stability of the closed-loop system. A more general approach is proposed in [24], where three architectures for dynamic output feedback controllers are presented. The event-triggering conditions depend only on the norms of the local variables and are obtained using Lyapunov arguments. Dynamic output feedback controllers are also addressed in [29], where the asymptotic stability of the resulting closed-loop system is guaranteed by a condition in terms of an LMI (Linear Matrix Inequality) and in [30] these results are extended to deal with uncertain systems. However, none of these works have addressed the problem of tuning key parameters of the event generator to minimize the number of triggering events. In the event-triggered control framework, one can consider two approaches. In the first one, which corresponds to an emulation problem, the controller is given a priori (see, for example, [13], [27], [19], [2] and references therein). The second approach, which is called a co-design problem, performs the design of both the controller and the event-triggering rule, simultaneously, and is addressed in a few papers. The control parameters design is then carried out by using a fully continuous-time approach [6, 21, 22, 23] or a fully discrete-time approach [12]. Of course, the attainable performances are affected by the choice of the continuous-time or the discrete-time synthesis approach, as for example discussed in [1, 5] where some optimality criteria are included in the synthesis phase.

In this work, inspired by [15] and [21], the design of the event-triggering strategy for observer-based state feedback is proposed based on the decrease of a Lyapunov function. In the context of event-triggered control, the plant evolves in continuous time, whereas the control signal is updated depending on discrete-time events, the resulting closed-loop system can be cast as hybrid or impulsive systems. Nevertheless, instead of considering the classical hybrid framework to study mixed continuous and discrete dynamics as defined in [11], we use an alternative direction as proposed in [20, 23]. The paper then deals with an observer-based event-triggered control strategy for linear systems using only local (that is available) variables. Let us emphasize that the design of event-triggered controllers based on measured signals is a challenging problem (see, for example, [2], [9] and references therein). Sufficient conditions based on LMI associated with convex optimization problems are proposed to ensure the asymptotic stability of the closed loop and the output convergence to a constant reference in both emulation (see, for example, [18, 19, 26, 28] and references therein) and co-design contexts (see, for example, [19] and references therein). Indeed, the proposed approach allows either to design the event-triggering rules or co-design the event-triggering rule with the controller gain. Moreover, the results we propose in the paper are complementary to those previously cited in the sense that we pursue an event-triggering strategy for observer-based controllers allowing to track a constant reference. The stability of the closed-loop sampled-data system under the event-triggering strategy is formally proven based on the Lyapunov theory. Furthermore, following the idea presented in [16], Zeno behaviors are avoided thanks to a minimum dwell-time, which is explicitly forced as a design parameter of the LMI conditions. The paper can then be considered as a comprehensive version of [20], where the event-triggering strategy was based on a simple and rough algorithm, without a guarantee of the absence of Zeno phenomenon. In addition, the LMI conditions are proposed both in emulation and co-design contexts and are expressed in a very simple and compact form, differently from those of [20]. The originality of the paper relies on the design of the event-triggered control based on two conditions: one to ensure the continuous-time stability conditions and the second one to adjust the co-design among all possible solutions of the first condition thanks to a tunable parameter. The problem of tuning the control strategy is also addressed from a simple optimization criterion, which cope with the implicit objective to reduce the number of updates by playing on the optimization of event-triggered rule and on the parameter related to the expected average sampling rate of the event-triggered implementation.

The paper is organized as follows. Section 2 presents the system under consideration, the sampled-data control implementation and then the event-triggered problem we intend to solve. In Section 3, upon giving basic ingredients on which the approach is based, the event-triggered strategy is proposed in a context of emulation. Then, Section 4 is dedicated to the co-design of event-triggered control, that is to the design of both the controller gain and the event-triggering rule. Section 5 proposes an optimization method to compute the parameters of the event-triggering rule and the controller gain (in the co-design approach). In Section 6, several simulations illustrate the application of the proposed methodology and the influence of different parameters such as the dwell-time and the observer gain. Section 7 ends the paper with concluding remarks and potential future works.

Notation. For any matrix A, A' denotes its transpose and $He\{A\} = A + A'$. For two symmetric matrices of the same dimensions, A and B, $A > B$ means that $A - B$ is symmetric positive definite. I and 0 stand respectively for the identity and the null matrix of appropriate dimensions. Let us point out the main definitions of states used throughout the paper: x_p, x_o, $\tilde{x}$, $(x_{p,eq}, x_{o,eq}, u_{eq})$, and ε_{eq} denote respectively the plant state, the observer state, the state estimation error, the equilibrium point for the plant state, the observer state and the input, and the difference between the observer state and its equilibrium point.

2 Problem Statement

2.1 System data

Consider the following continuous-time linear plant:

$$\begin{cases} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t), \\ y_p(t) &= C_p x_p(t), \end{cases} \tag{1}$$

where $x_p(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y_p(t) \in \mathbb{R}^p$ are the state, the input and the output of the plant, respectively. The matrices A_p, B_p and C_p are constant and of appropriate dimensions. In addition, the pairs (A_p, B_p) and (C_p, A_p) are controllable and observable. Moreover, system (1) satisfies the following assumption.

Assumption 1 *$m \geq p$ and rank$\left(\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix}\right) = n + p$.*

Assumption 1 means that the plant has no transmission zeros at zero [31].

An observer-based state feedback controller to drive the output to a given nonzero constant set-point r is defined by:

$$\begin{cases} \dot{x}_o(t) &= A_p x_o(t) + B_p u(t) - K_o(y_p(t) - y_o(t)), \\ y_o(t) &= C_p x_o(t), \\ u(t) &= K_c x_o(t) + K_r r, \end{cases} \tag{2}$$

where $x_o(t) \in \mathbb{R}^n$, $y_o(t) \in \mathbb{R}^p$ and $r \in \mathbb{R}^p$ are the state and the output of the observer and the constant reference signal, respectively. Furthermore, $K_r \in \mathbb{R}^{m \times p}$ is a feedforward gain, $K_o \in \mathbb{R}^{n \times p}$ and $K_c \in \mathbb{R}^{m \times n}$ are the observer and controller gains, respectively.

By considering the continuous-time system described by (2), the control design is carried out according to the separation principle. The observer gain K_o is then designed to make $A_p + K_o C_p$ Hurwitz and the state estimation error dynamics is given by:

$$\dot{e}(t) = (A_p + K_o C_p)e(t), \tag{3}$$

where the state estimation error, $e(t) = x_p(t) - x_o(t) \in \mathbb{R}^n$, is globally asymptotically stable, i.e. $\lim_{t\to\infty} e(t) = 0$. Consequently, the estimation output error $e_y(t) = C_p e(t)$ also asymptotically converges to zero, i.e. $\lim_{t\to\infty} e_y(t) = 0$. On the other hand, the dynamic of the observer with the state feedback controller $u(t) = K_c x_o(t) + K_r r$ is given by:

$$\dot{x}_o(t) = (A_p + B_p K_c)x_o(t) + B_p K_r r - K_o e_y(t). \tag{4}$$

For a given constant reference signal r, the equilibrium point $(x_{p,eq}, x_{o,eq})$ of (3) and (4) satisfies

$$x_{o,eq} = x_{p,eq}, \quad (A_p + B_p K_c) x_{o,eq} + B_p K_r r = 0, \quad C_p x_{o,eq} = r. \tag{5}$$

The two last equations of (5) can be rewritten as follows

$$\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ K_c & K_r \end{bmatrix} \begin{bmatrix} x_{o,eq} \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix}. \tag{6}$$

Thanks to Assumption 1, the pseudo-inverse $\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix}^{\#}$ exists such that $\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix}^{\#} = I_{n+p}$ and from equation (6) one gets

$$K_r = \begin{bmatrix} -K_c & I \end{bmatrix} \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix}^{\#} \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{7}$$

Such an expression of the feedforward control gain K_r is valid for any controller gain K_c. Let us define the error dynamics between the observer state $x_o(t)$ and its equilibrium point, $\varepsilon_{eq}(t) = x_o(t) - x_{o,eq}$, which satisfies

$$\dot{\varepsilon}_{eq}(t) = (A_p + B_p K_c)\varepsilon_{eq}(t) - K_o e_y(t). \tag{8}$$

where $e_y(t)$ can be interpreted as an input to this system. Assuming that the controller gain K_c is designed such that $(A_p + B_p K_c)$ is Hurwitz, system (8) is input-to-state stable with respect to e_y and the unforced linear system (i.e. with $e_y = 0$) is asymptotically stable. Since (4) is also supposed to be asymptotically stable, it follows that $\lim_{t\to\infty} e_y(t) = 0$, which implies that $\lim_{t\to\infty} \varepsilon_{eq}(t) = 0$. We can therefore conclude that if $(A_p + B_p K_c)$ and $(A_p + K_o C_p)$ are Hurwitz one has $\lim_{t\to\infty} x_o(t) = \lim_{t\to\infty} x(t) = x_{eq}$, i.e. the output y_p converges asymptotically to the desired reference r.

While the problem of continuous-time or periodic sampled-data implementation of such a class of controller has been widely studied in the literature, our objective is to address the problem of an aperiodic sampled-data implementation of such an observer based-controller based on an event-triggered control strategy.

2.2 Sampled-data implementation of the control input

In this paper, we consider that the control input, u, is not assumed to be continuously implemented but is updated at certain instants $\{t_k\}_{k\in\mathbb{N}}$, which form a sequence of strictly increasing positive scalar and which is defined in the sequel. We consider that the control action is held constant between two sampling instants (t_k and t_{k+1}) through a zero order hold. Note, however, that differently from classical digital control approaches, the sampling interval $t_{k+1} - t_k$ is not assumed to be constant. In such a situation, the closed-loop system can be represented by

$$\left\{ \begin{array}{rcl} \dot{x}_p(t) & = & A_p x_p(t) + B_p u(t_k), \\ \dot{x}_o(t) & = & A_p x_o(t) + B_p u(t_k) - K_o e_y(t), \qquad \forall t \in [t_k \ , \ t_{k+1}), \\ u(t_k) & = & K_c x_o(t_k) + K_r r, \end{array} \right. \tag{9}$$

where we recall that $e_y(t) = C_p e(t) \in \mathbb{R}^p$ and $e(t) = x_p(t) - x_o(t) \in \mathbb{R}^n$. Note that the error dynamics e between the plant and he observer states is still governed by (3), the error dynamics ε_{eq} between the observer state x_o and its equilibrium $x_{o,eq}$ is affected by the sampled-data implementation of the control input u. Thus, the closed-loop system can be re-written as:

$$\left\{ \begin{array}{rcl} \dot{\varepsilon}_{eq}(t) & = & (A_p + B_p K_c)\varepsilon_{eq}(t) + B_p \delta(t) - K_o e_y(t), \\ \dot{e}(t) & = & (A_p + K_o C_p) e(t), \end{array} \right. \tag{10}$$

where we use the same formulation as in [23] to define $\delta(t)$

$$\delta(t) = K_c(\varepsilon_{eq}(t_k) - \varepsilon_{eq}(t)).$$

The variable $\delta(t)$ can be seen as a measure of the difference between the continuous-time and the sampled control input. Note that $\delta(t)$ depends only on the observer variables and is therefore available at the controller node. The sensor and controller are supposed to be in different nodes of the network as depicted in Figure 1, the SW block representing the event-triggered sampling strategy.

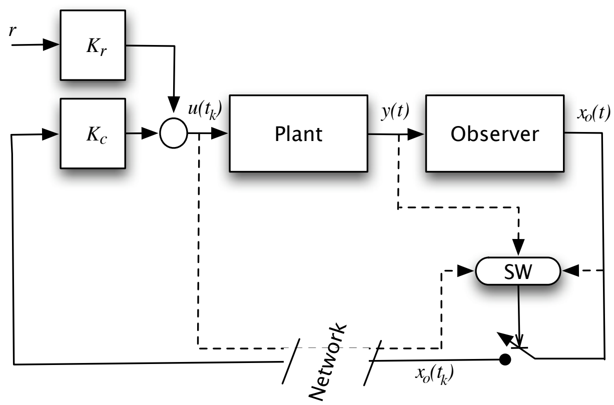


Figure 1: Observer based controller.

2.3 Problem Statement

In this paper, we are interested in the event-triggered implementation of the controller represented by (2). This means that an event generator algorithm is included in the controller to decide whether or not the control input has to be updated. The basic idea is therefore to decide when to sample based on the available information. Following the event-triggered control strategy proposed in [24, 25], the sampling instants are determined from the following logic:

$$t_{k+1} = \min\{t \geq t_k + T, \quad s.t. \quad f(\delta(t), y_a(t)) \geq 0\}. \tag{11}$$

where y_a represents the vector of available information to the controller (which corresponds in our case to $y_a(t) = [\varepsilon_{eq}(t)', e_y(t)']'$ and the function $f: \mathbb{R}^m \times \mathbb{R}^{(n+p)} \to \mathbb{R}$ has to be defined efficiently such that the asymptotic stability of the closed-loop system (10) under the event-triggered rule described in (11) is ensured.

The logic in (11) means that the next sampling time will occur at least T time units ahead the last one. In this case, T is the minimal dwell-time, which will be instrumental to prevent Zeno solutions. Moreover, note that for $t \geq t_k + T$ the control will not be updated until $f(\delta(t), y_a(t)) \geq 0$.

3 Event-triggered Strategy Design

The main objective of this work is to devise an event-triggered strategy to sample and to update the control signal applied to the plant based solely on available signals, that is, using only the available signals $u(t), x_o(t)$ and $y_p(t)$. In view of (11), this corresponds basically to design T and f in order to ensure the asymptotic stability of the sampled-data system (9).

3.1 Preliminary result

Let us first present a general formulation, inspired from [24], [25] and on which the main result of the event-triggered strategy developed below is based. Consider a generic linear system

$$\left\{\begin{array}{rcll} \dot{x}(t) & = & Ax(t)+BKy(t_k), & \forall t\in[t_k\ ,\ t_{k+1}),\\ y(t) & = & Cx(t), & \\ u(t) & = & Ky(t) & \end{array}\right. \tag{12}$$

where $x\in\mathbb{R}^{n_A}$, $u\in\mathbb{R}^{n_B}$ and $y\in\mathbb{R}^{n_C}$ represent the state, the input and the output of system (12) and where the matrices A,B,C and K have appropriate dimensions and are such that $A+BKC$ is Hurwitz. The following theorem constitutes the first contribution of the paper. It states a Lyapunov-based condition to ensure the asymptotic stability of the closed-loop system (12) under an event-triggering strategy and to prevent Zeno phenomena.

Theorem 1 *Consider a positive scalar T, a function $f:\mathbb{R}^{n_B}\times\mathbb{R}^{n_C}\to\mathbb{R}$ and the triggering rule*

$$t_{k+1}=\min\{t\geq t_k+T,\quad s.t.\quad f(K(y(t_k)-y(t)),y(t))\geq 0\}. \tag{13}$$

Consider a positive definite function $V(x)$, for which there exist two positive scalars ε_1 and ε_2 such that

$$\varepsilon_1\parallel x\parallel^2\leq V(x)\leq\varepsilon_2\parallel x\parallel^2 . \tag{14}$$

Assume that the function $V(x)$ satisfies:

$$\dot{V}(x(t))-f(K(y(t_k)-y(t)),y(t))\ <\ 0,\quad \forall t\in[t_k+T\ ,\ t_{k+1})\quad \forall k\in\mathbb{N}, \tag{15}$$

and

$$\Delta V_T(x)=V(x(t_k+T))-V(x(t_k))\ <\ 0,\quad \forall k\in\mathbb{N}. \tag{16}$$

Then, system (12) with the triggering rule (13) is asymptotically stable and the inter-sampling intervals are lower bounded by T.

Proof. Let us split the stability analysis in two intervals, namely $[t_k\ ,\ t_k+T]$ and $[t_k+T,\ t_{k+1})$. From (16), the function $V(x)$ satisfies

$$V(x(t_k+T))<V(x(t_k)),\qquad \forall k\in\mathbb{N}. \tag{17}$$

Consider now the interval $[t_k+T,\ t_{k+1})$. From (13) and the use of S-procedure, one obtains (15). Then, we conclude that $\dot{V}(x(t))<0$ and therefore

$$V(x(t))<V(x(t_k+T)),\qquad \forall t\in[t_k+T,\ t_{k+1}),\qquad \forall k\in\mathbb{N}. \tag{18}$$

Hence, from (17) and (18), we conclude that $V(x(t_{k+1}))<V(x(t_k))$. Moreover, since system (12) is linear and since the function $V(x)$ satisfies (14), there exists a positive scalar β such that $\max_{t\in[t_k,\ t_k+T]}V(x(t))\leq\beta V(x(t_k))$, for all $k\in\mathbb{N}$. Hence, associating this property to relations (13) and (15), i.e. to the fact that $\dot{V}(x)<0$ on the interval $[t_k+T\ ,\ t_{k+1})$, prevents the trajectories of the system from blowing up in between every two events. Therefore, asymptotic stability is ensured and a lower bound on the inter-sampling times is given by T by using similar arguments to Lemma 1 in [24]. ■

Remark 1 *The event-triggered rule* (13) *allows to avoid Zeno behavior since the inter-sampling times are lower bounded by the positive scalar T.*

Note that Theorem 1 does not induce real conservatism in the sense that if the continuous-time condition (15) holds then it is always possible to find a small enough T such that conditions (15)-(16) are verified at the same time.

3.2 Event-triggered strategy

Following the conditions presented in Theorem 1 by considering $n_A = n$, $n_B = m$, $n_C = n + p$ and a quadratic function f, a way to design the event-triggered rule using only the available signals is stated below.

Theorem 2 *For given controller and observer gains K_c and K_o and a positive scalar $T > 0$, suppose that there exist symmetric positive definite matrices P_ε, P, P_e, Q_ε and Q_δ of appropriate dimensions such that the following matrix inequalities*

$$\Phi_1 = \begin{bmatrix} \begin{bmatrix} He\{P_\varepsilon(A_p + B_pK_c)\} & (A_p + B_pK_c)'P - P_\varepsilon K_oC_p + P(A_p + K_oC_p) & P_\varepsilon B_p \\ * & He\{P_e(A_p + K_oC_p) - P'K_oC_p\} & P'B_p \\ * & * & -Q_\delta \end{bmatrix} & \begin{bmatrix} I & 0 \\ 0 & C_p' \\ 0 & 0 \end{bmatrix} \\ * & -Q_\varepsilon \end{bmatrix} < 0, \tag{19}$$

$$\Phi_2 = \begin{bmatrix} -\begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} & \Lambda(T)'\begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} \\ * & -\begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} \end{bmatrix} < 0, \tag{20}$$

are verified with

$$\Lambda(T) = e^{\left(\begin{bmatrix} A_p & -K_oC_p \\ 0 & A_p+K_oC_p \end{bmatrix} T\right)} + \int_0^T e^{\left(\begin{bmatrix} A_p & -K_oC_p \\ 0 & A_p+K_oC_p \end{bmatrix} s\right)} ds \begin{bmatrix} B_pK_c & 0 \\ 0 & 0 \end{bmatrix}.$$

Then, the event-triggered sampling rule defined by

$$t_{k+1} = \min\left\{ t \geq t_k + T, \quad s.t. \quad \delta(t)'Q_\delta\delta(t) - \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix} \geq 0 \right\} \tag{21}$$

is such that the origin of system (10) is globally asymptotically stable and, consequently, the output vectors y_p and y_o of both the plant and the observer converge to the reference signal r. Furthermore, the inter-sampling times are lower bounded by T.

Proof. Consider the Lyapunov candidate function for system (10) given by

$$V(\varepsilon_{eq}(t), e(t)) = \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \end{bmatrix}' \begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \end{bmatrix},$$

where the matrix $\begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix}$ is positive definite, which is ensured by the satisfaction of (20). The goal of this proof is to show that the LMI conditions (19) and (20) are sufficient conditions for the satisfaction of inequalities (15) and (16) of Theorem 1 respectively. Consider the time-derivative of V along the trajectories of system (10), for any $t \in [t_k + T,\ t_{k+1})$. The following expression is obtained :

$$\begin{aligned} \dot{V}(\varepsilon_{eq}(t), e(t)) - \delta(t)'Q_\delta\delta(t) + \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix} &= \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \\ \delta \end{bmatrix}' M \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \\ \delta \end{bmatrix} \\ &\quad - \delta(t)'Q_\delta\delta(t) + \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}, \end{aligned}$$

with

$$M = \begin{bmatrix} He\{P_\varepsilon(A_p + B_pK_c)\} & (A_p + B_pK_c)'P - P_\varepsilon K_oC_p + P(A_p + K_oC_p) & P_\varepsilon B_p \\ * & He\{P_e(A_p + K_oC_p) - P'K_oC_p\} & P'B_p \\ * & * & 0 \end{bmatrix}$$

Setting $e_y(t) = C_p e(t)$ in the previous equation leads to

$$\dot{V}(\varepsilon_{eq}(t), e(t)) - \delta(t)' Q_\delta \delta(t) + \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix} = \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \\ \delta(t) \end{bmatrix}' \Phi_0 \begin{bmatrix} \varepsilon_{eq}(t) \\ e(t) \\ \delta(t) \end{bmatrix},$$

with

$$\Phi_0 = M - diag(0, 0, Q_\delta) + \begin{bmatrix} I & 0 \\ 0 & C_p' \\ 0 & 0 \end{bmatrix} Q_\varepsilon^{-1} \begin{bmatrix} I & 0 \\ 0 & C_p' \\ 0 & 0 \end{bmatrix}'.$$

Therefore, by applying the Schur complement to Φ_0 one obtains Φ_1. If the LMI condition (19), i.e. $\Phi_1 < 0$, is satisfied, then the condition (15) of Theorem 1 with $f\left(\delta(t), \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}\right) = \delta(t)' Q_\delta \delta(t) - \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{bmatrix}$ is also satisfied. In order to prove that conditions (16) holds, note that solving the linear differential equation (10) over the interval $[t_k,\ t_k + T]$ yields

$$\begin{bmatrix} \varepsilon_{eq}(t_k + T) \\ e(t_k + T) \end{bmatrix} = \Lambda(T) \begin{bmatrix} \varepsilon_{eq}(t_k) \\ e(t_k) \end{bmatrix}. \tag{22}$$

Hence, condition (16) becomes

$$\begin{aligned} \Delta V_T(\varepsilon_{eq}, e) &= V(\varepsilon_{eq}(t_k + T), e(t_k + T)) - V(\varepsilon_{eq}(t_k), e(t_k)) \\ &= \begin{bmatrix} \varepsilon_{eq}(t_k) \\ e(t_k) \end{bmatrix}' \left(\Lambda(T)' \begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} \Lambda(T) - \begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix} \right) \begin{bmatrix} \varepsilon_{eq}(t_k) \\ e(t_k) \end{bmatrix}. \end{aligned}$$

Applying the Schur complement, it can be seen that the condition $\Phi_2 < 0$ ensures the satisfaction of (16). The proof is then concluded by virtue of Theorem 1. We can conclude that the solutions to system (10) converges asymptotically to 0, meaning that the plant output y_p converges asymptotically to the reference r. Moreover, the event-triggered strategy (21) implicitly ensures that the length between two successive sampling instants is lower bounded by T. ■

From the conditions of Theorem 2, it is important to observe that the lower bound on the inter-sampling times is directly obtained via the satisfaction of (20) without the need of additional calculations as required in [24]. Furthermore, one also gets the guarantee that $\begin{bmatrix} P_\varepsilon & P \\ P' & P_e \end{bmatrix}$ defines a quadratic Lyapunov function for the discrete-time system. Theorem 2 is then the second contribution of the paper.

Note that T appears as a tuning parameter of the event-triggered problem. Contrary to most of the approaches developed in the literature, where the dwell-time T is computed a posteriori. Moreover, for a sufficiently small T there will exist a solution to conditions (19)-(20). However if T is too small the number of control updates (events) tends to be larger as illustrated in Section 6. On the other hand, if T is too large, the conditions may be not feasible. Furthermore, a large T can lead to a significant performance degradation (when compared to the one obtained with a continuous-time implementation). This trade-off should be considered when choosing T.

4 Event-triggered control co-design

Based on the stability condition obtained in Theorem 2, the following constructive theorem provides conditions for the co-design of event-triggered control, that is for designing both the controller gain K_c and the matrices Q_δ and Q_ε defining the triggering rule.

Theorem 3 *Given an observer gain K_o and a parameter $T > 0$, suppose that there exist positive definite symmetric matrices W_ε, P_e, Q_ε and Q_δ and a matrix F_c of appropriate dimensions such that the following matrix inequalities*

$$\Psi_1 = \begin{bmatrix} \begin{bmatrix} He\{A_pW_\varepsilon + B_pF_c)\} & -K_oC_p & B_p \\ * & He\{P_e(A_p + K_oC_p)\} & 0 \\ * & * & -Q_\delta \end{bmatrix} & \begin{bmatrix} W_\varepsilon & 0 \\ 0 & C_p' \\ 0 & 0 \end{bmatrix} \\ * & -Q_\varepsilon \end{bmatrix} < 0, \tag{23}$$

$$\Psi_2 = \begin{bmatrix} -\left[\begin{smallmatrix} W_\varepsilon & 0 \\ 0 & P_e \end{smallmatrix}\right] & \left[\begin{smallmatrix} W_\varepsilon & 0 \\ 0 & P_e \end{smallmatrix}\right]\Lambda_1(T)' + \begin{bmatrix} F_c & 0 \end{bmatrix}'\Lambda_2(T)'\left[\begin{smallmatrix} I & 0 \\ 0 & P_e \end{smallmatrix}\right] \\ * & -\left[\begin{smallmatrix} W_\varepsilon & 0 \\ 0 & P_e \end{smallmatrix}\right] \end{bmatrix} < 0, \tag{24}$$

are verified with

$$\Lambda_1(T) = e^{\left(\left[\begin{smallmatrix} A_p & -K_oC_p \\ 0 & A_p+K_oC_p \end{smallmatrix}\right]T\right)}, \quad \Lambda_2(T) = \int_0^T e^{\left(\left[\begin{smallmatrix} A_p & -K_oC_p \\ 0 & A_p+K_oC_p \end{smallmatrix}\right]s\right)} ds \begin{bmatrix} B_p \\ 0 \end{bmatrix}.$$

Then, the controller gain $K_c = F_cW_\varepsilon^{-1}$ and the event-triggered sampling rule defined by

$$t_{k+1} = \min\left\{ t \geq t_k + T, \quad s.t. \quad \delta(t)'Q_\delta\delta(t) - \left[\begin{smallmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{smallmatrix}\right]' Q_\varepsilon^{-1} \left[\begin{smallmatrix} \varepsilon_{eq}(t) \\ e_y(t) \end{smallmatrix}\right] \geq 0 \right\} \tag{25}$$

is such that the origin is asymptotically stable for system (10) and, consequently, the output vectors of both the plant and the observer converge to the reference signal r. Furthermore, the inter-sampling times are lower bounded by T.

Proof. Based on Theorem 1, by considering $P = P' = 0$, $W_\varepsilon = P_\varepsilon^{-1}$ and $F_c = K_cW_\varepsilon$, then pre- and post-multiplying by $diag(W_\varepsilon; I; I; I)$ it follows that condition $\Phi_1 < 0$ is equivalent to the condition $\Psi_1 < 0$. Furthermore, pre- and post-multiplying Φ_2 by the matrix $diag(W_\varepsilon, I, W_\varepsilon, I)$ ensures that $\Phi_2 < 0$ is equivalent to $\Psi_2 < 0$. ■

Remark 2 *While Theorem 2 only provides the triggering rule for prescribed controller and observer gains, K_c and K_o and a positive scalar T, Theorem 3 provides both the triggering rule and the controller gain K_c for given a given observer gain K_o and a given $T > 0$. Then the conditions of Theorem 3 provide an event-triggered control co-design. Theorem 3 constitutes the third contribution of the paper. The design of the controller K_c could be completed by adding some performance constraints as, for example, LQ-performance, H_2, H_∞ performance or by considering some pole placement constraints to satisfy some rate of convergence requirement of the closed-loop system.*

5 Optimization of the event-triggered strategies

It can be checked that the conditions in Theorem 2 are LMIs provided that K_c, K_o and T are fixed, as classically in an emulation context. In order to choose K_c and K_o classical design technique, possibly adding performance criteria, can be used. It is important to note that provided that matrices $A_p + B_pK_c$ and $A_p + K_oC_p$ are Hurwitz, there always exists a small enough positive scalar T such that the LMIs of Theorem 2 are feasible. Similarly, conditions of Theorem 3 are linear as soon as K_o and T are given. The choice of the observer gain is done such that $A_p + K_oC_p$ is Hurwitz with fast enough eigenvalues. Then, in order to optimize the selection of the event-triggered parameters, we can consider the following convex optimization problems:

- Event-triggered rule design (Theorem 2). For given controller and observer gains K_c and K_o and a positive scalar $T > 0$:

$$\begin{array}{l} \min\limits_{P_\varepsilon, P, P_e, Q_\varepsilon, Q_\delta} trace(Q_\delta) + trace(Q_\varepsilon) \\ \text{s.t. } (19),\ (20). \end{array} \tag{26}$$

- Event-triggered control co-design (Theorem 3). For given observer gain K_o and a positive scalar $T > 0$:

$$\begin{array}{l} \min\limits_{W_\varepsilon, P_e, Q_\varepsilon, Q_\delta, F_c} trace(Q_\delta) + trace(Q_\varepsilon) \\ \text{s.t. } (23),\ (24),\ W_\varepsilon > 0,\ P_e > 0. \end{array} \tag{27}$$

The dwell-time T being also a design parameter, whose role is connected to the expected average sampling rate of the event triggered implementation one can seek for minimizing its value through problems (26) or (27) by iteratively increasing T and testing LMI conditions. Note that convex optimization problems proposed are aimed at reducing as much as possible the occurrences of sampling. This aspect is illustrated in the next section.

6 Numerical example

The considered example is inspired by the model of a magnetic levitation system as mentioned. These systems can be found in important applications such as precision bearing and transportation systems (see, for example, the Feedback Levitation System (http://www.feedback-instruments.com/)). The system suspends a small metal ball by means of an electromagnetic field generated by an electromagnet. The linearized model for this system can be written as:

$$m\ddot{x} = k_1 x + k_2 i$$

where i is the coil current, x is the ball vertical position, m is the ball mass [10]. In the paper we have chosen reasonable values at $x = 0$ for the parameters m, k_1 and k_2 in order to obtain the data A_p, B_p and C_p defining the plant (1). as follows:

$$A_p = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix},\ B_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C_p = \begin{bmatrix} 1 & 0 \end{bmatrix},\ r = 5. \tag{28}$$

Note that the open-loop system is unstable since has a pole in the right hand side of the complex plane.

We only address the problem of the design of both the controller gain K_c and the event-triggering rules through the matrices Q_δ and Q_σ, as presented in Theorem 3.

Two observer gains K_o are considered:

$$K_{o1} = \begin{bmatrix} -3.5 & -7 \end{bmatrix}',\ and\ K_{o2} = \begin{bmatrix} -42 & -444 \end{bmatrix}'. \tag{29}$$

These two observer gains have been selected to show the influence of the observer gain on the event-triggered control. Indeed while the first gain leads to eigenvalues of $A_p + K_o C_p$ equal to $-1.5, -2$, the second gain leads to eigenvalues equal to $-20, -22$.

Then, we apply Theorem 3 and the optimization problem (27) for several values of design parameter T. Figure 2 depicts six simulations showing, for each of them, the time evolution of the state of the plant and the observer, of a timer $\tau := t - t_k$ representing when the control input is updated and of the control signal obtained considering the initial conditions $x_p(0) = [-5\ \ 0]'$, $x_o(0) = [0\ \ 0]'$. We have considered here three values for the design parameter T: $T = 0.2, 0.4$ and 0.8.

T	K_o	K_c	Q_δ	Q_ε^{-1}
0.2	K_{o1}	$[-11.9240\ \ -7.0307]$	0.6484	$\begin{bmatrix} 4.201 & 0.302 & -0.143 \\ 0.302 & 3.519 & -0.202 \\ -0.143 & -0.202 & 4.665 \end{bmatrix}$
0.4	K_{o1}	$[-6.2664\ \ -3.1687]$	1.7767	$\begin{bmatrix} 2.929 & 0.282 & -0.0067 \\ 0.282 & 3.366 & -0.0007 \\ -0.067 & -0.007 & 0.3720 \end{bmatrix}$
0.8	K_{o1}	$[-5.4479\ \ -2.7445]$	1.623	$\begin{bmatrix} 2.595 & -0.885 & -0.032 \\ -0.885 & 4.052 & -0.018 \\ -0.032 & -0.018 & 6.446 \end{bmatrix}$
0.2	K_{o2}	$[-11.4222\ \ -6.9135]$	0.7464	$\begin{bmatrix} 4.001 & 0.342 & -0.002 \\ 0.342 & 3.353 & -0.006 \\ -0.002 & -0.0006 & 4.396 \end{bmatrix}$
0.4	K_{o2}	$[-7.3796\ \ -3.9756]$	1.1649	$\begin{bmatrix} 3.337 & -0.038 & -0.004 \\ -0.038 & 3.176 & -0.003 \\ -0.004 & -0.003 & 4.048 \end{bmatrix}$
0.8	K_{o2}	$[-5.0127\ \ -2.5115]$	9.3231	$\begin{bmatrix} 2.413 & 0.010 & -0.002 \\ 0.010 & 3.081 & -0.001 \\ -0.002 & -0.001 & 3.517 \end{bmatrix}$

Table 1: Table showing controller gains and the event-triggered rule parameters Q_δ and Q_ε for several values of T and of the observer gain K_o.

Table 1 summarizes the solution K_c, Q_δ and Q_ε obtained for each value of T and for each observer gain considered.

It is worth mentioning that the design parameter T corresponds to a guaranteed dwell-time for the event-triggered control system. Indeed, it can be seen in Figure 2, that the "peaks" of the timer are always greater than the corresponding value of T.

For both observer gains, the influence of T directly appears in the number of control updates, N_u, obtained during the $10s$-simulations. Increasing T leads to a notable reduction of N_u, for this particular initial condition. Nevertheless, increasing T also affects the performances of the responses. Indeed it can be seen that the simulation obtained with $T = 0.2$ converges more quickly than for $T = 0.4$ or 0.8. Moreover for large values of T (here 0.8), the response oscillates around the reference signal $r = 5$. The same behavior can also be observed for other initial conditions.

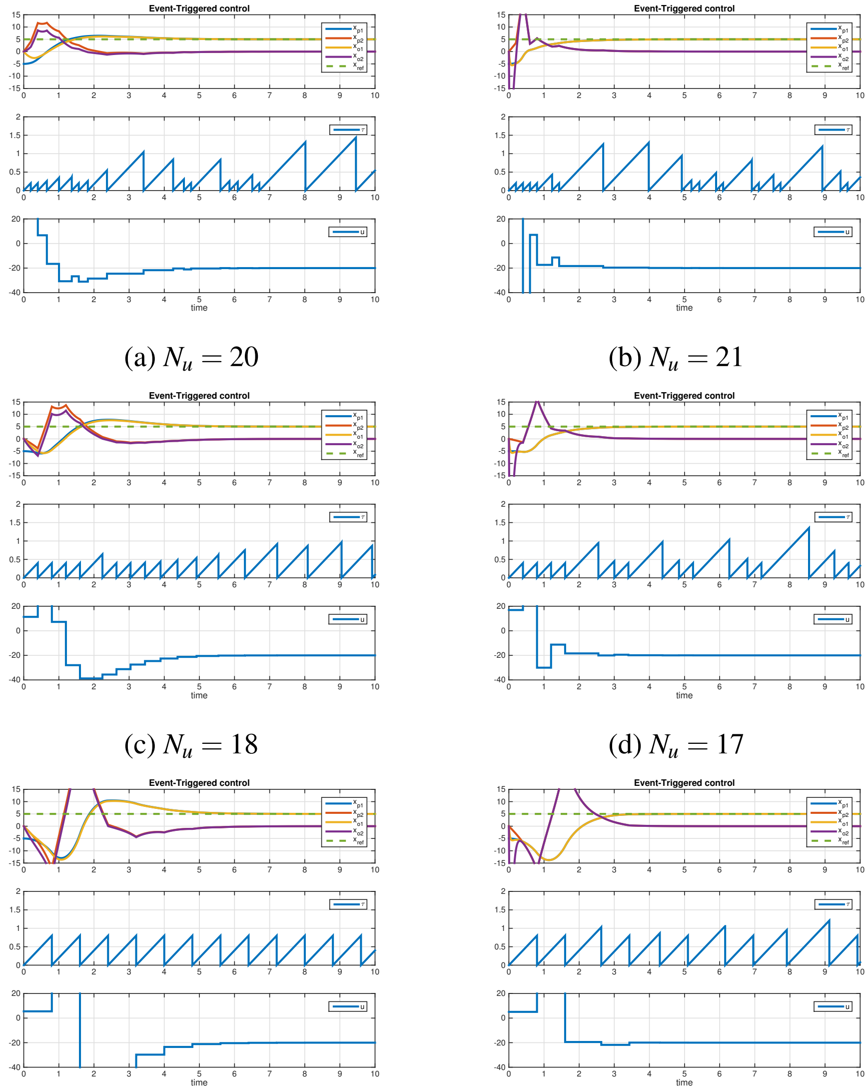
We also note that selecting the largest value for T may recover a periodic implementation. Indeed, this is due to the fact that at time $t_k + T$, triggering rule f is already positive, which does not allow enlarging the sampling interval. Therefore, the triggering rule only resumes to a periodic implementation of period T. This remark highlights the tradeoff between the reduction of the number of control updates between periodic and event-triggered implementations.

Comparing now the effects of considering "slow" and "fast" observer gains, Figure 2 also shows that employing "fast" observer gains help reducing the number of control updates N_u. This is an expected result since the triggering rule (25) depends explicitly on the estimation error e_y.

7 Conclusion

This paper presented a systematic method for designing event-triggering strategies considering observer-based controllers. The event-triggering design method is based on Lyapunov arguments and uses only information on available signals. Hence, since a Lyapunov-based approach is considered, stability under the event-triggered sampling strategy is formally guaranteed. The proposed implementation is parameterized by a dwell-time T, which prevents the Zeno phenomenon occurrence.

Considering an emulation approach (i.e. the state feedback and observer gains is a priori given), design



$T = 0.2$ (a) $N_u = 20$ (b) $N_u = 21$

$T = 0.4$ (c) $N_u = 18$ (d) $N_u = 17$

$T = 0.8$ (e) $N_u = 13$ (f) $N_u = 12$

$K_o = [-3.5 \;\; -7]'$ $K_o = [-42 \;\; -444]'$

Figure 2: Evolution of the plant and observer states with the reference r (top), the timer $t - t_k$ (middle) and the control input u (bottom) for $T = 0.2, 0.4$ and 0.8.

procedure consists in fixing, first, the design parameter T and then solving a convex optimization problem to determine, in a systematic way, the parameters of the event-triggering rule aiming at reducing as much as possible the occurrences of sampling. It is also shown that the conditions can be adapted for a co-design of the estimated state-feedback gain and the event-triggering rules parameters.

The work proposed lets the room for future developments. In particular, the co-design not only of the state feedback gain but also of the observer gain is an open problem. One of the difficulties in this case is the non-convexity of condition (24) if K_o is a free variable. One direction to overcome this problem is the application of the results proposed in [7, 8] about the equivalence of an inequality such as (20) (containing a matrix exponential) to an inequality where the argument of the exponential matrix appears linearly. The drawback in this case is that this equivalent inequality has infinite dimension, since one of the decision variables depends on the time between two impulses (events). At the expense of a substantially numerical complexity increasing, this can be overcome (for instance) by using polynomial matrices and applying Sum of Squares techniques. Note also that to apply this technique, a maximal bound on the dwell-time must be considered. On the other hand, there is another difficulty that prevents so far to obtain convex conditions for the synthesis of K_o. It should be observed that due to the the cross product $P_e K_o$, it is not possible to make convex (23) if K_o and P_e are decision variables. An ad hoc way to overcome this would be to apply some iterative algorithm. Another open issue regards the extension of the approach to cope with the perfect (or almost perfect) tracking of non constant signals (such as periodic ones, for instance). This problem seems more involved in the context of event-triggered control, since the reference would also be sampled aperiodically.

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