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# Observer-Based Fuzzy Control for Nonlinear Networked Systems Under Unmeasurable Premise Variables

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**Abstract**—The problem of fuzzy observer-based controller design is investigated for nonlinear networked control systems subject to imperfect communication links and parameter uncertainties. The nonlinear networked control systems with parameter uncertainties are modeled through an interval type-2 (IT2) Takagi-Sugeno (T-S) model, in which the uncertainties are handled via lower and upper membership functions. The measurement loss occurs randomly both in the sensor-to-observer and the controller-to-actuator communication links. Specially, a novel data compensation strategy is adopted in the controller-to-actuator channel. The observer is designed under the unmeasurable premise variables case, and then, the controller is designed with the estimated states. Moreover, the existence conditions for the controller can ensure that the resulting closed-loop system is stochastically stable with the predefined disturbance attenuation performance. Two examples are provided to illustrate the effectiveness of the proposed method.

**Index Terms**—Fuzzy observer-based control; Nonlinear networked control systems; IT2 T-S fuzzy model; Unmeasurable premise variables.

## I. INTRODUCTION

NETWORKED control systems (NCSs) consist of numerous connected subsystems [1]–[3], which exchange information through the communication network. The subsystems are distributed geographically. Subsystems transmit data via the communication network rather than the point-to-point wire. NCSs have some advantages, such as low cost, ease of maintenance, installation and flexibility. Meantime, some network-induced problems, including data packet dropouts [4] and network-induced delays [5] have been considered in the network environment. Recently, some results about stability and stabilization problems [6], [7], filter design problem [8], fault detection problem [9] for NCSs have been reported in the literatures.

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In practical systems, the state variables are usually unavailable, which makes it difficult to utilize them to stabilize the unstable systems. Since the observer can estimate the unmeasurable states effectively, it plays an important role in designing the observer-based controller [10]–[15]. In order to handle the nonlinearity of the system, Takagi-Sugeno (T-S) fuzzy model was proposed in [16], which describes nonlinear systems through a group of “IF-THEN” rules. It can approximate any smooth nonlinear terms to any satisfying accuracy via “blending” all sub-systems with membership functions [17]. Based on the T-S fuzzy model approach, considerable results about the observer-based control for nonlinear NCSs have been reported. To mention a few, the authors in [18] proposed a sliding mode observer-based fault estimation approach for a class of nonlinear NCSs subject to Markov transfer delays. The authors in [19] investigated the observer-based controller design for a class of NCSs with multiple packet losses. The authors in [20] studied the observer-based control problem for a class of discrete-time systems with random measurement loss and multiplicative noises. The problem of output tracking for NCSs subject to network-induced delays, packet disorder, and packet dropout was investigated in [21]. The affine dynamic systems with quantized measurements were stabilized based on the observer in [22]. However, the membership functions of the observer designed in the above literatures depended on the state variables of the plant, which are not available in practice. To some degree, it brings some difficulties to transmit theoretical results into practical applications. Furthermore, even the data packet dropouts model was established in the above papers, the successive measurement loss case can not be handled well, which may lead to the instability of the controlled systems.

On the other hand, the uncertainties often exist in systems since the external environment and system components may change. Due to using the fixed grades of membership, the type-1 T-S fuzzy model-based approach is not able to capture the uncertainty effectively. The type-2 fuzzy set was proposed to extend the type-1 fuzzy set theory [23]. Based on such theory, the interval type-2 (IT2) fuzzy set was proposed to decrease the computational complexity [24], [25]. In view of the advantages of the IT2 fuzzy set theory, considerable results have been reported, such as autonomous mobile robots [26], adaptive filter design [27], controller design for delta parallel robot [28] and modular and reconfigurable robots [29]. The authors in [30] combined the IT2 fuzzy set with the T-S fuzzy model to deal with the IT2 controller design problem

for nonlinear systems subject to parameter uncertainties, and the lower and upper membership functions were introduced to capture the uncertainties. By using the inverted pendulum example, the effectiveness of the designed IT2 controller was illustrated. To consider more uncertain information, the footprint of uncertainty (FOU) was considered and an IT2 controller was designed to stabilize the nonlinear system in the framework of the IT2 T-S fuzzy model [31]. Both state-feedback and output-feedback control schemes were developed on the basis of IT2 T-S fuzzy model in [32]. The authors in [33] proposed an IT2 filter design method with D-stability under a unified frame. Using IT2 T-S fuzzy model, the problem of model reduction for nonlinear systems with D-stability constraint was investigated in [34]. The IT2 controller designed for NCSs with data packet loss was investigated in [35]. The authors in [36] proposed an IT2 filter scheme for nonlinear NCSs subject to data loss and uncertainties. However, there exist few results concerning the observer-based controller design under the unmeasurable premise variables for NCSs based on the IT2 T-S fuzzy model, which motivates our work.

The problem of IT2 observer-based control is investigated for nonlinear NCSs under the unmeasurable premise variables on the basis of the IT2 T-S fuzzy model in this paper. The considered nonlinear NCSs are subject to random measurement loss and parameter uncertainties. The IT2 T-S fuzzy model is used to model the nonlinear NCSs, capturing and expressing the uncertainties via lower and upper membership functions. The key contributions of this paper can be summarized as follows. 1. The membership functions of the observer-based controller to be designed depend on the states estimated by the observer rather than those of the plant. 2. A novel data compensation and regulation scheme is adopted, which facilitates improving the system performance. 3. A novel observer-based fuzzy controller is designed for nonlinear NSCs under unmeasurable premise variables. Finally, some simulation results are provided to demonstrate the effectiveness of the method proposed.

The remainder of the paper is organized as follows. Section II describes the considered problem. The main results concerning the controller design conditions are presented in Section III. Simulation results are given to show the usefulness of the proposed controller design strategy in Section IV. Finally, Section V concludes the paper.

**Notation:** The notation utilized throughout this paper is standard. The superscript “ $T$ ” denotes matrix transposition and the identity matrix and zero matrix with compatible dimensions are represented by  $I$  and  $0$ , respectively. The notation  $P > 0$  ( $\geq 0$ ) stands for that  $P$  is positive definite (semi-definite). The notation  $\|A\|$  indicates the norm of a matrix  $A$  and is defined by  $\|A\| = \sqrt{\text{tr}(A^T A)}$ .  $l_2[0, \infty)$  means the space of square-integrable vector functions over  $[0, \infty)$ ,  $R^n$  describes the  $n$ -dimensional Euclidean space,  $R^{m \times n}$  indicates the set of all real matrices of dimension  $m \times n$ , and  $\|\cdot\|_2$  shows the usual  $l_2[0, \infty)$  norm. In the complex matrix, the symbol  $(*)$  is utilized to represent a symmetric term, and we utilize  $\text{diag}\{\dots\}$  to describe the matrix with block diagonal structure. In addition,  $E\{x|y\}$  and  $E\{x\}$  signifies expectation of  $x$

conditional on  $y$  and expectation of  $x$ , respectively. Matrices in this paper without dimensions explicitly stated, we assume they have compatible dimensions.

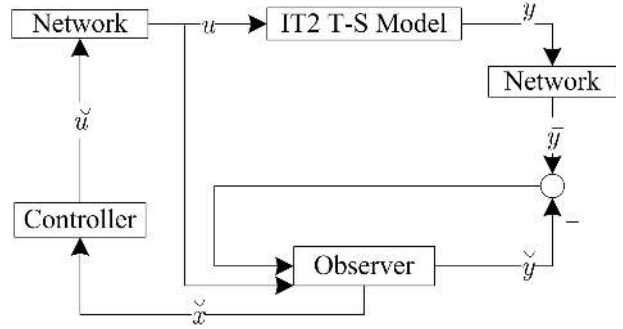


Fig. 1. The block diagram of the closed-loop networked control system.

## II. PROBLEM FORMULATION

Fig. 1 provides the prototype of the nonlinear NCSs consisting of different modules. It can be seen that communication networks are employed to transmit data packets since the sensor, the controller and the actuator are assumed to be distributed in different places.

### A. IT2 T-S Model

This section starts with giving the nonlinear NCSs in the framework of the IT2 T-S fuzzy model, in which the number of fuzzy rules is  $r$ . The T-S model corresponding to the  $i$ th rule is given as follows:

**Plant Rule  $i$ :** IF  $f_1(x(k))$  is  $M_{i1}$ , and  $f_2(x(k))$  is  $M_{i2}$  and, ..., and  $f_\theta(x(k))$  is  $M_{i\theta}$ , THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + E_i w(k), \\ z(k) &= C_{1i} x(k) + D_i u(k) + F_i w(k), \\ y(k) &= C_{2i} x(k), \quad i = 1, 2, \dots, r, \end{aligned} \quad (1)$$

where  $M_{ip}$  ( $i = 1, 2, \dots, r$ ,  $p = 1, 2, \dots, \theta$ ) is the fuzzy set,  $f(x(k)) = [f_1(x(k)), f_2(x(k)), \dots, f_\theta(x(k))]$  stands for the premise variable,  $x(k) \in R^{n_x}$  is the state,  $y(k) \in R^{n_y}$  is the measured output,  $z(k) \in R^{n_z}$  is the controlled output,  $u(k) \in R^{n_u}$  is the control input and  $w(t) \in R^{n_w}$  is the disturbance input which belongs to  $l_2[0, \infty)$ .  $A_i$ ,  $B_i$ ,  $E_i$ ,  $C_{1i}$ ,  $D_i$ ,  $F_i$  and  $C_{2i}$  are system matrices with appropriate dimensions. The scalar  $r$  is the number of “IF-THEN” rules of the system. The interval sets listed below present the firing strength corresponding to the  $i$ th rule:

$$W_i(x(k)) = [\underline{m}_i(x(k)), \overline{m}_i(x(k))],$$

where

$$\begin{aligned} \underline{m}_i(x(k)) &= \prod_{p=1}^{\theta} \underline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{m}_i(x(k)) &= \prod_{p=1}^{\theta} \overline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{u}_{M_{ip}}(f_p(x(k))) &\geq \underline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{m}_i(x(k)) &\geq \underline{m}_i(x(k)) \geq 0, \end{aligned}$$

$\underline{u}_{M_{ip}}(f_p(x(k)))$ ,  $\bar{u}_{M_{ip}}(f_p(x(k)))$ ,  $\underline{m}_i(x(k))$  and  $\bar{m}_i(x(k))$  denote the lower membership functions, upper membership functions, lower grades of membership and upper grades of membership, respectively.

The dynamics of system (1) is inferred as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^r m_i(x(k)) [A_i x(k) + B_i u(k) \\ &\quad + E_i w(k)], \\ z(k) &= \sum_{i=1}^r m_i(x(k)) [C_{1i} x(k) + D_i u(k) \\ &\quad + F_i w(k)], \\ y(k) &= \sum_{i=1}^r m_i(x(k)) C_{2i} x(k), \end{aligned} \quad (2)$$

where

$$\begin{aligned} m_i(x(k)) &= \frac{\phi_i(x(k))}{\sum_{\varrho=1}^r \phi_{\varrho}(x(k))} \geq 0, \\ \phi_i(x(k)) &= \underline{a}_i(x(k)) \underline{m}_i(x(k)) + \bar{a}_i(x(k)) \bar{m}_i(x(k)), \\ 0 &\leq \underline{a}_i(x(k)) \leq 1, \quad 0 \leq \bar{a}_i(x(k)) \leq 1, \\ \sum_{i=1}^r m_i(x(k)) &= 1, \quad \underline{a}_i(x(k)) + \bar{a}_i(x(k)) = 1 \end{aligned} \quad (3)$$

with  $\underline{a}_i(x(k))$  and  $\bar{a}_i(x(k))$  are nonlinear weighting functions and  $m_i(x(k))$  is regarded as the grade of membership.

*Remark 1:* The coefficients  $\bar{a}_i(x(k))$  and  $\underline{a}_i(x(k))$  play an important role in computing the actual membership functions  $m_i(x(k))$ . When the parameter perturbation occurs in the system, the actual membership function could be adjusted timely via them and membership functions with lower and upper bounds. Both coefficients should be able to reflect the change of the membership function resulted from the uncertain parameters and regulate the actual membership function timely and exactly. However, an appropriate method to determine them is still an open problem. In [30], the authors have demonstrated that these coefficients are affected by the uncertain parameters and a group of coefficients correspond to a group of fixed values of uncertain parameters. These weighting functions are assumed to be a function of states and uncertainties in both [30] and [31].

### B. Observer and Controller Design

In this paper, the observer shares the same membership function expressions with the plant. However, the membership functions of the observer depend on the estimated state variables instead of those of the plant. The details of the observer are as follows:

**Observer Rule  $j$ :** IF  $f_1(\tilde{x}(k))$  is  $M_{j1}$ , and  $f_2(\tilde{x}(k))$  is  $M_{j2}$  and, ..., and  $f_{\theta}(\tilde{x}(k))$  is  $M_{j\theta}$ , THEN

$$\begin{aligned} \tilde{x}(k+1) &= A_j \tilde{x}(k) + B_j u(k) + L_j (\bar{y}(k) - \tilde{y}(k)), \\ \tilde{y}(k) &= C_{2j} \tilde{x}(k), \quad j = 1, 2, \dots, r, \end{aligned} \quad (5)$$

where  $\tilde{x}(k) \in R^{n_x}$  denotes the state,  $\bar{y}(k) \in R^{n_y}$  and  $\tilde{y}(k) \in R^{n_y}$  denote the measured signal transmitted through

the communication network and the measured output, respectively.  $L_j$  are observer gains with appropriate dimensions to be determined. The dynamics fuzzy observer (5) is inferred as follows:

$$\begin{aligned} \tilde{x}(k+1) &= \sum_{j=1}^r m_j(\tilde{x}(k)) [A_j \tilde{x}(k) + B_j u(k) \\ &\quad + L_j (\bar{y}(k) - \tilde{y}(k))], \\ \tilde{y}(k) &= \sum_{j=1}^r m_j(\tilde{x}(k)) C_{2j} \tilde{x}(k). \end{aligned} \quad (6)$$

In order to design the controller more flexibly and decrease the complexity of the controller design, it has its own membership functions. Additionally, it uses the estimated state variables in membership functions.

**Controller Rule  $l$ :** IF  $g_1(\tilde{x}(k))$  is  $N_{l1}$ , and  $g_2(\tilde{x}(k))$  is  $N_{l2}$  and, ..., and  $g_{\bar{\Psi}}(\tilde{x}(k))$  is  $N_{l\bar{\Psi}}$ , THEN

$$\tilde{u}(k) = K_l \tilde{x}(k), \quad l = 1, 2, \dots, r, \quad (7)$$

where  $\tilde{u}(k) \in R^{n_u}$  is the output of the controller and  $K_l$  are controller parameters to be determined. The following interval sets describe the firing strength of the  $l$ th rule:

$$\Omega_l(\tilde{x}(k)) = [\underline{\omega}_l(\tilde{x}(k)), \bar{\omega}_l(\tilde{x}(k))],$$

where

$$\begin{aligned} \underline{\omega}_l(\tilde{x}(k)) &= \prod_{q=1}^{\bar{\Psi}} \underline{u}_{N_{lq}}(g_q(\tilde{x}(k))) \geq 0, \\ \bar{\omega}_l(\tilde{x}(k)) &= \prod_{q=1}^{\bar{\Psi}} \bar{u}_{N_{lq}}(g_q(\tilde{x}(k))) \geq 0, \\ \underline{u}_{N_{lq}}(g_q(\tilde{x}(k))) &\geq \underline{u}_{N_{lq}}(g_q(\tilde{x}(k))) \geq 0, \\ \bar{u}_{N_{lq}}(g_q(\tilde{x}(k))) &\geq \bar{u}_{N_{lq}}(g_q(\tilde{x}(k))) \geq 0, \end{aligned}$$

$\underline{u}_{N_{lq}}(g_q(\tilde{x}(k)))$ ,  $\bar{u}_{N_{lq}}(g_q(\tilde{x}(k)))$ ,  $\underline{\omega}_l(\tilde{x}(k))$  and  $\bar{\omega}_l(\tilde{x}(k))$  denotes the lower membership functions, upper membership functions, lower grades of membership and upper grades of membership, respectively.

Then, the overall fuzzy controller is as follows:

$$\tilde{u}(k) = \sum_{l=1}^r \omega_l(\tilde{x}(k)) K_l \tilde{x}(k), \quad (8)$$

where

$$\begin{aligned} \omega_l(\tilde{x}(k)) &= \frac{\varphi_l(x(k))}{\sum_{\rho=1}^r \varphi_{\rho}(x(k))} \geq 0, \\ \varphi_l(x(k)) &= \underline{b}_l(\tilde{x}(k)) \underline{\omega}_l(\tilde{x}(k)) + \bar{b}_l(\tilde{x}(k)) \bar{\omega}_l(\tilde{x}(k)), \\ 0 &\leq \underline{b}_l(\tilde{x}(k)) \leq 1, \quad 0 \leq \bar{b}_l(\tilde{x}(k)) \leq 1, \end{aligned} \quad (9)$$

$$\sum_{l=1}^r \omega_l(\tilde{x}(k)) = 1, \quad \underline{b}_l(\tilde{x}(k)) + \bar{b}_l(\tilde{x}(k)) = 1, \quad (10)$$

$\underline{b}_l(x(k))$  and  $\bar{b}_l(x(k))$  are nonlinear functions and  $\omega_l(x(k))$  is regarded as the grade of membership.

It can be seen that the membership functions of the observer and the controller use the estimated state  $\tilde{x}(k)$  rather than the

state  $x(k)$  of the plant. It, to some degree, can result in less conservative results.

*Remark 2:* In the IT2 controller model, the weighting functions  $\bar{b}_l(x(k))$  and  $\underline{b}_l(x(k))$  could influence the stability capacity. That was illustrated via simulation results in [31]. In order to enhance the stability capacity, they are defined as nonlinear functions regard to state variables, which can ensure them to be dynamic.

*Remark 3:* It is obvious that weighting functions in (3) and (9) can present the uncertain membership functions. By reconstructing the membership function with lower and upper membership functions and corresponding weighting functions, the information of parameter uncertainties can be captured.

### C. Communication Channels

Due to the insertion of the communication network, the measurement loss happens intermittently and unavoidably. For the sensor-to-observer link and the controller-to-actuator link, the following strategies are provided to model the phenomenon of measurement loss.

$$\begin{aligned}\bar{y}(k) &= \alpha(k)y(k), \\ u(k) &= \beta(k)\tilde{u}(k) + (1-\beta(k))\bar{K}_l u(k-1),\end{aligned}\quad (11)$$

in which  $\alpha(k)$  and  $\beta(k)$  satisfy the Bernoulli stochastic process.  $\alpha(k)$  and  $\beta(k)$  describe the imperfect sensor-to-observer link and the imperfect controller-to-actuator link, respectively. An assumption concerning  $\alpha(k)$  and  $\beta(k)$  is as follows:

$$\begin{aligned}\text{Prob}\{\alpha(k)\} &= \begin{cases} E\{\alpha(k)\} = \bar{\alpha}, & \alpha(k) = 1, \\ 1 - \bar{\alpha}, & \alpha(k) = 0, \end{cases} \\ \text{Prob}\{\beta(k)\} &= \begin{cases} E\{\beta(k)\} = \bar{\beta}, & \beta(k) = 1, \\ 1 - \bar{\beta}, & \beta(k) = 0. \end{cases}\end{aligned}$$

*Remark 4:* As can be seen from (11), the data loss model for the controller-to-actuator channel has two main functions. One is that the last time constant data packet transmitted successfully can be used as the present input to stabilize the plant when data packet dropouts occur randomly. It can compensate the lost data by itself rather than the external input. The other is that the gains  $\bar{K}_l$  can regulate the compensation data when the measurement loss occurs successively. It can update the data timely and decrease the unstable source. When  $\bar{K}_l = 0$ , (11) can reduce to the model we have used in our previous work [35]. When  $\bar{K}_l = I$ , it can reduce to the model used in [19]. In [37], the state used in controller is an estimated value instead of the real one when the data packets are transmitted successfully, which is different from the model used in this paper. In [38], [39], the Markov chains were introduced to model the imperfect communication links, accounting late arrival packets and obtaining nice results.

Define the estimation error as  $e(k) = x(k) - \tilde{x}(k)$ . According to (2), (6) and (8), the closed-loop system is represented

as

$$\begin{aligned}x(k+1) &= \sum_{i=1}^r \sum_{l=1}^r m_i(x(k)) \omega_l(\tilde{x}(k)) \\ &\quad \times [(A_i + \beta(k) B_i K_l) x(k) \\ &\quad + (1 - \beta(k)) B_i \bar{K}_l u(k-1) \\ &\quad - \beta(k) B_i K_l e(k) + E_i w(k)],\end{aligned}\quad (12)$$

$$\begin{aligned}\tilde{x}(k+1) &= \sum_{j=1}^r \sum_{l=1}^r m_j(\tilde{x}(k)) \omega_l(\tilde{x}(k)) \\ &\quad \times [(A_j + \beta(k) B_j K_l + \alpha(k) L_j C_{2i} \\ &\quad - L_j C_{2j}) \tilde{x}(k) + \alpha(k) L_j C_{2i} e(k) \\ &\quad + (1 - \beta(k)) B_i \bar{K}_l u(k-1)],\end{aligned}\quad (13)$$

$$\begin{aligned}e(k+1) &= x(k+1) - \tilde{x}(k+1) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\tilde{x}(k)) \omega_l(\tilde{x}(k)) \\ &\quad \times [(A_i - A_j + \beta(k) (B_i - B_j) K_l - L_j \\ &\quad \times (\alpha(k) C_{2i} - C_{2j}) \tilde{x}(k) + (A_i - \alpha(k) L_j C_{2i}) e(k) \\ &\quad + (1 - \beta(k)) (B_i - B_j) \bar{K}_l u(k-1) \\ &\quad + E_i w(k)].\end{aligned}\quad (14)$$

Then, the augmented system for (13) and (14) is as follows:

$$\begin{aligned}\xi(k+1) &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\tilde{x}(k)) \omega_l(\tilde{x}(k)) \\ &\quad \times [(A_{1ijl} + A_{2ijl}) \xi(k) + \bar{E}_i w(k)] \\ z(k) &= \sum_{i=1}^r \sum_{l=1}^r m_i(x(k)) \omega_l(\tilde{x}(k)) \\ &\quad [(C_{1il} + C_{2il}) \xi(k) + F_i w(k)],\end{aligned}\quad (15)$$

where

$$\begin{aligned}\xi(k) &= [\tilde{x}^T(k) \quad e^T(k) \quad u^T(k-1)]^T, \\ \bar{E}_i &= [0 \quad E_i^T \quad 0]^T, \\ A_{1ijl} &= \begin{bmatrix} \Theta_{1ijl} & \bar{\alpha} L_j C_{2i} & (1 - \bar{\beta}) B_j \bar{K}_l \\ \Theta_{2ijl} & A_i - \bar{\alpha} L_j C_{2i} & (1 - \bar{\beta}) \bar{B}_{ij} \bar{K}_l \\ \bar{\beta} K_l & 0 & (1 - \bar{\beta}) \bar{K}_l \end{bmatrix}, \\ A_{2ijl} &= \begin{bmatrix} \Theta_{3ijl} & \bar{\alpha}(k) L_j C_{2i} & -\bar{\beta}(k) B_j \bar{K}_l \\ \Theta_{4ijl} & -\bar{\alpha}(k) L_j C_{2i} & -\bar{\beta}(k) \bar{B}_{ij} \bar{K}_l \\ \bar{\beta}(k) K_l & 0 & -\bar{\beta}(k) \bar{K}_l \end{bmatrix}, \\ \bar{B}_{ij} &= B_i - B_j, \\ \Theta_{1ijl} &= A_j + \bar{\beta} B_j K_l + L_j (\bar{\alpha} C_{2i} - C_{2j}), \\ \Theta_{2ijl} &= A_i - A_j + \bar{\beta} (B_i - B_j) K_l - L_j (\bar{\alpha} C_{2i} - C_{2j}), \\ \Theta_{3ijl} &= \bar{\beta}(k) B_j K_l + \bar{\alpha}(k) L_j C_{2i}, \\ \Theta_{4ijl} &= \bar{\beta}(k) (B_i - B_j) K_l - \bar{\alpha}(k) L_j C_{2i}, \\ C_{1il} &= [C_{1i} + \bar{\beta} D_i K_l \quad C_{1i} \quad (1 - \bar{\beta}) D_i \bar{K}_l], \\ C_{2il} &= [\bar{\beta}(k) D_i K_l \quad 0 \quad -\bar{\beta}(k) D_i \bar{K}_l].\end{aligned}$$

It is obvious that

$$\begin{aligned}E\{\bar{\alpha}(k)\} &= 0, \quad E\{\bar{\alpha}(k)\bar{\alpha}(k)\} = \bar{\alpha}(1 - \bar{\alpha}), \\ E\{\bar{\beta}(k)\} &= 0, \quad E\{\bar{\beta}(k)\bar{\beta}(k)\} = \bar{\beta}(1 - \bar{\beta}).\end{aligned}$$

In order to develop the main results, the following definition is introduced.

**Definition 1.** [37] For any initial condition  $\xi(0)$ , when  $w(k) \equiv 0$ , the system (15) is stochastically stable if there exist a matrix  $W > 0$  such that

$$E \left\{ \sum_{k=0}^{\infty} |\xi(k)|^2 \mid \xi(0) \right\} < \xi^T(0)W\xi(0).$$

The purpose of this paper is to design the observer-based controller for nonlinear NCSs satisfying the following two conditions.

- 1) The system in (15) is stochastically stable,
- 2) Under zero initial condition, with a given positive scalar  $\gamma$ , the controlled output  $z(k)$  satisfies

$$E \left\{ \sqrt{\sum_{k=0}^{\infty} z^T(k)z(k)} \right\} \leq \gamma \|w\|_2. \quad (16)$$

### III. MAIN RESULTS

In this section, the sufficient stability conditions and observer-based controller design conditions are obtained in the following theorem, and the conditions can guarantee the closed-system to be stochastically stable with the the given disturbance attenuation level.

*Theorem 1:* With the given scalar  $\gamma$ , matrices  $K_l$ ,  $\bar{K}_l$  and  $L_j$  ( $l, j = 1, 2, \dots, r$ ) and  $\omega_l(\check{x}(k)) - \sigma_l m_l(\check{x}(k)) > 0$  ( $\sigma_l > 0$ ), the closed-loop system (15) is stochastically stable with the predefined disturbance attenuation performance, if there exist symmetric matrix  $P > 0$  and matrix  $G$  with appropriate dimensions satisfying the following inequalities

$$\bar{\Lambda}_{ijl} + \bar{\Lambda}_{ilj} - 2G < 0, \quad j \leq l, \quad (17)$$

$$\sigma_l \bar{\Lambda}_{ijl} + \sigma_j \bar{\Lambda}_{ilj} - \sigma_l G - \sigma_j G + 2G < 0, \quad j \leq l, \quad (18)$$

where

$$\begin{aligned} \bar{\Lambda}_{ijl} &= \Lambda_{ijl} + \Phi_{il}, \quad \Phi_{il} = \begin{bmatrix} \Pi_{il} & C_{1il}^T F_i \\ * & F_i^T F_i - \gamma^2 I \end{bmatrix}, \\ \Lambda_{ijl} &= \begin{bmatrix} \bar{\Theta}_{1ijl} & A_{1ijl}^T P^{-1} \bar{E}_i \\ * & \bar{E}_i^T P^{-1} \bar{E}_i \end{bmatrix}, \\ \bar{C}_{2il} &= \begin{bmatrix} tD_i K_l & 0 & -tD_i \bar{K}_l \end{bmatrix}, \\ \bar{A}_{2ijl} &= \begin{bmatrix} tB_j K_l + fL_j C & fL_j C_{2i} & -tB_j \bar{K}_l \\ \bar{\Theta}_{2ijl} & -fL_j C_{2i} & -t\bar{B}_{ij} \bar{K}_l \\ tK_l & 0 & -t\bar{K}_l \end{bmatrix}, \\ \bar{\Theta}_{1ijl} &= A_{1ijl}^T P^{-1} A_{1ijl} + \bar{A}_{2ijl}^T P^{-1} \bar{A}_{2ijl} - P^{-1}, \\ \bar{\Theta}_{2ijl} &= t\bar{B}_{ij} K_l - fL_j C_{2i}, \quad f = \sqrt{\bar{\alpha}(1-\bar{\alpha})}, \\ \Pi_{il} &= C_{1il}^T C_{1il} + \bar{C}_{2il}^T \bar{C}_{2il}, \quad t = \sqrt{\bar{\beta}(1-\bar{\beta})}. \end{aligned}$$

*Proof:* Before proceeding further, the following common quadratic Lyapunov function for system (15) is introduced.

$$V(k) = \xi^T(k) P^{-1} \xi(k),$$

where  $P$  is a symmetric positive definite matrix to be determined. Define  $\eta^T(k) = [\xi^T(k) \quad w^T(k)]$ .

Then, we get

$$\begin{aligned} \Delta V(k) &\leq E \left\{ \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) \omega_l(\check{x}(k)) \right. \\ &\quad \times [(A_{1ijl} + A_{2ijl}) \xi(k) + \bar{E}_i w(k)]^T P^{-1} \\ &\quad \times [(A_{1ijl} + A_{2ijl}) \xi(k) + \bar{E}_i w(k)] \left. \right\} \\ &\quad - \xi^T(k) P^{-1} \xi(k) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) \omega_l(\check{x}(k)) \\ &\quad \times \eta^T(k) \Lambda_{ijl} \eta(k). \end{aligned} \quad (19)$$

To obtain less conservative results, a slack matrix is introduced with the following expression

$$\begin{aligned} &\sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) \\ &\quad \times [(m_l(\check{x}(k)) - \omega_l(\check{x}(k)))G] = 0 \end{aligned}$$

Substituting (20) into (19), we can get

$$\begin{aligned} \Delta V(k) &\leq \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) \\ &\quad \times \eta^T(k) [\omega_l(\check{x}(k)) \Lambda_{ijl} + (m_l(\check{x}(k)) \\ &\quad - \omega_l(\check{x}(k)))G] \eta(k) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) \eta^T(k) \\ &\quad \times [m_l(\check{x}(k)) (\sigma_l \Lambda_{ijl} - \sigma_l G + G) \\ &\quad + (\omega_l(\check{x}(k)) - \sigma_l m_l(\check{x}(k))) (\Lambda_{ijl} - G)] \eta(k). \end{aligned}$$

When  $w(k) = 0$ , according to (17) and (18), we can get  $F < 0$ , where

$$\begin{aligned} F &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\check{x}(k)) [(\sigma_l \bar{\Theta}_{1ijl} - \sigma_l G_1 \\ &\quad + G_1) + (\omega_l(\check{x}(k)) - \sigma_l m_l(\check{x}(k))) (\bar{\Theta}_{1ijl} - G_1)], \end{aligned}$$

and  $G_1$  is the upper-left block of  $G$ , that is,  $G = \begin{bmatrix} G_1 & G_2 \\ * & G_3 \end{bmatrix}$ .

Then, we can get

$$\begin{aligned} &E \{ \xi^T(k+1) P^{-1} \xi(k+1) \} - \xi^T(k) P^{-1} \xi(k) \\ &\leq -\lambda_{\min}(-F) \xi^T(k) \xi(k). \end{aligned}$$

Computing the mathematical expectation for both sides and summing up the both sides of the inequality from  $k = 0, 1, \dots, c$  with any  $c \geq 1$ , we get

$$\begin{aligned} &E \{ \xi^T(c+1) P^{-1} \xi(c+1) \} - \xi^T(0) P^{-1} \xi(0) \\ &\leq -\lambda_{\min}(-F) E \left\{ \sum_{k=0}^c |\xi(k)|^2 \right\}, \end{aligned}$$

which yields

$$\begin{aligned} E \left\{ \sum_{k=0}^c |\xi(k)|^2 \right\} &\leq (\lambda_{\min}(-F))^{-1} \{ \xi^T(0) P^{-1} \xi(0) \\ &\quad - E \{ \xi^T(c+1) P^{-1} \xi(c+1) \} \} \\ &\leq (\lambda_{\min}(-F))^{-1} \xi^T(0) P^{-1} \xi(0), \end{aligned}$$

where  $\xi(0)$  is the initial condition. When  $c = 1, \dots, \infty$ , considering  $E \{ \xi^T(\infty) P \xi(\infty) \} \geq 0$ , we have

$$\begin{aligned} E \left\{ \sum_{k=0}^c |\xi(k)|^2 \right\} &\leq (\lambda_{\min}(-F))^{-1} \xi^T(0) P^{-1} \xi(0) \\ &= \xi^T(0) W \xi(0), \end{aligned}$$

in which

$$W \triangleq (\lambda_{\min}(-F))^{-1} P^{-1},$$

which means  $W > 0$ . Thus, according to Definition 1, the resulting closed-loop system in (15) is stochastically stable. In the following part, disturbance rejection performance of the closed-loop fuzzy system in (15) will be considered. Under the zero initial condition, introduce the following performance index.

$$\begin{aligned} J &\triangleq E \{ V(k+1) | \eta(k) \} - V(k) \\ &\quad + E \{ z^T(k) z(k) | \eta(k) \} - \gamma^2 w^T(k) w(k). \end{aligned}$$

Then, we get

$$\begin{aligned} J &= E \{ z^T(k) z(k) | \eta(k) \} - \gamma^2 w^T(k) w(k) + \Delta V(k) \\ &\leq E \left\{ \sum_{i=1}^r \sum_{l=1}^r m_i(x(k)) \omega_l(\tilde{x}(k)) [(C_{1il} + C_{2il}) \xi(k) \right. \\ &\quad \left. + F_i w(k)]^T [(C_{1il} + C_{2il}) \xi(k) + F_i w(k)] \right\} \\ &\quad - \gamma^2 w^T(k) w(k) + \Delta V(k) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\tilde{x}(k)) \eta^T(k) [m_l(\tilde{x}(k)) \\ &\quad \times (\sigma_l \bar{\Lambda}_{ijl} - \sigma_l G + G) + (\omega_l(\tilde{x}(k)) \\ &\quad - \sigma_l m_l(\tilde{x}(k))) (\bar{\Lambda}_{ijl} - G)] \eta(k) \\ &= \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r m_i(x(k)) m_j(\tilde{x}(k)) \eta^T(k) \{ m_l(\tilde{x}(k)) \\ &\quad \times (\sigma_l \bar{\Lambda}_{ijl} + \sigma_j \bar{\Lambda}_{ilj} - \sigma_l G - \sigma_j G + 2G) \\ &\quad + (\omega_l(\tilde{x}(k)) - \sigma_l m_l(\tilde{x}(k))) \\ &\quad \times (\bar{\Lambda}_{ijl} + \bar{\Lambda}_{ilj} - 2G) \} \eta(k). \end{aligned} \quad (20)$$

According to (17) and (18), one can have

$$E \{ z^T(k) z(k) \} - \gamma^2 w^T(k) w(k) + \Delta V(k) \leq 0,$$

which suggests  $J \leq 0$ . Then, we can obtain  $E \left\{ \sqrt{\sum_{k=0}^{\infty} z^2(k)} \right\} \leq \gamma \|w\|_2$ , and the proof of Theorem 1 is completed. ■

The conditions in Theorem 1 are coupled, which makes it difficult to design the observer and the controller. Next, the congruence transformation and singular value decomposition are utilized to obtain the desired conditions. Here, the output matrices  $C_{2i}$  ( $i = 1, 2, \dots, r$ ) are chosen as a common one

(i.e.,  $C_{2i} = C$ ) with the constraint of full row rank. Based on this constraint, it facilitates designing the observer and the controller, which can be seen in the proof of Theorem 2,

*Theorem 2:* Under  $\omega_l(\tilde{x}(k)) - \sigma_l m_l(\tilde{x}(k)) > 0$  ( $\sigma_l > 0$ ), the closed-loop fuzzy system in (15) is stochastically stable and satisfies the predefined disturbance attenuation performance  $\gamma$ , if there exist symmetric matrix  $P > 0$  and matrices  $Y_{11} > 0, Y_{12} > 0, Y_{13} > 0, Y_2 > 0, \bar{G}_\zeta$  ( $\zeta = 1, 2, \dots, 9$ ),  $G_{10}, \bar{L}_j > 0, \bar{K}_l, \bar{K}_l$  satisfying the following conditions:

$$\begin{bmatrix} 2\tilde{P} & \Upsilon_{ijl} + \Upsilon_{ilj} \\ * & 2\bar{\Xi} \end{bmatrix} < 0, \quad j \leq l, \quad (21)$$

$$\begin{bmatrix} 2\tilde{P} & \sqrt{\sigma_l} \Upsilon_{ijl} + \sqrt{\sigma_j} \Upsilon_{ilj} \\ * & \bar{\Xi} \end{bmatrix} < 0, \quad j \leq l, \quad (22)$$

where

$$\begin{aligned} \tilde{P} &= \text{diag} \{-P, -P, -I, -I\}, \\ Y &= \text{diag} \{Y_1, Y_1, Y_2\}, \\ \Upsilon_{ijl} &= \begin{bmatrix} \tilde{A}_{1ijl}^T & \tilde{A}_{2ijl}^T & \tilde{C}_{1il}^T & \tilde{C}_{2il}^T \\ \tilde{E}_i^T & 0 & F_i^T & 0 \end{bmatrix}^T, \\ P &= \begin{bmatrix} P_1 & P_2 & P_3 \\ * & P_4 & P_5 \\ * & * & P_6 \end{bmatrix}, \\ \tilde{A}_{1ijl} &= \begin{bmatrix} \tilde{\Theta}_{1jl} & \bar{\alpha} \bar{L}_j J_3^T & (1 - \bar{\beta}) B_j \bar{K}_l \\ \tilde{\Theta}_{2ijl} & A_i Y_1 - \bar{\alpha} \bar{L}_j J_3^T & (1 - \bar{\beta}) \bar{B}_{ij} \bar{K}_l \\ \bar{\beta} \bar{K}_l & 0 & (1 - \bar{\beta}) \bar{K}_l \end{bmatrix}, \\ \tilde{A}_{2ijl} &= \begin{bmatrix} t B_j \bar{K}_l + f \bar{L}_j J_3^T & f \bar{L}_j J_3^T & -t B_j \bar{K}_l \\ \tilde{\Theta}_{3ijl} & -f \bar{L}_j J_3^T & -t \bar{B}_{ij} \bar{K}_l \\ t \bar{K}_l & 0 & -t \bar{K}_l \end{bmatrix}, \\ \tilde{C}_{1il} &= [C_{1i} Y_1 + \bar{\beta} D_i \bar{K}_l \quad C_{1i} Y_1 \quad (1 - \bar{\beta}) D_i \bar{K}_l], \\ \tilde{C}_{2il} &= [t D_i \bar{K}_l \quad 0 \quad -t D_i \bar{K}_l], \\ \tilde{\Theta}_{1jl} &= A_j Y_1 + \bar{\beta} B_j \bar{K}_l + \bar{L}_j (\bar{\alpha} J_3^T - J_3^T), \\ \tilde{\Theta}_{2ijl} &= A_i Y_1 - A_j Y_1 + \bar{\beta} \bar{B}_{ij} \bar{K}_l - \bar{L}_j (\bar{\alpha} J_3^T - J_3^T), \\ \tilde{\Theta}_{3ijl} &= t(B_i - B_j) \bar{K}_l - f \bar{L}_j J_3^T, \\ \bar{L}_j &= [\bar{L}_j \quad 0], \quad Y_1 = J_3 \begin{bmatrix} J_2^{-1} J_1^T Y_{11} & 0 \\ Y_{12} & Y_{13} \end{bmatrix} J_3^T, \\ \bar{\Xi} &= \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \\ * & \Xi_5 & \Xi_6 & \Xi_7 \\ * & * & \Xi_8 & \Xi_9 \\ * & * & * & \Xi_{10} \end{bmatrix}, \\ \bar{\Xi} &= \begin{bmatrix} \bar{\Xi}_1 & \bar{\Xi}_2 & \bar{\Xi}_3 & \bar{\Xi}_4 \\ * & \bar{\Xi}_5 & \bar{\Xi}_6 & \bar{\Xi}_7 \\ * & * & \bar{\Xi}_8 & \bar{\Xi}_9 \\ * & * & * & \bar{\Xi}_{10} \end{bmatrix}, \end{aligned}$$

$$\Xi_1 = 2(P_1 - Y_1 - Y_1^T) - \sigma_l \bar{G}_1 - \sigma_j \bar{G}_1 + 2\bar{G}_1,$$

$$\Xi_2 = 2P_2 - \sigma_l \bar{G}_2 - \sigma_j \bar{G}_2 + 2\bar{G}_2,$$

$$\Xi_3 = 2P_3 - \sigma_l \bar{G}_3 - \sigma_j \bar{G}_3 + 2\bar{G}_3,$$

$$\Xi_4 = -\sigma_l \bar{G}_4 - \sigma_j \bar{G}_4 + 2\bar{G}_4,$$

$$\Xi_5 = 2(P_4 - Y_1 - Y_1^T) - \sigma_l \bar{G}_5 - \sigma_j \bar{G}_5 + 2\bar{G}_5,$$

$$\Xi_6 = 2P_5 - \sigma_l \bar{G}_6 - \sigma_j \bar{G}_6 + 2\bar{G}_6,$$

$$\Xi_7 = -\sigma_l \bar{G}_7 - \sigma_j \bar{G}_7 + 2\bar{G}_7,$$

$$\begin{aligned}
\bar{\Xi}_8 &= 2(P_6 - Y_2 - Y_2^T) - \sigma_l \bar{G}_8 - \sigma_j \bar{G}_8 + 2\bar{G}_8, \\
\bar{\Xi}_9 &= -\sigma_l \bar{G}_9 - \sigma_j \bar{G}_9 + 2\bar{G}_9, \\
\bar{\Xi}_{10} &= -2\gamma^2 I - \sigma_l G_{10} - \sigma_j G_{10} + 2G_{10}, \\
\bar{\Xi}_1 &= P_1 - Y_1 - Y_1^T - \bar{G}_1, \\
\bar{\Xi}_2 &= P_2 - \bar{G}_2, \quad \bar{\Xi}_3 = P_3 - \bar{G}_3, \\
\bar{\Xi}_4 &= -\bar{G}_4, \\
\bar{\Xi}_5 &= (P_4 - Y_1 - Y_1^T) - \bar{G}_5, \quad \bar{\Xi}_6 = P_5 - \bar{G}_6, \\
\bar{\Xi}_7 &= -\bar{G}_7, \quad \bar{\Xi}_8 = P_6 - Y_2 - Y_2^T - \bar{G}_8, \\
\bar{\Xi}_9 &= -\bar{G}_9, \quad \bar{\Xi}_{10} = -\gamma^2 I - G_{10},
\end{aligned}$$

$J_1$ ,  $J_2$  and  $J_3$  are obtained via the singular value decomposition of the output matrix  $C$ , that is,  $C = J_1 [ J_2 \ 0 ] J_3^T$  with  $J_1 J_1^T = I$  and  $J_3 J_3^T = I$ . The observer, controller and compensator gains can be obtained as follows:

$$L_j = \bar{L}_j Y_{11}^{-1}, \quad K_l = \bar{K}_l Y_1^{-1}, \quad \bar{K}_l = \check{K}_l Y_2^{-1}.$$

*Proof:* If there exist a nonsingular matrix  $Y$ , the following inequality holds

$$(P - Y)^T P^{-1} (P - Y) \geq 0,$$

which yields

$$P - Y - Y^T \geq -Y^T P^{-1} Y.$$

Thus, according to (21), the following inequality holds

$$\begin{bmatrix} 2\bar{P} & \Upsilon_{ijl} + \Upsilon_{ilj} \\ * & 2\check{\Xi} \end{bmatrix} < 0, \quad (23)$$

where

$$\begin{aligned}
\check{\Xi} &= \begin{bmatrix} \check{\Xi}_1 & \check{\Xi}_2 & \check{\Xi}_3 & \check{\Xi}_4 \\ * & \check{\Xi}_5 & \check{\Xi}_6 & \check{\Xi}_7 \\ * & * & \check{\Xi}_8 & \check{\Xi}_9 \\ * & * & * & \check{\Xi}_{10} \end{bmatrix}, \\
\check{\Xi}_1 &= -Y_1^T P_1^{-1} Y_1 - \bar{G}_1, \quad \check{\Xi}_2 = -Y_1^T P_2^{-1} Y_1 - \bar{G}_2, \\
\check{\Xi}_3 &= -Y_1^T P_3^{-1} Y_2 - \bar{G}_3, \quad \check{\Xi}_4 = \bar{\Xi}_4, \\
\check{\Xi}_5 &= -Y_1^T P_4^{-1} Y_1 - \bar{G}_5, \quad \check{\Xi}_6 = -Y_1^T P_5^{-1} Y_2 - \bar{G}_6, \\
\check{\Xi}_7 &= \bar{\Xi}_7, \quad \check{\Xi}_8 = -Y_2^T P_6^{-1} Y_2 - \bar{G}_8, \\
\check{\Xi}_9 &= \bar{\Xi}_9, \quad \check{\Xi}_{10} = \bar{\Xi}_{10}.
\end{aligned}$$

Define

$$\begin{aligned}
\bar{G}_1 &= Y_1^T G_1 Y_1, \quad \bar{G}_2 = Y_1^T G_2 Y_1, \quad \bar{G}_3 = Y_1^T G_3 Y_2, \\
\bar{G}_4 &= Y_1^T G_4, \quad \bar{G}_5 = Y_1^T G_5 Y_1, \quad \bar{G}_6 = Y_1^T G_6 Y_2, \\
\bar{G}_7 &= Y_1^T G_7, \quad \bar{G}_8 = Y_2^T G_8 Y_2, \quad \bar{G}_9 = Y_2^T G_9, \\
G &= \begin{bmatrix} G_1 & G_2 & G_3 & G_4 \\ * & G_5 & G_6 & G_7 \\ * & * & G_8 & G_9 \\ * & * & * & G_{10} \end{bmatrix}. \quad (24)
\end{aligned}$$

Pre- and post-multiplying (23) by  $\text{diag} \{I, I, I, I, Y^{-T}, I\}$  and its transpose and considering (24), the inequality (17) can be obtained. Similarly, the inequality (18) is obtained from (22). The proof is completed. ■

*Remark 5:* The congruence transformation and singular value decomposition techniques are utilized to obtain Theorem 2. Besides, the cone complementarity linearization technique

TABLE I  
MEMBERSHIP FUNCTIONS  
WITH LOWER AND UPPER BOUNDS OF THE PLANT

Lower bounds
$\underline{u}_{M_{11}}(x_1(k)) = 1 - e^{-\frac{x_1^2(k)}{1.2}}$
$\underline{u}_{M_{12}}(x_1(k)) = 1 - \bar{u}_{M_{11}}(x_1(k))$
Upper bounds
$\bar{u}_{M_{11}}(x_1(k)) = 1 - 0.23e^{-\frac{x_1^2(k)}{1.2}}$
$\bar{u}_{M_{12}}(x_1(k)) = 1 - \underline{u}_{M_{11}}(x_1(k))$

also can be used to developed the conditions to design the controller. The authors in [19] have pointed out the drawbacks and advantages. Recently, the problem of observer-based sliding model control for the system subject to the actuator fault, input disturbances and sensor fault was investigated in [40]. The authors employed the descriptor operator to assemble the state, disturbance and fault into the state of the new system and nice criteria were obtained. Such idea can be used to design the observer-based controller for the system with the actuator fault, input disturbances and sensor fault in the IT2 T-S fuzzy model framework.

#### IV. SIMULATION RESULTS

In this section, the effectiveness of the observer-based controller design method is demonstrated via two numerical examples. First, we consider the following numerical example.

*Example 1:* Consider the following discrete-time T-S fuzzy model of the form (1):

**Plant Rule  $i$ :** IF  $f(x(k))$  is  $M_i$ , THEN

$$\begin{aligned}
x(k+1) &= A_i x(k) + B_i u(k) + E_i w(k), \\
z(k) &= C_i x(k) + D_i u(k) + F_i w(k), \quad i = 1, 2, \\
y(k) &= Cx(k),
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.7238 & 0.4972 \\ 0.5148 & 0.0877 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0504 \\ 0.1042 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.9652 & 0.0969 \\ 0.4813 & 0.0027 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0684 \\ 0.0643 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} 0.0005 \\ 0.0124 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.0011 \\ 0.0031 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} -0.0749 & -0.0830 \end{bmatrix}, \quad D_1 = -0.0188, \\
C_2 &= \begin{bmatrix} -0.1052 & -0.0532 \end{bmatrix}, \quad D_2 = -0.0425, \\
F_1 &= -0.0172, \quad F_2 = -0.0080, \\
C &= \begin{bmatrix} -0.1187 & 0 \end{bmatrix}.
\end{aligned}$$

*Remark 6:* As we have stated above, the uncertainties can be captured and expressed via the membership functions with lower and upper bounds, and thus, there exist few uncertainties in the given system matrices. More details are provided in Example 2.

Table I and Table II provide the membership functions with lower and upper bounds of the plant and those of the controller, respectively.



TABLE II  
MEMBERSHIP FUNCTIONS  
WITH LOWER AND UPPER BOUNDS OF THE CONTROLLER

Lower bounds	Upper bounds
$\underline{u}_{N_{11}}(\tilde{x}_1(k)) = e^{-\frac{\tilde{x}_1^2(k)}{0.35}}$	$\bar{u}_{N_{11}}(\tilde{x}_1(k)) = \underline{u}_{N_{11}}(\tilde{x}_1(k))$
$\underline{u}_{N_{12}}(\tilde{x}_1(k)) = 1 - \underline{u}_{N_{11}}(\tilde{x}_1(k))$	$\bar{u}_{N_{12}}(\tilde{x}_1(k)) = \underline{u}_{N_{12}}(\tilde{x}_1(k))$

To ensure  $\omega_l(\tilde{x}(k)) - \sigma_l m_l(\tilde{x}(k)) > 0$  ( $l = 1, 2$ ), we define  $\sigma_1 = 0.3$  and  $\sigma_2 = 0.2$ . For demonstration, the initial condition and external disturbance are given as follows:

$$\begin{aligned} x(0) &= [0.5 \ 0]^T, \quad \tilde{x}(0) = [0 \ 0]^T, \\ w(k) &= e^{-0.1k} \cos(2k). \end{aligned}$$

Fig. 2 plots the state responses of the open-loop system, from which we can see that the system is unstable without considering the controller. Next, the effectiveness of the controller designed in this paper is demonstrated.

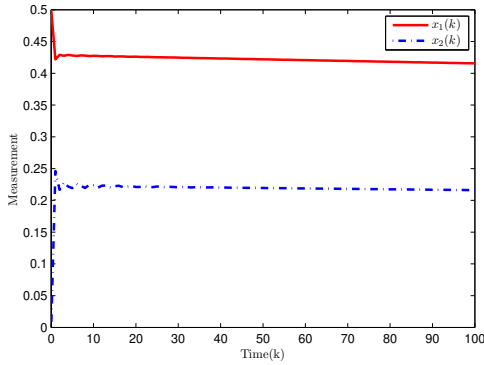


Fig. 2. State responses of the open-loop system

Applying Theorem 2 with  $\gamma = 0.35$  and  $\bar{\alpha} = \bar{\beta} = 0.8$ , the observer, controller and compensation gains can be computed as follows:

$$\begin{aligned} L_1 &= [-2.3112 \ -2.1052]^T, \\ L_2 &= [-1.4308 \ -1.4523]^T, \\ K_1 &= [-3.1263 \ 0.5134], \\ K_2 &= [-2.3665 \ 0.7762], \\ \bar{K}_1 &= 0.0020, \quad \bar{K}_2 = 0.0038. \end{aligned}$$

Fig. 3 depicts the phenomena of stochastic measurement loss with the probabilities  $\bar{\alpha} = \bar{\beta} = 0.8$ . Therein, the probability can be determined through a prior experience. Fig. 4 and Fig. 5 plot the state estimation for unmeasurable state variables of the plant, respectively. It can be seen that the closed-loop system can be stabilized with the designed controller and the designed observer also can estimate the unmeasurable states effectively. In addition, Fig. 6 and Fig. 7 are provided to show the responses of the input  $u(k)$  and the controller output  $z(k)$ , respectively. Fig. 8 plots the practical response of the disturbance attenuation performance, which illustrates that the practical response is far less than the predefined level (i.e., 0.35).

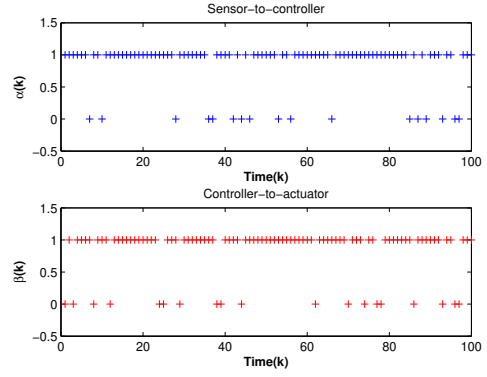


Fig. 3. Data packet loss with the probability  $\bar{\alpha} = \bar{\beta} = 0.8$

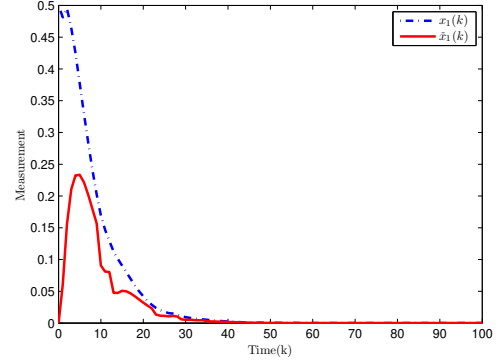


Fig. 4. State responses of  $x_1(k)$  and  $\hat{x}_1(k)$

When  $\bar{K}_l = I$ , there exist no feasible solutions to obtain the observer, controller and compensator gains to stabilize the system, which can demonstrate that it is necessary to adopt the compensation scheme proposed in this paper when NCSs are modeled. Additionally, in [37], the comparisons of  $\bar{K}_l = I$  and  $\bar{K}_l = 0$  have been carried out, and the advantages of the compensation strategy (i.e.,  $\bar{K}_l = I$ ) over the common one (i.e.,  $\bar{K}_l = 0$ ) have been demonstrated through the disturbance attenuation level.

*Example 2:* To further illustrate the effectiveness of the proposed control scheme, a single-link rigid robot system well

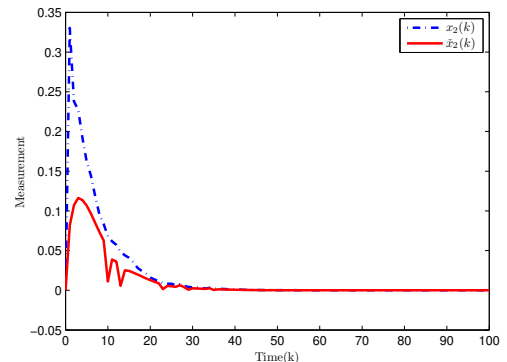
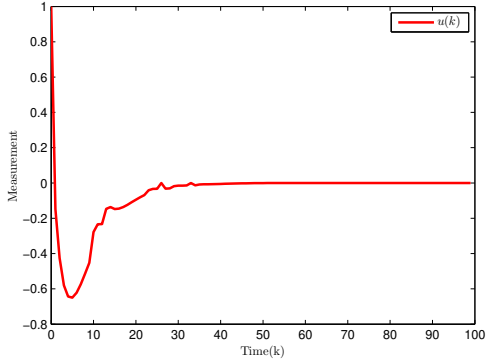
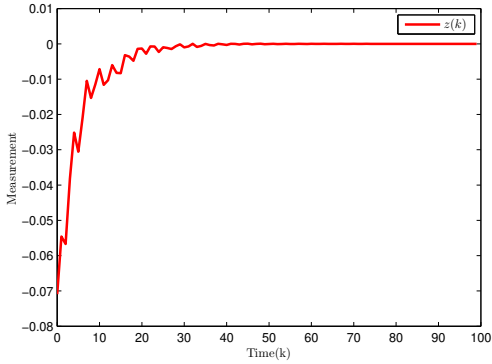


Fig. 5. State responses of  $x_2(k)$  and  $\hat{x}_2(k)$

Fig. 6. Response of  $u(k)$ Fig. 7. Response of  $z(k)$ 

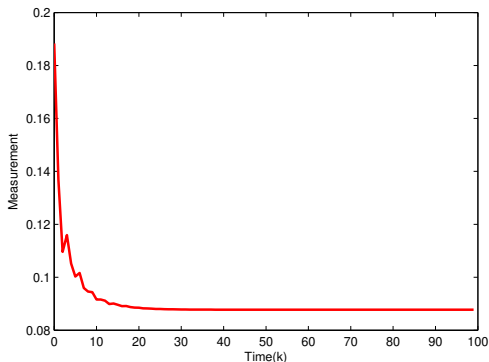
studied in [41] is considered. It is governed by the following equation.

$$\mathcal{J}\ddot{\mathcal{V}} = -(0.5mgl + Mgl) \sin(\mathcal{V}) + u,$$

where

$$\mathcal{J} = Ml^2 + \frac{1}{3}ml^2,$$

and  $\mathcal{J}$  denotes the moment of inertia,  $\mathcal{V} \in [0, \frac{\pi}{2}]$  stands for the joint rotation angle,  $m = 1.5$  kg presents the mass of the

Fig. 8. Response of  $\sqrt{\frac{\sum_{k=0}^{\infty} z^T(k)z(k)}{\sum_{k=0}^{\infty} w^T(k)w(k)}}$ 

load,  $g = 9.8$  m/s<sup>2</sup> is the gravity coefficient and  $M = 3$  kg means the mass of the rigid link.

Considering uncertainties existing in systems, there exist two main methods to obtain the IT2 T-S fuzzy model. One is in [30], [31] and [35] and the other is in [42]. The former starts with giving the range of uncertain parameters, and then the maximum and minimum values of nonlinear characteristics with parameter uncertainties are calculated. According to the approach in [43], membership functions can be obtained. Then, lower and upper ones are computed with uncertain information. Based on the above information, an IT2 T-S fuzzy model is obtained. The latter is similar to traditional type-1 T-S fuzzy model. To begin with, it neglects the uncertain parameters and model nonlinear systems with the approach in [43]. Then, the uncertain parameters are considered and the range of perturbation is given. Based on the given parameter perturbation, we define lower and upper membership functions. Next, we use the latter to handle the uncertain single-link rigid robot system. The detailed steps are as follows:

*Step 1.* Construct the T-S fuzzy model with sampling time  $T = 0.04$  neglecting uncertainties through defining  $x(k) = [x_1^T(k) \ x_2^T(k)]^T = [\mathcal{V}^T \ \dot{\mathcal{V}}^T]^T$ .

**Plant Rule 1:** *IF*  $x_1(k)$  is about 0, *THEN*

$$x(k+1) = A_1x(k) + B_1u(k),$$

**Plant Rule 2:** *IF*  $x_1(k)$  is about  $\frac{\pi}{2}$ , *THEN*

$$x(k+1) = A_2x(k) + B_2u(k),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{-(0.5mgl+Mgl)}{\mathcal{J}} & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{1}{\mathcal{J}} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{-2(0.5mgl+Mgl)}{\mathcal{J}\pi} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{1}{\mathcal{J}} \end{bmatrix}.$$

*Step 2.* Determine membership functions without uncertainties.  $f_1 = 1 - \frac{2|x_1(k)|}{\pi}$ , and  $f_2 = 1 - f_1$ .

*Step 3.* Consider perturbation in  $\mathcal{J}$  up and down 20%. Lower and upper membership functions are as follows:  $\underline{m}_1 = 0.8 - \frac{1.6|x_1(k)|}{\pi}$ ,  $\bar{m}_1 = 1 - \frac{1.6|x_1(k)|}{\pi}$ ,  $\underline{m}_2 = 1 - \bar{m}_1$ , and  $\bar{m}_2 = 1 - \underline{m}_1$ .

In this example, membership functions of the controller are defined as same as those in Example 1. Fig. 9 plots lower and upper membership functions. Obviously, uncertain  $\mathcal{J}$  can be repressed through lower and upper membership functions with corresponding weighting coefficients. With sampling period  $T = 0.04$ , we can get

$$A_1 = \begin{bmatrix} 0.9832 & 0.0398 \\ -0.8353 & 0.9832 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0009 \\ 0.0455 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.9893 & 0.0399 \\ -0.5329 & 0.9893 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0009 \\ 0.0456 \end{bmatrix}.$$

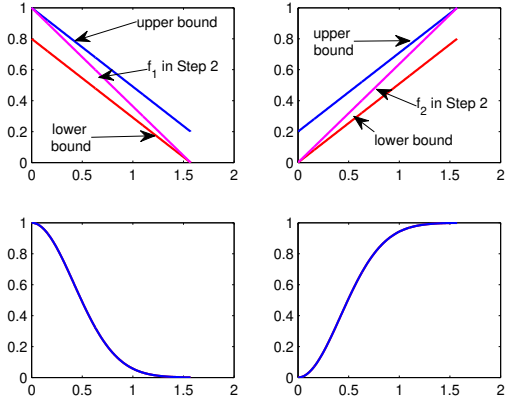


Fig. 9. Lower and upper membership functions

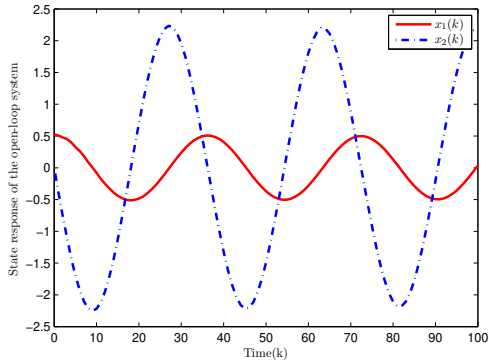
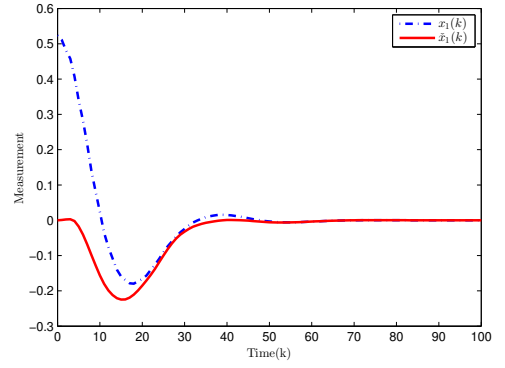
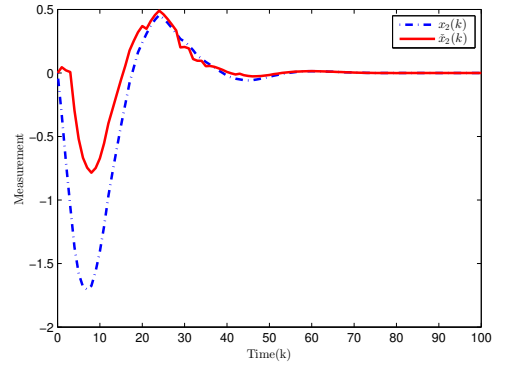
Other relative matrices are given as follows:

$$\begin{aligned} E_1 &= \begin{bmatrix} 0.0202 \\ -0.0004 \end{bmatrix}, E_2 = \begin{bmatrix} 0.0100 \\ -0.0076 \end{bmatrix}, \\ C &= [0 \ 1], C_1 = [0.0060 \ 0.0212], \\ C_2 &= [0.0131 \ -0.0070], \\ D_1 &= -0.0102, D_2 = 0.0005, \\ F_1 &= -0.0024, F_2 = -0.0006. \end{aligned}$$

Applying Theorem 2, and giving  $\gamma = 1.35$ , the observer, controller and compensator gains can be computed as follows:

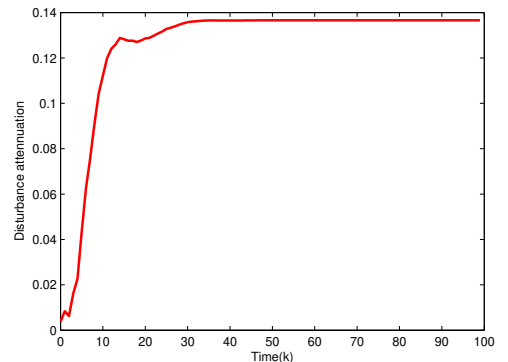
$$\begin{aligned} L_1 &= [0.0048 \ 0.2882]^T, \\ L_2 &= [0.0051 \ 0.2742]^T, \\ K_1 &= [-2.7521 \ -5.3111], \\ K_2 &= [-1.6277 \ -4.8504], \\ \bar{K}_1 &= 0.1502, \bar{K}_2 = 0.2064. \end{aligned}$$

Other parameters and conditions without detailed definition are same to those in Example 1. For simulation, the initial conditions are given as  $x(0) = [\frac{\pi}{6} \ 0]$  and  $\dot{x}(0) = [0 \ 0]$ . Figs. 10-13 demonstrate the simulation results for the single-link rigid robot system with uncertainties, from which we can see that the proposed control approach is effective.


 Fig. 10. Trajectories of  $x_1(k)$  and  $x_2(k)$ 

 Fig. 11. Trajectories of  $x_1(k)$  and  $\hat{x}_1(k)$ 

 Fig. 12. Trajectories of  $x_2(k)$  and  $\hat{x}_2(k)$ 

## V. CONCLUSIONS

In this paper, the problem of the observer-based controller design for a class of nonlinear NCSs subject to data loss and parameter uncertainties and unmeasurable states has been investigated in the framework of the IT2 T-S fuzzy model. By using the IT2 T-S fuzzy model, the nonlinear NCSs with uncertainties were established, in which the uncertainties are captured and expressed via the lower and upper membership functions and relative weighting coefficients. By designing the observer, the unmeasurable states were estimated and


 Fig. 13. Trajectory of  $\sqrt{\frac{\sum_{k=0}^{\infty} z^T(k)z(k)}{\sum_{k=0}^{\infty} w^T(k)w(k)}}$

used to design the observer and the controller. A novel data loss model was considered to compensate and regulate the lost data effectively and facilitate maintaining the system stability. Using a slack matrix, less conservative results are obtained. Moreover, conditions are developed to design the IT2 observer-based controller ensuring the closed-loop system to be stochastically stable with disturbance attenuation performance. Finally, two illustrative examples are provided to demonstrate the effectiveness of the method proposed in this paper.

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