INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust. Nonlinear Control 2015; **00**:1–16
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/rnc

Observer-based Leader-Following Consensus of Uncertain Nonlinear Multi-Agent Systems†

P. Shi[1]*, Q. K. Shen[2]

[1] *School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia*
[2] *College of Information Engineering, Yangzhou University, Yangzhou 225127, China*

SUMMARY

In this paper, the leader-following consensus problem of uncertain high-order nonlinear multi-agent systems on directed graph with a fixed topology is studied, where it is assumed that the relative states of a follower and its neighbors are immeasurable and only the relative outputs are available. Nonlinear adaptive observers are firstly proposed for each follower to estimate the states of it and its neighbors, and an observer-based distributed adaptive control scheme is constructed to guarantee that all followers asymptotically synchronize to a leader with tracking errors being semi-globally uniform ultimate bounded. Based on algebraic graph theory and Lyapunov theory, the closed-loop system stability analysis is conducted. Finally, numerical simulations are presented to illustrate the effectiveness and potential of the proposed new design techniques. Copyright © 2015 John Wiley & Sons, Ltd.

Received . . .

KEY WORDS: leader-following consensus; multi-agent systems; cooperative control

1. INTRODUCTION

Leader-following consensus of multi-agent systems (MASs), as one of the distributed control problems of MASs [1-20, 25-28, 30,31], has attracted extensive attention from the control community for the last two decades. Based on the condition that the states of a follower and its neighbors are assumed to be measurable, the distributed state-feedback controller is constructed for each follower in communication network. However, in practical applications, the full states of an agent and its neighbors can not always be observed by a follower, but only their outputs are available. Hence, how to obtain the consensus only using the output information becomes more practical and challenging, which motivates us for this work.

On the other hand, the observer-based control approach is a common and important approach in the case where system states are not available [21-23]. In recent years, this approach has been extended to MASs [16]. In [16], the synchronization of identical general linear systems on a digraph was studied, and a framework for cooperative tracking control was proposed, including full state feedback control, observer design and dynamic output feedback control. Very recently, the robust cooperative tracking problem was investigated for nonlinear MASs with the bounded external disturbances in [25] where two state observers were designed. However, the aforementioned works only focused on second-order systems [25] or linear systems [16]. Recently, the distributed control problem of high-order nonlinear MASs has attracted attention [30]. For high-order nonlinear multi-agent systems, the design of distributed state observer and controller still is an open and important problem, which is another motivation of this work.

*Correspondence to: E-mail: peng.shi@adelaide.edu.au

Prepared using rncauth.cls *[Version: 2010/03/27 v2.00]*

In this paper, we investigate the leader-following consensus control problem of uncertain nonlinear high-order MASs on the directed graph with a fixed topology. Compared with existing works, the novelty and significance of our work are as follows:

i) Compared with the results in [7, 10-13, 18], the relative states of a follower and its neighbors considered in this paper are not measurable, but only the relative outputs are available. Besides, the systems considered in this paper is high-order uncertain nonlinear systems;

ii) Different from [23], the design of the proposed distributed adaptive observer and controller does not require the assumption that the upper bounds of actual and optimal approximation errors should be known exactly. In addition, it is not necessary to know the upper bounds of the leader node's nonlinear dynamics, external disturbance, while these bounds are assumed to be known in [7]; and

iii) In order to avoid the controller singularity problem, the stability analysis is performed in two cases, and a novel adaptive distributed control scheme is proposed.

The rest of this note is organized as follows. In Section 2, basic graph theory and notations, the problem formulation, and mathematical description of neural-networks (NNs) are introduced. In Section 3, the main technical results of this note are given, which include the design of local state observer for each follower, and distributed adaptive neural controllers. A numerical simulation is presented in Section 4. These simulation results demonstrate the effectiveness of the proposed technique. Finally, Section 5 draws the conclusion.

Notations: Throughout this note, R, R^n and $R^{n\times m}$ denote, respectively, the real numbers, the real n-vectors and the real $n\times m$ matrices; $|\cdot|$ is the absolute value of a real number; $||\cdot||$ is the Euclidean norm of a vector; $||\cdot||_F$ is the Frobenius norm of a matrix; $tr\{\cdot\}$ is the trace of a matrix; $\sigma(\cdot)$ is the set of singular values of a matrix, with the maximum singular value $\bar{\sigma}(\cdot)$ and the minimum singular value $\underline{\sigma}(\cdot)$; $\lambda(\cdot)$ is the set of eigenvalues of a matrix, with the smallest eigenvalue $\lambda_{\min}(\cdot)$ and the largest eigenvalue $\lambda_{\max}(\cdot)$; matrix $P>0$ ($P\geq 0$) means P is positive definite (positive semi-definite); I denotes the identity matrix with appropriate dimensions.

2. PRELIMINARIES AND COOPERATIVE TRACKING CONTROL FORMULATION

2.1. Basic Graph Theory and Notations

Consider a multi-agent system consisting of a leader and the followers. To solve the consensus problem and describe the information exchange between agents, basic graph theory and some notations are introduced as in [7].

$G=(v,E)$ denotes a weighted graph, where $v=(v_1,\cdots,v_N)$ is the nonempty set of nodes/agents, $E\subseteq v\times v$ is the set of edges/arcs. $(v_i,v_j)\in E$ means there is an edge from node i to node j. Its weighted adjacency matrix $A=[a_{ij}]\in R^{N\times N}$, and $a_{ij}>0$ if $(v_j,v_i)\in E$; otherwise $a_{ij}=0$. Throughout this paper, it is assumed that $a_{ii}=0$, the topology is fixed and G is a directed graph. Define $d_i=\sum_{j=1}^{N}a_{ij}$ as the weighted in-degree of node i and $D=diag(d_1,\cdots,d_N)\in R^{N\times N}$ as in-degree matrix. The graph Laplacian matrix is $L=[l_{ij}]=D-A\in R^{N\times N}$. Let $\underline{\mathbf{1}}=[1,\cdots,1]^T\in R^{N\times 1}$; then $L\underline{\mathbf{1}}=0$. The set of neighbors of node i is denoted as $\mathcal{N}_i=\{j|(v_j,v_i)\in E\}$. If node j is a neighbor of node i, then node i can get information from node j, not necessarily vice versa for directed graph. [7]

2.2. Cooperative Tracking Problem

Consider N $(N\geq 2)$ agents with distinct dynamics. Dynamics of the kth agent is described in Brunovsky form:

$$\begin{cases} \dot{x}_{k,i}=x_{k,i+1}, \quad i=1,\cdots,n-1 \\ \dot{x}_{k,n}=f_k(\bar{x}_k)+u_k+h_k(\bar{x}_k,t) \\ y_k=x_{k,1} \end{cases} \tag{1}$$

where $k=1,\cdots,N$; $\bar{x}_k=[x_{k,1},\cdots,x_{k,n}]^T\in R^n$ and $y_k\in R$ denote the state vector and output of node k, respectively; $f_k(\bar{x}_k)\in R$ is an unknown smooth function; $u_k\in R$ is the control

DOI: 10.1002/rnc

input/protocol; and $h_k(\bar{x}_k, t) \in R$ is an external disturbance, which is unknown but bounded. In this paper, it is assumed that $\bar{x}_k$ is immeasurable and only y_k is available.

Dynamics of the leader node, labeled 0, is described as :

$$\begin{cases} \dot{x}_{0,i} = x_{0,i+1}, \ \ i = 1, \cdots, n-1 \\ \dot{x}_{0,n} = f_0(\bar{x}_0, t) \\ y_0 = x_{0,1} \end{cases} \tag{2}$$

where $\bar{x}_0 = [x_{0,1}, \cdots, x_{0,n}]^T \in R^n$ and $y_0 \in R$ denote the state vector and output of leader node 0, respectively; $f_0(\bar{x}_0, t) \in R$ is piecewise continuous in time t and locally Lipschitz in $\bar{x}_0$ for $\forall t \geq 0$ and $\bar{x}_0 \in R^n$, and it is unknown for all follower nodes.

In the following, for notational simplicity, we use $\bullet$ to denote $\bullet(\cdot)$. For example, h_k is the abbreviation of $h_k(\bar{x}_k, t)$.

Define

$$x_i = [x_{1,i}, \cdots, x_{N,i}]^T, \ \ i = 1, \cdots, n$$

$$f = [f_1, \cdots, f_N]^T, \ u = [u_1, \cdots, u_N]^T, \ h = [h_1, \cdots, h_N]^T$$

then the above agents' dynamics can be re-written as:

$$\begin{cases} \dot{x}_i = x_{i+1}, \ \ i = 1, \cdots, n-1 \\ \dot{x}_n = f + u + h \end{cases} \tag{3}$$

Define the ith order tracking error for node k as follows:

$$\delta_{k,i} = x_{k,i} - x_{0,i}, i = 1, \cdots, n \tag{4}$$

Let $\delta_i = [\delta_{1,i}, \cdots, \delta_{N,i}]^T \in R^N$, $i = 1, \cdots, n$, then

$$\delta_i = x_i - \underline{x}_{0,i} \tag{5}$$

where $\underline{x}_{0,i} = [x_{0,i}, \cdots, x_{0,i}]^T \in R^N$.

For the kth node, the neighborhood synchronization error is defined as

$$e_{k,i} = \sum\nolimits_{j \in \mathcal{N}_k} a_{kj}(x_{j,i} - x_{k,i}) + b_k(x_{0,i} - x_{k,i}) \tag{6}$$

where $i = 1, \cdots, n$, $k = 1, \cdots, N$ and $b_k \geq 0$ is the weight of edge from the leader node to node k ($k = 1, \cdots, N$), $b_k > 0$ if there is an edge from the leader node to node k, otherwise $b_k = 0$.

Let

$$e_i = [e_{1,i}, \cdots, e_{N,i}]^T, \ \underline{f}_0 = [f_0, \cdots, f_0]^T \in R^N, \ B = diag\{b_1, \cdots, b_N\} \in R^{N \times N}$$

Similar to (3), the above tracking error can be re-written as follows:

$$\begin{cases} \dot{e}_i = e_{i+1}, \ \ i = 1, \cdots, n-1 \\ \dot{e}_n = -(L + B)(f + u + h - \underline{f}_0) \end{cases} \tag{7}$$

Define the augmented graph as $\bar{G} = \{\bar{v}, \bar{E}\}$, $\bar{v} = \{v_0, v_1, \cdots, v_N\}$ and $\bar{E} \subseteq \bar{v} \times \bar{v}$.

Note that, it is assumed that only relative output information can be used for the followers' controllers design in this paper. Hence, state observers should be designed for each follower to estimate the states of it and its neighbors. Correspondingly, the practical ith order tracking error and neighborhood synchronization error can be described as follows:

$$\hat{\delta}_{k,i} = \hat{x}_{k,i} - \hat{x}_{0,i} \tag{8}$$

$$\hat{e}_{k,i} = \sum\nolimits_{j \in N_k} a_{kj}(\hat{x}_{j,i} - \hat{x}_{k,i}) + b_k(\hat{x}_{0,i} - \hat{x}_{k,i}) \tag{9}$$

where $\hat{x}_{k,i}$ is the estimate of $x_{k,i}$, $i=1,\cdots,n$, $k=1,\cdots,N$.

Let $\hat{\delta}_i=[\hat{\delta}_{1,i},\cdots,\hat{\delta}_{N,i}]^T\in R^N$, $i=1,\cdots,n$, then

$$\hat{\delta}_i=\hat{x}_i-\underline{\hat{x}}_{0,i} \tag{10}$$

where $\underline{\hat{x}}_{0,i}=[\hat{x}_{0,i},\cdots,\hat{x}_{0,i}]^T\in R^N$.

The cooperative tracking control problem considered herein is as follows:

i) Adaptive nonlinear observers are constructed for each follower in graph $\bar{G}$ to estimate as accurately as possible the states of it and its neighbors;

ii) Based on the estimated states, the corresponding distributed adaptive neural-networks-based controllers are designed for all followers in the graph such that all followers synchronize as far as possible a leader with tracking errors being semi-globally uniform ultimate bounded (SGUUB).

Since $f_k(\bar{x}_k)$ in (1) is unknown, neural networks (NNs) presented in [24] shall be used to approximate it as

$$f_k(\bar{x}_k)=W_k^{*T}S_k(\bar{Z}_k)+\varepsilon_k(\bar{Z}_k)$$

where $\varepsilon_k(\bar{Z}_k)$ denotes the optimal approximation error,

$$\bar{Z}_k=[z_{k,1},\cdots,z_{k,p_k}]^T=[\bar{x}_k^T,1]^T$$

$$S_k(\bar{Z}_k)=[s_{k,1}(\bar{Z}_k),\cdots,s_{k,N_W}(\bar{Z}_k)]^T$$

$$s_{k,i}(\bar{Z}_k)=\exp(-\frac{\sum_{j=1}^{p_k}(z_{k,j}-v_{k,i,j})^2}{(c_{k,i})^2})$$

p_k is the dimension of Z_k, $c_{k,i}>0$ is the width of the receptive field, and $v_{k,i,j}\in R$, $i=1,2,\cdots,N_W$ are the center of the Gaussian function, N_W is the number of the NNs. The ideal weight W_k^* is defined as

$$W_i^*=\arg\min_{W\in\Omega_W}[\sup_{z\in\Omega_Z}\left|W_k^TS_k(\bar{Z}_k)-f_k(\bar{x}_k)\right|]$$

where $\Omega_W=\{W_k|\ ||W_k||\le w_m$ with a design parameter $w_m>0$, Ω_Z denotes an enough large compact set.

From the universal approximation results, we know, NNs can approximate any continuous function to any accuracy on compact set Ω_Z [24].

Define the actual approximate error as follows:

$$\omega_k(\hat{\bar{Z}}_k)=f_k(\bar{x}_k)-\hat{W}_k^TS_k(\hat{\bar{Z}}_k)$$

where $\hat{\bar{Z}}_k=[\hat{\bar{x}}_k^T,1]^T$, $\hat{\bar{x}}_k=[\hat{x}_{k,1},\cdots,\hat{x}_{k,n}]^T$.

To design appropriate controllers for the followers, the following assumptions and lemmas are given.

Assumption 1: The augmented graph $\bar{G}$ contains a spanning tree, where the leader node is the root with no incoming edges from the follower nodes.

Assumption 2: There exists an unknown real constant $M_{f_0}>0$ such that $|f_0(\bar{x}_0,t)|\le M_{f_0}$.

Assumption 3: For each node k, there exists an unknown real constant $M_{k,h}>0$ such that $|h_k(\bar{x}_k,t)|\le M_{k,h}$.

Assumption 4: There exist unknown real constants $M_{k,\varepsilon}>0$ and $M_{k,\omega}>0$, $k=1,2,\cdots,N$, such that $|\varepsilon_k(\bar{Z}_k)|\le M_{k,\varepsilon}$ and $|\omega_k(\hat{\bar{Z}}_k)|\le M_{k,\omega}$ over a compact set.

Lemma 1 [30]: Suppose that Assumption 1 holds. Let

$$\kappa=[\kappa_1,\cdots,\kappa_N]^T=(L+B)^{-1}\underline{\mathbf{1}},$$

$$q=[q_1,\cdots,q_N]=(L+B)^{-1}\underline{\mathbf{1}},$$

DOI: 10.1002/rnc

$$P = diag\{p_i\} = diag\{\frac{\kappa_i}{q_i}\},$$
$$Q = P(L+B) + (L+B)^T P,$$

Then $P > 0$ and $Q > 0$.

Lemma 2 [7]: $||\delta_i|| \leq ||e_i||/\underline{\sigma}(L+B), i = 1, \cdots, n$, where $\underline{\sigma}(L+B)$ is a minimum singular value of matrix $L+B$.

Remark 1: If the full sates of an agent and the nodes in its neighborhood are available, the control objective is to design distributed state feedback controllers for all follower nodes in the graph such that tracking error δ_i is SGUUB, for all i $(i = 1, \cdots, n)$. The detailed design can be seen in [7] and [26].

3. MAIN RESULTS

3.1. *Design of Local Observers*

In order to design the observers, (1) can be re-written as:

$$\begin{cases} \dot{\bar{x}}_k = (A_o + KC)\bar{x}_k + G(f_k + u_k + h_k) \\ y_k = C\bar{x}_k \end{cases}$$

where

$$A_o = \begin{bmatrix} -k_1 & & & \\ \vdots & & I & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, \ G = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, \ K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}, \ C = [1, 0, \cdots, 0]$$

and $k_i \in R$, $i = 1, \cdots, n$, are chosen such that A_o is a strict Hurwitz matrix. Thus, given a matrix $Q_o > 0$, there exists a matrix $P_o = P_o^T > 0$ satisfying

$$P_o A_o + A^T{}_o P_o + P_o G G^T P_o = -Q_o \tag{11}$$

The local observer for each follower k $(i = 1, \cdots, N)$ is designed in the following form:

$$\begin{cases} \dot{\hat{\bar{x}}}_k = A_o \hat{\bar{x}}_k + K y_k + G(\hat{f}_k + u_k) \\ \hat{y}_k = C\hat{\bar{x}}_k \end{cases} \tag{12}$$

where $\hat{\bar{x}}_k = [\hat{x}_{k,1}, \cdots, \hat{x}_{k,n}]^T$ is the state vector of the kth observer used to estimate the state of the kth agent, $\hat{f}_k = \hat{W}_k^T S_k(\hat{\bar{Z}}_k)$ is the estimate of unknown function f_k.

Let us define the state estimation error $\tilde{\bar{x}}_k = \bar{x}_k - \hat{\bar{x}}_k$, the state estimation error dynamics can be described as follow:

$$\dot{\tilde{\bar{x}}}_k = A_o \tilde{\bar{x}}_k + G(\omega_k + h_k)$$

where $\omega_k = f_k - \hat{W}_k^T S_k(\hat{\bar{Z}}_k)$, $\hat{\bar{Z}}_k = [\hat{\bar{x}}_k^T, 1]^T$.

Define the following Lyapunov function

$$V_{k,o} = \tilde{\bar{x}}_k^T P_o \tilde{\bar{x}}_k \tag{13}$$

Differentiating $V_{k,o}$ with respect to time t, one has

$$\begin{aligned} \dot{V}_{k,o} &= \tilde{\bar{x}}_k^T [P_o A_o + A^T{}_o P_o] \tilde{\bar{x}}_k + 2\tilde{\bar{x}}_k P_o G(\omega_k + h_k) \\ &\leq \tilde{\bar{x}}_k^T (P_o A_o + A^T{}_o P_o + P_o G G^T P_o) \tilde{\bar{x}}_k + (\omega_k + h_k)^2 \\ &\leq -\tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k + M_{k,\omega h}^2 = -\tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k + \mu_{k,\omega h} \end{aligned} \tag{14}$$

where $\mu_{k,\omega h} = M_{k,\omega h}^2$ is an unknown real constant, $M_{k,\omega h} = M_{k,\omega} + M_{k,h}$ $(|\omega_k| \leq M_{k,\omega}$, $|h_k| \leq M_{k,h})$.

DOI: 10.1002/rnc

Remark 2. Clearly, if $\mu_{k,\omega h} = 0$, the above error dynamics is stable. This fact will be shown in the controller design. It is necessary to point out that, in [23], the term $\mu_{k,\omega h}$ in (14) is assumed to be known. In our work, such assumption is removed by using a suitable adaptive compensation term defined later.

3.2. *Design of Controllers and Stability Analysis*

Starting with the design of the distributed adaptive controller, it is assumed that the state of the leader can be accessed by its neighbors as in [16].

From the local observer (12), one has

$$\begin{cases} \dot{\hat{x}}_{k,i} = \hat{x}_{k,i+1} + k_i y_k, i = 1, \cdots, n-1 \\ \dot{\hat{x}}_{k,n} = \hat{f}_k + u_k + k_i y_k \\ \hat{y}_k = C\hat{\bar{x}}_k \end{cases}$$

Define $\hat{e}_i = [\hat{e}_{1,i}, \cdots, \hat{e}_{N,i}]^T$, where $\hat{e}_{k,i}$, $k = 1, \cdots, N$ are defined in (9), then one has

$$\begin{cases} \dot{\hat{e}}_i = \hat{e}_{i+1} - (L+B)y, i = 1, \cdots, n-1 \\ \dot{\hat{e}}_n = -(L+B)(\hat{f} + u - \underline{f}_0) \end{cases} \tag{15}$$

where $y = [y_1, \cdots, y_N]^T$.

Define the filtered tracking error $\hat{s}_k$ for the kth node as follows:

$$\hat{s}_k = \sum\nolimits_{i=1}^{n-1} \beta_{k,i}\hat{e}_{k,i} + \hat{e}_{k,n} \tag{16}$$

where $\beta_{k,i} = C_{n-1}^{i-1}\alpha_k^{n-i}, i = 1, \cdots, n-1, \alpha_k > 0$ denotes a design parameter,

$$C_{n-1}^{i-1} = (n-1)(n-2)\cdots(n-i+1)/((i-1)(i-2)\cdots 1)$$

Let

$$\hat{\bar{e}}_k(t) = [\hat{e}_{k,1}(t), \cdots, \hat{e}_{k,n}(t)]^T$$

Lemma 3 [29]: Let $\hat{s}_k$ be defined by (16), and then,

1) if $\hat{s}_k = 0$, then $\lim_{t\to\infty}\hat{\bar{e}}_{k,1}(t) = 0$;

2) if $|\hat{s}_k| \le \epsilon_k$, $\hat{\bar{e}}_k(0) \in \Omega_{\epsilon_k}$, then $\hat{\bar{e}}_k(t) \in \Omega_{\epsilon_k}$, $\forall t \ge 0$;

3) if $|\hat{s}_k| \le \epsilon_k$, $\hat{\bar{e}}_k(0) \notin \Omega_{\epsilon_k}$, then $\exists T_k = (n-1)/\alpha_k$, $\ni \forall t \ge T_k$, $\hat{\bar{e}}_k(t) \in \Omega_{\epsilon_k}$, where $\Omega_{\epsilon_k} = \{\hat{\bar{e}}_k(t) \big| |\hat{e}_{k,i}| \le 2^{(i-1)}\alpha_k^{i-n}\epsilon_k, i = 1, \cdots, n, k = 1, 2, \cdots N$, ϵ_k is a design parameter.

For simplification, let

$$\beta_{1,i} = \cdots = \beta_{N,i} = \lambda_i, \lambda_n = 1,\ i = 1, \cdots, n$$

then

$$\hat{s}_k = \lambda_1\hat{e}_{k,1} + \cdots + \lambda_n\hat{e}_{k,n}$$

Let

$$\hat{s} = [\hat{s}_1, \cdots, \hat{s}_N]^T$$

then it follows

$$\hat{s} = \lambda_1\hat{e}_1 + \cdots + \lambda_n\hat{e}_n$$

Further, one has

$$\begin{aligned} \dot{\hat{s}} &= \sum\nolimits_{i=1}^{n-1} \lambda_i\hat{e}_{i+1} - \sum\nolimits_{i=1}^{n-1} \lambda_i(L+B)y + \dot{\hat{e}}_n \\ &= \gamma - \sum\nolimits_{i=1}^{n} \lambda_i(L+B)y - (L+B)(\hat{f} + u - \underline{f}_0) \end{aligned} \tag{17}$$

where $\gamma = \sum_{i=1}^{n-1} \lambda_i\hat{e}_{i+1}$.

Since

$$\hat{f}_k = f_k - (f_k - \hat{f}_k) = f_k - \omega_k$$

one has

$$\hat{f} = f - (f - \hat{f}) = W^{*T}S + \varepsilon - \omega$$

where

$$W^{*T} = diag\{W_1^{*T}, \cdots, W_N^{*T}\},\ S = [S_1^T(\bar{Z}_1)\cdots, S_N^T(\bar{Z}_N)]^T$$

$$\varepsilon = [\varepsilon_1, \cdots, \varepsilon_N]^T,\ \omega = [\omega_1, \cdots, \omega_N]^T$$

Hence, (17) can be thus re-written as

$$\dot{\hat{s}} = \gamma - \sum\nolimits_{i=1}^{n} \lambda_i(L+B)y - (L+B)(W^{*T}S + \varepsilon - \omega + u - \underline{f}_0)$$

Define

$$V_s = \sum\nolimits_{k=1}^{N} V_{k,o} + \frac{\hat{s}^T P \hat{s}}{2}$$

where $P = P^T$ and $V_{k,o}$ are respectively defined as in Lemma 1 and (13), respectively.

Differentiating V_s with respect to time t, it yields

$$\begin{aligned}\dot{V}_s \leq & -\sum\nolimits_{k=1}^{N} \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k + \sum\nolimits_{k=1}^{N} \mu_{k,\omega h} + \hat{s}^T P\gamma - \\ & \hat{s}^T P \sum\nolimits_{i=1}^{n} \lambda_i(L+B)y - \hat{s}^T P(D+B)W^{*T}S - \\ & \hat{s}^T P(D+B)(\varepsilon - \omega - \underline{f}_0) - s^T P(L+B)u + \\ & \hat{s}^T PAW^{*T}S + s^T PA(\varepsilon - \omega - \underline{f}_0)\end{aligned} \tag{18}$$

Now, let us consider $\sum_{k=1}^{N} \mu_{k,\omega h}$ in (18).

First, let us choose a design real constant $\varpi > 0$.

If $||s|| \geq \sqrt{\varpi}$, then one has

$$\begin{aligned}\sum\nolimits_{k=1}^{N} \mu_{k,\omega h} &= \sum\nolimits_{k=1}^{N} ||\hat{s}||^2 \frac{\mu_{k,\omega h}}{||\hat{s}||^2} \\ &\leq \sum\nolimits_{k=1}^{N} ||\hat{s}||^2 \frac{\mu_{k,\omega h}}{\varpi} \\ &= \sum\nolimits_{k=1}^{N} \hat{s}^T \hat{s} \frac{\mu_{k,\omega h}}{\varpi} \\ &= \hat{s}^T \Delta_s \bar{M}_\mu\end{aligned} \tag{19}$$

where $k = 1, \cdots, N$,

$$\Delta_s = diag\{(\hat{s}_1), \cdots, (\hat{s}_N)\},\ \bar{M}_\mu = [M_{1,\mu}, \cdots, M_{N,\mu}]^T,\ M_{k,\mu} = \frac{\mu_{k,\omega h}}{\varpi}$$

Since $P > 0$ and $D + B > 0$ are diagonal matrices, then

$$\begin{aligned}\sum\nolimits_{k=1}^{N} \mu_{k,\omega h} &\leq \frac{\hat{s}^T P(D+B)\Delta_s \bar{M}_\mu}{\underline{\sigma}(P)\underline{\sigma}(D+B)} \\ &\leq g_2 \frac{\hat{s}^T P(D+B)\Delta_s \bar{M}_\mu}{\underline{\sigma}(D+B)}\end{aligned}$$

where

$$g_2 \geq \frac{1}{\underline{\sigma}(P)} \tag{20}$$

DOI: 10.1002/rnc

is a design parameter. Hence, (18) can be developed as follows:

$$\begin{aligned}\dot{V}_s \le & -\sum\nolimits_{k=1}^{N} \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k + g_2 \frac{\hat{s}^T P(D+B)\Delta_s \bar{M}_\mu}{\underline{\sigma}(D+B)} - \\ & \hat{s}^T P \sum\nolimits_{i=1}^{n} \lambda_i (L+B) y - \hat{s}^T P(D+B) W^{*T} S - \\ & \hat{s}^T P(D+B)(\varepsilon - \omega - \underline{f}_0) - s^T P(L+B) u + \\ & \hat{s}^T P A W^{*T} S + s^T P A(\varepsilon - \omega - \underline{f}_0) + \hat{s}^T P \gamma\end{aligned} \tag{21}$$

In the following, define the notation: $\tilde{\bullet} = \bullet - \hat{\bullet}$.

Let $\hat{W}^T = diag\{\hat{W}_1^T, \cdots, \hat{W}_N^T\}$, which denotes the estimate of $W^{*T} = diag\{W_1^{*T}, \cdots, W_N^{*T}\}$.

Define control law as follows:

$$u = \begin{cases} 0, & if\ ||\hat{s}|| < \sqrt{\varpi}; \\ (D+B)^{-1}\gamma - \sum\nolimits_{i=1}^{n} \lambda_i y - \hat{f} - \Delta_s[\hat{\bar{M}}_{\varepsilon\omega f} + \\ \quad g_2 \hat{\bar{M}}_\mu / (\underline{\sigma}(D+B))] - g_1 \hat{s}, & otherwise \end{cases} \tag{22}$$

where $\varpi > 0$ is a design parameter,

$$\hat{f} = [\hat{f}_1, \cdots, \hat{f}_N]^T,\ \hat{f}_k = \hat{W}_k^T S_k(\bar{Z}_k)$$

$\hat{f}_k$ and $\hat{W}_k$ are the estimates of f_k and W_k, respectively; $\bar{Z}_k = [\bar{x}_k^T, 1]^T$,

$$\hat{\bar{M}}_{\varepsilon\omega f} = [\hat{M}_{1,\varepsilon\omega f}, \cdots, \hat{M}_{N,\varepsilon\omega f}]^T$$

$\hat{M}_{k,\varepsilon\omega f}$ is the estimate of $M_{k,\varepsilon\omega f}$,

$$M_{k,\varepsilon\omega f} = M_{k,\varepsilon} + M_{k,\omega} + M_{f_0},\ k = 1, \cdots, N$$

$\hat{\bar{M}}_\mu$ is the estimate of $\bar{M}_\mu$, $g_1 > 0$ and g_2 are design parameters, which satisfies

$$\frac{g_1 \underline{\sigma}(Q)}{2} - (5r + \frac{\bar{\lambda}^2}{4r\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A) > 0 \tag{23}$$

where $r > 0$ is a parameter,

$$\bar{\lambda} = \max\{\lambda_1, \cdots, \lambda_n\} \text{ and } \underline{\lambda} = \min\{\lambda_1, \cdots, \lambda_n\}$$

Define the adaptive laws as follows:

$$\dot{\hat{W}} = \begin{cases} \vartheta_w,\ if\ ||\hat{W}|| < w_m\ or\ ||\hat{W}|| = w_m\ and\ \hat{W}^T\vartheta_w \le 0; \\ \vartheta_w - \dfrac{\hat{W}\hat{W}^T\vartheta_w}{||\hat{W}||^2},\ if\ ||\hat{W}|| = w_m\ and\ \hat{W}^T\vartheta_w > 0 \end{cases} \tag{24}$$

$$\dot{\hat{\bar{M}}}_{\varepsilon\omega f} = \begin{cases} \vartheta_{\varepsilon\omega f},\ if\ ||\hat{\bar{M}}_{\varepsilon\omega f}|| < w_{\varepsilon\omega f}\ or\ ||\hat{\bar{M}}_{\varepsilon\omega f}|| = w_{\varepsilon\omega f} \\ \qquad and\ \hat{\bar{M}}_{\varepsilon\omega f}\vartheta_{\varepsilon\omega f} \le 0; \\ \vartheta_{\varepsilon\omega f} - \dfrac{\hat{\bar{M}}_{\varepsilon\omega f}\hat{\bar{M}}_{\varepsilon\omega f}^T\vartheta_{\varepsilon\omega f}}{||\hat{\bar{M}}_{\varepsilon\omega f}||^2},\ if\ ||\hat{\bar{M}}_{\varepsilon\omega f}|| = w_{\varepsilon\omega f} \\ \qquad and\ \hat{\bar{M}}_{\varepsilon\omega f}\vartheta_{\varepsilon\omega f} > 0 \end{cases} \tag{25}$$

$$\dot{\hat{\bar{M}}}_\mu = \begin{cases} \vartheta_\mu,\ if\ ||\hat{\bar{M}}_\mu|| < w_\mu\ or\ ||\hat{\bar{M}}_\mu|| = w_\mu\ and\ \hat{\bar{M}}_\mu\vartheta_\mu \le 0; \\ \vartheta_\mu - \dfrac{\hat{\bar{M}}_\mu\hat{\bar{M}}_\mu^T\vartheta_\mu}{||\hat{\bar{M}}_\mu||^2},\ if\ ||\hat{\bar{M}}_\mu|| = w_\mu\ and\ \hat{\bar{M}}_\mu\vartheta_\mu > 0 \end{cases} \tag{26}$$

where

$$\vartheta_w = \eta_1 S \hat{s}^T P(D+B) - \eta_W \hat{W}$$

$$\vartheta_{\varepsilon\omega f} = \eta_2 \hat{s}^T P(D+B)\Delta_s - \eta_{Md\varepsilon} \hat{\bar{M}}_{\varepsilon\omega f}$$

$$\vartheta_\mu = \frac{\eta_3 g_2 \hat{s}^T P(D+B)\Delta_s}{\underline{\sigma}(D+B)} - \eta_\mu \hat{\bar{M}}_\mu$$

$w_m > 0$, $w_{\varepsilon\omega f} > 0$, $w_\mu > 0$, $\eta_W > 0$, $\eta_{Md\varepsilon} > 0$ and $\eta_\mu > 0$ are design parameters.

The following theorem is presented to prove the stability of the closed-loop system.

Theorem 1: Consider distributed system (1) with leader node (2) under Assumptions 1-4, local state observer (12), distributed control law (22) and adaptive laws (24)-(26), if there exist matrices K, $P_o = P_o^T > 0$ and $Q_o > 0$ satisfying (11) and design parameters g_1, g_2 and r are chosen such that (20) and (23) hold, then one has the following results:

(i) The observation errors $\tilde{x}_{k,i}$, $k = 1, 2, \cdots, N$, $i = 1, 2, \cdots, n$ are semi-globally uniform ultimate bounded and belong to an adjustable set $\Omega_{\tilde{x}}$; and

(ii) All followers in the graph synchronize to the leader node 0 with the tracking errors $\delta_{k,i}(t)$, $k = 1, 2, \cdots, N$, $i = 1, \cdots, n$ being semi-globally uniform ultimate bounded and belonging to an adjustable set Ω_δ, where

$$\Omega_{\tilde{x}} = \left\{ \tilde{x}_{k,i} \mid |\tilde{x}_{k,i}| \le \sqrt{\frac{\mu_0}{\lambda_0 \lambda_{min}(P_0)}} \right\}$$

$$\Omega_\delta = \left\{ \delta_{k,i} | \ |\delta_{k,i}| \le \frac{\sqrt{N} 2^{(i-1)} \alpha_k^{(i-n)} \sqrt{\frac{2\mu_0}{\lambda_0 \underline{\sigma}(P)}}}{\underline{\sigma}(L+B)} + \sqrt{\frac{\mu_0}{\lambda_0 \lambda_{min}(P_0)}} \right\}$$

where $\mu_0 = \frac{\mu_0'}{\lambda_0} + V(0)$, $\mu_0' = \mu_{WM} + \mu_s$,

$$\mu_s = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r}(S^T \tilde{W}\tilde{W}^T S + \bar{M}_{\varepsilon\omega f}^T \bar{M}_{\varepsilon\omega f} + \hat{\bar{M}}_{\varepsilon\omega f}^T \hat{\bar{M}}_{\varepsilon\omega f} + \frac{g_2^2 \hat{\bar{M}}_\mu^T \hat{\bar{M}}_\mu}{\underline{\sigma}^2(D+B)})$$

$$\mu_{WM} = \frac{\eta_W}{2\eta_1} tr(W^{*T} W^*) + \frac{\eta_{Md\varepsilon}}{2\eta_2} \bar{M}_{\varepsilon\omega f}^T \bar{M}_{\varepsilon\omega f} + \frac{\eta_\mu}{2\eta_3} \bar{M}_\mu^T \bar{M}_\mu$$

$$\lambda_0 = \min\{\frac{\lambda_{\min}(Q_o)}{\lambda_{\max}(P_o)}, \frac{\frac{1}{2} g_1 \underline{\sigma}(Q) - (5r + \frac{1}{4r}\frac{\bar{\lambda}^2}{\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)}{\bar{\sigma}(P)}, \frac{\eta_W}{2\eta_1}, \frac{\eta_{Md\varepsilon}}{2\eta_2}, \frac{\eta_\mu}{2\eta_3}\}$$

$$V(0) = \sum_{k=1}^N \tilde{\bar{x}}_k^T(0) P \tilde{\bar{x}}_k(0) + \frac{\hat{s}^T(0) P \hat{s}(0)}{2} + \frac{tr\{\tilde{W}^T(0)\tilde{W}(0)\}}{2\eta_1} + \frac{\tilde{\bar{M}}_{\varepsilon\omega f}^T(0)\tilde{\bar{M}}_{\varepsilon\omega f}(0)}{2\eta_2} + \frac{\tilde{\bar{M}}_\mu^T(0)\tilde{\bar{M}}_\mu(0)}{2\eta_3}$$

Proof:

Noting that, the following development will be given for two cases.

Case 1. $||\hat{s}|| \ge \sqrt{\varpi}$

Substituting the control law (22) into (21), one has

$$\begin{aligned}\dot{V}_s =& -g_1 \hat{s}^T P(L+B)\hat{s} + \hat{s}^T P(D+B)\tilde{W}^T S + \\ & \hat{s}^T P(D+B)\Delta_s(\tilde{\bar{M}}_{\varepsilon\omega f} + \frac{g_2 \tilde{\bar{M}}_\mu}{\underline{\sigma}(D+B)}) + \hat{s}^T PA\tilde{W}^T S + \hat{s}^T PA(\varepsilon + \omega - \underline{f}_0) - \\ & \hat{s}^T PA\Delta_s(\hat{\bar{M}}_{\varepsilon\omega f} + \frac{g_2 \hat{\bar{M}}_\mu}{\underline{\sigma}(D+B)}) + \hat{s}^T PA(D+B)^{-1}\gamma - \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned}$$

Since

$$\hat{s}^T PA\tilde{W}^T S \le \bar{\sigma}(P)\bar{\sigma}(A) r \hat{s}^T \hat{s} + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r} S^T \tilde{W}\tilde{W}^T S$$

$$\hat{s}^T PA(\varepsilon + \omega - \underline{f}_0) \leq \bar{\sigma}(P)\bar{\sigma}(A) r \hat{s}^T \hat{s} + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r} \bar{M}_{\varepsilon\omega f}^T \bar{M}_{\varepsilon\omega f}$$
$$- \hat{s}^T PA\Delta_s(\hat{\bar{M}}_{\varepsilon\omega f} + \frac{g_2 \hat{\bar{M}}_\mu}{\underline{\sigma}(D+B)})$$
$$\leq 2\bar{\sigma}(P)\bar{\sigma}(A) r \hat{s}^T \hat{s} + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r} (\hat{\bar{M}}_{\varepsilon\omega f}^T \hat{\bar{M}}_{\varepsilon\omega f} + \frac{g_2^2 \hat{\bar{M}}_\mu^T \hat{\bar{M}}_\mu}{\underline{\sigma}^2(D+B)})$$
$$\hat{s}^T PA(D+B)\gamma \leq \bar{\sigma}(P)\bar{\sigma}(A) r \hat{s}^T \hat{s} + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r} \gamma^T \gamma$$

one has

$$\begin{aligned}\dot{V}_s \leq & - g_1 \hat{s}^T P(L+B)\hat{s} + \hat{s}^T P(D+B)\tilde{W}^T S + \\ & \hat{s}^T P(D+B)\Delta_s(\tilde{\bar{M}}_{\varepsilon\omega f} + g_2 \tilde{\bar{M}}_\mu / \underline{\sigma}(D+B)) + \\ & 5r\bar{\sigma}(P)\bar{\sigma}(A)\hat{s}^T \hat{s} + (\bar{\sigma}(P)\bar{\sigma}(A)/(4r))(S^T \tilde{W}\tilde{W}^T S + \\ & \bar{M}_{\varepsilon\omega f}^T \bar{M}_{\varepsilon\omega f} + \hat{\bar{M}}_{\varepsilon\omega f}^T \hat{\bar{M}}_{\varepsilon\omega f} + \\ & \frac{g_2^2 \hat{\bar{M}}_\mu^T \hat{\bar{M}}_\mu}{\underline{\sigma}^2(D+B)} + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r} \gamma^T \gamma - \\ & \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned}$$

Since

$$\gamma_k^2 = \sum_{i=1}^{n-1} \lambda_i^2 e_{k,i+1}^2 \leq \frac{\bar{\lambda}^2}{\underline{\lambda}^2} \hat{s}_k^2$$

we have

$$\gamma^T \gamma = \sum_{k=1}^N \gamma_k^2 \leq \sum_{k=1}^N \frac{\bar{\lambda}^2}{\underline{\lambda}^2} \hat{s}_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2} \sum_{k=1}^N \hat{s}_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2} \hat{s}^T \hat{s}$$

and

$$\begin{aligned}\dot{V}_s \leq & - g_1 \hat{s}^T P(L+B)\hat{s} + \hat{s}^T P(D+B)\tilde{W}^T S + \\ & \hat{s}^T P(D+B)\Delta_s(\tilde{\bar{M}}_{\varepsilon\omega f} + \frac{g_2 \tilde{\bar{M}}_\mu}{\underline{\sigma}(D+B)}) + \\ & (5r + \frac{\bar{\lambda}^2}{4r\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)\hat{s}^T \hat{s} + \mu_s - \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned} \tag{27}$$

where

$$\mu_s = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{4r}(S^T \tilde{W}\tilde{W}^T S + \bar{M}_{\varepsilon\omega f}^T \bar{M}_{\varepsilon\omega f} + \hat{\bar{M}}_{\varepsilon\omega f}^T \hat{\bar{M}}_{\varepsilon\omega f} + \frac{g_2^2 \hat{\bar{M}}_\mu^T \hat{\bar{M}}_\mu}{\underline{\sigma}^2(D+B)})$$

Define

$$V = V_s + V_W$$

where

$$V_W = \frac{tr\{\tilde{W}^T \tilde{W}\}}{2\eta_1} + \frac{\tilde{\bar{M}}_{\varepsilon\omega f}^T \tilde{\bar{M}}_{\varepsilon\omega f}}{2\eta_2} + \frac{\tilde{\bar{M}}_\mu^T \tilde{\bar{M}}_\mu}{2\eta_3}$$

and $\eta_l > 0, l = 1, 2, 3$ are design parameters.

Differentiating V with respect to time t and considering (27), we have

$$\begin{aligned}\dot{V} \leq & - g_1 \hat{s}^T P(L+B)\hat{s} + (5r + \frac{1}{4r}\frac{\bar{\lambda}^2}{\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)\hat{s}^T \hat{s} + \\ & \mu_s + \hat{s}^T P(D+B)\tilde{W}^T S + \hat{s}^T P(D+B)\Delta_s(\tilde{\bar{M}}_{\varepsilon\omega f} + \\ & \frac{g_2 \tilde{\bar{M}}_\mu}{\underline{\sigma}(D+B)}) - \frac{1}{\eta_1} tr\{\tilde{W}^T \dot{\hat{W}}\} - \frac{1}{\eta_2}\tilde{\bar{M}}_{\varepsilon\omega f}^T \dot{\hat{\bar{M}}}_{\varepsilon\omega f} - \\ & \frac{1}{\eta_3}\tilde{\bar{M}}_\mu^T \dot{\hat{\bar{M}}}_\mu - \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned} \tag{28}$$

DOI: 10.1002/rnc

Substituting adaptive laws (24)-(26) into (28), we have

$$\begin{aligned}\dot{V} \leq & -g_1\hat{s}^T P(L+B)\hat{s} + (5r + \frac{1}{4r}\frac{\bar{\lambda}^2}{\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)\hat{s}^T\hat{s} + \\ & \frac{\eta_W}{\eta_1}tr(\tilde{W}^T\hat{W}) + \frac{\eta_{Md\varepsilon}}{\eta_2}\tilde{\bar{M}}_{\varepsilon\omega f}^T\hat{\bar{M}}_{\varepsilon\omega f} + \\ & \frac{\eta_\mu}{\eta_3}\tilde{\bar{M}}_\mu^T\hat{\bar{M}}_\mu + \mu_s - \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned}$$

Since

$$\begin{aligned}tr(\tilde{W}^T\hat{W}) &\leq -\frac{tr(\tilde{W}^T\tilde{W})}{2} + \frac{tr(W^{*T}W^*)}{2} \\ \tilde{\bar{M}}_{\varepsilon\omega f}^T\hat{\bar{M}}_{\varepsilon\omega f} &\leq -\frac{\tilde{\bar{M}}_{\varepsilon\omega f}^T\tilde{\bar{M}}_{\varepsilon\omega f}}{2} + \frac{\bar{M}_{\varepsilon\omega f}^T\bar{M}_{\varepsilon\omega f}}{2} \\ \tilde{\bar{M}}_\mu^T\hat{\bar{M}}_\mu &\leq -\frac{\tilde{\bar{M}}_\mu^T\tilde{\bar{M}}_\mu}{2} + \frac{\bar{M}_\mu^T\bar{M}_\mu}{2}\end{aligned}$$

then we have

$$\begin{aligned}\dot{V} \leq & -g_1\hat{s}^T P(L+B)\hat{s} + (5r + \frac{1}{4r}\frac{\bar{\lambda}^2}{\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)\hat{s}^T\hat{s} - \\ & \frac{\eta_W}{2\eta_1}tr(\tilde{W}^T\tilde{W}) - \frac{\eta_{Md\varepsilon}}{2\eta_2}\tilde{\bar{M}}_{\varepsilon\omega f}^T\tilde{\bar{M}}_{\varepsilon\omega f} - \frac{\eta_\mu}{2\eta_3}\tilde{\bar{M}}_\mu^T\tilde{\bar{M}}_\mu + \\ & \mu_s + \mu_{WM} - \sum_{k=1}^N \tilde{\bar{x}}_k^T Q_o \tilde{\bar{x}}_k\end{aligned} \tag{29}$$

where

$$\mu_{WM} = \frac{\eta_W}{2\eta_1}tr(W^{*T}W^*) + \frac{\eta_{Md\varepsilon}}{2\eta_2}\bar{M}_{\varepsilon\omega f}^T\bar{M}_{\varepsilon\omega f} + \frac{\eta_\mu}{2\eta_3}\bar{M}_\mu^T\bar{M}_\mu$$

Let $\mu_0' = \mu_{WM} + \mu_s$ and

$$\lambda_0 = \min\{\frac{\lambda_{\min}(Q_o)}{\lambda_{\max}(P_o)}, \frac{\frac{1}{2}g_1\underline{\sigma}(Q) - (5r + \frac{1}{4r}\frac{\bar{\lambda}^2}{\underline{\lambda}^2})\bar{\sigma}(P)\bar{\sigma}(A)}{\bar{\sigma}(P)}, \frac{\eta_W}{2\eta_1}, \frac{\eta_{Md\varepsilon}}{2\eta_2}, \frac{\eta_\mu}{2\eta_3}\}$$

then one has

$$\dot{V} \leq -\lambda_0 V + \mu_0'$$

Further, we have

$$0 \leq V(t) \leq \frac{\mu_0'}{\lambda_0} + [V(0) - \frac{\mu_0'}{\lambda_0}]e^{-\lambda_0 t} \leq \mu_0 \tag{30}$$

where $\mu_0 = \frac{\mu_0'}{\lambda_0} + V(0)$.

Because

$$\lambda_{\min}(P_o)\tilde{\bar{x}}_k^T\tilde{\bar{x}}_k \leq V_{k,o} = \tilde{\bar{x}}_k^T P_o \tilde{\bar{x}}_k \leq V$$

one has

$$||\tilde{\bar{x}}_k|| \leq \sqrt{\frac{\mu_0}{\lambda_0\lambda_{min}(P_0)}}$$

Since $\tilde{\bar{x}}_k = [\tilde{x}_{1,i}, \cdots, \tilde{x}_{N,i}]^T$, we have further

$$|\tilde{x}_{k,i}| \leq \sqrt{\mu_0/(\lambda_0\lambda_{min}(P_0))} \tag{31}$$

This means that the state estimation errors $\tilde{x}_{k,i}$, $k = 1, \cdots, N$, $i = 1, \cdots, n$ are SGUUB, belong to set $\Omega_{\tilde{x}}$.

Since $\underline{\sigma}(P)||\hat{s}||^2 \leq 2V_s \leq 2V$, then $||\hat{s}|| \leq \sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}$. Further, one has

$$|\hat{s}_k| \leq \sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}$$

DOI: 10.1002/rnc

Similarly, we have,

$$||\tilde{W}_k|| \leq \sqrt{\frac{2\eta_1\mu_0}{\lambda_0}}, \ \ |\tilde{M}_{k,\varepsilon\omega f}| \leq \sqrt{\frac{2\eta_2\mu_0}{\lambda_0}}, \ \ |\tilde{M}_{k,\mu}| \leq \sqrt{\frac{2\eta_3\mu_0}{\lambda_0}}$$

This means that all signals in the closed-loop system are SGUUB.

In the following, we will show that the tracking error $\delta_{k,i}$, $k = 1, \cdots, N$, $i = 1, \cdots, n$ are SGUUB.

Since $|\hat{s}_k| \leq \sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}$, from Lemma 3, we have

$$|\hat{e}_{k,i}| \leq 2^{(i-1)}\alpha_k^{(i-n)}\sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}$$

Since $\hat{e}_i = [\hat{e}_{1,i}, \cdots, \hat{e}_{N,i}]^T$, one has

$$||\hat{e}_i|| \leq \sqrt{N}2^{(i-1)}\alpha_k^{(i-n)}\sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}$$

Further, from Lemma 2, we have the following result:

$$||\hat{\delta}_i|| \leq \frac{\sqrt{N}2^{(i-1)}\alpha_k^{(i-n)}\sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}}{\underline{\sigma}(L+B)} \tag{32}$$

Since $\delta_i = x_i - \underline{x}_{0,i}$ and $\hat{\delta}_i = \hat{x}_i - \underline{x}_{0,i}$, one has $\delta_i = \hat{\delta}_i + x_i - \hat{x}_i = \hat{\delta}_i + \tilde{\bar{x}}_i$ and $||\delta_i|| \leq ||\hat{\delta}_i|| + ||\tilde{\bar{x}}_i||$. Further, one has

$$||\delta_i|| \leq \frac{\sqrt{N}2^{(i-1)}\alpha_k^{(i-n)}\sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}}{\underline{\sigma}(L+B)} + \sqrt{\frac{\mu_0}{\lambda_0\lambda_{min}(P_0)}}$$

Since $\delta_i = [\delta_{1,i}, \cdots, \delta_{N,i}]^T$, then

$$|\delta_{k,i}| \leq \frac{\sqrt{N}2^{(i-1)}\alpha_k^{(i-n)}\sqrt{\frac{2\mu_0}{\lambda_0\underline{\sigma}(P)}}}{\underline{\sigma}(L+B)} + \sqrt{\frac{\mu_0}{\lambda_0\lambda_{min}(P_0)}} \tag{33}$$

which means that the practical tracking error $\delta_{k,i}$ is SGUUB, belong to set Ω_δ.

Case 2: $||\hat{s}|| < \sqrt{\varpi}$

Since $\hat{s} = [\hat{s}_1, \cdots, \hat{s}_N]^T$, it follows $||\hat{s}|| < \sqrt{\varpi}$ that

$$||\hat{s}_k|| < \sqrt{\varpi}, \ \ \text{for} \ \ k = 1, 2\cdots, N$$

which implies that $\hat{s}_k$ is SGUUB. Similar to the analysis in Case 1, we can obtain from Lemma 3 that, $\hat{e}_{k,i}$ $(i = 1, 2, \cdots, n)$ is also SGUUB. Since $\hat{e}_i = [\hat{e}_{1,i}, \cdots, \hat{e}_{N,i}]^T$, we further obtain that $||\hat{e}_i||$ is SGUUB. Furthermore, from Lemma 2, we finally obtain that $\hat{\delta}_k$ is SGUUB.

From the above analysis, we know, if $||\hat{s}|| < \sqrt{\varpi}$ and $|\hat{s}_k| < \sqrt{\varpi}$, then δ_k is bounded, which means that the control objective has been achieved. This implies that no control action should be taken for less power consumption.

From the analysis in Cases 1 and 2, it follows that, under distributed control laws (22) and adaptive laws (24)-(26), both the observation errors and the tracking errors are SGUUB, which means the consensus control objective defined in Section 2 has been obtained. This proof is completed.

Remark 3: Similar to [24], it is assumed that W_k^* is bounded. From the adaptive law defined by (24), $\hat{W}_k$ is bounded. Since W_k^* and $\hat{W}_k$ are bounded, and $S_k(\bar{Z}_k) \leq 1$, $\tilde{W}^TS$ is also bounded. From Assumptions 2-4, i.e., ε_k, ω_k and f_0 are bounded, $\bar{M}_{\varepsilon\omega f}$ and M_μ thus are bounded. In addition, the adaptive law defined by (25) ensures that $\hat{\bar{M}}_{\varepsilon\omega f}$ is also bounded. Furthermore, $\tilde{\bar{M}}_{\varepsilon\omega f}$ is bounded, too. Because $\tilde{W}^T S$, $\bar{M}_{\varepsilon\omega f}$ and $\hat{\bar{M}}^T_{\varepsilon\omega f}$ are bounded, if $r > 0$ is chosen to be sufficiently large, then μ_s becomes sufficiently small. Similarly, since W^*, $\bar{M}_{\varepsilon\omega f}$ and $\bar{M}_\mu$ are bounded. Hence, if η_W, $\eta_{Md\varepsilon}$, η_μ, η_1, η_2 and η_3 are chosen appropriately, then μ_{WM} can become small enough. Hence, μ_0 $(= \mu_s + \mu_{WM})$ can become sufficiently small by choosing appropriately the above parameters, which means all signals in the closed-loop system can converge to a sufficiently small neighborhood of the origin, and implies that the control scheme presented in this paper can guarantee that the underlying system has a desired control performance.

Remark 4: Recall (19). Clearly, singularity takes place at the point $||\hat{s}|| = 0$, where the control objective is supposed to be achieved. However, the main task in this paper is to design a distributed controller for each follower such that it can asymptotically synchronize to a leader with synchronizing errors being SGUUB. In addition, $\hat{s} = 0$ is difficult to obtain due to the external disturbances. Hence, from a practical point of view, when $||\hat{s}|| \leq \sqrt{\varpi}$, which implies $|\delta_{k,i}|$ is less than another constant shown in (33), the control objective is supposed to be achieved. Therefore, it is more practical that, once the system reaches its origin, no control action needs to be taken to reduce power consumption.

4. SIMULATION RESULTS

Consider a 5-node digraph G and a leader node described in Figure 1, where the numbers on the edge denote the weights between corresponding two nodes. The dynamics of the leader node is described as follows:

$$\begin{cases} \dot{x}_{0,1}(t) = x_{0,2}(t) \\ \dot{x}_{0,2}(t) = x_{0,3}(t) \\ \dot{x}_{0,3}(t) = -x_{0,2} - 2x_{0,3} + 3\sin(2t) + 6\cos(2t) - \\ \ (x_{0,1} + x_{0,2} - 1)^2 (x_{0,1} + 4x_{0,2} + 3x_{0,3} - 1)/3 \end{cases}$$

The follower nodes are described by third-order nonlinear systems in the form of (1) with

$$\begin{aligned} &\dot{x}_{1,3}(t) = x_{1,2}\sin(x_{1,1}) + \cos(x_{1,3})^2 + u_1 + h_1 \\ &\dot{x}_{2,3}(t) = -x_{2,1}x_{2,2} + 0.01x_{2,1} - 0.01(x_{2,1})^2 + u_2 + h_2 \\ &\dot{x}_{3,3}(t) = x_{3,2} + \sin(x_{3,3}) + u_3 + h_3 \\ &\dot{x}_{4,3}(t) = -3(x_{4,1} + x_{4,2} - 1)^2 (x_{4,1} + x_{4,2} + x_{4,3} - 1) - \\ &\ x_{4,2} - x_{4,3} + 0.5\sin(2t) + \cos(2t) + u_4 + h_4 \\ &\dot{x}_{5,3}(t) = \cos(x_{5,1}) + u_5 + d_5 \end{aligned}$$

where the disturbance $h_k (k = 1, \cdots, N)$ is random and bounded by $|h_k| \leq 1$. The initial states are chosen as follows: $x_0 = [0,0,0]^T$, $x_1 = [3,-3,3]^T$, $x_2 = [2,-2,2]^T$, $x_3 = [1,-1,1]^T$, $x_4 = [-2,2,-2]^T$, $x_5 = [-3,3,-3]^T$, $\hat{x}_1 = [3,2.5,-2]^T$, $\hat{x}_2 = [2,2,-1.5]^T$, $\hat{x}_3 = [1,1.5,-1]^T$, $\hat{x}_4 = [-2,1,-0.5]^T$, $\hat{x}_5 = [-3,0.5,1]^T$. The weight $\hat{W}_k \in R^{10}$, $k = 1, \cdots, 5$ are taken randomly in interval $(0,1]$. The sample time 0.08s. From Figure 1, we know

$$A = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}, \ D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 2 & 0 & -2 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 \\ -3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ L + B = \begin{bmatrix} 7 & 0 & -2 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 \\ -3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

DOI: 10.1002/rnc

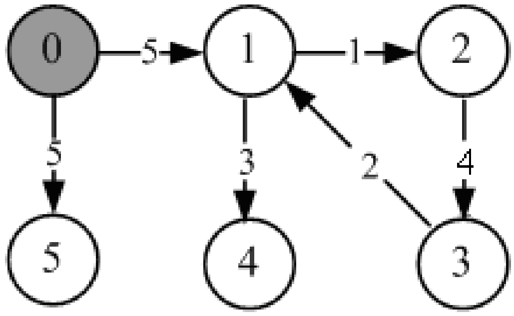

Figure 1. Communication topology $\overline{G}$

Then, we can obtain $q,\ P$ and Q in Lemma 2.1,

$$q = \begin{bmatrix} 0.7000 \\ 1.7000 \\ 1.9500 \\ 1.0333 \\ 0.2000 \end{bmatrix},\ P = \begin{bmatrix} 1.4286 & 0 & 0 & 0 & 0 \\ 0 & 0.5882 & 0 & 0 & 0 \\ 0 & 0 & 0.5128 & 0 & 0 \\ 0 & 0 & 0 & 0.9681 & 0 \\ 0 & 0 & 0 & 0 & 5.0000 \end{bmatrix}$$

$$Q = \begin{bmatrix} 20.0000 & -0.5882 & -2.8571 & -2.9042 & 0 \\ -0.5882 & 1.1765 & -2.0513 & 0 & 0 \\ -2.8571 & -2.0513 & 4.1026 & 0 & 0 \\ -2.9042 & 0 & 0 & 5.8083 & 0 \\ 0 & 0 & 0 & 0 & 50.0000 \end{bmatrix}$$

Further, we have the singular values of $A,\ D+B,\ P$ and Q: the singular values of A: 4.0000, 3.1623, 2.0000, 0, 0; the singular values of $D+B$: 7, 5, 4, 3, 1; the singular values of P: 5.0000, 1.4286, 0.9681, 0.5882, 0.5128; and the singular values of Q: 50.0000, 21.0388, 5.5209, 4.5977, 0.0701. Hence, $\bar{\sigma}(P) = 5.0000$, $\underline{\sigma}(P) = 0.5128$, $\bar{\sigma}(Q) = 50.000$, $\underline{\sigma}(Q) = 0.0701$, $\bar{\sigma}(A) = 4.0000$, $\underline{\sigma}(D+B) = 1.0000$.

In this simulation study, we choose the design parameters as follows: $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 1$, $\eta_1 = \eta_2 = \eta_3 = 100$, $\eta_W = \eta_{Md\varepsilon} = \eta_\mu = 0.01$, $\mu_s = \mu_{WM} = 0.01$. In addition, we set $r = 100$, $g_1 = 4000$ and $g_2 = 3$, which guarantee (3.10) and (3.12) holds.

The simulation results are presented in Figures 2 and 3. From Figure 2, it can be seen that the states of the observers can asymptotically converge to the actual states with bounded observation errors. As shown in Figure 3, it is shown that, under the distributed control laws defined by (3.24), tracking errors asymptotically converge to a small neighborhood of the origin and a desired tracking control performance is obtained. The simulation results demonstrate the effectiveness of the adaptive consensus control scheme proposed in this paper.

5. CONCLUSION

This paper investigates the leader-following consensus problem of uncertain high-order nonlinear MASs on directed graph. Nonlinear adaptive observers are proposed for each follower in the graph to estimate the relative states. Using the estimated states, observer-based distributed adaptive controllers are designed to guarantee that all followers asymptotically synchronize a leader with tracking errors being semi-globally uniform ultimate bounded.

ACKNOWLEDGEMENT

This work was partially supported by the National Natural Science Foundation of China (61473250,

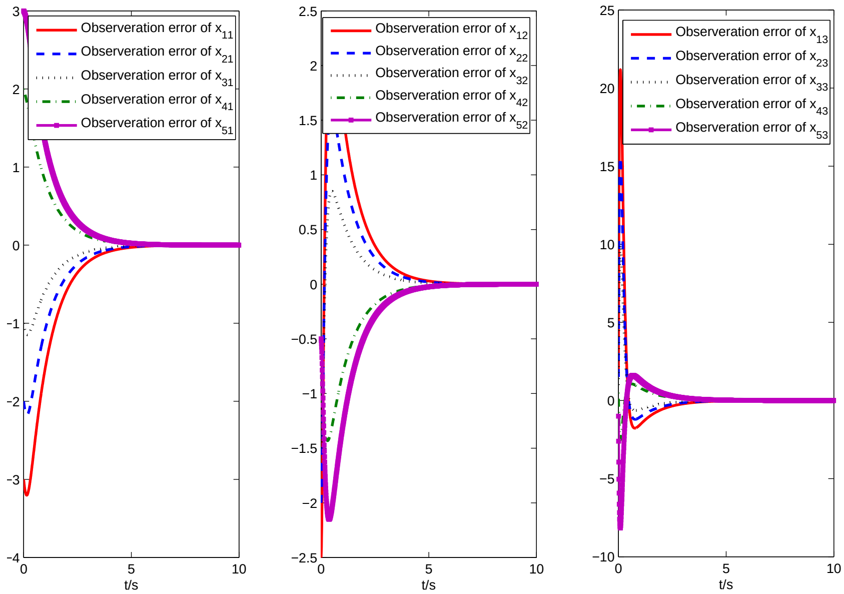

Figure 2. The time profiles of the observation errors

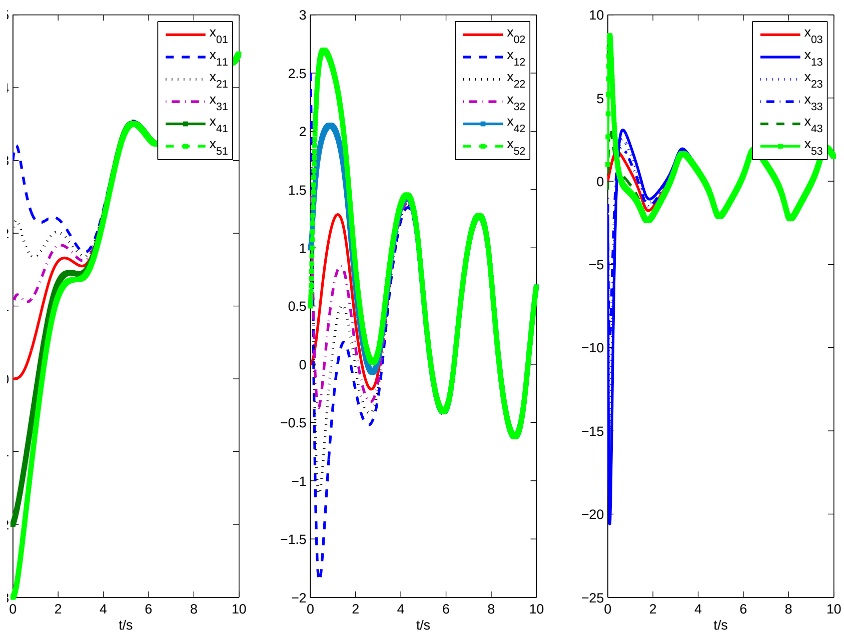

Figure 3. The time profiles of the states of the followers and the leader

61573112, 61503156), the Australian Research Council (DP140102180, LP140100471), the 111 Project (B12018), and the Natural Science Foundation of Jiangsu Higher Education Institution (14KJB120013).

REFERENCES

1. Defoort M, Floquet T, Kokosy A, and Perruquetti W, 2008. *Sliding-mode formation control for cooperative autonomous mobile robots*, IEEE Transactions on Industrial Electronics, 55, pp. 3944-3953.
2. Cruz D, McClintock J, Perteet B, Orqueda O, Cao Y, and Fierro R, 2007. *Decentralized cooperative control-a multivehicle platform for research in networked embedded systems*, IEEE Control Systems Magazine, 27, pp. 58-78.
3. Wang X, Yadav V, and Balakrishnan S, *Cooperative UAV formation flying with obstacle/collision avoidance*, 2007. IEEE Transactions on Control Systems Technology, 15, pp. 672-679.
4. Chen J, Cao X, Cheng P, Xiao Y, and Sun Y, 2010. *Distributed collaborative control for industrial automation with wireless sensor and actuator networks*, IEEE Transactions on Industrial Electronics, 57, pp. 4219-4230.
5. Murray R, 2007. *Recent research in cooperative control of multivehicle systems*, ASME Journal of Dynamic Systems, Measurement, and Control, 129, pp. 571-583.
6. Ren W and Cao C, 2011. *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. London: Springer-Verlag.
7. Zhang H and Lewis F L, 2012. *Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics*, Automatica, 48, pp. 1432-1439.
8. Ren W, Beard R, and Atkins E, 2007. *Information consensus in multivehicle cooperative control*, IEEE Control Systems Magazine, 27, pp. 71-82.
9. Cheng L, Hou Z G, Tan M, Lin Y, and Zhang W J, 2011. *Solving a modified consensus problem of linear multi-agent systems*, Automatica, 47, pp. 2218-2223.
10. Zhu W and Cheng D, 2010. *Leader-following consensus of second-order agents with multiple time-varying delays*, Automatica, 46, pp. 1994-1999.
11. Wang W and Slotine J, 2006. *A theoretical study of different leader roles in networks*, IEEE Transactions on Automatic Control, 51, pp. 1156-1161.
12. Ren W, Moore K, and Chen Y, 2007. *High-order and model reference consensus algorithms in cooperative control of multivehicle systems*, ASME Journal of Dynamic Systems, Measurement, and Control, 129, pp. 678-688.
13. Hong Y, Chen G, and Bushnell L, 2008. *Distributed observers design for leader-following control of multi-agent networks*, Automatica, 44, pp. 846-850.
14. Li Z, Duan Z, and Lewis F L, 2010. *Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties*, Automatica, http://dx.doi.org/10.1016/j.automatica.2013.12.008, 2013.
15. Cheng L, Hou Z G, Tan M, Y. Lin, and W. J. Zhang, 2010. *Neural-network-based adaptive leader-following control for multi-agent systems with uncertainties*, IEEE Transactions on Neural Networks, 21, pp. 1351-1358.
16. Zhang H, Lewis F L, and Das A, 2011. *Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback*, IEEE Transactions on Automatic Control, 56, pp. 1948-1952.
17. Zhang H, Lewis F L, and Qu Z, 2012. *Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs*, IEEE Transactions on Industrial Electronics, 59, pp. 3026-3041.
18. Yu H and Xia X, 2012. *Adaptive consensus of multi-agents in networks with jointly connected topologies*, Automatica, 48, pp.1783-1790.
19. Semsar-Kazerooni E and Khorasani K, 2008. *Optimal consensus algorithms for cooperative team of agents subject to partial information*, Automatica, 44, pp. 2766-2777.
20. Du H, Li S and Shi P, 2012. *Robust consensus algorithm for second-order multi-agent systems with external disturbances*, International Journal of Control, Correctly, 85, pp.1913-1928.
21. Zhou Q, Shi P, Lu J, and Xu S, 2011. *Adaptive output feedback fuzzy tracking control for a class of nonlinear systems*, IEEE Transactions on Fuzzy Systems, 19, pp.972-982.
22. Zhang J, Shi P, and Xia Y, 2010. *Robust adaptive sliding mode control for fuzzy systems with mismatched uncertainties*, IEEE Transactions on Fuzzy Systems, 18, pp.700-711.
23. Tong S C and Li Y, 2009. *Observer-based fuzzy adaptive control for strict-feedback nonlinear systems*, Fuzzy Sets and Systems, 160, pp.1749-1764.
24. Ge S S, Hong F, and Lee T H, 2005. *Robust adaptive control of nonlinear systems with unknown time delays*, Automatica, 41, pp. 1181-1190.
25. Meng Z, Lin Z, and Ren W, 2013. *Robust cooperative tracking for multi non-identical second-order nonlinear systems*, Automatica, 49, pp. 2363-2372.
26. Shen Q K, Jiang B, Shi P and Zhao J, 2014. *Cooperative adaptive fuzzy tracking control for networked unknown nonlinear multi-agent systems with time-varying actuator faults*, IEEE Transactions on Fuzzy Systems, 22, pp. 494-504.
27. Shi G and Johansson K H, 2013. *Robust consensus for continuous-time multiagent dynamics*, SIAM Journal on Control and Optimization, 51, pp. 3673-3691.
28. Qiao W and Sipahi R, 2013. *A linear time-invariant consensusdynamics with homogeneous delays: analytical study and synthesis of rightmost eigenvalues*, SIAM Journal on Control and Optimization, 51, pp. 3971-3992.
29. Slotine J E and Li W, 1991. *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentine-Hall.
30. Peng Z, Wang D, Zhang H and Sun G, 2014. *Distributed neural network control for adaptive synchronization of uncertain dynamical multiagent systems*, IEEE Transactions on Neural Networks and Learning Systems, 25, pp. 1508-1519.
31. Zhang H, Li Z, Qu Z and Lewis F L, 2015. *On constructing Lyapunov functions for multi-agent systems*, Automatica, 58, pp.39-42.

DOI: 10.1002/rnc