OFDM Power Loading Using Limited Feedback

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is a practical broadband signaling technique for use in multipath fading channels. Over the past ten years, research has shown that power loading, where the power allocations on the OFDM frequency tones are jointly optimized, can improve error rate or capacity performance. The implementation of power loading, however, is dependent on the presence of complete forward link channel knowledge at the transmitter. In systems using frequency division duplexing (FDD), this assumption is unrealistic. In this paper, we propose power loading for OFDM symbols using a limited number of feedback bits sent from the receiver to the transmitter. The power loading vector is designed at the receiver, which is assumed to have perfect knowledge of the forward link channel, and conveyed back to the transmitter over a limited rate feedback channel. To allow for the vector to be represented by a small number of bits, the power loading vector is restricted to lie in a finite set, or codebook, of power loading vectors. This codebook is designed offline and known *a priori* to both the transmitter and receiver. We present two power allocation selection algorithms that optimize the probability of symbol error and capacity, respectively. Simulation results show that the proposed limited feedback techniques provide performance close to full channel knowledge power loading.**

***Index Terms*—Limited feedback, orthogonal frequency division multiplexing, power loading.**

I. Introduction

RESEARCH has consistently shown that multicarrier modulation (MCM) is a practical and powerful technique for combatting intersymbol interference in multipath communication channels. A popular implementation of MCM, called orthogonal frequency division multiplexing (OFDM), has been or is expected to be included in current and next generation wireless standards such as IEEE 802.16 [1], IEEE 802.11a [2], and IEEE 802.20. In OFDM, a broadband signal is constructed from narrowband signals or tones. The broadband signal is constructed in the frequency domain, converted to a time domain signal using an orthogonal transformation, and then appended with a cyclic prefix before transmission.

One simple method for improving system performance is by optimally allocating the transmit power among the different frequency tones, commonly called power loading. Power loading can be used to optimize error rate performance [3]–[7], capacity [8]–[12], and transmit power [13] when knowledge of the forward link channel is available at the transmitter. Power loading particularly enhances performance when it is combined with rate adaptation to optimize some performance criterion [4], [6]–[13].

While it is sometimes possible to use the reverse link channel estimate to design power loading for the forward link in time division duplexing (TDD) systems (see, for example, the imperfect channel estimate work in [14]–[17]), problems arise when power loading is implemented with frequency division duplexing (FDD). FDD systems lack forward and reverse link channel reciprocity because the forward and reverse links fade independently. There are three methods for implementing power loading that overcome this channel reciprocity problem. Linear precoding techniques (see [18]–[20]) provide excellent performance without any form of channel knowledge at the transmitter. The second option is to use statistical or partial channel information to design the power loading allocation [21]–[24]. Alternatively, FDD systems can perform power loading using channel information conveyed from the receiver to the transmitter over a low rate feedback channel. Signaling techniques that use a low rate feedback channel, often called limited feedback signaling, have been previously studied in the multiple antenna literature for use with channel quantization feedback [25], [26], limited feedback beamforming [27]–[30], limited feedback precoding [31]–[34] and limited feedback covariance optimization [35]–[37].

In this paper, we propose power loading in OFDM wireless systems using limited feedback sent from the receiver to the transmitter. We consider limited feedback schemes that optimize an error rate or capacity criterion. Our limited feedback power loading framework works by using a codebook of power loading vectors designed offline and known to both the transmitter and receiver. The receiver, which has full forward link channel knowledge, chooses one of the codebook power loading vectors and conveys the vector to the transmitter over a lowrate feedback channel. We present codebook designs for both error rate criterion and capacity criterion loading. For each criterion, we characterize the optimal full channel knowledge loading algorithm and then use the vector quantization Lloyd algorithm (see, for example, [38]) to generate a codebook. Other recent work in limited feedback for multicarrier systems includes [39]–[41].

We also present a capacity optimizing limited feedback strategy called multimode power loading which operates similarly to the algorithm proposed by Leke and Cioffi in [12]. In multimode power loading, the codebook is designed by choosing all possible on-off tone configurations. For an M tone OFDM system, the multi-mode power loading codebook would consist of $2^M - 1$ unit vectors constructed by normalizing all possible nonzero vector combinations of 1's and 0's. This technique was perviously considered for limited feedback linear

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precoding and adaptive modulation in multiple antenna wireless systems [42], [43].

The organization of this paper is as follows. Section II gives an overview of the general system model for limited feedback power loaded OFDM systems. Power loading for use with an error rate criterion is presented in Section III. Section IV proposes methods for power loading under a capacity performance criterion. Section V presents Monte Carlo simulation results. Section VI presents some conclusions and discusses future research areas.

II. System Overview

Consider an M tone OFDM system described by Fig. 1. The frequency domain input-output relationship for the ith tone, assuming perfect pulse shaping, equalization, sampling, and synchronization, is given by

$$y_i = h_i x_i + n_i \tag{1}$$

where x_i is the tone's transmitted signal, h_i is the complex channel response, n_i is additive complex Gaussian noise, and y_i is the postprocessing received signal. Collecting all the OFDM tone input-output expressions together in vector notation gives

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

where[1] $\mathbf{y} = [y_1\ y_2\ \cdots\ y_M]^T, \mathbf{x} = [x_1\ x_2\ \cdots\ x_M]^T, \mathbf{n} = [n_1\ n_2\ \cdots\ n_M]^T$, and $\mathbf{H} = \mathrm{diag}(h_1, h_2, \ldots, h_M)$. We will assume that the noise is distributed according to $\mathcal{CN}(0, N_0)$ for all i and that $E[n_i^* n_j] = 0$ when $i \neq j$. We will use $\mathbf{h}$ to refer to the vector $[h_1\ h_2\ \cdots\ h_M]^T$; thus $\mathbf{H} = \mathrm{diag}(\mathbf{h})$.

In power loading systems, x_i can be decomposed as

$$x_i = a_i s_i$$

where a_i is a power loading weight on the ith tone and s_i is the transmitted symbol. The key is to design the vector $\mathbf{a} = [a_1\ a_2\ \cdots\ a_M]^T$ as a function of the channel responses to optimize some performance criterion. Because of the parallel channel relationship created by the orthogonal transformations in OFDM, we will assume that $\mathbf{a} \in \mathbb{R}_+^M$. This follows from the fact that, given our assumptions, the performance is independent of a constant phase-offset on each subcarrier. For power constraint reasons, we will assume that $E[|s_i|^2] = \mathcal{E}_s$.

The total instantaneous transmit power is given by

$$P = \mathbf{x}^*\mathbf{x} = \sum_{i=1}^{M} |a_i s_i|^2.$$

We will fix a total average power constraint of

$$E\left[\sum_{i=1}^{M} |a_i s_i|^2\right] = \sum_{i=1}^{M} |a_i|^2 E[|s_i|^2] = \mathcal{E}_s. \tag{3}$$

[1]We use ^{T} to denote transposition, * to denote conjugate-transposition, $\mathrm{diag}(h_1, h_2, \ldots, h_M)$ to denote a function that returns a matrix with $h_1, h_2, \ldots, h_M$ on the diagonal, $\mathrm{diag}(\mathbf{h})$ to denote a function that returns a matrix with the entries in the vector $\mathbf{h}$ on the diagonal, $E[\cdot]$ to denote expectation, $\mathbb{R}$ to denote the set of real numbers, $\mathbb{R}_+$ to denote the set of nonnegative real numbers, $\|\cdot\|_2$ to denote the vector two-norm, $\mathrm{argmax}(\mathrm{argmin})$ to denote a function that returns a global maximizer (minimizer), and $\mathrm{card}(\cdot)$ to denote the cardinality of a set.

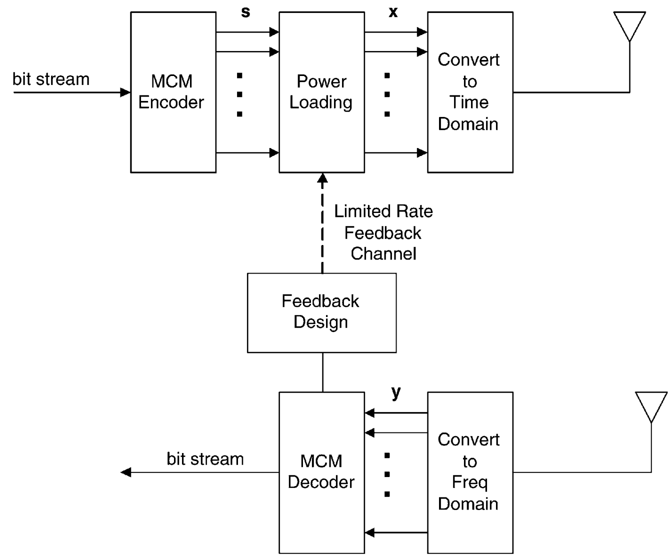

Fig. 1. Block diagram of a limited feedback power loading OFDM system.

This constraint simplifies to $\|\mathbf{a}\|_2 = 1$ by applying the constellation's average transmit power constraint.

We will assume that the transmitter and receiver lack channel reciprocity, meaning that the forward and reverse link channel responses are independent. This means that the receiver has perfect knowledge of $\mathbf{h}$, while the transmitter has no knowledge of $\mathbf{h}$. Therefore, the vector $\mathbf{a}$ will be designed at the receiver and fed back to the transmitter over a low rate feedback channel. In order for the vector to be conveyed using limited feedback, we will assume that $\mathbf{a}$ is restricted to lie in a finite set, or codebook, $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N\}$. This codebook will be designed offline (i.e., not as a function of the instantaneous channel conditions) and fixed for all transmissions. The receiver will choose one of the vectors in $\mathcal{A}$ to optimize a performance criterion and convey it back to the transmitter using $B = \lceil \log_2 N \rceil$ bits of feedback. We will consider both error rate and capacity performance criteria.

Limited feedback power loading has two fundamental problems that we solve in this paper. First, it must be determined how the receiver chooses the optimal $\mathbf{a}$ from $\mathcal{A}$ given the current channel realization $\mathbf{h}$. To solve this problem, we will leverage previous work on full channel knowledge power loading. Second, we must derive codebook design methods to construct $\mathcal{A}$. The design of $\mathcal{A}$ will depend on the performance criterion chosen.

III. Minimum Error Rate Power Loading

In this section, we will consider the design of $\mathbf{a}$ that minimizes the average symbol error rate. We will assume that all subcarriers carry the same number of bits and use the same constellation. Therefore, we use the model that for each frequency tone $s_i \in \mathcal{S}$ where $\mathcal{S}$ is some constellation.

Let $P_e(\gamma)$ denote the probability of symbol error for a narrowband system transmitting symbols from $\mathcal{S}$ with average signal-to-noise ratio (SNR) $\gamma = \mathcal{E}_s/N_0$ over an additive white Gaussian noise (AWGN) channel. Using results from [44], the probability of symbol error nearest neighbor union bound (NNUB) is given by

$$P_e(\gamma) \leq N_e Q\left(\sqrt{\frac{1}{2}d_{\min}^2\gamma}\right) \quad (4)$$

where N_e is the average number of nearest neighbors to points in $\mathcal{S}$, $Q(\cdot)$ is the Q-function, and $d_{\min}$ is the minimum distance of $\mathcal{S}$.

For a given power loading allocation, the probability of vector symbol error (i.e., the probability that at least one symbol is in error) can be bounded by

$$1-\prod_{i=1}^{M}(1-P_e(|a_ih_i|^2\gamma)) \leq 1-\prod_{i=1}^{M}\left(1-N_eQ\left(\sqrt{\frac{1}{2}d_{\min}^2|a_ih_i|^2\gamma}\right)\right). \quad (5)$$

We will thus choose $\mathbf{a}$ to minimize this probability of vector symbol error bound using the following selection criterion.

Error Rate Criterion: Choose $\mathbf{a} \in \mathcal{A}$ using

$$\mathbf{a} = \operatorname*{argmin}_{\mathbf{a}'\in\mathcal{A}} 1-\prod_{i=1}^{M}\left(1-N_eQ\left(\sqrt{\frac{1}{2}d_{\min}^2|a_i'h_i|^2\gamma}\right)\right). \quad (6)$$

It was shown in [5] that

$$a_i = \sqrt{\frac{|h_i|^2d_{\min}^2\gamma}{1+(|h_i|^2d_{\min}^2\gamma)^2}\left(\sum_{k=1}^{M}\frac{|h_k|^2d_{\min}^2\gamma}{1+(|h_k|^2d_{\min}^2\gamma)^2}\right)^{-1}} \quad (7)$$

obtains excellent error rate performance compared to other power loading techniques and approximates the optimal power loading solution. The optimal power loading scheme requires M transcendental equations to be solved, and the expression in (7) is derived as an approximate solution to these M equations. Interestingly, as $\gamma \rightarrow \infty$, (7) approaches

$$a_{\text{eq},i} = \frac{|h_i|^{-1}}{\sqrt{\sum_{k=1}^{M}|h_k|^{-2}}}. \quad (8)$$

The asymptotic power loading solution in (8) is known as equalized power loading [3]. With equalized power loading, the power loading weights are designed to force all tones to encounter what looks like an AWGN channel. Implementing the power loading vectors described by (7) and (8), however, require full transmitter channel knowledge.

Equalized power loading is also the solution to maximizing the minimum receive SNR over all subcarriers. This follows because we can bound

$$1-\prod_{i=1}^{M}\left(1-N_eQ\left(\sqrt{\frac{1}{2}d_{\min}^2|a_ih_i|^2\gamma}\right)\right) \leq MN_eQ\left(\sqrt{\frac{1}{2}d_{\min}^2\min_{1\leq i\leq M}|a_ih_i|^2\gamma}\right).$$

Using this high SNR power loading allocation, we will design a codebook that quantizes the equalized power loading vector that uses the inverted channel vector

$$\mathbf{g} = [|h_1|^{-1}\ |h_2|^{-1}\ \cdots\ |h_M|^{-1}]^T.$$

For this, the equalized power loading vector is given by $\mathbf{a}_{\text{eq}} = \mathbf{g}/\|\mathbf{g}\|_2$.

We will design the codebook to minimize the distortion

$$E\left[\min_{\mathbf{a}\in\mathcal{A}}\left(\min_{1\leq i\leq M}|a_{\text{eq},i}h_i| - \min_{1\leq j\leq M}|a_jh_j|\right)^2\right]. \quad (9)$$

This can be bounded as

$$E\left[\min_{\mathbf{a}\in\mathcal{A}}\left(\min_{1\leq i\leq M}|a_{\text{eq},i}h_i| - \min_{1\leq j\leq M}|a_jh_j|\right)^2\right] \leq ME\left[\|\mathbf{h}\|_2^2\right]E\left[\min_{\mathbf{a}\in\mathcal{A}}\|\mathbf{a}-\mathbf{a}_{\text{eq}}\|_2^2\right] \quad (10)$$

by i) noting that $\min_i|v_i| - \min_j|w_j| \leq \sum_i|v_i-w_i|$ for arbitrary vectors $\mathbf{v}$ and $\mathbf{w}$ and ii) using the relationship between the vector one norm and two norm.

Note that

$$\begin{aligned}\|\mathbf{a}-\mathbf{a}_{\text{eq}}\|_2^2 &= \|\mathbf{a}-\mathbf{g}/\|\mathbf{g}\|_2\|_2^2 \\ &= (\mathbf{a}-\mathbf{g}/\|\mathbf{g}\|_2)^T(\mathbf{a}-\mathbf{g}/\|\mathbf{g}\|_2) \\ &= \|\mathbf{a}\|_2^2 + \|\mathbf{g}/\|\mathbf{g}\|_2\|_2^2 - 2\mathbf{a}^T\mathbf{g}/\|\mathbf{g}\|_2 \\ &= 2-2\mathbf{a}^T\mathbf{g}/\|\mathbf{g}\|_2.\end{aligned}$$

Therefore, minimizing a bound on (9) is equivalent to minimizing an average distortion function

$$D_{\text{SER}}(\mathcal{A}) = E\left[\min_{\mathbf{a}\in\mathcal{A}}\|\mathbf{a}-\mathbf{g}/\|\mathbf{g}\|_2\|_2^2\right]. \quad (11)$$

Using (11), we can design the codebook using the vector quantization Lloyd algorithm [38, p. 188]. The algorithm can be summarized by the following steps:

1) Randomly generate a codebook of power loading vectors $\mathcal{A}_0 = \{\mathbf{a}_{0,1}, \mathbf{a}_{0,2}, \ldots, \mathbf{a}_{0,N}\}$.
2) Set $i = 1$.
3) Divide the set of possible channel magnitudes $\mathbb{R}_+^M$ into N quantization regions with the kth region defined as
$$\mathcal{C}_k = \{\mathbf{g}\in\mathbb{R}_+^M \mid \mathbf{g}^T\mathbf{a}_{i-1,l} \leq \mathbf{g}^T\mathbf{a}_{i-1,k}\forall l\neq k\}.$$
4) Construct a new codebook $\mathcal{A}_i$ with the kth power loading vector $\mathbf{a}_{i,k}$ given by
$$\mathbf{a}_{i,k} = \operatorname*{argmin}_{\mathbf{a}:\|\mathbf{a}\|_2\leq 1}E[2-2\mathbf{a}^T\mathbf{g}/\|\mathbf{g}\|_2 \mid \mathbf{g}\in\mathcal{C}_k]. \quad (12)$$
5) If $D_{\text{SER}}(\mathcal{A}_{i-1}) - D_{\text{SER}}(\mathcal{A}_i) > \epsilon$, set $i = i+1$ and go back to Step 3. Otherwise, set $\mathcal{A} = \mathcal{A}_i$ and terminate algorithm. The value of ϵ in Step 5 should be chosen to minimize the error, but practically the algorithm converges quickly even for small values of ϵ.

The Lloyd algorithm can be easily implemented by generating Q test channels $\{\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_Q\}$ according to the assumed complex Gaussian distribution. When using a set of test channels, the set of vectors in the kth quantization cell can be written as

$$\mathcal{C}_k = \left\{ \mathbf{g}_1^{(k)}, \mathbf{g}_2^{(k)}, \ldots, \mathbf{g}_{Q_k}^{(k)} \right\}$$

where Q_k is the cardinality of the cell. To obtain an approximate solution to (12), we would like to solve for $\mathbf{a}$ that maximizes

$$\sum_{i=1}^{Q_k} \left(\frac{\mathbf{g}_i^{(k)}}{\left\| \mathbf{g}_i^{(k)} \right\|_2} \right)^T \mathbf{a} \geq \| \mathbf{G}_k \mathbf{a} \|_2$$

where

$$\mathbf{G}_k = \left[\frac{\mathbf{g}_1^{(k)}}{\left\| \mathbf{g}_1^{(k)} \right\|_2} \; \frac{\mathbf{g}_2^{(k)}}{\left\| \mathbf{g}_2^{(k)} \right\|_2} \; \cdots \; \frac{\mathbf{g}_{Q_k}^{(k)}}{\left\| \mathbf{g}_{Q_k}^{(k)} \right\|_2} \right]^T .$$

This follows because

$$\underset{\mathbf{a}:\|\mathbf{a}\|_2 \leq 1}{\operatorname{argmin}} E[2 - 2\mathbf{a}^T \mathbf{g} / \|\mathbf{g}\|_2 \mid \mathbf{g} \in \mathcal{C}_k]$$
$$\approx \underset{\mathbf{a}:\|\mathbf{a}\|_2 \leq 1}{\operatorname{argmin}} \sum_{i=1}^{Q_k} \frac{1}{Q_k} \left(2 - 2\mathbf{a}^T \mathbf{g}_i^{(k)} / \|\mathbf{g}_i^{(k)}\|_2 \right).$$

The expression $\|\mathbf{G}_k \mathbf{a}\|_2$ is maximized by setting $\mathbf{a}$ equal to the dominant right singular vector of $\mathbf{G}_k$. Therefore, Step 4 can be implemented by setting $\mathbf{a}_{i,k}$ equal to the dominant right singular vector of the matrix $\mathbf{G}_k$.

One approximate method for decreasing the complexity would be to partition the M tones into L different M/L tone groups. The Lloyd algorithm can then be performed once to obtain an M/L group power loading vector codebook of size $2^{B/L}$. From the channel model assumption, each of the L different groups will be statistically equivalent. Therefore, we can construct a size 2^B codebook from the L repetition product set of a group codebook.

IV. Capacity Maximizing Power Loading

Unlike probability of error criterion power loading, capacity based power loading is usually combined with bit loading (i.e., varying the number of bits transmitted on each tone) in order to match the per-subcarrier capacity. In this section, we will design limited feedback power loading algorithms. The limited feedback algorithms will be chosen to maximize the average sum capacity across the effective parallel subcarrier channels after power loading.

The instantaneous capacity of an OFDM system with power loading is

$$C(\mathbf{a} \mid \mathbf{h}) = \sum_{i=1}^{M} \log_2(1 + \gamma |a_i h_i|^2) \tag{13}$$

where $\gamma = \mathcal{E}_s / N_0$. We will choose the power loading vector from the codebook $\mathcal{A}$ in order to maximize the capacity for a given channel realization. Thus, we will use the criterion.

Capacity Criterion: Choose $\mathbf{a} \in \mathcal{A}$ such that

$$\mathbf{a} = \underset{\mathbf{a}' \in \mathcal{A}}{\operatorname{argmax}} \, C(\mathbf{a}' \mid \mathbf{h}). \tag{14}$$

When full channel knowledge is available at the transmitter, the optimal unquantized solution for power loading design is waterfilling [9], [45]. In general, waterfilling requires a computationally complex iterative optimization, but this optimization simplifies asymptotically with the SNR. The following lemma summarizes the high and low SNR results.

Lemma 1: Bliss *et al.* [46] As $\gamma \rightarrow \infty$, the optimal power loading vector sets $a_i = 1/\sqrt{M}$ for all i. As $\gamma \rightarrow 0$, the optimal power loading vector sets

$$a_i = \begin{cases} 1 & \text{if } i = i_0; \\ 0 & \text{else} \end{cases}.$$

where $i_0 = \operatorname{argmax}_{1 \leq i \leq M} |h_i|^2$.

Lemma 1 fully characterizes both the high SNR and low SNR design criteria. At high SNR, no feedback is needed. This matches similarly with the high SNR MIMO results cited in the spatial multiplexing literature [47]–[49]. It also allows us to characterize the optimal low SNR feedback scheme. We will discuss feedback design for three cases: high SNR, low SNR, and nonextreme SNR.

A. High SNR

Using the results of Lemma 1, we can see that the equal power allocation loading technique first proposed by Bingham in [3] is optimal for high SNR. Thus, feedback at high SNR is unnecessary, and the number of feedback bits B should be set to zero.

B. Low SNR

Low SNR systems, in contrast, definitely benefit from feedback. Pouring power on smaller gain channels wastes valuable power budget resources and causes a significant capacity loss relative to adaptive loading techniques. Using the low SNR result in Lemma 1 gives a simple, optimal feedback technique.

Optimal Low SNR Feedback Design: At low SNR, use $B = \lceil \log_2 M \rceil$ bits of feedback and set

$$\mathbf{a} = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_M\}$$

where $\mathbf{e}_i$ is the ith column of the $M \times M$ identity matrix. Then the receiver can send back

$$\mathbf{a} = \mathbf{e}_{i_o}$$

where $i_0 = \operatorname{argmax}_{1 \leq i \leq M} |h_i|^2$.

At low SNR, the optimal (waterfilling) solution transmits all data bits on the tone with the best channel magnitude. This leads to an efficient codebook design that can be easily stored in real-time OFDM systems such as those being used in next generation wireless local area network systems. The optimal vector (or equivalently optimal tone) can be located using a brute force search over the M subcarrier tones.

C. Nonextreme SNR

When the SNR takes a nonextreme value, the simple asymptotic results are not optimal. We propose two codebook design methods for moderate SNR limited feedback power loading.

The first algorithm is based on the Lloyd algorithm, while the second extends the high and low SNR loading algorithms.

The first algorithm we propose is an extension of the MIMO covariance quantization techniques derived in [27], [36], [37]. We will design a low average distortion quantizer for the optimal waterfilling power allocation. The optimal waterfilling power loading vector, denoted $\mathbf{a}_{w\text{ fill}}$, satisfies

$$\|\mathbf{a}_{w\text{ fill}}\|_2^2 = 1 \quad \text{and} \quad a_{w\text{ fill},i} = \sqrt{\max\left(0, \mu - \frac{1}{\gamma|h_i|^2}\right)}$$

where μ is a waterlevel constant. Using (13) and the analysis in [37], an average distortion $D_{\text{cap}}(\mathcal{A})$ can then be defined as

$$D_{\text{cap}}(\mathcal{A}) = E\left[\min_{1\le i\le N} -C(\mathbf{a}_i \,|\, \mathbf{h})\right]. \tag{15}$$

Using this distortion, the Lloyd algorithm can be implemented as follows.

1) Randomly generate a codebook of power loading vectors $\mathcal{A}_0 = \{\mathbf{a}_{0,1}, \mathbf{a}_{0,2}, \ldots, \mathbf{a}_{0,N}\}$.
2) Set $i = 1$.
3) Divide the set of possible channel magnitude vectors $\mathbb{R}_+^M$ into N quantization regions with the kth region defined as

$$\mathcal{C}_k = \left\{\mathbf{h} \in \mathbb{R}_+^M \,|\, C(\mathbf{a}_{i-1,l} \,|\, \mathbf{h}) \le C(\mathbf{a}_{i-1,k} \,|\, \mathbf{h}) \forall l \ne k\right\}.$$

4) Construct a new codebook $\mathcal{A}_i$ with the kth power loading allocation $\mathbf{a}_{i,k}$ given by

$$\mathbf{a}_{i,k} = \operatorname*{argmax}_{\mathbf{a}:\|\mathbf{a}\|_2\le 1} E[C(\mathbf{a} \,|\, \mathbf{h}) \,|\, \mathbf{h} \in \mathcal{C}_k]. \tag{16}$$

It was shown in [37] that this step can be easily implemented by setting $\mathbf{a}_{i,k}$ equal to the waterfilling allocation of the channel vector $\mathbf{h}_k = [h_{k,1} \; h_{k,2} \; \cdots \; h_{k,M}]^T$ with $|h_{k,i}| = E[|h_i| \,|\, \mathbf{h} \in \mathcal{C}_k]$.
5) If $D_{\text{cap}}(\mathcal{A}_{i-1}) - D_{\text{cap}}(\mathcal{A}_i) > \epsilon$, set $i = i + 1$ and go back to Step 3. Otherwise, set $\mathcal{A} = \mathcal{A}_i$ and terminate the algorithm.

Just as in the probability of symbol error codebook design, a codebook that locally minimizes the distortion in (15) can be designed using the Lloyd algorithm [38]. This algorithm is easily implemented using a slightly modified version of the algorithm given in Section III.

While the Lloyd algorithm can design efficient codebooks, it does not yield any form of closed form codebook design. Previous work by Leke and Cioffi in [12] motivated power loading algorithms that turn off tones that have channel gains that yield a negative waterfilling value. We propose a modified version of this approach that we call *multi-mode power loading*.

D. Multi-Mode Power Loading

Let $\mathcal{J}$ be the set of all non-empty subsets of $\{1, 2, \ldots, M\}$. Design the codebook $\mathcal{A}$ as

$$\mathcal{A} = \left\{\frac{1}{\sqrt{\text{card}(\mathcal{I})}} \sum_{i\in\mathcal{I}} \mathbf{e}_i \,|\, \forall \mathcal{I} \in \mathcal{J}\right\}. \tag{17}$$

The receiver then chooses

$$\mathbf{a} = \operatorname*{argmax}_{\mathbf{a}'\in\mathcal{A}} \sum_{i=1}^{M} \log_2(1 + \gamma|a_i' h_i|^2) \tag{18}$$

and conveys the power loading vector to the transmitter using M bits of feedback.

It is important to note that multi-mode power loading will become impractical for large M. Both the brute force search over the codebook and the storage requirements grow exponentially with M. For large M, vector quantization schemes such as the Lloyd algorithm could possibly be employed.

Multi-mode power loading is guaranteed to provide performance equal to or exceeding the performance of the algorithm proposed in [12]. The algorithm in [12] sets the power loading vector equal to

$$a_{LC,i} = \begin{cases} 1/\sqrt{M_{w\text{ fill}}} & \text{if } a_{w\text{ fill},i} > 0, \\ 0 & \text{if } a_{w\text{ fill},i} = 0 \end{cases}$$

where $M_{w\text{ fill}}$ is the number of tones with nonzero power allocations after waterfilling. This solution (denoted $\mathbf{a}_{LC}$) is a member of the multi-mode codebook $\mathcal{A}$, and thus

$$C(\mathbf{a}_{LC} \,|\, \mathbf{h}) \le \max_{\mathbf{a}\in\mathcal{A}} C(\mathbf{a} \,|\, \mathbf{h})$$

for any channel $\mathbf{h}$.

Note that the capacity criterion is not of much interest in a fixed rate uncoded system. Systems using capacity power loading will, in general, provide poor uncoded probability of error performance compared to the probability of error selection criterion. When coding and rate adaptation are allowed, the capacity criterion should be used. The probability of error selection will only be *ad hoc* for a coded system.

The codebooks designed from the capacity and probability of error criterion will be much different. Specifically, channel inversion actually allocates power inversely proportionally to the tone's channel magnitude. This is counter to capacity allocation which allocates power proportionally to channel magnitude.

E. Feedback Amount

It is also of interest to understand the amount of feedback that can be allocated. The design of limited feedback deals with the tradeoff between performance and feedback amount. The feedback amount should be as large as possible to maximize performance, but the feedback amount should be as small as possible to minimize system overhead.

A bound on the amount of feedback that can be supported can be obtained using the results in [50], [51]. The idea is to assume that the forward and reverse links in the point-to-point system i) fade independently, ii) use the same number of tones, and iii) support the same feedback rate. For a feedback amount of B, a feedback rate of at least B/T_c bits per second is required for a block fading channel with coherence time T_c. Using results from [52], we must have that

$$C(\mathbf{a}_{w\text{ fill}} \,|\, \mathbf{h}) - C\left(\frac{1}{\sqrt{M}}\mathbf{o} \,|\, \mathbf{h}\right) \le M e^{-1} \log_2(e)$$

where $\mathbf{o}$ is the M-dimensional vector of all ones.

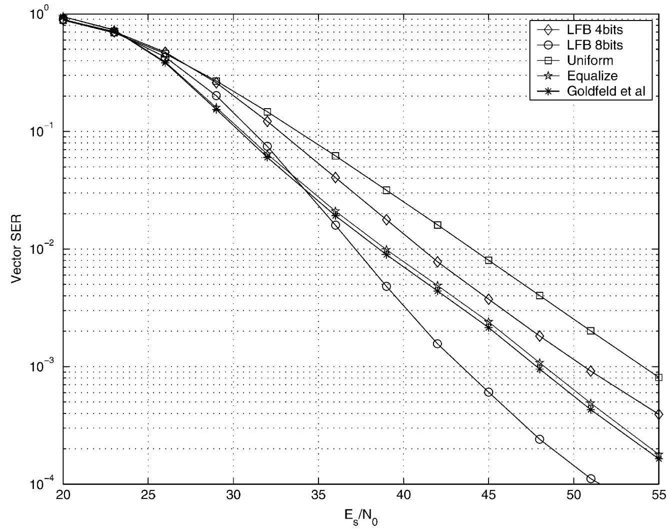

Fig. 2. Error rate performance of a 32 tone limited feedback power loading OFDM system.

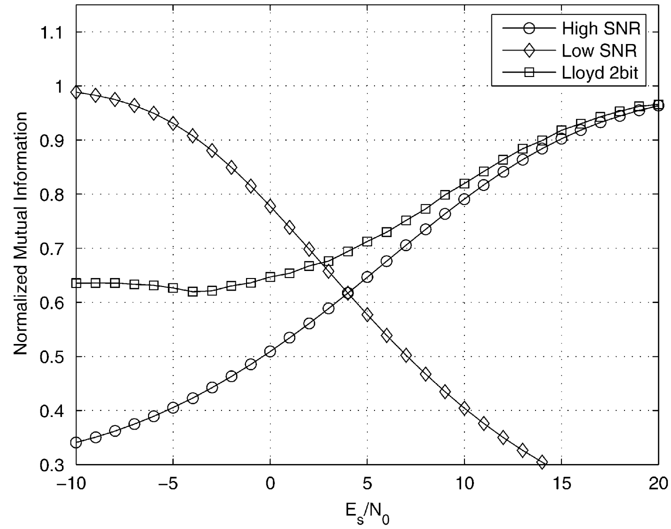

Fig. 3. Capacity performance of a 16 tone limited feedback power loading OFDM system.

The feedback rate then should satisfy

$$B/T_C \leq W\left(C(\mathbf{a}_{w\,\text{fill}}\,|\,\mathbf{h}) - C\left(\frac{1}{\sqrt{M}}\mathbf{o}\,|\,\mathbf{h}\right)\right) \leq MWe^{-1}\log_2(e)$$

where W is the subcarrier spacing. Manipulation shows that we should have $B \leq MWT_c e^{-1}\log_2(e)$. This gives a bound also on the applicability of multi-mode power loading which requires $B = M$. In this case we must have that $M \leq MWT_c e^{-1}\log_2(e)$ or rather $1 \leq WT_c e^{-1}\log_2(e)$. For small coherence times or small subcarrier spacings, multi-mode precoding can not be applied. In this situation, as mentioned before, a smaller B can be obtained by using the Lloyd algorithm.

V. Simulations

Simulations were performed using the independent Rayleigh fading channel model. The simulations tested the probability of symbol vector error and capacity criteria limited feedback schemes.

Experiment 1: The first experiment simulated limited feedback power loading on a 32 tone OFDM system using the error rate criterion and binary phase shift keying (BPSK) modulation. The vector symbol error rate (SER), the probability that at least one subcarrier symbol is in error, simulation results are shown in Fig. 2. Four bit and eight bit codebooks were designed using the Lloyd algorithm. Uniform power loading, equalized power loading, and the algorithm from [5] were simulated for comparison. Note that four bits of feedback provides a 3.4 dB improvement over uniform power allocation at a vector SER of 10^{-3}. Eight bit limited feedback power loading gives a 4.4 dB gain over the approximately optimal scheme in [5] at an error rate of 10^{-3}. Note that the subcarrier equalization and the power loading in [5] require perfect channel knowledge. The plot also clearly demonstrates that the equalized allocation and the allocation in [5] are suboptimal and are only rough approximations to the optimal solution. Limited feedback power loading provides a dramatic performance advantage that comes with only a few bits of feedback.

Experiment 2: This experiment examined the performance of the asymptotic feedback schemes for a 16 tone OFDM system. Fig. 3 presents the performance when the capacity of the asymptotic allocations are normalized by the perfect channel knowledge waterfilling capacity. The high SNR assumption technique, which uses no feedback, obtains excellent performance at high SNR but dips below 80% of the full channel knowledge capacity at $\mathcal{E}_s/N_0 = 10$ dB. In contrast, sending $\lceil \log_2 16 \rceil = 4$ bits of feedback with the low SNR feedback scheme is optimal at very low SNRs. It falls below 80% of the capacity for SNRs above 0 dB. A two bits of feedback power loading system (i.e., $\mathcal{A}$ has four elements) using the capacity criterion is also presented. The codebook was designed using the Lloyd algorithm. Note that the two bits of feedback outperforms high SNR uniform power loading by 2.5 dB when obtaining 70% of the waterfilling capacity. The two bit limited feedback design also outperforms low SNR power loading (which requires four feedback bits) for SNRs above 2.65 dB.

Experiment 3: The third simulation, see Fig. 4, studied the capacity performance of the asymptotic feedback techniques and the multi-mode feedback technique for an eight tone OFDM system. Once again, the mutual informations were normalized by the perfect channel knowledge capacity. The 80% cross-over points are now 7 dB for the zero feedback high SNR assumption technique and 1.5 dB for the three bit feedback low SNR technique. Note that the multi-mode technique, which requires eight bits of feedback, obtains more than 98.5% of the waterfilling capacity for all SNRs. This high performance comes at the expense of switching the subcarriers off and on.

Experiment 4: Finally, we considered the capacity tradeoffs of limited feedback with varying feedback rates. Fig. 5 shows the results for high SNR, low SNR, and various feedback rate power loadings for a 32 tone system. The high rate

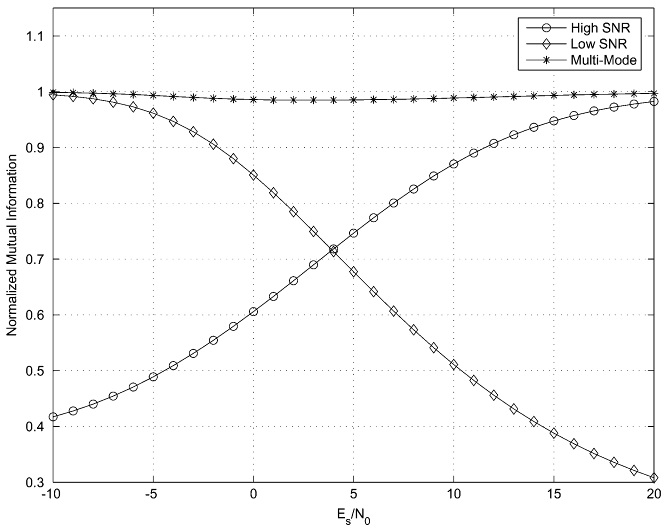

Fig. 4. Capacity performance of an eight tone limited feedback power loading OFDM system.

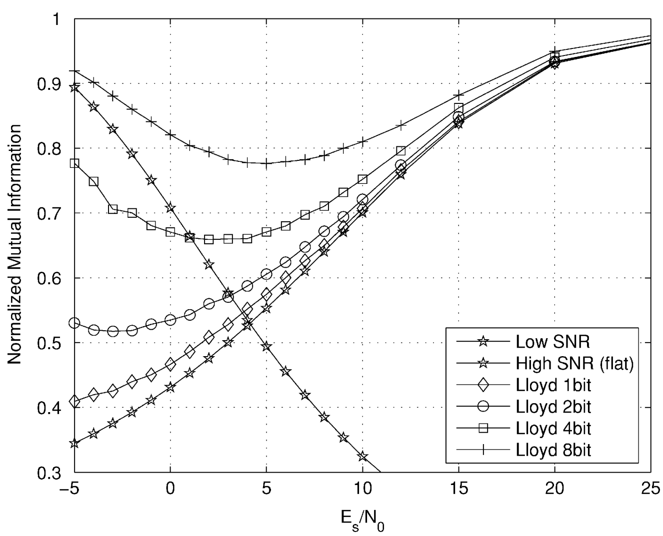

Fig. 5. Capacity performance of a 32 tone limited feedback power loading OFDM system.

codebook uses uniform power loading, while the low SNR codebook sends all data on the tone with the largest channel magnitude which requires five bits of feedback. One, two, four, and eight bit codebooks were designed using the Lloyd algorithm. Note that four bits obtains more than 65% of the waterfilling capacity for all SNRs. In addition, using two bits of feedback instead of one bit adds approximately a 1.4 dB performance improvement when operating at 60% of the waterfilling capacity. The eight bit codebook outperforms low and high SNR power allocations for all SNRs. This would be expected because the eight bit codebook requires the most feedback out of any of the power loading schemes.

VI. Conclusion

We proposed limited feedback power loading for OFDM wireless systems in this paper. The limited feedback approach uses a codebook of power loading vectors known to both the transmitter and receiver. The receiver chooses an optimal power loading vector from the codebook and conveys the vector to the transmitter over a low rate feedback channel. We derived power allocation selection functions that minimize the probability of vector symbol error and the capacity. We showed that the Lloyd algorithm can be successfully used to design high performance codebooks. As well, we characterized asymptotically optimal codebooks for capacity selection. These asymptotic feedback strategies motivated a technique called multi-mode precoding for use with capacity selection.

This paper serves only as an introduction to the applications of limited feedback in OFDM. Much work has been published over the last few years about the application of limited feedback to multi-antenna wireless systems (see for example [25]–[37], [53], [54]), and we conjecture that there is also a wide array of applications of limited feedback in OFDM that can provide practical performance improvements. The application of limited feedback to multi-antenna OFDM systems remains an open problem. Possible applications of limited feedback multi-antenna OFDM include future broadband wireless networks, wireless local loop, and fourth generation cellular.

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