Off-axis parabolic mirrors: A method of adjusting them and of measuring and correcting their aberrations

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This paper proposes a method of adjusting off-axis parabolic mirrors, based on the minimization of beam-quality parameter M^2. This method requires no additional adjustment elements and possesses adequate accuracy: 1% for the angular orientation of the mirror and 2.3% for the rms deviation of the M^2 value. An experimental technique is given for measuring and correcting the aberrations of strongly focusing off-axis optical elements. The technique is based on measuring the wave front by the Shack–Hartmann method. The aberrations were corrected using a controllable flexible mirror in the manual and automatic regimes by the aperture-probing and phase-conjugation methods, respectively. © *2005 Optical Society of America*

INTRODUCTION

Powerful terawatt devices that make it possible to obtain laser radiation with an intensity of the order of 10^{21} W/cm^2 and femtosecond pulse widths are currently being used to study the interaction of light with matter in the relativistic regime, as well as to create x-ray sources, to accelerate electrons, and for experiments on nonlinear quantum electrodynamics, the initiation of photonuclear reactions, and rapid triggering of thermonuclear reactions. In particular, apparatus created at Livermore National Laboratory (USA) makes it possible to obtain laser radiation pulses with an energy of 660 J and a pulse width of 440 fs, which gives a peak power of the order of 1 PW and, with focusing of the beam, a radiation intensity greater than 10^{21} W/cm^2. The ATLAS 8-TW titanium–sapphire laser system at the Max Planck Institute (Germany) also makes it possible to achieve high intensities by focusing a 150-fs pulse.[1] Such intensities are achieved with rigorous focusing of the radiation, accomplished by an off-axis parabolic mirror.[2] Unlike ordinary lenses, an ideal parabolic mirror makes it possible to solve the problems associated with the distortion of laser radiation caused by spherical aberration and by spreading of the laser pulse. However, like any optical element, a parabolic mirror in reality introduces aberrations into the wave front, and this reduces the possibility of rigorous focusing of the laser radiation. This article proposes techniques for measuring and correcting aberrations that can be used not only for short-focus parabolic mirrors, but also for other cases—for example, for correcting aberrations associated with atmospheric turbulence.

A METHOD OF ADJUSTING PARABOLIC MIRRORS USING THE M^2 PARAMETER

Before proceeding to measure the aberrations of an optical element, it is necessary to align it correctly. In an off-axis parabolic mirror, the back face is usually perpendicular to the axis of rotation of the parabola. Therefore, one way to adjust it is to set the back face of the mirror perpendicular to the optical axis. However, this method requires additional adjustment elements to be introduced into the system, and this is not always convenient. We propose to use a method of adjusting off-axis parabolic mirrors in which the minimum value of parameter M^2 will be considered the alignment criterion of the system.

The physical meaning of parameter M^2 is simple—it is the ratio of the divergence angle θ_0 of a Gaussian beam and the divergence angle θ of an arbitrary beam (with identical beam diameters in the near field): $M^2=\theta/\theta_0$.[3] This formula can be rewritten as follows: $M^2=d_f/d_{f(\mathrm{Gauss})}$, where $d_{f(\mathrm{Gauss})}$ is the diameter of a Gaussian beam in the focal plane of a converging lens with focal length f. If the spatial transverse intensity distribution has the form of a circle, parameter M^2 can also be computed from[3]

$$M^2=\pi d d_0/(4\lambda f), \tag{1}$$

where d_0 is the beam diameter in the near field, d is the beam diameter at the focus f of the lens, and λ is the wavelength of the laser radiation. In the case of a beam whose spatial transverse intensity distribution is in the form of an ellipse, two values of parameter M^2 are measured (along the x and y axes): $M_x^2=\pi d_x d_{0x}/(4\lambda f)$, $M_y^2=\pi d_y d_{0y}/(4\lambda f)$, where d_{0x} and d_{0y} are the beam diameters in the near field along the x and y coordinates, respectively, and d_x and d_y are the beam diameters at the focus f of the lens.

An optical element that is not aligned accurately enough will introduce distortions into the wave front of the recorded beam, and this will increase the size of the focal spot and increase parameter M^2 by comparison with its value for a correctly aligned element. Thus, the quality of the adjustment itself can be judged by tracking the value of M^2 during the adjustment.

To determine the angle of best position of an off-axis parabolic mirror relative to the incident radiation, the layout

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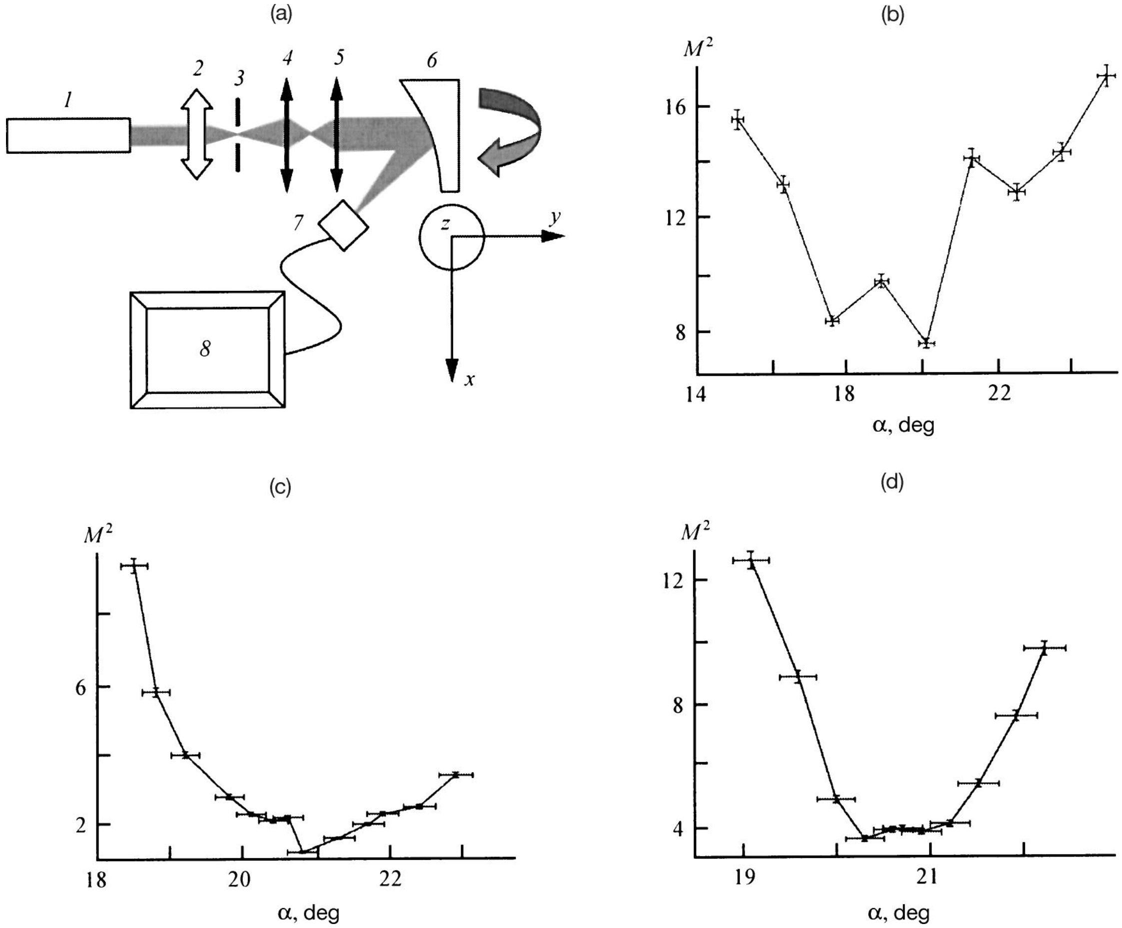


FIG. 1. Finding angle of the best position of a parabola. (a) Layout of the experimental apparatus: *1*—LGN-303 laser, *2*—microscope objective (20×, focal length 8.55 mm), *3*—20-μm stop, *4*—lens with 45-mm focal length, *5*—lens with 300-mm focal length, *6*—off-axis parabolic mirror, *7*—CV-M50 8-bit CCD camera, *8*—personal computer, *7* and *8*—M^2 sensor; (b)–(d) dependences of parameter M^2 on angle of rotation of parabolic mirror. See text for explanation.

shown in Fig. 1a was assembled for the experimental apparatus. The radiation from an LGN-303 helium–neon laser was used (a stabilized single-mode, single-frequency laser with a wavelength of 0.63 μm and a power of 1 mW), collimated by lenses *2*, *4*, and *5* and having a quality parameter of $M^2 = 1.1-1.2$. The radiation was directed toward parabolic mirror *6* of interest, was reflected from it, and was focused onto CCD camera *7*. The CV-M50 8-bit CCD camera that was used had a 6.4×4.8-mm exit window, a spatial resolution of 752×582 points, SNR $S/N > 59$ dB, and spectral range 350–1100 nm. The analog signal from the CCD camera went to a computer. The M^2 sensor consists of elements *7* and *8* in Fig. 1a.

Measurements were made for off-axis parabolic mirrors with focal lengths 150 and 50 mm and apertures 50 and 20 mm, respectively. The beam diameter in the near field on the parabolic mirrors (at the $1/e^2$ level) was 9 mm for the first mirror and 11 mm for the second. The measurements of the beam diameter in the far field (in the focal plane of the parabolic mirror), and consequently the measurements of M^2, were carried out for each rotation of the mirror by a definite angle α in the xy plane (Fig. 1a).

The minimum value of M^2 for the first mirror was 7.6 ±0.2 for an angle of rotation of 20.1°±0.2° (Fig. 1b), and for the second 3.6±0.1 for an angle of rotation of 20.8° ±0.2° (Fig. 1d).

The quality of the parabolic mirror can be judged from the minimum values obtained for parameter M^2. It can be seen from Fig. 1c that M^2 is close to unity for the second mirror, with a beam diameter of 4 mm ($M^2 = 1.2$), and this indicates that its surface quality is very good on the area occupied by the beam.

As a result of these measurements, the position corresponding to the correct alignment was found for each of the off-axis parabolic mirrors investigated here.

STUDY OF THE SURFACE QUALITY OF OFF-AXIS MIRRORS, USING A SHACK–HARTMANN WAVE-FRONT SENSOR

There are various methods of measuring wave-front aberrations;[4] for example, the interferometric method,[5] the Foucault knife-edge method,[5] the Zernike filtering method,[5] and so on. For measurements in real time, it seems most suitable to us to use a Hartmann sensor.[5,6]

A photograph of the sensor, its optical layout, and the operating principle are shown in Figs. 2a, 2b, and 2c, respectively. The main elements of the sensor are a lens raster and

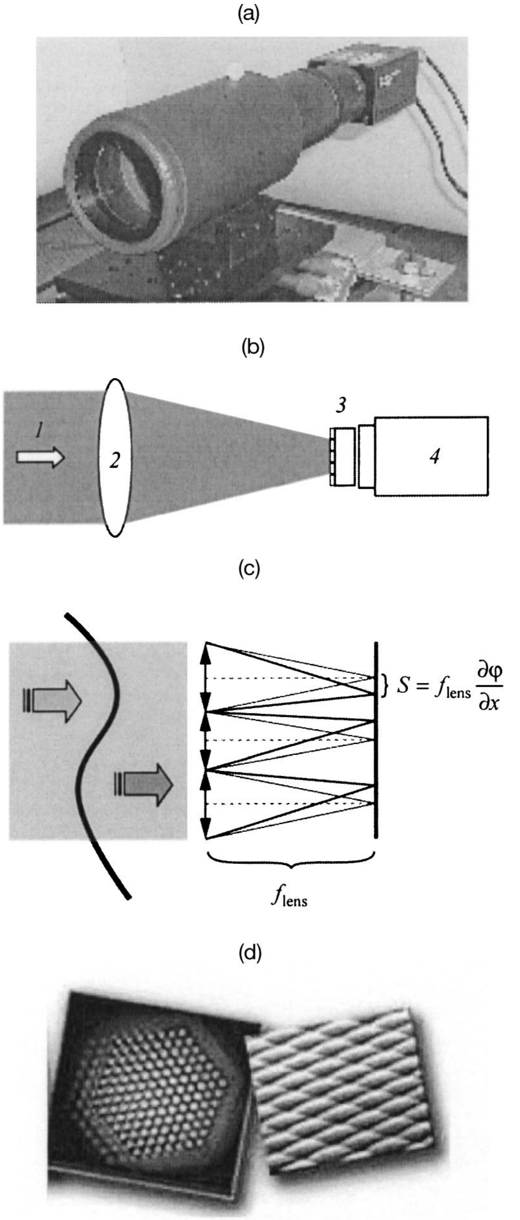


FIG. 2. Shack–Hartmann wave-front sensor. (a) Photograph of sensor; (b) layout of sensor: *1*—laser radiation, *2*—scaling lens ($f=250$ mm), *3*—microlens raster, *4*—CCD camera; (c) operating principle of sensor: f_{lens} is the focus of the microlens, S is the displacement of the focal spot of the microlens; (d) varieties of microlens rasters.

a CV-M50 CCD camera. The microlens raster (Fig. 2d) consisted of 15×15 microlenses and had the following parameters: size 4.5×4.5 mm, focal length of the microlenses 8 mm, and aperture of the microlenses 0.3 mm. A supplementary focusing lens ($f=250$ mm, diameter 50 mm) was used to match the entrance aperture of the sensor with the aperture of the CCD camera. Table I shows the main parameters of the Shack–Hartmann wave-front sensor. When it is used to make measurements, the radiation of an LGN-303 helium–neon laser, passing through a system of lenses, was expanded so that, after being reflected from the parabolic mirror, a collimated beam was formed that covers virtually the entire effective diameter of the mirror.

TABLE I. Main parameters of the Shack–Hartmann wave-front sensor.

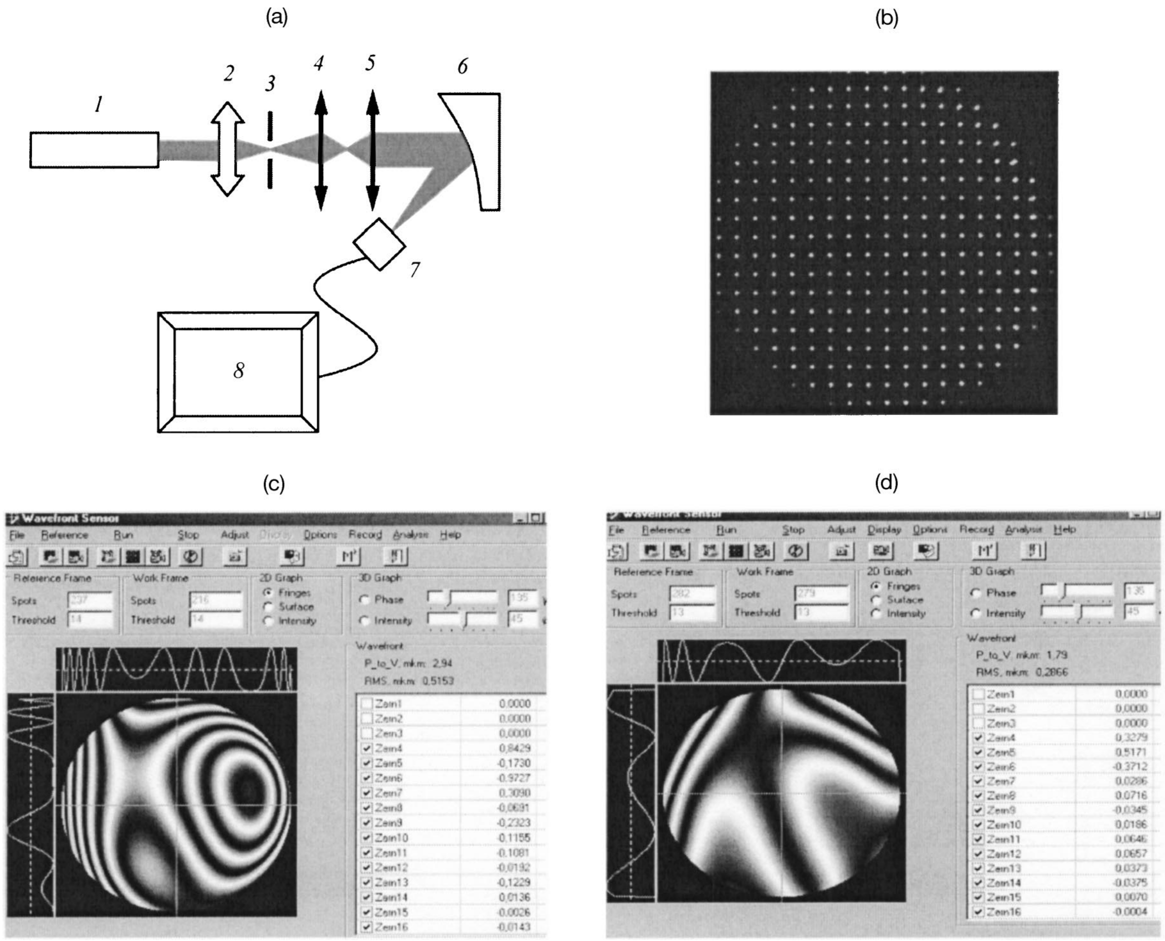
Parameter	Value
Spectral range, nm	350–1100
Dynamic range of phase measurement, μm	more than 15
Measurement accuracy (calculated) for $P-V=15$ μm, μm	0.08
Response rate, Hz	about 12
Input beam diameter, mm	20–40
Photo input	Matrox Meteor II

In order to avoid losses in the accuracy of the wave-front reconstruction, it is necessary for the Hartmannogram (Fig. 3b) to have the same number of focal spots for the reference and measured beams. It is fairly difficult to experimentally achieve such coincidence, and therefore Hartmannograms were determined for various diameters of the reference beam. To do this, parabolic mirror *6* was eliminated from the layout of the experimental apparatus (Fig. 3a), a collimated beam was formed by displacing lenses *4* and *5*, and a stop having its diameter was placed in front of the Shack–Hartmann sensor. In analyzing the distortions introduced into the wave front by the mirrors of interest, a reference beam with exactly the same number of points as the measurement beam was generated by means of a special program.

The Shack–Hartmann wave-front sensor and the processing program that we used allow us to simultaneously observe (in real time) the calculated interferogram of the beam (an analog of a Fizeau interferogram) and the values of the coefficients of the Zernike polynomials (Figs. 3c and 3d). The software that we created can be switched to different viewing regimes: (1) a two-dimensional image of the interferogram of the beam, the amplitudes of the wave-front aberrations [the differences of the maximum and minimum phase distortions, denoted below as $P-V$ (peak-to-valley)], and the intensity; (2) a three-dimensional image of the phase and the intensity, with the possibility of rotating the image. The rms errors of the measurements is also shown. The values of either all the Zernike coefficients or some of them, chosen in a definite way, may be viewed during the measurements. For instance, it is possible in our case to neglect the coefficients of the polynomials corresponding to tilt and defocusing, since it is easy to compensate such distortions by tilting the CCD camera and by displacing it along the propagation axis of the radiation.

For the off-axis parabolic mirror with focal length 150 mm and aperture diameter 50 mm (with a beam diameter at the mirror of 11 mm), the measured amplitude of the wave-front aberrations was $P-V=2.94\pm0.52$ μm. The largest contribution to the wave-front distortions is from astigmatism along the x axis (0.64 ± 0.11 μm) and coma along the y axis (0.97 ± 0.17 μm) (Fig. 3c).

For the off-axis parabolic mirror with focal length 50 mm and aperture diameter 20 mm, when the beam diameter at the mirror was 6 mm, the amplitude of the wave-front aberrations was $P-V=1.79\pm0.29$ μm. The contribution to the wave-front distortions is from astigmatism along the x



(c)

Reference Frame: Spots 237, Threshold 14

Work Frame: Spots 216, Threshold 14

3D Graph: 135, 45

P_to_V, mkm: 2,94

RMS, mkm: 0,5153

☐ Zern1	0,0000
☐ Zern2	0,0000
☐ Zern3	0,0000
☑ Zern4	0,8429
☑ Zern5	-0,1730
☑ Zern6	-0.9727
☑ Zern7	0,3090
☑ Zern8	-0,0691
☑ Zern9	-0,2323
☑ Zern10	-0,1155
☑ Zern11	-0.1081
☑ Zern12	-0,0192
☑ Zern13	-0,1229
☑ Zern14	0,0136
☑ Zern15	-0.0026
☑ Zern16	-0,0143

(d)

Reference Frame: Spots 282, Threshold 13

Work Frame: Spots 279, Threshold 13

3D Graph: 135, 45

P_to_V, mkm: 1,79

RMS, mkm: 0,2866

☐ Zern1	0,0000
☐ Zern2	0,0000
☐ Zern3	0,0000
☑ Zern4	0,3279
☑ Zern5	0,5171
☑ Zern6	-0.3712
☑ Zern7	0,0286
☑ Zern8	0,0716
☑ Zern9	-0,0345
☑ Zern10	0,0186
☑ Zern11	0,0646
☑ Zern12	0,0657
☑ Zern13	0,0373
☑ Zern14	-0,0375
☑ Zern15	0,0070
☑ Zern16	-0,0004

FIG. 3. Study of the aberrations of a parabola with a Shack–Hartmann sensor. (a) layout of experimental apparatus: *1*—He–Ne laser, *2*—microscope objective, *3*—10-μm stop, *4*—lens with focal length 45 mm, *5*—lens with 300-mm focal length, *6*—off-axis parabolic mirror, *7*—Shack–Hartmann sensor, *8*—personal computer; (b) Hartmannogram; (c) results of measurements for the parabola with $f = 150$ mm; (d) results of measurements for the parabola with $f = 50$ mm.

axis (0.33 ± 0.05 μm) and along the y axis (0.51 ± 0.08 μm) as well as coma along the x axis (0.37 ± 0.06 μm) (Fig. 3d).

CORRECTING DISTORTIONS BY MEANS OF A FLEXIBLE CONTROLLABLE BIMORPH MIRROR

The presence of the detected and measured "smooth" aberrations (astigmatism and coma) makes it impossible to achieve rigorous focusing of the laser radiation on the target. Therefore, the next problem that faces us is to correct these distortions. It is most suitable to use a controllable flexible bimorph mirror.[7–12] The layout of the flexible corrector that we used, a so-called semipassive bimorph mirror,[7,9–12] is shown in Fig. 4a. A bimorph mirror with the electrode configuration shown in Fig. 4b was used. The radii of the sectioned electrodes equal, respectively, 12 and 26 mm, the aperture of the entire mirror is 33 mm, the thickness is 3 mm, and the maximum deformation amplitude of the mirror is 7 μm. For effective correction, it is necessary that the beam fill at least 90% of the mirror's surface.

To correct the aberrations of the parabolic mirror in the manual regime, we implemented the method of coordinate descent, which we chose because it is easy to implement. Voltages with a step of $+1$ V were successively applied to a chosen electrode of the mirror, while the $P-V$ amplitude of the wave-front aberrations was recorded. If the $P-V$ value decreased, the voltage continued to be applied in the chosen direction, but, if the $P-V$ value increased, the step of the voltage increment was reversed (to -1 V). Thus, the voltage that corresponded to the minimum $P-V$ amplitude of the aberrations was first found by controlling a single electrode, and then the control process was repeated for the next electrode, and so on, until the smallest $P-V$ parameter was reached.

The method of coordinate descent was used to correct the aberrations introduced into the wave front by both mirrors investigated here. The flexible mirror was corrected and the wave front was monitored using the software that we created, which made it possible to observe the following:

- a calculated interferogram of the beam corresponding to a Fizeau interferogram,
- a calculated intensity distribution in the far field,
- the applied voltages,
- the $P-V$ amplitude of the aberrations and the rms error of the measurements,
- the coefficients of the Zernike polynomials.

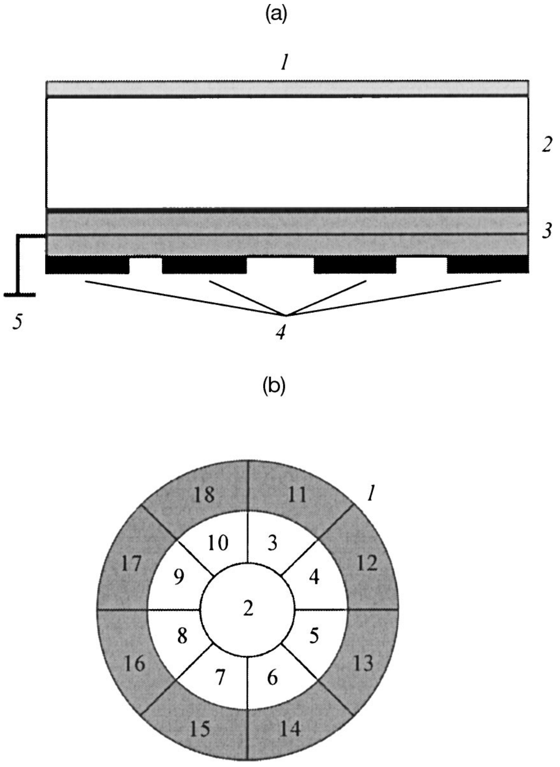


FIG. 4. (a) layout of a flexible semipassive bimorph mirror: *1*—reflective coating, *2*—glass substrate, *3*—piezoceramic disks, *4*—controlling electrodes, *5*—common ground electrode; (b) configuration of the mirror electrodes. Number *1* denotes the common solid electrode, which is located between the glass substrate and the second piezoceramic disk.

A schematic diagram of an experimental apparatus for compensating the aberrations of an off-axis parabolic mirror is shown in Fig. 5a. In the process of correction, the $P-V$ value was minimized while simultaneously recording the calculated interference pattern and the calculated intensity distribution in the far field.

For the mirror with a focal length of $f=150$ mm, the $P-V$ value before correction was 3.35 ± 0.49 μm (Fig. 5b). The diameter of the laser beam at the parabolic mirror was 11 mm (at the $1/e^2$ level), with a mirror diameter of 50 mm. It can be seen from the values of the Zernike coefficients that the main contribution to the wave-front distortion is from astigmatism along the x axis (-1.02 ± 0.15 μm) and coma along the y axis (0.66 ± 0.10 μm). After carrying out the correction in the manual regime by the aperture-probing method, it was possible to reduce the aberration amplitude to 1.30 ± 0.18 μm. The calculated interferograms, the intensity distributions in the far field, and also the Zernike coefficients and the voltage values before and after correction are shown in Figs. 5b and 5c. It can be seen that correcting the aberrations with a flexible bimorph mirror made it possible to reduce the $P-V$ value by a factor of 3.

Next the aberrations of the off-axis parabolic mirror with focal length 50 mm and mirror aperture 20 mm were measured and corrected. The diameter of the expanded beam of laser radiation in the plane of the parabolic mirror was 30 mm. The defocusing acquired during the expansion of the beam was compensated by applying a voltage of -130 V to the first electrode of the adaptive mirror.

The amplitude of the aberrations introduced by the parabolic mirror into the beam, as shown by the measurements, was 1.81 ± 0.25 μm. It can be seen from the table of Zernike

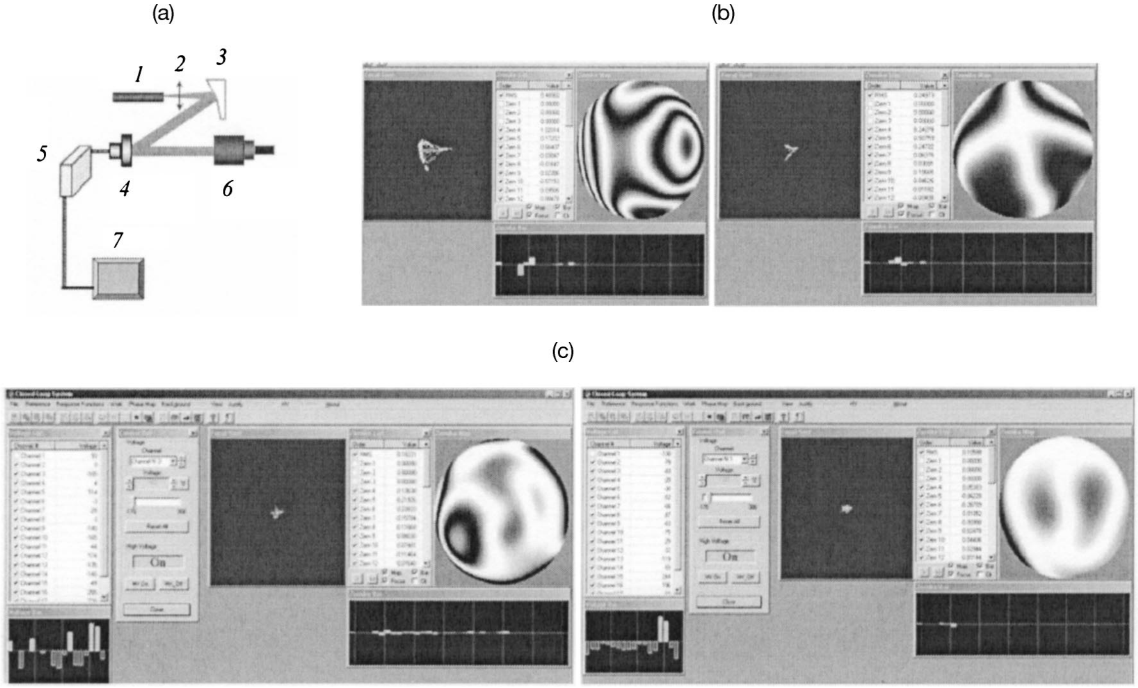


FIG. 5. Corrections of wave-front distortions by the method of coordinate descent. (a) schematic diagram of experimental apparatus: *1*—radiation source (laser), *2*—scaling optics, *3*—parabola, *4*—flexible bimorph mirror, *5*—control unit for flexible bimorph mirror, *6*—Shack–Hartmann wave-front sensor, *7*—personal computer; (b) before correction [for the first ($f=150$ mm) and the second ($f=50$ mm) parabolic mirrors]; (c) after correction.

TABLE II. Comparison of the effectiveness of the aperture-probing and phase-conjugation methods ($P-V$, μm).

Methods	Parabola (f=150 mm)		Parabola (f=50 mm)	
	before correction	after correction	before correction	after correction
Aperture-probing method	3.35 ± 0.49	1.30 ± 0.18	1.81 ± 0.25	0.750 ± 0.11
Phase-conjungtion method		1.50 ± 0.21		0.62 ± 0.08

coefficients that the main contribution to the wave-front distortion is from astigmatism along the y axis, 0.51 ± 0.07 μm. After making a correction by the aperture-probing method in the manual regime, the $P-V$ value decreased from 1.81 ± 0.25 μm to 0.75 ± 0.11 μm (Fig. 5c).

It can be seen from these data that this correction improved the intensity distribution of the beam in the far field, as well as reducing the amplitude of the wave-front aberrations by a factor of 2. In this case, the amplitude of the individual aberrations, for example, astigmatism along the y axis, was reduced by about a factor of 10 when the second parabolic mirror was used.

COMPARISON OF THE APERTURE-PROBING METHOD AND THE PHASE-CONJUGATION METHOD

It is well known that the phase-conjugation method is fast but not always effective, since an adaptive mirror reproduces a wave front with a certain error. It was therefore decided to compare it with the aperture-probing method, which is inferior in response rate but gives an advantage in effectiveness.[13]

The basic operating principle of the phase-conjugation method is as follows: The measured wave front is expanded in the response functions of the wave-front corrector. The weighting factors of the expansion are determined by minimizing the residual error of the expansion, for example, by the method of least squares. The resulting coefficients are the voltages that are applied to the flexible mirror. Thus, when these voltages are supplied, the adaptive mirror reproduces the wave front with the minimum rms error.

The response functions of the mirror were measured before beginning the correction and then used as expansion functions for the wave front. Each response function describes the mirror's surface deformation (expressed in micrometers) when unit voltage is applied to one chosen electrode.

The layout of the experimental apparatus of the adaptive optical system that compensates the phase distortions of the parabolic mirror (using the phase-conjugation method) is similar to the layout of Fig. 5a, but with the difference that feedback is formed in this case (elements *6* and *7* are connected). The flexible mirror is controlled in the automatic regime on the basis of the phase-conjugation method. The diameters of the laser beams on the adaptive mirrors and the test elements remained as before in order to compare the results of correcting the aberrations by the two methods: phase conjugation and aperture probing.

The following values of the $P-V$ parameter are obtained when the phase-conjugation method is used:

(1) For the parabolic mirror with focal length 150 mm, the amplitude of the wave-front aberrations decreased from 3.35 ± 0.49 μm to 1.5 ± 0.21 μm when the beam diameter is 11 mm.
(2) For the parabolic mirror with focal length of 50 m, the amplitude of the wave-front aberrations decreased from 1.81 ± 0.25 μm to 0.62 ± 0.08 μm when the beam diameter is 6 mm.

The results of the correction obtained using the phase-conjugation method and the aperture-probing method are shown in Table II. It can be seen that the two correction algorithms give results that approximately coincide (taking into account the degree of accuracy). This indicates that both algorithms can be used with approximately equal effectiveness to solve such problems as correcting slowly varying large-scale aberrations. However, the phase-conjugation algorithm is preferable if high response rate is required in the system.

CONCLUSIONS

An original method has been proposed for adjusting parabolic mirrors, based on minimizing the beam-quality parameter M^2. This method requires no additional adjustment elements and possesses a fair amount of accuracy (rms error$=2.3\%$). An experimental technique is described for measuring the surface distortions of short-focus off-axis parabolic mirrors, tested on two samples mirrors with focal lengths of 150 and 50 mm and apertures of 50 and 20 mm, respectively. The technique is based on measuring the wave front by the Shack–Hartmann method. The measured aberration amplitudes were $P-V=2.94\pm 0.52$ μm for the first mirror ($f=150$ mm) and $P-V=1.79\pm 0.29$ μm for the second ($f=50$ mm). The measured aberrations were corrected by means of a flexible bimorph mirror. The adaptive mirror was controlled by two different methods: phase conjugation and aperture probing. The two algorithms under the conditions of the experiments gave similar results—they reduced the aberrations of the optical elements by a factor of 2–3.

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