THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47 St., New York, N.Y. 10017

91-GT-389



# Off-Design Performance Prediction of Turbofans using Gasdynamics

F. S. MIRZA-BAIG*
H. I. H. SARAVANAMUTTOO†
Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Canada

## ABSTRACT

This paper describes a mathematical model by which the off-design performance of turbofans can be predicted, knowing just the design point parameters. Off-design performance has been estimated by using gasdynamic properties of the exhaust nozzles, which regulate the aerothermodynamic behaviour of the upstream components. Approximate overall performance of a turbofan at typical cruising conditions can be estimated, making the use of compressor and turbine maps redundant.

## NOMENCLATURE

$A_h$ — exit area of hot exhaust nozzle
$A_c$ — exit area of cold exhaust nozzle
a — velocity of sound
$C_p$ — specific heat at constant pressure
Ma — Mach number
m — mass flow
$P_t$ — total pressure
R — gas constant
$T_t$ — total temperature
$\gamma$ — specific heat ratio
$\epsilon_c$ — pressure ratio of compressor
$\epsilon_f$ — fan pressure ratio
$\eta_f$ — polytropic fan efficiency
$\eta_c$ — polytropic compressor efficiency
$\eta_t$ — polytropic turbine efficiency
$\eta_n$ — isentropic nozzle efficiency
$\eta_i$ — isentropic intake efficiency
$\eta_m$ — mechanical efficiency
Q — $m\sqrt{T_t}/P_t$
$\psi_1$ — $\frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_d}$

### Subscripts

o — ambient atmosphere; free stream
oo — Sea level conditions
a — air
d,des — design
h — hot
c — cold
t — total

### Acronyms

SLS — Sea Level Static
ISA — International Standard Atmosphere
TET — Turbine Entry Temperature
FPR — Fan Pressure Ratio
BPR — Bypass Ratio
OPR — Overall Pressure Ratio

## INTRODUCTION

The question arises as to why a new mathematical model for off-design performance is needed, when a method based on component matching (Saravanamuttoo, 1972) is already available for both turbojets and turbofans. It is because the standard method requires compressor and turbine characteristics, which are usually unavailable. The new model can be used for preliminary performance calculations for parametric studies, particularly for aircraft design purposes.

Off design performance prediction methods have been investigated by NASA, Royal Aircraft Establishment and others; they developed programs such as DYNGEN, SMOTE, GENENG, DIGISIM. The only problem was that all of them relied on compressor and turbine performance maps.

*Graduate Student
†Professor, Fellow ASME

Presented at the International Gas Turbine and Aeroengine Congress and Exposition
Orlando, FL June 3-6, 1991

A major contribution made by Wittenberg (1976), basing his work on that of Saravanamuttoo (1972), showed that gasdynamic relationships can be used for matching of nozzles and turbines. Wittenberg's use of generalized turbine characteristics and gasdynamic relations makes performance maps of compressors redundant.

If the compressor maps are available, they can be used to take into account the effect of changes of compressor efficiency with engine conditions. The aim of this paper is to develop a method based on Wittenberg's work, which can be used to give a reasonable prediction of typical cruise performance from a knowledge of design point parameters.

## DIFFICULTIES IN SIMULATION

The basic reason why is it so difficult to predict the performance parameters, is because of the *unavailability of information.* Other factors include

1. Inherent complexity of the problem.
2. Abundance of independent input parameters, and the intricate way these factors affect each other.
3. Secondary effects.

In the case of turbofans, FPR, OPR, BPR, SLS thrust, air mass flow and fuel flow are the only specified parameters. The turbine entry temperature at the design point may not be known to the user, nor is any information regarding the component efficiencies available. Therefore the availability of information is very critical in predicting the design and off-design steady state performance of a gas turbine.

A major task facing a gas turbine modeller is the number of variable input and output parameters. According to Mattingly et al.,(1987), considering all the intermediate and supporting parameters, there are at least 35 such parameters, but out of these 7 parameters form the crux of the design and off-design analysis of an engine.

## CHOICE OF DESIGN VARIABLES

The fixing of the four main thermodynamic variables, TET, OPR, BPR and FPR defines the design cycle completely because these four variables determine the design parameters such as flow areas of the exhaust nozzle and of the turbine for a fixed geometry engine. In the ensuing section on off-design analysis we will discuss how the nozzles regulate the aerothermodynamic properties of the upstream components.

The rest of the design point analysis of a turbofan is straightforward and can be found in any text such as Gas Turbine Theory by Cohen, Rogers and Saravanamuttoo (1987). Figure 1 shows a schematic view of a twin spool turbofan with the station numbering.

## SIMPLIFYING ASSUMPTIONS

1. The LP and HP compressors are assumed to have equal polytropic efficiencies.
2. The polytropic efficiency of the turbine is assumed to be constant for all operating conditions.
3. The high pressure and low pressure turbines have equal polytropic efficiencies.
4. The expansion in the exhaust nozzle(s) is assumed to be ideal, i.e. there is no under or overexpansion, hence the area of the fixed nozzle remains constant throughout.
5. The mass of fuel added is taken equal to the amount of air bled for cooling of disc and bearings.

## OFF-DESIGN ANALYSIS

A turbofan engine is more complicated than a turbojet because it has two nozzles – one for the cold flow and the other for the hot flow; depending on the throttle settings and the flight conditions, the nozzles may be unchoked or choked. The method of estimating the off-design performance has thus to be based on the assumption that the nozzles may be either choked or unchoked. When studying off-design performance, the design variables are already selected and the independent variables are basically the flight conditions and the throttle settings.

### Matching of turbines with exhaust nozzle

For off-design analysis it is necessary to understand the matching procedures between the turbine and exhaust nozzle. In this study, the mass flow characteristics of turbines have been approximated by nozzle characteristics, i.e. *a single mass flow curve with a constant turbine efficiency is assumed.* The effect of turbine rotational speed on mass flow and efficiency has been assumed to be of secondary nature because of the very restricted range of the turbine when operating in front of a nozzle (Cohen et al.,1987). To match the turbine mass flow and hot exhaust nozzle mass flow we have the relation,

$$\frac{m_h\sqrt{T_{t_7}}}{P_{t_7}} = \frac{m_h\sqrt{T_{t_6}}}{P_{t_6}}\frac{P_{t_6}}{P_{t_7}}\sqrt{\frac{T_{t_7}}{T_{t_6}}} \qquad (1)$$

where $m_h\sqrt{T_t}/P_t$ is derived from the non dimensional mass flow term $m\sqrt{RT}/AP$ ; if we assume fixed areas of cold and hot nozzles $A_c$ and $A_h$ respectively ,we can get $m_h\sqrt{T_{t_7}}/P_{t_7}$ and $m_a\sqrt{T_{t_3}}/P_{t_3}$. At the design point we know $m\sqrt{T_t}/P_t$ at the nozzles, and since $m\sqrt{T_tR}/AP_t$ is a function of nozzle pressure ratio, $A_c$ and $A_h$ can be easily calculated. The temperature ratio across the LP turbine is given by $T_{t_7}/T_{t_6} = (P_{t_7}/P_{t_6})^{\eta_t(\gamma_g-1)/\gamma_g}$,

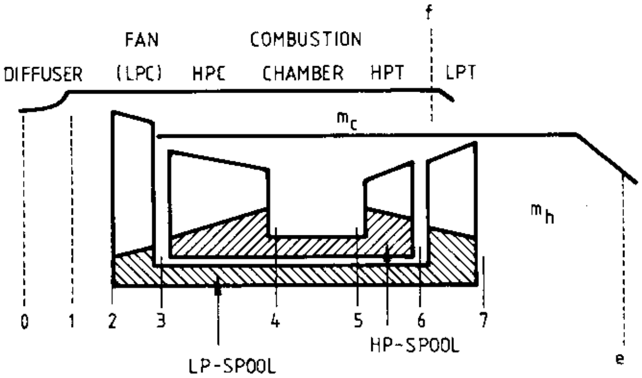


FIGURE 1. SCHEMATIC VIEW OF A TWIN SPOOL TURBOFAN.

hence equation (1) can be written as,

$$\frac{m_h\sqrt{T_{t_7}}}{P_{t_7}} = \frac{m_h\sqrt{T_{t_6}}}{P_{t_6}}\left(\frac{P_{t_6}}{P_{t_7}}\right)^{1-\frac{\eta_t(\gamma_g-1)}{2\gamma_g}} \qquad (2)$$

where $m_h\sqrt{T_{t_6}}/P_{t_6}$ is the LP turbine mass flow function. The mass flow through the nozzle is a function of pressure ratio across the nozzle, i.e. $m_h\sqrt{T_{t_7}}/P_{t_7} = f(P_{t_7}/P_O)$, and hence by equation (2) we can find the turbine pressure ratio for any nozzle pressure ratio $P_{t_7}/P_O$. The matching of turbines and a choked exhaust nozzle is shown in Figure 2.

## Analysis of HP Spool

The off-design performance calculations in the case of turbofans is based on the assumption that *the low pressure turbine (LPT) is always choked.* The assumption of choked LPT fixes the operating point of the high pressure turbine (HPT). Moreover it simplifies the analysis of turbofan performance, as now the behaviour of the high pressure spool can be estimated in the same way as for a single spool turbojet engine with a choked nozzle flow.

When the LPT is choked, it can be seen from Figure 2 that there is a *fixed HP turbine operating point,* which is independent of LPT pressure ratio. Thus the maximum pressure ratio across the HP turbine is controlled by choking of the LPT. Due to the fixed operating point of the HP turbine, the temperature ratio $T_{t_6}/T_{t_5}$ is fixed, and is the same for both design and off-design conditions. When we deal with the *compatibility of work* between the various components, we know that power supplied by the HP turbine to drive the HP compressor is given as,

$$m_h C_{p_a}(T_{t_4} - T_{t_3}) = \eta_m m_h C_{p_g}(T_{t_5} - T_{t_6}) \qquad (3)$$

Now, the compressor temperature ratio is given as $T_{t_4}/T_{t_3} = \epsilon^{(\gamma_a-1)/\eta_c\gamma_a}$ hence equation (3) can also be written as

$$m_h C_{p_a} T_{t_3}(T_{t_4}/T_{t_3} - 1) = \eta_m m_h C_{p_g} T_{t_5}(1 - T_{t_6}/T_{t_5})$$

With further modification equation (3) becomes

$$\frac{T_{t_4}}{T_{t_3}} - 1 = \eta_m \frac{C_{p_g}}{C_{p_a}}\,\frac{T_{t_5}}{T_{t_3}}\,\left(1 - \frac{T_{t_6}}{T_{t_5}}\right) \qquad (4)$$

The compressor temperature ratio is given as $T_{t_4}/T_{t_3} = \epsilon^{(\gamma_a-1)/\eta_c\gamma_a}$ and since mechanical efficiency $\eta_m$, $C_{p_g}$ and $C_{p_a}$ are assumed constant, equation (4) can be written as

$$\frac{T_{t_4}/T_{t_3} - 1}{(T_{t_4}/T_{t_3})_d - 1} = \frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_d}$$

because $T_{t_6}/T_{t_5}$ is fixed, due to the fixed operating point of the HP turbine.

The above equation can be further modified to give

$$\frac{\epsilon_c^{(\gamma_a-1)/\eta_c\gamma_a} - 1}{(\epsilon_c)_d^{(\gamma_a-1)/(\eta_c)_d\gamma_a} - 1} = \frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_d}$$

The compressor pressure ratio at a new throttle setting $(T_{t_5}/T_{t_3})$ becomes

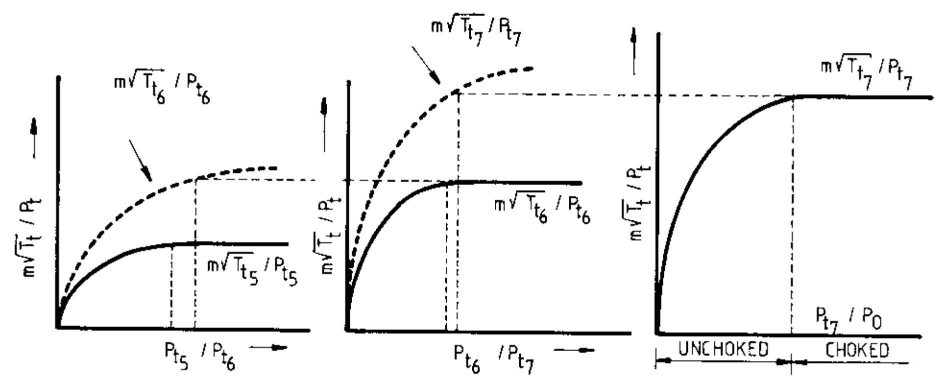


FIGURE 2. MATCHING OF CHOKED TURBINES WITH THE CHOKED EXHAUST NOZZLE OF A TWIN SPOOL TURBOFAN.

$$\epsilon_c = \left(1 + \frac{(T_{t_5}/T_{t_3})}{(T_{t_5}/T_{t_3})_d} \left((\epsilon_c)_d^{\frac{\gamma_a-1}{(\eta_c)_d\gamma_a}} - 1\right)\right)^{\frac{\eta_c\gamma_a}{\gamma_a-1}} \qquad (5)$$

functions of the same variable $P_{t_4}/P_O$ one can get the equilibrium point of the LP spool. The matching of compressor and turbine/nozzle characteristic curves is shown in Figure 3.

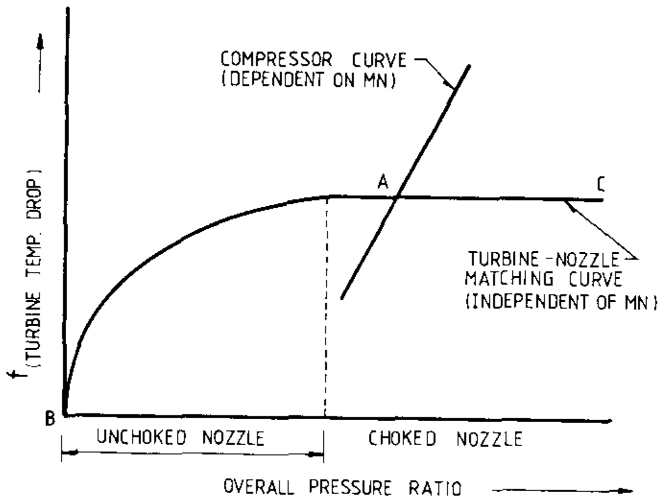


FIGURE 3. MATCHING OF COMPRESSOR AND TURBINE-NOZZLE CHARACTERISTIC CURVES.

## Programming Aspect

The basic requirement is estimation of the performance of an aircraft at varying flight Mach numbers and altitudes. For a given flight Mach number and turbine entry temperature $T_{t_5}$, a range of $P_{t_3}/P_O$ is assumed which gives a range of fan pressure ratios $\epsilon_f$ because

$$\frac{P_{t_3}}{P_O} = \frac{P_{t_3}}{P_{t_2}}\frac{P_{t_2}}{P_{t_O}}\frac{P_{t_O}}{P_O} = \epsilon_f(1+\eta_i\frac{(\gamma_a-1)}{2} * M_O^2)^{\frac{\gamma_a}{\gamma_a-1}} \qquad (10)$$

Now from the known turbine inlet temperature $T_{t_5}$ and $T_{t_2}$ one can get a range of $T_{t_5}/T_{t_3}$ because

$$\frac{T_{t_5}}{T_{t_2}} = \frac{T_{t_5}}{T_{t_3}}\frac{T_{t_3}}{T_{t_2}} = \frac{T_{t_5}}{T_{t_3}} * \epsilon_f^{\frac{(\gamma_a-1)}{\eta_f\gamma_a}} \qquad (11)$$

Knowing $T_{t_5}/T_{t_3}$ it is easy to get a range of $\epsilon_{HPC}$ and $m_h\sqrt{T_{t_3}}/P_{t_3}$ from equations (5) and (7). Hence the operating point on the HPC working line now varies with $P_{t_3}/P_O$.

For the same range of $P_{t_3}/P_O$ one can get a range of $m_c\sqrt{T_{t_3}}/P_{t_3}$ from the cold nozzle characteristics because the cold nozzle exhaust area $A_c$ is assumed to be fixed. Knowing $m_c\sqrt{T_{t_3}}/P_{t_3}$ and $m_h\sqrt{T_{t_3}}/P_{t_3}$, a range of bypass ratios is calculated, with $\lambda = \frac{m_c\sqrt{T_{t_3}}/P_{t_3}}{m_h\sqrt{T_{t_3}}/P_{t_3}}$. A range of $T_{t_3}/T_{t_2}$ is obtained from various values of $\epsilon_f$ that have been calculated previously, because $T_{t_3}/T_{t_2} = \epsilon_f^{\gamma_a-1/\eta_f\gamma_a}$. Moreover a range of overall pressure ratios $P_{t_4}/P_O$ is obtained because $P_{t_4}/P_O = (\epsilon_{HPC})(P_{t_3}/P_O)$. Hence all the terms of the left hand side of equation (9) can be

On satisfying the essential condition of *compatibility of flow* between the turbine and compressor we get

$$\frac{m_h\sqrt{T_{t_3}}}{P_{t_3}} = \frac{m_h\sqrt{T_{t_5}}}{P_{t_5}}\sqrt{\frac{T_{t_3}}{T_{t_5}}}\frac{P_{t_5}}{P_{t_4}}\frac{P_{t_4}}{P_{t_3}} \qquad (6)$$

In this expression the turbine mass flow is given by $Q_5$, where $Q_5 = m_h\sqrt{T_{t_5}}/P_{t_5}$ is determined by the HP turbine operating point and is the same for both design and off-design conditions. Both $m_h\sqrt{T_{t_5}}/P_{t_5}$ and $P_{t_5}/P_{t_4}$ can be assumed to be the same for both design and off-design conditions.

Hence equation (6) can be modified to give

$$\frac{m_h\sqrt{T_{t_3}}}{P_{t_3}} = (Q_{3_d})\frac{(\sqrt{T_{t_3}/T_{t_5}})}{(\sqrt{T_{t_3}/T_{t_5}})_d}\frac{\epsilon_c}{(\epsilon_c)_d} \qquad (7)$$

where $Q_{3_d} = (\frac{m_h\sqrt{T_{t_3}}}{P_{t_3}})_d$.Thus by using equation (7), a new value of compressor mass flow can be found, and since we have already determined the new compressor pressure ratio from equation (5), hence we can get a working line by varying $T_{t_5}/T_{t_3}$. This means that the working line on the high pressure compressor map is fixed and is given by the relationship

$$\epsilon_{HPC} = f(m_h\sqrt{T_{t_3}}/P_{t_3}, T_{t_5}/T_{t_3})$$

## Analysis of LP Spool

Having analysed the HP spool, we can now consider the power balance for the low-pressure spool which is given below

$$(m_c+m_h)C_{p_a}(T_{t_3}-T_{t_2}) = \eta_m m_h C_{p_g}(T_{t_6}-T_{t_7}) \qquad (8)$$

The assumption that the amount of bled air (for disc cooling) is equal to the mass of fuel added is implicit in equation (8) On further modification we get

$$(\lambda+1)(1-T_{t_2}/T_{t_3}) = \eta_m\frac{C_{p_g}}{C_{p_a}}\frac{T_{t_5}}{T_{t_3}}\frac{T_{t_6}}{T_{t_5}}(1-T_{t_7}/T_{t_6}) \qquad (8a)$$

But the temperature ratio $T_{t_6}/T_{t_5}$ is fixed, due to the fixed working point of HPT. From equation (8*a*) we can get

$$\left(\frac{\lambda+1}{\lambda_{des}+1}\right)\frac{(1-T_{t_2}/T_{t_3})}{(1-(T_{t_2}/T_{t_3})_{des})} = \psi\frac{(1-T_{t_7}/T_{t_6})}{(1-(T_{t_7}/T_{t_6})_{des})} \qquad (8b)$$

where $\psi = \frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_{des}}$, hence (8b) can be further modified to

$$\frac{1}{\psi}\left(\frac{\lambda+1}{\lambda_{des}+1}\right)\frac{(1-T_{t_2}/T_{t_3})}{1-(T_{t_2}/T_{t_3})_{des}} = \frac{(1-T_{t_7}/T_{t_6})}{1-(T_{t_7}/T_{t_6})_{des}} \qquad (9)$$

Now the left hand side of the equation (9) is a unique function of cold nozzle pressure ratio $P_{t_3}/P_O$ for a given flight Mach number $M_O$, and $\psi$ and can be plotted against $P_{t_4}/P_O$ to give a compressor curve. Similarly the right hand side of the equation (9) is a function of hot nozzle pressure ratio $P_{t_7}/P_O$, and when plotted against overall pressure ratio $P_{t_4}/P_O$ is known as turbine/nozzle characteristic curve. By estimating both the left hand side and the right hand side of (9) as calculated as functions of $P_{t_4}/P_O$ and a curve drawn between the left hand side of equation (9) vs $P_{t_4}/P_O$ for a given flight Mach number, altitude and turbine entry temperature $T_{t_5}$.

For the LPT and the hot exhaust nozzle, the mass flow matching equation is given as

$$\frac{m_h\sqrt{T_{t_7}}}{P_{t_7}} = \frac{m_h\sqrt{T_{t_6}}}{P_{t_6}}\frac{P_{t_6}}{P_{t_7}}\sqrt{\frac{T_{t_7}}{T_{t_6}}} \quad (12)$$

Since the LPT is assumed to be choked throughout the design and off-design conditions, the mass flow parameter $\frac{m_h\sqrt{T_{t_6}}}{P_{t_6}}$ is constant. Hence

$$\frac{m_h\sqrt{T_{t_7}}}{P_{t_7}} = \left(\frac{m_h\sqrt{T_{t_7}}}{P_{t_7}}\right)_{des}\left(\frac{P_{t_6}/P_{t_7}}{(P_{t_6}/P_{t_7})_{des}}\right)^{1-\frac{\eta_t(\gamma_g-1)}{2\gamma_g}} \quad (13)$$

because

$$T_{t_7}/T_{t_6} = (P_{t_7}/P_{t_6})^{(\gamma_g-1)\eta_t/\gamma_g} \quad (14)$$

On assuming a range of $P_{t_7}/P_O$ we can get a range of $m_h\sqrt{T_{t_7}}/P_{t_7}$ from the hot nozzle characteristics assuming the hot nozzle exit area $A_h$ is fixed. Then from equation (12) it is easy to get a range of LP turbine pressure ratios $P_{t_7}/P_{t_6}$. From (14) we can get a range of $T_{t_7}/T_{t_6}$ values. Moreover a range of overall pressure ratios $P_{t_4}/P_O$ is obtained because

$$\frac{P_{t_4}}{P_O} = \frac{P_{t_7}}{P_O}\ \frac{P_{t_6}}{P_{t_7}}\ \frac{P_{t_5}}{P_{t_6}}\ \frac{P_{t_4}}{P_{t_5}}$$

where $P_{t_6}/P_{t_5}$ is the fixed pressure ratio of HPT, while $P_{t_5}/P_{t_4}$ is the assumed combustor pressure loss (remains constant throughout). Hence all the terms of the right hand side of the equation (9) are known, and a curve between the right hand side and $P_{t_4}/P_O$ can be drawn. It is important to note that the right hand side of equation (9) is independent of $\psi$, Mach number and altitude.The equilibrium condition of the low pressure spool is determined by the intersection of the two curves representing the left hand and right hand sides of equation (9) as functions of $P_{t_4}/P_O$.

At the equilibrium point, the corresponding values of *compressor pressure ratio, fan pressure ratio, bypass ratio* and $m_h\sqrt{T_{t_3}}/P_{t_3}$ are selected. From the fan pressure ratio the values of $T_{t_3}$ and $P_{t_3}$ are easily determined. Knowing the compressor pressure ratio, it is easy to get the values of $T_{t_4}$ and $P_{t_4}$, as $T_{t_3}$ and $P_{t_3}$ are already known. Since the turbine inlet temperature is specified beforehand, the fuel flow can also be determined. Since the values of $m_h\sqrt{T_{t_3}}/P_{t_3}$, $T_{t_3}$ and $P_{t_3}$ are known, it is easy to get the value of hot (core) mass flow because

$$m_h = \frac{m_h\sqrt{T_{t_3}}}{P_{t_3}}\frac{P_{t_3}}{\sqrt{T_{t_3}}}$$

Knowing the value of $m_h$, the mass flow of cold air can also be found as $m_c = m_h\lambda$.

All these parameters can then be substituted in the power equation of the HP spool,

$$m_h C_{p_a} T_{t_3}(T_{t_4}/T_{t_3} - 1) = \eta_m m_h C_{p_g} T_{t_5}(T_{t_6}/T_{t_5} - 1)$$

to get $T_{t_6}$. Knowing $T_{t_6}/T_{t_5}$ we can get the pressure ratio across the HP turbine. Again, if the known parameters are substituted in the LP spool power equation,

$$(m_c + m_h)C_{p_a}(T_{t_3} - T_{t_2}) = \eta_m m_h C_{p_g}(T_{t_6} - T_{t_7})$$

then $T_{t_7}$ can be found as it is the only unknown. Hence the pressure ratio across the LP turbine can also be calculated, and if one knows $P_{t_7}$ then the pressure ratio across the hot nozzle can easily be calculated.

Since $P_{t_3}$ and $P_{t_7}$ are known, the thrusts for the hot and cold nozzle can be calculated as shown in the design point analysis. Thus all the important performance parameters such as fuel flow, thrust and specific fuel consumption can be calculated by the procedure described.

## VALIDATION OF MODEL

The purpose of this analysis is to validate the mathematical model with published values. The overall parameters we wish to estimate are fuel flow and gross thrust developed at various flight conditions. We can also determine compressor pressure ratio, air flow rate, gross thrust parameter, exhaust gas parameter etc; but they are not of primary importance as compared to fuel flow and net thrust.

### Twin Spool Turbofans

The Garrett TFE 731-2 is a twin spool turbofan and at its design point (ISA SLS) it develops a thrust of 15.57 KN (3500 lbf) at takeoff conditions, with an sfc of 0.491 lb/hr-lb. It is a medium bypass ratio, two-spool turbofan engine, incorporating a single-stage gear-driven fan.

An important thing to be considered is that the efficiencies ( whether polytropic or isentropic) of the components are to be selected such that at the design point the correct design thrust, fuel flow and exit nozzle areas can be matched. It is necessary to iterate until a satisfactory solution is obtained.

When the aircraft is operating at maximum cruise conditions, one can vary the flight Mach number at a particular altitude and predict values of net thrust and fuel flow from the developed computer program. Figure 4, 5 and 6 show the predicted values of thrust and fuel flow at different altitudes as compared with the published data (Garrett 731 brochure). It can be seen that even when the component efficiencies (polytropic) are assumed constant throughout, there is a good relationship between the predicted and published data.

An important point noticed was that bypass ratio increases as the thrust is cut back. The reason why bypass ratio increases can be found by simple logical reasoning as shown. Since bypass ratio is given as $m_c/m_h$ it can be written in terms of exit nozzle mass flow parameters as

$$\frac{m_c}{m_h} = \left(\frac{m_c\sqrt{T_{t_2}}}{A_c P_{t_2}}\right)\left(\frac{A_h\sqrt{P_{t_7}}}{m_h T_{t_7}}\right)\frac{A_c}{A_h}\frac{P_{t_2}}{P_{t_7}}\sqrt{\frac{T_{t_7}}{T_{t_2}}}$$

Now in the cruise conditions the nozzles are choked and so $\frac{m_c\sqrt{T_{t_2}}}{A_c P_{t_2}} = 0.396$ and $\frac{m_h\sqrt{T_{t_7}}}{A_h P_{t_7}} = 0.389$. So bypass ratio now depends only on

$$\frac{m_c}{m_h} \simeq 1 * (A_c/A_h) * \frac{P_{t_2}}{P_{t_7}}\sqrt{\frac{T_{t_7}}{T_{t_2}}}$$

As the engine is throttled back there is more decrease in $P_{t_7}$ than in any other parameter, which pushes up the term $\frac{P_{t_2}}{P_{t_7}}$ and so the bypass ratio rises as thrust is cut back in cruise.

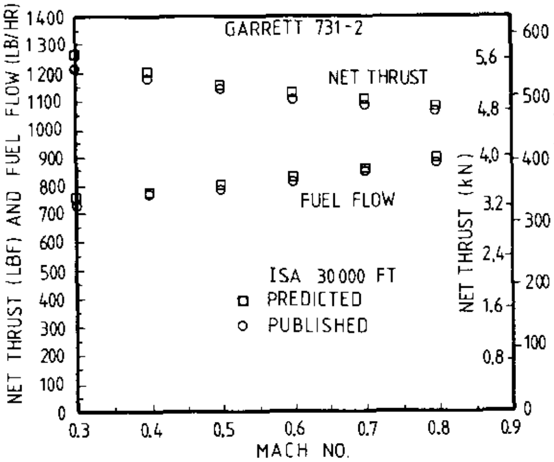


FIGURE 4. COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA.

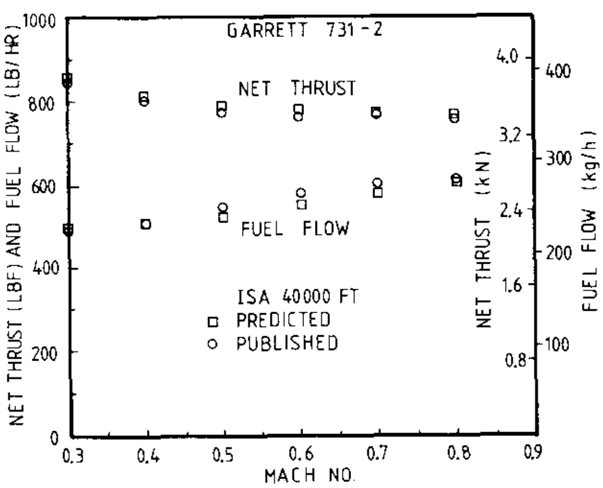


FIGURE 5. COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA.

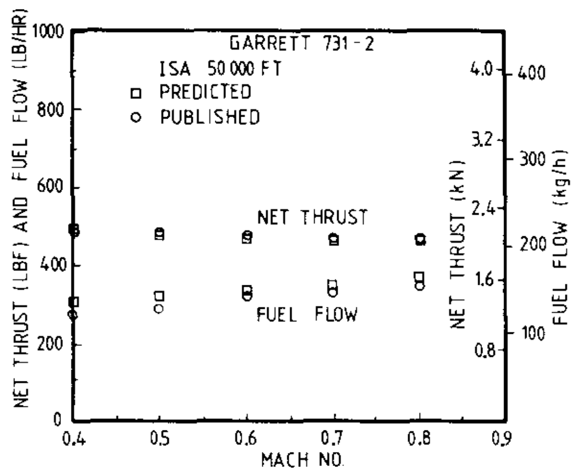


FIGURE 6. COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA.

## Three Spool Turbofans

The IP and HP spools can be considered as a single spool in the design and off-design conditions, if the polytropic efficiencies of the compressors and turbines are assumed to be equal. Hence a three-spool turbofan becomes mechanically equivalent to a twin spool turbofan using this assumption.

The added assumption that the LP turbine is choked, makes the three-spool case exactly similar to a twin-spool engine. The design point calculations are nearly the same as those for the twin spool. So one can effectively use the same program for three-spool and twin-spool engines, with the only difference that the inter-turbine and inter-compressor temperatures and pressures calculated will be totally different from the actual values.

The designation RB211 applies to a family of advanced technology three- shaft turbofans of high bypass ratio and high pressure ratios, with thrusts ranging from 166KN (37,400 lb) to 249 KN (56,000 lb). The RB211-22B fitted to the L-1011-1 and -100 Tristar is flat rated at 187 KN (42,000 lb) to 28.9°$C$ . At the cruise condition (35,000 ft and Ma=0.8), the RB211-22B produces 43.1 KN (9,700 lb) thrust and gives a specific fuel consumption (sfc)=0.618 lb/hr-lb. (Jane's All The World Aircrafts, 1987-88).

## Performance at SLS

Figure 7 shows a very good simulation of the predicted net thrust and the hot nozzle thrust with the published data. It can be seen that as the mass flow increases, the fan thrust increases much faster than the hot nozzle thrust. The fuel consumption simulation in Figure 8 is good, and the deviation between the predicted and published values is of the order of 4-5%.

From the program we can obtain non-dimensional total mass flow $\frac{m_a\sqrt{T_{t_3}/T_{O_O}}}{P_{t_3}/P_{O_O}}$ and also get the values of fan pressure ratio $\epsilon_f$ and bypass ratio $\lambda$. These values are then compared with the published data (AN: Performance Report RB211-22, 1968), and the graphical representation can be seen in Figure 9.

The fan pressure ratio simulation in Figure 9 is good but the bypass ratio does not match well with the published data.

Performance at Cruise Conditions

The aircraft is assumed flying at 30,000 feet and at Mach no.=0.8 and the program is run for various tur-

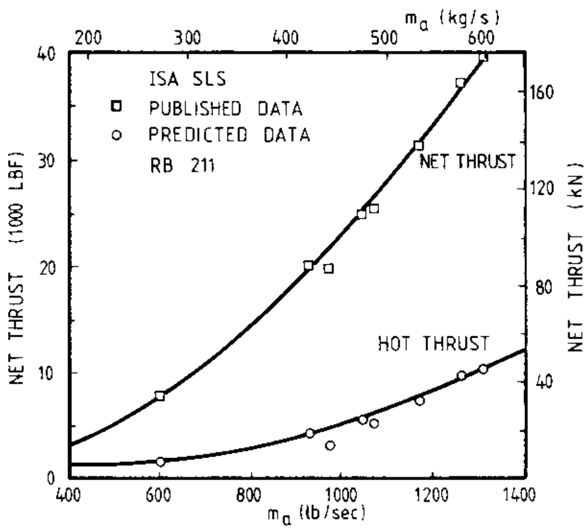


FIGURE 7. COMPARISON OF PREDICTED NET THRUST AND HOT NOZZLE THRUST WITH PUBLISHED DATA.

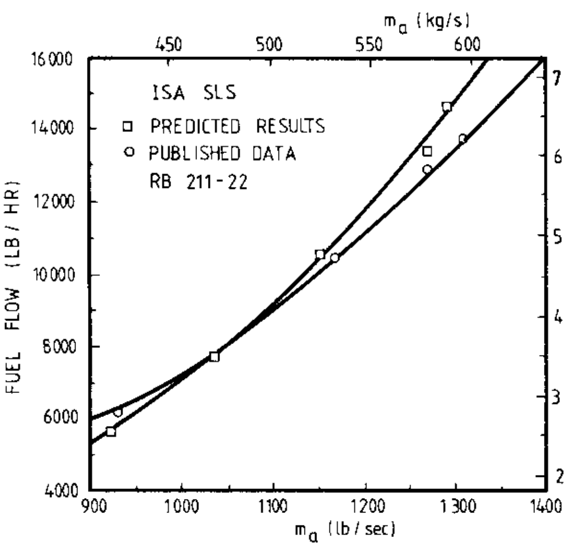


FIGURE 8. COMPARISON OF PREDICTED AND PUBLISHED FUEL FLOW.

bine inlet temperatures and the predicted values are compared with the published data. From Figure 10 one can see the invariance of the working line of IP+HP spool as it remains nearly the same as that found at ISA SLS. The reason for this is that the LP turbine always remains choked, so the IP+HP spool is shielded from the effects of varying Mach number as well as well as fr

bine

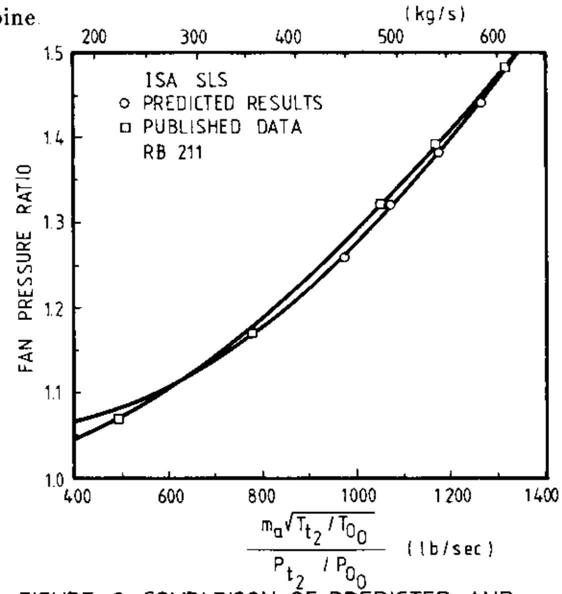


FIGURE 9. COMPARISON OF PREDICTED AND PUBLISHED FAN PRESSURE RATIO.

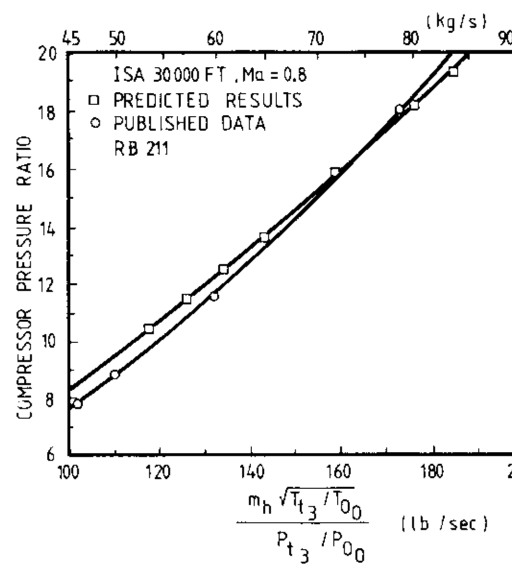


FIGURE 10. COMPARISON OF PREDICTED WORKING LINE OF IP+HP SPOOL WITH PUBLISHED DATA.

## CONCLUSIONS

1. Performance calculations as done for Garrett TFE 731-2 agree quite well with the published data, even when constant fan or compressor efficiency is assumed.

2. The simulation as performed on RB211-22 shows good agreement with published data. The values at sea-level static condition show good agreement down to 50% of the maximum thrust, after that the agreement is not reasonable. For the flight condition at a typical Mach number and altitude (30000 ft, Ma=0.8) the simulation is quite good, even at low values of thrust.
3. The aerodynamic throats formed at the propelling nozzle(s) and at the turbine nozzles dictate the behaviour of components upstream of the nozzles.
4. The mass flow parameter $m\sqrt{T_t}/P_t$ at the throat of the HP turbine remains constant for all off-design cruise conditions, and is the only parameter which remains constant at a particular cruising speed.
5. It was seen that the net hot nozzle thrust for Garrett 731-2 is nearly independent of the Mach number, while the net fan thrust decreases as the Mach number is increased. In other words it means that the effect of the Mach number is being borne by the LP spool only, as the HP spool is shielded by the LP spool from the downstream conditions.
6. The program may be useful for preliminary design analysis in aircraft design offices, and may also be used as a pedagogical aid for students interested in understanding the complexity of off-design analysis of gas turbines.

## ACKNOWLEDGEMENTS

Financial support for this study was provided by the Natural Sciences and Engineering Research Council of Canada.

## REFERENCES

AN: Performance Specification and Installation Notes, 1968, RB.211-22, Rolls-Royce Lim.(Taken from Wittenberg, H., 1976).

Cohen. H., Rogers, G.F.C., and Saravanamuttoo, H.I.H., 1987 Gas Turbine Theory, 3rd ed., Longman, London.

Garrett TFE 731-2 brochure, Allied Signal Aero- space Company, 1988.

Mattingly J.D., Heiser W., and Daley D.H., 1987, Aircraft Engine Design, AIAA Education series.

Saravanamuttoo, H.I.H., 1972,"A rapid method for the matching of two spool turbojets", Canadian Aeronautics and Space Journal.

Saravanamuttoo, H.I.H., 1972, "A rapid matching procedure for twin spool turbofans", Canadian Aeronautics and Space Journal.

Taylor J.W.R., 1988, Jane's All The World Aircraft 1987-88, Jane's Publishing Inc.

Wittenberg, H., 1976, "Prediction of off design performance of turbojet and turbofan engines", AGARD CP-242-76 Proceedings on Performance Prediction Methods.