

**Title: Offshore floating vertical axis wind turbines, dynamics modelling state of the art. Part II: Mooring Line & Structural Dynamics**

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Keywords: floating offshore wind turbines; VAWT; vertical axis wind turbine; coupled dynamics; mooring; structural dynamics

## **Abstract**

The need to exploit enhanced wind resources far offshore as well as in deep waters requires the use of floating support structures to become economically viable. The conventional three-bladed horizontal axis wind turbine may not continue to be the optimal design for floating applications. Therefore it is important to assess alternative concepts in this context that may be more suitable. Vertical axis wind turbines (VAWTs) are a promising concept, and it is important to first understand the coupled and relatively complex dynamics of floating VAWTs to assess their technical feasibility. As part of this task, a series of articles have been developed to present a comprehensive literature review covering the various areas of engineering expertise required to understand the coupled dynamics involved in floating VAWTs. This second article focuses on the modelling of mooring systems and structural behaviour of floating VAWTs, discussing various mathematical models and their suitability within the context of developing a model of coupled dynamics for. Emphasis is placed on computational aspects of model selection and development as computational efficiency is an important aspect during preliminary design stages. This paper has been written both for researchers new to this research area, outlining underlying theory whilst providing a comprehensive review of the latest work, and for experts in this area, providing a comprehensive list of the relevant references where the details of modelling approaches may be found.

## **Introduction**

The need to increase renewable energy's share in global energy production and to exploit offshore wind resources is moving wind farms further offshore and into deeper waters. In depths greater than 50 metres, bottom-mounted (i.e. fixed) support structures for offshore wind turbines may not remain the most economically viable option [1; 2]. A transition from fixed to floating support structures is essential for deep offshore wind farms to become economically viable in the near future.

The onshore wind industry has reached a relatively mature level, with the majority of large scale wind turbines sharing the same configuration: horizontal axis of rotation, three blades, upwind, variable-speed and variable blade pitch (with feathering capability). This has been the result of several decades of research and development; originally several configurations had been considered, including horizontal axis wind turbines (HAWTs) with a different number of blades, but also vertical axis wind turbine (VAWT) configurations [3]. The conventional design emerged as the optimum techno-economic trade-off for the onshore large scale wind market.

The same "evolutionary process" did not take place for the offshore wind market, substituted by a "marinisation" of the trusted configurations used for the onshore market. It has been implicitly assumed that, despite the very different environmental conditions of an offshore environment, the optimum configuration for the wind turbine is the same i.e. the conventional three bladed, upwind,

horizontal axis wind turbine. This has been implicitly assumed not only for the bottom-mounted offshore wind turbine configurations, but also for proposed floating systems. The proven technology of HAWTs aids in de-risking commercial-scale offshore wind farms.

It is therefore important to assess the technical and economic feasibilities of alternative wind turbine configurations for the offshore floating wind industry in order to ensure that the most suitable configurations are employed, with VAWTs being one promising category of wind turbines. The first step is to understand the complex dynamics of such a floating system subjected to the harsh offshore environment. As part of this task, a series of articles have been developed to present a comprehensive literature review covering the various areas of engineering expertise required to understand the coupled dynamics involved in floating VAWTs.

In part one of this series [4], an in depth review of different aerodynamic engineering models for VAWTs and their suitability for floating applications is presented. In the present second article, approaches to adequately model mooring systems and the structural behaviour of floating offshore wind turbines (FOWTs) during preliminary design stages are described and discussed, with particular emphasis on floating VAWT characteristics and computational aspects. Examples of current implementations by research groups are also reviewed. The third part in this series [5] focuses on appropriate hydrodynamic models and coupled modelling methodologies for investigating the coupled dynamics of floating VAWTs during preliminary design stages.

This paper aims to review appropriate mooring line and structural engineering models, outlining relative advantages and limitations considering floating VAWTs and impacts of model fidelity. A discussion is presented concerning the computational aspects when developing efficient coupled dynamics models and a review of current implementations used by research groups for the analysis of both floating HAWTs and VAWTs.

## **2. Mooring line dynamics**

Mooring lines act as the station keeping system for FOWTs, maintaining the position of the FOWT on the sea surface as well as contributing to platform stability under environmental loading. Whilst the importance of mooring line modelling may be sometimes overlooked in FOWT coupled dynamics modelling, mooring lines are critical to platform dynamic response. In the wider offshore industry two different types of mooring lines have been used predominantly [6]:

1. Catenary mooring lines: are freely hanging chain lines or wires connecting a floating surface platform to anchors on the seabed some distance from the platform. A combination of the mooring line mass and anchor horizontal forces maintain station-keeping of the platform (restricting the surge, sway, and yaw degrees of freedom). An example is depicted in Figure 1a.

2. Tensioned mooring lines: Taut, lightweight elastic lines that are connected vertically (Figure 1b) or at an incline (Figure 1c, also known as spread moorings) to the platform maintain platform position through elastic forces when the platform perturbs from its equilibrium position. Differently from the previous, it generates significant restoring forces in all the six DOF.

Figure 1 – Simplified floating VAWT schematic highlighting: a) catenary mooring lines; b) inclined tensioned moorings; c) vertical tensioned moorings.

In very deep waters, mooring systems can represent a significant cost of the total system, and whilst this is not such a significant issue with the offshore oil & gas industry where profit margins are very large, it is an important factor for FOWTs where achieving a cost-effective and profitable system is one of the main challengers facing developers [7]. Whilst chain catenary moorings proven in the oil & gas industry have been used for floating HAWT prototypes WindFloat [8] and Hywind [9], cost reductions through the use of novel taut fibre moorings (see Robertson et al. [10] for such systems) is possible. Table 1 provides a brief comparison of the advantages and disadvantages of catenary and tensioned mooring lines for station-keeping of FOWTs. The development of novel mooring line configurations and materials, as well as combinations of the abovementioned mooring systems (see e.g. Ren et al. [11]) is required to lower the eventual cost of energy from FOWTs. Following on from this, there is a need for models that adequately represent these novel mooring systems within FOWT coupled dynamics design models.

In the following subsections a number of models shall be described that are suitable for use during the preliminary design of a FOWT (for both catenary and tensioned mooring systems), with particular attention to floating VAWTs.

Table 1 – Comparison of catenary and tensioned mooring systems

## 2.1 Linear Force-Displacement-Velocity Model

The simplest model for mooring lines is the linear force-displacement-velocity (FDV) relationship, mathematically represented in Eqn 1.

$$\mathbf{F}_{mooring}(t) = \mathbf{C} \cdot \dot{\mathbf{x}}(t) + \mathbf{K} \cdot \mathbf{x}(t) \quad (\text{Eqn 1})$$

In this model resistances to motion in some or all degrees of freedom (DOFs), depending on the specific mooring design, are introduced to represent the station-keeping characteristics. Mooring stiffness ( $\mathbf{K}$ ) is usually a function of the augmentation of the direction of mooring line forces and

mooring line elasticity, whilst mooring damping ( $\mathbf{C}$ ) is a function of structural and hydrodynamic damping present in the mooring system, but they are considered constrained (linear approach). This model is not very accurate, but may be used in the initial analysis of a moored floating structure as it can adequately represent the mooring system characteristics on the global motions of FOWTs. Since there is no direct representation of individual mooring lines, it is not possible to investigate individual mooring line forces. On the other hand, it may be quickly applied in a numerical model and produces clear performance trends for engineers to assess the preliminary requirements for a mooring system.

An extension of this approach is to use a nonlinear force-displacement-velocity relation that can more accurately represent the behaviour of the mooring lines, given by Eqn 2 where the mooring damping and stiffness matrices are now a function of the 6 DOF platform motion (non linear approach). One challenge in using this method is to identify the appropriate values for stiffness and damping in the relevant degrees of freedom and any couplings, as this would usually require reverting to more complex models to derive the final values. On the other hand the linearity of the above model allows it to be used during control system design, optimisation and implementation.

$$\mathbf{F}_{mooring}(t) = \mathbf{C}(\mathbf{x}) \cdot \dot{\mathbf{x}}(t) + \mathbf{K}(\mathbf{x}) \cdot \mathbf{x}(t) \quad (\text{Eqn 2})$$

## 2.2 Quasi-Static Model

An improvement to the linear FDV model is the quasi-static (Q-S) approach. The Q-S model (see, for example [12]) involves an analytical derivation of the mooring line tensions and is usually applied to catenary mooring systems, although modifications are possible to include tensioned mooring systems. Although it can include buoyancy forces, elastic stretching and seabed friction, it does not consider the inertia, hydrodynamic and elastic damping of the mooring lines. This approach has been adopted in some studies, but may yield unsatisfactory results in particular operating conditions and a move towards nonlinear dynamic models is preferable beyond initial design stages [13-16].

An expansion of the Q-S model called the multi-segmented, quasi-static (MSQS) model [17], has been developed to allow for multiple combinations of mooring lines within three-dimensional space. In this modification, the analysis of a three-dimensional mooring system is possible, although each mooring line ‘segment’ must lie within a two-dimensional plane. By applying the conventional Q-S model to each of these ‘segments’, the corresponding motions and forces at the mooring line-platform connection may be solved through spatial transformations.

## 2.3 Dynamic Models

Taking a step further, dynamic models incorporate the inertial characteristics of the mooring lines and do not make use of the force equilibrium assumption.

### 2.3.1 Multibody approach

One promising approach is the multibody formulation as described by Kreuzer and Wilke [18], and in relation to FOWTs by Cordle [13], Muskulus [14] and Matha et al. [15]. This approach discretizes each mooring line into a number of rigid or flexible elements that are each connected by a spring-damper system, as shown in Figure 2.

Figure 2- Multibody representation (adapted from [15])

The multibody formulation is known also as Lumped Mass Method (LMM) that involves ‘lumping’ all force contributions from external forces, mass and internal reactions at a specific number of nodes along the length of the mooring line. This results in obtaining a set of differential equations by assuming dynamic equilibrium and stress/strain continuity at each node (see Kurian et al. [19]). Its ability to accommodate large-amplitude, three-dimensional motions and hydrodynamic drag forces, whilst not being too computationally expensive, makes this approach very attractive. A limitation with most LMM implementations is that line structural damping and torsional stiffnesses are not included [18], although the consideration of these mooring line characteristics would only be required during the detailed design stage of the mooring system. An advantage is that the multibody formulation uses the same underlying mathematics that describes the structural dynamics (see below), thereby using common modules for simulation execution. This will lead to a shorter development time and a more streamlined and robust model.

### 2.3.2 Finite-element approach

More detailed dynamic models utilising the finite-element (FE) approach are also possible, but the finer discretisation and more complex mathematical formulation of the mooring lines may lead to more computationally intensive simulations that is undesirable for FOWT coupled dynamics codes, as will be discussed in §4. As highlighted by Kurian et al. [19], the FE approach makes use of interpolation functions to calculate an internal system state in terms of the mooring line nodal displacements within a generalised coordinate system. Hence by applying these interpolation functions to the established kinematic relations, and also dynamic equilibrium, the motion of each mooring line element may be obtained. Although this results in more computationally intensive

algorithms, the FE approach allows for the analysis of systems with considerable torsional and bending stiffnesses. Hall et al. [20] utilised the FE approach to assess the accuracy of lower-order models, finding that the selection of appropriate models should be done on a case-by-case basis. Table 2 provides a brief overview of the capabilities of the models outlined above.

Table 2 – Mooring line models capabilities

## **2.4 Considerations for Tensioned Moorings**

Whilst all the above mentioned models are applicable to catenary mooring systems, tensioned moorings usually require a different mathematical treatment. One of the first mathematical models for tensioned moorings, in particular tension-leg-platforms (TLPs), was developed by Morgan and Malaeb [21], where couplings between different DOFs as well as nonlinearities are modelled in the time domain. This model falls within the nonlinear force-displacement classification described above. Recently there have been a number of studies investigating TLP support structures for FOWTs [22-28].

Obviously the higher-fidelity dynamic models described in §2.3 are still applicable for tensioned moorings, although care must be taken to accommodate phenomena specific to this type of mooring system. One characteristic seen in tensioned moorings when one or more of the tethers goes slack (that is, tension within the line falls to zero) and quickly becomes taut again due to wave-induced platform motion, imparting shock loads on the FOWT. Another issue is resonant motion of the tensioned moorings, which may be encountered through hydrodynamic forces (drag and vortex-induced forces) and support structure excitation (platform motion and aerodynamic loading) [29]. Hence it is important to consider these aspects during the development of computational models such that they are not overlooked during the design of a FOWT. For instance, the linear force-displacement relation is not adequate to take into account slack tethers, but could be modified with some conditional statements to accommodate snap loads. The use of dynamic models can include a wide variety of nonlinear phenomena pertaining to tensioned moorings, but require more development and computational resources.

## **2.5 Considerations for Floating VAWTs**

One issue that has not arisen with floating HAWTs due to their inherent design is the effect of the rotor-generator interaction. As the torque from an operational VAWT assembly is generated about the vertical axis, an oscillatory moment in yaw is imparted on the floating support. With no fixed structure to compensate for this torque, the mooring lines will have to accommodate this additional load in a VAWT configuration. When using the Q-S model to analyse potential novel mooring

configurations, in particular with discontinuities (e.g. buoyancy aids), the numerical stability of the computational procedure becomes an issue, particularly with the implementation of the conventional force-based approach to obtaining system equations. One alternative that may greatly reduce computational instabilities is to apply the energy variation principle, as first introduced to ocean engineering by Ma et al. [30]. By deriving the mooring line system equations based on the energy transfer within the system, a more robust computational scheme can be employed to solve these system equations (see e.g. Collu et al. [31]).

Another aspect that should be considered when selecting a model for the mooring lines is the possibility of distributed mooring systems for multiple floating turbines, i.e. in a wind farm. This type of mooring system would require a model that was flexible enough to accommodate the various possible configurations whilst maintaining computational efficiency.

Table 3 gives a summary of the different approaches for modelling the effects of mooring lines on the dynamics of floating VAWTs.

Table 3 - Mooring line models comparison

## 2.6 Consequences of Limiting Model Complexity

When using a linear or nonlinear force-displacement model, only the global mooring system forces are calculated and hence information about individual mooring line tensions cannot be obtained. On the contrary, these two models may be implemented very easily and quickly when compared to the higher fidelity models, as long as the mooring stiffnesses in all six DOFs are known. For catenary systems, the Q-S model is usually linearised about the equilibrium position of the FOWT to obtain these mooring stiffnesses, whilst analytical derivations from first principles may be used for tensioned moorings. The benefit of linearised models is that they can be implemented during the design and deployment of FOWT control systems.

Karimirad [32] found that the use of full dynamic models rather than static or Q-S models had a significant effect on the predicted tensions of individual mooring lines and that mooring damping does not have an effect on global platform motions.

Kallesøe and Hansen [33] investigated the differences between the Q-S and dynamic mooring line models on the turbine loads of a floating HAWT, finding that whilst both fatigue and extreme blade loads are unaffected by the choice of mooring model, the results indicated that the Q-S approach leads to more conservative FOWT designs.

Furthermore, Hall et al. [20] carried out a thorough study to understand the importance of mooring line model fidelity for different FOWT platform designs. It was found that whilst statically the Q-S model is in very good agreement with a dynamic FEM-based model, there are marked differences in certain stochastic conditions and for different platform designs. Particularly the mooring system in



some cases accounted for a significant portion of the total platform damping, which is not captured by the Q-S model but can drastically alter maximum loads and platform motion.

Hence it can be seen that the choice of mooring line model is closely linked to the type of mooring system, FOWT platform design being used and current design stage, as well as the computational resources available and design stage.

### **3 Structural dynamics**

The simplest dynamical representation of a body (e.g. FOWT) is to assume it is a single rigid body with a point mass and inertial characteristics. The next approach would be to discretise the system into a small number of rigid bodies (e.g. divide FOWT into support structure, tower and blades), allowing for the engineer to investigate the forces acting on connections between these rigid bodies. To further improve the numerical predictions of such a model, it is possible to integrate the flexibility of slender components (e.g. tower and blades) through a linear modal representation that would characterise such flexibilities (assuming small local deflections) [34].

The limitations of these models are that they are linear and are not strictly valid for large-amplitude displacements and deflections, as experienced by FOWTs [13]. Further discretisation and use of different model formulations is required to more accurately represent the structural behaviour of a FOWT within a coupled model of dynamics. The two main methods in this regards are the multibody formulation and the finite element method (FEM), described in §2.3. FEM can be very computationally intensive since it usually results in thousands of equations to be solved simultaneously. To reduce this large computational requirement, reduced order models may be implemented that still represent system components sufficiently [35]. The multibody formulation fits into this niche.

To model the motion and flexible behaviour, the multibody formulation introduces a moving frame of reference to each substructure [35]. This allows for elastic deformations of each component to be solved linearly since the relative displacements (to the moving reference frame) are small. A more in-depth review is given by [35] and subsequent references.

The choice of the optimum method is heavily dependent on the application; the amount of detail required (preliminary versus detailed design), the type of system being analysed and the computational resources available. Table 4 summarizes the three main approaches and their characteristics.

Table 4 - Comparison of structural modelling approaches

### 3.1 Gyroscopic effects

Gyroscopic effects due to the rotating wind turbine rotor are an important aspect to consider. The gyroscopic effect is that any couple, apparently tending to incline the axis of a rotating body in a given direction, actually causes an inclination of the axis in a plane perpendicular to that given direction [36]. There are different ways to integrate gyroscopic effects into the coupled dynamics model, depending on the kinetic formulation employed. With the lower-order structural models, gyroscopic effects would have to be explicitly included as external forces in the equations of motion. On the other hand with some multibody formulations such as that found in [37], the rotor gyroscopic effect may be implicitly included in the equations of motion.

The VAWT gyroscopic moments induced by the platform motion in roll ( $M_{44}^G$ ) and pitch ( $M_{55}^G$ ) [38], are analytically given by Eqn 3:

$$M_{44}^G = -I_{zz}\dot{\psi}\dot{x}_5 \qquad M_{55}^G = -I_{zz}\dot{\psi}\dot{x}_4 \qquad (Eqn 3)$$

where  $I_{zz}$  is the rotor inertia about its rotational axis,  $\dot{\psi}$  is the rotor rotational speed,  $\dot{x}_4$  is the platform roll velocity and  $\dot{x}_5$  is the platform pitch velocity. As can be deduced from the above definition and equations roll and pitch motions and gyroscopic forces are coupled, which should be considered when carrying out long-crested sea simulations. As yet there has been very little research into gyroscopic effects on floating VAWTs. Blusseau and Patel [38] conducted a frequency-domain analysis of the gyroscopic effect on a VAWT mounted on a semi-submersible floating platform. It was found that the roll and pitch motions were adversely affected, with significant increases in peak amplitudes. In this analysis the gyroscopic effect was represented by a damping matrix in the equations of motion.

### 3.2 Aeroelasticity

Aeroelasticity may play a major role in the structural loads of the rotor blades, and there appear to be two levels of aeroelastic modelling that may be implemented. The first is to only include deformations of the structure as a whole, i.e., modelling the blades as flexible beams with rigid aerofoil cross-sections [39-41]. The second is to also include deformations to the cross-section of the blade as mentioned in [42; 43]. It has yet to be seen whether the latter would affect the global motions of the floating wind turbine and be relevant during the preliminary design of FOWTs, although Vorpahl et al. [44] found that the inclusion of tower and blade flexibilities (i.e., the first level described above) does not have a significant impact on platform motions. Hansen et al. [45] provide an overview of how aeroelasticity is integrated into different aerodynamic models.

### **3.3 Hydroelasticity**

Hydroelasticity is defined as the study of motion and distortion of deformable bodies responding to environmental excitations in the sea [46; 47], in particular the interaction between hydrodynamic, inertial and elastic forces. A number of theories have been developed to implement hydroelasticity for marine structures, ranging from linear two-dimensional models to nonlinear three-dimensional models. A review of these theories was carried out by Chen et al. [46].

Once again there are different levels of analysis available. Simple finite element or multibody representations of the floating body as interconnected beams allows for a rudimentary inclusion of hydroelasticity. On the other hand, a detailed finite element model may be used to accurately predict the body deformations. The amount of detail required does not only depend on the stage of design, but also on the hydrodynamic and structural models being used. Without detailed distributed hydrodynamic pressure data, one may not use a detailed structural finite element model appropriately. Therefore careful consideration should be used when selecting the modelling requirements for the design of a floating structure.

Over recent years the importance of hydroelasticity in very large floating structures, such as floating airports, has generated a number of investigations to efficiently model this phenomenon [47]. Such research has the potential to be exploited for the development of an efficient code for modelling floating VAWTs. One particular study by Taghipour et al. [48] presented the results of including hydroelasticity in a hydrodynamic state space model for a flexible barge with good agreement between predicted and experimental results. As this type of hydrodynamic model is already established as being computationally very efficient (see e.g. [49]), this method of incorporating hydroelasticity is very promising.

### **3.4 Discussion**

In a comparative study on floating HAWTs by Karimirad et al. [50], it was found that there are differences in motions of a floating wind turbine between a rigid model and elastic model. The rigid body model produced larger motions in certain degrees of freedom than the elastic model due to the lack of structural damping. This is indicative of the importance of including hydroelasticity in a coupled dynamics model as such differences in motion will affect the fatigue and reliability analysis of the system.

As is evident throughout this paper, different models are required at different stages of the design cycle. When considering the elasticity of the structure, it might be beneficial to prioritise which of the above phenomena should be included. Since wind turbine blades are usually considerably more flexible than the floating support structures, the effects of aeroelasticity will be more significant than the effects of hydroelasticity. Therefore it would aid the design cycle more if aeroelasticity is first

considered rather than hydroelasticity. The effect of hydroelasticity will be more pronounced in larger structures, such as a multi-turbine unit (that is, having multiple turbines on a single floating support structure). Then the prioritisation would shift somewhat depending on the particular system design.

#### **4 Computational issues**

With the need for more computationally efficient design codes, a review of implementing the different structural models efficiently is required. The level of modelling detail used dictates the computational resources required, and sometimes increased levels of detail do not warrant the additional computational effort for small changes in the results. This was experienced by Brommundt et al. [51] where the authors found that a full nonlinear mooring model greatly slows simulations (by about 30 times).

As mentioned before, there is a trade-off between model accuracy and computational effort. For the case of simulating the global motions of floating wind turbines, a compromise in model fidelity is required to model the structural characteristics. Following on from this, the selection of a mathematical model is heavily dependent on what the modeller is interested in. For instance Karimirad [32] found that the global motions of a spar-type floating HAWT were not affected by the damping contribution of mooring lines, but the tension in the mooring lines was critically dependent on this and the mooring inertial contributions. This was also seen in a study by Kvittem and Moan [52]. On the contrary, Hall et al. [20] found that selection of mooring line model is FOWT design specific (cf. §2.6), but corroborates the fact that higher-fidelity models are required to investigate mooring designs. Thereby if one were investigating mooring line dynamics specifically, a higher-fidelity model is warranted.

The multibody formulation is one of the most promising approaches and there are a number of publications outlining computational methods to accelerate the execution of such models. In [53] one may find an extensive review of modelling flexible multibody systems. Here the authors discuss the characteristics of different multibody modelling approaches and their computational advantages. Particular attention is directed to [53], where the authors review strategies for including fluid-structure interactions in flexible multibody systems. Other publications such as [54-56] describe various methodologies to increase the computational efficiency based on system identification and order reduction.

As desktop computing resources available to researchers are ever increasing, a shift to parallel computing is more viable. It has been implemented successfully in computational fluid dynamics as well as finite element analysis and has the potential to significantly reduce computational times in flexible multibody models [57; 58].

## 5 Current implementations

The trend in current design codes has been to implement the multibody formulation [14; 59; 60], and in some cases coupling this with a finite element model. Following is a list of some examples of structural and mooring line models implemented in coupled dynamics design codes currently being developed for FOWTs:

- The National Renewable Energy Laboratory in Colorado, USA has developed one of the most widely-used floating HAWT design codes, called FAST [61]. In the FAST code, a modal finite element approach is used to establish the natural modes of vibration for the wind turbine blades and tower which are in turn used to represent the deflections of these components. Interfacing to the MSC ADAMS software has also been carried out to perform multi-body simulations. Progress has also been made to allow other aero-structural models to be implemented with the FAST framework [62], even possibly to include floating VAWTs [63]. Furthermore, this code utilises the conventional Q-S static approach described in §2.2 that can also accommodate tensioned moorings (see Jonkman [12]), although now the MSQS modification is being included through the Mooring Analysis Program (MAP), see Masciola et al. [17; 64].

Due to the open-source and modular nature of FAST, it is possible to interface with other software. This was done by Hall et al.[20] to implement a FEM-based mooring model using the ProteusDS software with FAST, as well as by Masciola et al. [65] to couple OrcaFlex (another industrial FE mooring modelling software) with FAST, assessing the performance of lower-order mooring line models.

- The Risø National Laboratory at the Technical University of Denmark has long been developing HAWC2, a coupled dynamics code for floating HAWTs [66]. Recently it has been adapted to accommodate floating VAWTs as part of the DeepWind project [67; 68]. The multibody formulation is implemented for both the structural model and mooring line model (although the use of a linear/nonlinear mooring line force-displacement relation is also present).
- A research group at the Norwegian Research Centre for Offshore Wind Technology (NOWITECH) based in Trondheim, Norway have recently started to develop a dedicated coupled dynamics model for floating VAWTs (see Wang et al. [69]). This model interfaces the SIMO/RIFLEX software with a custom dynamic link library to account for the VAWT aerodynamic forces. Within the SIMO/RIFLEX software, the FOWT and mooring lines are modelled as flexible elements based on a nonlinear finite element approach.
- A research group at Cranfield University, UK is also developing the first generic coupled dynamics model dedicated to floating VAWTs called FloVAWT within the

MATLAB/Simulink programming environment (see Collu et al. [31; 70]). Whilst substantial effort has so far been placed on developing aerodynamic and hydrodynamic modules, mooring lines are currently represented through a variant of the MSQS model with the energy-based solution approach and a simplified finite element model is implemented to accommodate turbine deformations.

- Owens et al.[63; 71] have developed a dedicated finite element structural code for floating VAWTs. This code has been designed to be very flexible and modular such that it can be easily interfaced with other aerodynamic and hydrodynamic models developed by others. In particular it is envisaged to be coupled with the FAST code described above as well as high-fidelity VAWT aerodynamic models such as CACTUS [72].

Besides dedicated floating wind turbine codes, there are a number of general-use codes that have the potential to be integrated into a coupled dynamics code. Two examples are SIMPACK and MBDyn, with applications to wind turbines presented in [73; 74] and [75], respectively.

One of the main reasons that the multibody formulation is so popular is that it accommodates large-amplitude motions. Previously this inherent characteristic of floating wind turbines contributed to errors in model predictions. It has also been postulated that since the system is undergoing large-amplitude motions, some of the assumptions taken in the hydrodynamic model are invalidated [60].

A number of publications have presented variations of the multibody formulation for floating wind turbines [37; 76]. Wang et al. [37] in particular produced a method which requires only six equations of motion to compute the general motion, no matter how many DOFs are present in the system.

The adaptability of the multibody formulation allows it to be used to model not only the main structural components of the floating wind turbine, but also the mechanical subsystems within, in particular the drive train and generator [77-80]. This would also improve code execution as the same subroutines may be applied to more than one section of the model.

A review on current state of the art floating wind turbine design codes by Cordle and Jonkman [59] found that almost all major codes implement the multibody formulation or a modification thereof. Although the multibody formulation might seem to be the ideal solution, the finite element method still has applications in certain design stages. For example in [54], the need to establish localised hydrodynamic loads on the floating structure resulted in a coupled boundary-element-method and FEM model, such that the distribution of hydrodynamic pressures calculated by the boundary-element-method routine is directly translated to the FEM procedure. In this case FEM was the optimal approach to use.

## 6 Conclusions

Modelling of mooring lines and FOWT structural dynamics are closely related, sharing common underlying mathematical models at times. In §2, a number of increasingly complex mooring line models were described, from the simple force-displacement relation to the nonlinear FE model. The advantages and downfalls of each model were mentioned (Table 2, Table 3), and careful considerations for the type of mooring system being studied and phenomena specific to VAWTs were discussed. The trade-offs between computational efficiency and model fidelity were also reviewed. The force-displacement-velocity relation is the simplest approach, calculating only global mooring system static forces, whilst implementing the multibody formulation or finite element approach allows for detailed dynamic analysis of individual mooring lines and the interaction with the surrounding environment.

Following this, a review of modelling the turbine structure within coupled dynamics design codes was presented. Table 4 summarises the advantages and disadvantages of the different modelling approaches, with use of a rigid body model providing global platform performance predictions and more complex models such as FEM allow for detailed analyses of structural components of the FOWT. Aspects relating to physical phenomena of operational floating VAWTs, namely gyroscopic forces and system inertia variation were discussed. Issues relating to the modelling and inclusion of aero- and hydroelasticity in preliminary design stages of FOWTs were briefly reviewed, with the recommendation of prioritising aeroelasticity rather hydroelasticity due to the flexible nature of blades versus the relatively more rigid construction of typical FOWT support structures.

Current efforts by various research groups to develop structural and mooring line models with coupled dynamics design tools for FOWTs (with focus on floating VAWTs) have been documented in §6. As previously seen in a review of design tools by Cordle and Jonkman [59], the general trend is to implement the multibody formulation to model the turbine and mooring lines, whilst the interest in FEMs is also increasing for coupled dynamics design tools.

Different models are suitable for different stages of design. Simplified, lower-order models can estimate quickly the global performance of a relatively high number of configurations considered for a FOWT system, allowing to narrow down the design space and therefore using the higher fidelity/computationally expensive models only on a limited number of configuration, to delve into the finer details.

## Acknowledgements

The research leading to these results has been performed in the frame of the H2OCEAN project ([www.h2ocean-project.eu](http://www.h2ocean-project.eu)) and has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 288145.

It reflects only the views of the author(s) and the European Union is not liable for any use that may be made of the information contained herein.

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