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A Multivariate Financial Approach

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## **Oil Price Dynamics and Speculation. A Multivariate Financial Approach**

Giulio Cifarelli\* and Giovanna Paladino<sup>°</sup> +

### **Abstract**

This paper assesses empirically whether speculation affects oil price dynamics. The growing presence of financial operators in the oil markets has led to the diffusion of trading techniques based on extrapolative expectations. Strategies of this kind foster feedback trading that may cause large departures of prices from their fundamental values. We investigate this hypothesis using a modified CAPM that follows Shiller (1984) and Sentana and Wadhvani (1992). At first, a univariate GARCH(1,1)-M is estimated assuming that the risk premium is a function of the conditional oil price volatility. The single factor model, however, is outperformed by the multifactor ICAPM (Merton, 1973) which takes into account a larger investment opportunity set. The analysis is then carried out using a trivariate CCC GARCH-M model with complex nonlinear conditional mean equations where oil price dynamics are associated with both stock market and exchange rate behavior. We find strong evidence that oil price shifts are negatively related to stock price and exchange rate changes and that a complex web of time varying first and second order conditional moment interactions affect both the CAPM and feedback trading components of the model. Despite the difficulties, we identify a significant role of speculation in the oil market which is consistent with the observed large daily upward and downward shifts in prices. A clear evidence that it is not a fundamentals-driven market. Thus, from a policy point of view - given the impact of volatile oil prices on global inflation and growth - actions that monitor more effectively speculative activities on commodity markets are to be welcomed.

**Keywords:** oil price dynamics; feedback trading; speculation; multivariate GARCH-M.

**JEL Classification:** G11 G12 G18 Q40  
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## Introduction

Investment funds have recently poured large amounts of money in the commodity markets and have raised their holdings to \$260 billions as of mid 2008 from \$13 billions in 2003. During that period the price of crude oil, among other commodities, rose relentlessly, fostering the debate on the role of speculation on oil prices.<sup>1</sup>

For many a decade regulators did impose limits on the behavior of financial agents in order to prevent them from manipulating commodity exchanges, which were much smaller than the bond or stock markets. Commercial operators only, such as farms, airlines or manufacturers (and the corresponding middlemen that handled their trading activities) were allowed to buy nearly unlimited amounts of oil. In 1991, however, the Commodity Futures Trading Commission (CFTC) granted a similar status also to financial firms as the latter successfully argued that trading commodities on behalf of investors was tantamount to brokering commodity transactions for commercial firms.

Empirical evidence on the relevance of speculation is not clearcut. At the end of July 2008 a CFTC report concluded that speculators were not systematically driving oil prices.<sup>2</sup> A few days later, however, a data revision showed that just four swap dealers held 49 percent of all NYMEX oil contracts that bet on oil price increases, providing clear evidence of concentration of power in the market.<sup>3</sup> Indeed, it is quite difficult to distinguish between pure speculation and commercial trading, which may involve the need to hedge the risk of adverse price shifts. If many investment banks, hedge funds and private equity firms have invested in physical assets, such as pipelines and storage terminals and hedge their business exposures, commercial traders behave as speculators whenever they hedge risk in excess of their actual needs.

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<sup>1</sup> The price of WTI crude oil rose by over 170 percent between January 2007 and June 2008.

<sup>2</sup> Produced by the CFTC task force the 22<sup>nd</sup> of July.

<sup>3</sup> Dealers make trades that forecast either price increases or price decreases.

The aim of this paper is to assess empirically whether speculation does affect oil price dynamics. If the lack of reliable data on speculative positions in the oil futures markets prevents direct studies, the role of speculators can still be analyzed in an indirect way, with the help of heterogeneous agents models, based on the interaction between two stylized types of traders, viz. fundamentalists and noise/feedback/chartists. Oil supply is relatively inelastic and its price is mostly influenced by (excess) demand shifts that stem from the two categories of traders mentioned above.

The growing presence of financial operators in the oil markets has led to the diffusion of trading techniques based on extrapolative expectations, where a price trend is assumed to be lasting. Strategies of this kind tend to foster feedback trading: "positive" whenever investors buy when prices rise and sell when they fall and "negative" if investors buy when prices fall and sell if they rise. The literature has typically focused on positive feedback trading, seen as an irrational strategy that moves prices away from their fundamentals related values, raises uncertainty and contributes to market fragility. Its presence is typically associated with a negative autocorrelation of returns. Indeed, if prices overshoot their fundamental values because of the behavior of noise traders (possibly anticipated by rational ones, as in De Long et al., 1990), the market corrects for the overreaction in the following periods, shifting prices in the opposite direction, and generates in this way a negative return autocorrelation pattern. Feedback trading seems to be a stylized aspect of stock market behavior. Cutler et al. (1991) and Sentana and Wadhvani (1992) find evidence of feedback trading in the US stock market whilst Koutmos (1997) and Koutmos and Saidi (2001) detect its presence in, respectively, several European and emerging equity markets. The impact of feedback trading by specific groups of operators – such as foreign or institutional investors - is finally examined in Lakonishok et al. (1992), Hyuk et al. (1999) and Nofsinger and Sias (1999), among many others.

We investigate at first the hypothesis that also some participants in the crude oil market engage in feedback trading activities, using a behavioral CAPM that follows Shiller (1984) and Sentana and Wadhvani (1992). We use an

univariate GARCH(1,1) setting where the risk premium is a function of the conditional oil price volatility.

The single factor model, despite its attractiveness, misses some relevant aspects of financial market pricing and is outperformed by the multifactor ICAPM, which takes into account a larger investment opportunity set. Indeed, Scruggs' (1998) two-factor parameterization introduces an additional measure of risk and allows the covariance between the asset under investigation and the variable that proxies for the state of the investment opportunities to influence the behavior of returns over time. Such a framework can be used to model the role of oil in financial portfolio hedging decisions.

Oil price dynamics are often associated with both stock market and exchange rate behavior. A number of studies, based on different data and estimation procedures, find a negative financial linkage between oil and stock prices i.e. a large negative covariance risk between oil and a widely diversified portfolio of assets. A substantial body of literature, however, claims that there is a predominant real linkage between the value of equities and oil via production and the business cycle, expansionary periods (in turn related to stock increases) being closely associated with oil price rises.

As for the dollar, it has traditionally influenced the price of oil and of other commodities, including gold and base metals, which are mostly priced in the green currency. Here too we have two channels of transmission, a real and a financial one. From a macroeconomic point of view higher oil prices lead to higher trade deficits which, weakening the dollar, bring about compensatory oil price increases. The financial channel has become more relevant in recent years, with the entry of hedge funds, banks and other financial institutions in the commodity markets. As noted by Roache (2008), commodities behave differently from stocks and bonds and offer diversification. Traders that are bearish on the dollar will sell a dollar labelled (stock) asset and buy oil (and vice versa if they are bullish on the dollar) in order to diversify their portfolio. Indeed, crude oil has attracted funds away from financial markets during the recent bouts of turmoil.

This paper investigates the behavior, from October 1992 to June 2008, of weekly changes in the WTI oil price, in the Dow Jones stock index and in the US dollar effective exchange rate. The analysis improves upon previous work in several respects.

- (i) It carefully examines the relevance of feedback trading in the spot oil market using long and homogeneous time series which span more than fifteen years and encompass large shifts in market sentiment. The short run dynamics of oil price changes and its interaction with the corresponding futures price are parameterized with the help of models of growing complexity which identify a convincing common pattern. To the best of our of knowledge, there is but little empirical work documenting the interaction between noise and informed trading by oil market participants.
- (ii) While there is a large body of literature dealing with feedback trading in stocks and other types of assets in an univariate setting, very little research has been done in a multivariate framework. Our investigation builds on a bivariate approach, originally set out by Dean and Faff (2008), that introduces feedback trading in a two factor ICAPM model of stock and bond returns interaction by Scruggs (1998). Oil prices, exchange rate and stock index rates of change are simultaneously modelled with the help of a GARCH-M approach which parameterizes their conditional second moments. The complex dynamics of feedback trading behavior in periods of stress are carefully set out, in a bivariate context at first, involving the WTI oil price and the Dow Jones stock market index and successively, adding the US dollar effective exchange rate, in a trivariate one.

Speculative behavior seems to affect the crude oil and stock exchange pricing in the time period analyzed in this paper. Indeed, we find convincing evidence of positive feedback trading in the oil and stock markets. As expected the corresponding price overshooting correction brings about serial correlation of the returns, the magnitude of which increases with the level of

volatility within and across markets. Oil price shifts are negatively related to stock price and exchange rate variation and our estimates unravel a complex web of time varying first and second order conditional moment interactions that affect both the CAPM and feedback trading components of the model and justify the use of a multivariate approach.

The analysis is organised as follows. Section 1 introduces the theoretical framework, based on the multifactor inter-temporal CAPM developed by Merton (1973), where the presence of noise traders allows to account for behavioral asset pricing mechanisms such as feedback trading. The empirical evidence is presented in Section 2 where the relevance of feedback trading in the crude oil market is investigated using GARCH parameterizations. Section 2.1 provides a basic estimation of the oil price dynamics in an univariate context. The analysis is then extended to a multivariate approach. Section 2.2 investigates the links between oil and stock prices via a two-factor ICAPM parameterized by a CCC bivariate GARCH-M with complex nonlinear conditional mean equations. Section 2.3 introduces the exchange rate in the previous model and provides a comprehensive picture of the dynamic interrelation between the conditional moments of the three time series. Section 3 concludes the paper.

## **1. A multifactor ICAPM with feedback trading**

The relationship between returns and volatility is central for the pricing of an asset or a commodity. Indeed, as suggested by the Capital Asset Pricing Model (CAPM), the greater the uncertainty about the future price, which increases with its volatility, the higher is the return that is required in order to compensate for the non-diversifiable risk. In a major breakthrough Merton (1973) points out in the Intertemporal Capital Asset Pricing Model (ICAPM) that investors will price an asset in relation not only to the systematic risk

but also to the expectations of future changes in the investment opportunity set, proxied by various factors or “state variables”.<sup>4</sup>

Both models, however, are unable to account for the serial correlation of the returns, a stylized pricing characteristic of several asset and commodity markets. In this paper the feedback trading interpretation by Cutler et al. (1991) is adopted. Following Dean and Faff (2008), it is combined with the ICAPM, while Shiller (1984) and Sentana and Wadhvani (1992) insert the feedback behavior in the CAPM.

The latter propose a model with two types of agents, smart money investors who maximize expected utility subject to a wealth constraint and feedback traders who follow the market. They show that when traders adopt a positive feedback strategy, buying assets when their prices are high and selling when they are low, the corresponding returns exhibit negative serial correlation. They also find that positive feedback trading raises the overall volatility of returns. Conversely, the opposite strategy of negative feedback trading makes returns less volatile.

The demand for oil by informed traders is governed by a simplified risk return consideration. They invest on the basis of rational forecasts of future returns and hold a larger fraction of their wealth in oil when they expect higher returns, in line with the tenets of the CAPM.

Their demand for oil reads as follows

$$Q_t = \frac{E_{t-1}(r_t) - \alpha}{\mu_t} \quad (1)$$

where  $Q_t$  is the fraction of the oil demand held by the first group of traders,  $r_t = \Delta S_t$  is the ex-post oil return in period  $t$ , the first log difference of the spot oil price  $S_t$  and  $\alpha$  is the risk free rate. The risk premium  $\mu_t$  is assumed to be a function of the conditional variance of the oil returns  $\sigma_t^2$ , and the following relationship holds

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<sup>4</sup> A recent application of the ICAPM to the commodity futures markets is provided by Roache (2008). In this paper we apply the model to spot prices, this decision is justified by the “de facto” integration of the oil market in financial portfolios. As a consequence the pricing behavior has acquired financial characteristics.



$$\mu_t = \mu(\sigma_t^2) \quad (2)$$

where  $\mu' > 0$ . If  $Q_t = 1$  then equation (1) reverts to the standard CAPM and

$$E_{t-1}(r_t) = \alpha + \mu(\sigma_t^2).$$

The second group of agents demands oil according to the following function

$$I_t = \gamma r_{t-1} \quad (3)$$

where  $I_t$  is the share of oil demand they hold. If  $\gamma > 0$  there is positive feedback trading, agents buy (sell) when the rate of change of prices of the previous period is positive (negative). When  $\gamma < 0$ , with negative feedback trading, agents sell (buy) when the prices are rising (falling) in the previous time period.

Market equilibrium implies that  $I_t + Q_t = 1$  and

$$E_{t-1}(r_t) - \alpha = \mu(\sigma_t^2) - \gamma [\mu(\sigma_t^2)] r_{t-1} \quad (4)$$

becomes the CAPM with feedback trading or behavioral CAPM asset pricing relationship tested in various empirical studies.

In Merton's (1973) multifactor ICAPM, relaxing the hypothesis that the opportunity set is static, the asset demand adjusts as risk averse investors update their exposure to the portfolio built to hedge inter-temporal (stochastic) future shifts in the opportunity set. The expectations of future changes in the opportunity set are captured by the a vector of  $n$  state variables which influence the expected risk premium demanded by investors, assumed to take decisions in a dynamic world that responds to news. The prices of the assets thus reflect, besides the systematic risk, quantified by their covariance with the market returns, their covariances with the  $n$  state variables.

The ICAPM is set in a continuous time framework where both the returns and state variables are assumed to follow standard diffusion processes. Agents are risk adverse, with a utility of wealth function  $J(W(t), \bar{F}(t), t)$  where  $W(t)$  is wealth and  $\bar{F}(t)$  is a  $n \times 1$  vector of state variables  $(F_1, F_2, \dots, F_n)$  that describe the behavior over time of the investment opportunity set.

In equilibrium the expected market risk premium for asset M is given by <sup>5</sup>

$$E_{t-1}[r_{M,t} - \alpha] = \left[ \frac{-J_{WW}W}{J_W} \right] \sigma_{M,t}^2 + \left[ \frac{-J_{WF_1}}{J_W} \right] \sigma_{MF_1,t} + \dots + \left[ \frac{-J_{WF_n}}{J_W} \right] \sigma_{MF_n,t} \quad (5)$$

where  $E_{t-1}[\cdot]$  is the expectation operator conditional on information available at time  $t-1$ ,  $r_{M,t}$  is the return of asset  $M$ ,  $\sigma_{M,t}^2$  and  $\sigma_{MF_i,t}$  are the corresponding conditional variance and covariance with the state variable  $F_i$ ,

where  $i=1,\dots,n$ . The first coefficient  $\left[ \frac{-J_{WW}W}{J_W} \right]$  is the Arrow Pratt coefficient

of relative risk aversion.<sup>6</sup> It is always positive since  $J_W > 0$  and  $J_{WW} < 0$ , which suggests a positive relationship between risk premium and conditional variance. If  $J_{WF_i} = 0, \forall i$ , the marginal utility of wealth is independent from the state variables and the equation reverts to the standard CAPM. If a  $J_{WF_i} \neq 0$ , the sign of the impact of the corresponding  $i^{th}$  state variable will depend upon the interaction of the signs of  $J_{WF_i}$  and  $\sigma_{MF_i,t}$ , which are both a priori indeterminate. If  $J_{WF_i}$  and  $\sigma_{MF_i,t}$  are of the same sign, i.e. either both positive or both negative,  $J_{WF_i} \sigma_{MF_i,t}$  is positive and investors will demand a lower risk premium. If  $J_{WF_i}$  and  $\sigma_{MF_i,t}$  are of the opposite sign,  $J_{WF_i} \sigma_{MF_i,t}$  is negative and investors will demand a higher risk premium.

In order to introduce feedback trading, we assume that the demand of the informed traders can be parameterized by the ICAPM. Since the risk premium is now affected by the state variables, equation (2) can be rewritten as

$$\mu_t = \mu(\sigma_{M,t}^2, \sigma_{MF_1,t}, \dots, \sigma_{MF_n,t}) \quad (2')$$

In equilibrium  $I_t + Q_t = 1$  and the ICAPM with feedback trading asset pricing relationship becomes

$$E_{t-1}(r_t) - \alpha = \mu(\sigma_{M,t}^2, \sigma_{MF_1,t}, \dots, \sigma_{MF_n,t}) - \gamma \left[ \mu(\sigma_{M,t}^2, \sigma_{MF_1,t}, \dots, \sigma_{MF_n,t}) \right]_{t-1} \quad (6)$$

<sup>5</sup> Equation (5) is derived from Merton's first order conditions. See Merton (1973, equation (15), page 876).

<sup>6</sup> Low case letters indicate partial derivatives.

In the empirical investigation it will be further assumed that the risk premium  $\mu$  is a linear function of market volatility and of the covariances between the return and the state variables. We thus rewrite equation (6) as follows

$$E_{t-1}(r_t) - \alpha = \mu_1(\sigma_{M,t}^2) + \mu_2(\sigma_{MF_{1,t}}) + \dots + \mu_{n+1}(\sigma_{MF_{n,t}}) - \gamma [\mu_1(\sigma_{M,t}^2) + \mu_2(\sigma_{MF_{1,t}}) + \dots + \mu_{n+1}(\sigma_{MF_{n,t}})] r_{t-1} \quad (7)$$

With respect to the standard ICAPM, the ICAPM with feedback trading has an additional term  $r_{t-1}$ , with a nonlinear coefficient. Its sign depends upon (i) the dominant type of feedback trading (i.e. the sign of  $\gamma$ ), (ii) the sign of the conditional covariances  $\sigma_{MF_{i,t}}$ , and (iii) the sign of the corresponding  $\mu_2, \dots, \mu_{n+1}$  functions.

## 2. Empirical evidence

Despite a large body of empirical evidence on the ICAPM, the focus has mainly been on equities and little has been done on the alternative asset class represented by commodities.<sup>7</sup>

Our weekly data spans from 6 October 1992 to 24 June 2008. Oil spot prices ( $S_t$ ) are the WTI Spot Price fob (US dollars per Barrel), futures oil prices ( $F_t$ ) are provided by the EIA database<sup>8</sup>, the speculative position on the futures oil market ( $SPC_t$ ) is proxied by the net CFCT non commercial position.<sup>9</sup> The US stock return - the first difference of the logarithm of the Dow Jones industrial

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<sup>7</sup> Recently Khan et al. (2008) model the expected commodity futures return, including oil, as a linear function of systemic risk and two specific factors, the hedging pressure and a proxy for the scarcity of the commodity.

<sup>8</sup> Futures contract 1 expires on the 3<sup>rd</sup> business day prior to the 25<sup>th</sup> calendar day of the month preceding the delivery month. If the 25<sup>th</sup> calendar day of the month is a non-business day, trading ceases on the third business day prior to the business day preceding the 25<sup>th</sup> calendar day. Contracts 2 to 4 correspond to the successive delivery months following contract 1.

<sup>9</sup> This index is computed as the difference between short and long non commercial positions. It is similar to the hedging pressure measure used by Khan et al. (2008).

Index ( $J_t$ ) - and the US dollar nominal effective exchange rate<sup>10</sup> ( $Z_t$ ) are taken from Bloomberg and Fred Database, respectively.

Summary statistics are presented in Table 1. Over the sample period the average return on oil is higher than on equity, and the standard deviation of the oil market is significantly greater than the one associated to equity and exchange rate returns. Oil, equity and currency returns distributions are mildly skewed and leptokurtic. The stationarity of the series, tested with the ADF procedure, stands out clearly both for commodity and financial returns. Finally inter-temporal dependency of weekly returns (with the exception of the effective exchange rate changes) and squared weekly returns is confirmed by the Ljung Box Q-statistics. Volatility clustering affects all the markets (i.e. oil, equity and currency).

## 2.1 Univariate approach: feedback trading on the oil market

In order to estimate the model (4) we need a linear transformation of the coefficients in the feedback trading demand. We thus introduce the linearization  $\gamma[\mu(\sigma_t^2)] = b_2 + b_3 h_t^2$ , where  $h_t^2 = \hat{\sigma}_t^2$  is the conditional variance obtained with a GARCH-M model.

Empirically we compute the following GARCH(1,1)-M with feedback trading, where equation (4') is the conditional mean and equation (8) is the conditional variance specification

$$\Delta s_t = b_0 + b_1 h_t^2 + (b_2 + b_3 h_t^2) \Delta s_{t-1} + u_t \quad (4')$$

$$h_t^2 = \varpi + \alpha u_{t-1}^2 + \beta h_{t-1}^2 \quad (8)$$

$\Delta s_t = 100 * \Delta \log S_t$ ,  $b_1 h_t^2$  is the risk premium and  $u_t$  is the residual of the conditional mean. (4') becomes a simple CAPM if  $b_2$  and  $b_3$  are both zero and a CAPM with autocorrelated returns if  $b_2 \neq 0$  and  $b_3 = 0$ . In order to account for the impact on oil spot prices of some other exogenous factors, such as

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<sup>10</sup> The Trade Weighted Exchange Index for the major currencies (TWEXM) comes from the Federal Reserve of Saint Louis data base. Its weekly frequency is synchronized (same day of the week) with the frequency of the oil prices and of the stock index.

shifts in the previous period future oil return ( $\Delta f_t$ ) or in the speculative position on the futures oil markets ( $\Delta SPC_t$ ), i.e. in the net CFCT non commercial position, the following system is estimated

$$\Delta s_t = b_0 + (b_1 + b_3 \Delta s_{t-1}) h_t^2 + b_4 D_1 + b_5 \Delta f_{t-1} + b_6 \Delta SPC_{t-1} + u_t \quad (4'')$$

$$h_t^2 = \varpi + \alpha u_{t-1}^2 + \beta h_{t-1}^2 \quad (8)$$

where  $\Delta f_t = 100 * \Delta \log F_t$ ,  $\Delta s_t = 100 * \Delta \log S_t$  and  $\Delta SPC_t = (SPC_t - SPC_{t-1}) / SPC_{t-1}$ .<sup>11</sup>

$D_1$  is a dummy accounting for the steep price rise in the years 2007-2008 and could be interpreted as the expectation of a strong increase in demand associated with fundamental factors (due e.g. to the role of the BRIC countries in the global economy).

Table 2 presents the ML estimations of (4'') and (8) obtained with an univariate GARCH(1,1)-M procedure. A number of results stand out. First, there is positive feedback trading since  $b_3$  is always negative and significant. Second, the impact of the expected increase in oil demand due to fundamentals seems to be relevant since  $b_4$  is always significant and positive (in the range of 1.41-1.5). Third, the impact of the lagged rate of change of the futures oil prices is significantly positive and provides a boost (by threefold) to the absolute value of the feedback trading coefficient  $b_3$ .<sup>12</sup> Fourth, the speculative position  $SPC_t$  - proxied by the net short non commercial position in the futures oil market - does not affect the oil spot price dynamics even when oil futures prices are excluded from the specification. This is line with the doubts - mentioned above - on the reliability of data on speculative positions.<sup>13</sup> Indeed, a mere visual inspection of the behavior of the series over time (the graph is available upon request)

<sup>11</sup>  $b_2 = 0$  in the specification of equation (4'') and reflects a systematic empirical finding of the unrestricted estimates.

<sup>12</sup> In our analysis, we use the future contracts lagged by one period to avoid a simultaneity bias. We also tried futures prices for contracts that expire in 2, 3 and 4 months. Their informational content has a much smaller impact on oil spot price dynamics.

<sup>13</sup> This finding, however, contradicts the results of Khan et al. (2008) where the hedging pressure (a variable similar to  $SPC_t$ ) is significant in the equation of the nearest term futures oil return.

suggests that the recent surge in oil prices is not accompanied by a simultaneous significant increase in the net non commercial position.

Finally, in order to check for reverse causality, we repeated the GARCH(1,1)-M model estimation using as dependent variable the rate of change of the futures oil prices and found that the lagged rate of change of the spot prices had no effect on the dependent variable. We conclude that the presence of feedback trading on oil spot markets and the significant impact of the lagged rate of change of futures oil prices point out to an active role of uninformed noise operators in the oil market.

## 2.2 Bivariate GARCH-M: oil price and US stock index

Recent anecdotal evidence shows that oil prices co-move with other financial variables, oil price increases often going along with US stock price decreases. Informationally linked markets, such as oil and stock markets, are likely to react to the same information set and their movements are bound to be somehow correlated. By including the equity market into the analysis, we investigate if oil and equity stocks are part of a common hedging strategy and if the presence of feedback trading in both markets affects the dynamic structure of oil returns.

Equation (7) is estimated including a single state variable and replacing  $\gamma[\mu_1(\sigma_{Mt}^2) + \mu_2(\sigma_{MF_1,t})]$  by  $b_{3s} + b_{4s}h_{s,t}^2 + b_{5s}h_{sj,t}$  in the conditional mean equation of the oil price return and  $\gamma[\mu_1(\sigma_{Mt}^2) + \mu_2(\sigma_{MF_1,t})]$  with  $b_{3j} + b_{4j}h_{j,t}^2 + b_{5j}h_{sj,t}$  in the conditional mean equation of the Dow Jones Industrial index return. Following Scruggs (1998) and Dean and Faff (2008) the two factor ICAPM is then modelled as the bivariate non linear GARCH(1,1)-M with feedback trading system (9). The parameterization of  $H_t$  - to eliminate further complexities - is symmetrically<sup>14</sup> modelled as a CCC GARCH, despite the possible criticisms on the constant correlation assumption.<sup>15</sup>

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<sup>14</sup> Due to convergence problems, we disregard conditional variance asymmetries in the equity market (as in Koutmos, 1997), which would require an appropriate parameterization (e.g. a

$$\begin{aligned}
\Delta s_t &= b_{0s} + b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + (b_{4s}h_{s,t}^2 + b_{5s}h_{sj,t})\Delta s_{t-1} + b_{6s}D_1 + b_{7s}\Delta f_{t-1} + u_{s,t} \\
\Delta j_t &= b_{0j} + b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t} + (b_{3j} + b_{4j}h_{j,t}^2 + b_{5j}h_{sj,t})\Delta j_{t-1} + b_{6j}D_2 + u_{j,t} \\
u_t &= \begin{bmatrix} u_{s,t} \\ u_{j,t} \end{bmatrix} \\
u_t | \Omega_{t-1} &\sim N(0, H_t) \\
H_t &= \Delta R \Delta \\
R &= \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} \quad \Delta_t = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{j,t} \end{bmatrix} \\
h_{s,t}^2 &= \varpi_s + \alpha_s u_{s,t-1}^2 + \beta_s h_{s,t-1}^2; \quad h_{j,t}^2 = \varpi_j + \alpha_j u_{j,t-1}^2 + \beta_j h_{j,t-1}^2
\end{aligned} \tag{9}$$

where according to the previous section's result we assume a priori that  $b_{3s} = 0$ .  $\Delta j = 100 * \Delta \log J_t$ , where  $J_t$  is the Dow Jones Industrial Index and  $D_2$  is a dummy accounting for the stock bubble crash in 2000. It is set equal to 0 before 4/18/2000 and 1 thereafter. The estimates of the bivariate GARCH(1,1)-M model are set out in Table 3, section *a* and may be summarized as follows. The conditional variance equation coefficients are significant and of the expect signs and size. The conditional mean coefficients provide some original insights on oil and stock price dynamics. The coefficient  $b_{1s}$ , that relates oil returns with oil price volatility is significantly greater than in the univariate case; the introduction of a second factor, the stock market index, in the ICAPM magnifies the effect of the relation between risk premium and volatility directly via  $b_{2s}$ , the covariance coefficient, and indirectly affecting the size of  $b_{1s}$ . If the negative covariance rises<sup>16</sup> ( $\Delta h_{sj,t} > 0$ ) then oil price returns rise too. The impact of the oil price variance

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TGARCH). According to the standard asymmetry diagnostics of Engle and Ng (1993), available from the authors upon request, the symmetric framework does not seem to be seriously misspecified.

<sup>15</sup> The effect of shifts in volatility is accounted for by the joint ML conditional correlation and variance estimation (see Bollerslev, 1990, equations 6-7, page 500).

<sup>16</sup> The mean of  $h_{sj,t}$  is -0.37 (and is significantly different from zero) and its standard deviation is 0.159.

$h_{s,t}^2$  on the feedback trading coefficient is not modified by the presence of an external factor (the covariance between oil and stock returns) in terms of both size and sign. The overall effect of the covariance between oil and stocks on the conditional mean equations is stronger on stock returns than on oil price changes. However in the stock returns mean equation - given the negative sign of  $b_{3j}$  and  $b_{4j}$  - there could be a switch from positive to negative feedback trading.<sup>17</sup> As for the impact of the futures oil price changes, in this case too, their lagged value affects the spot prices whereas the reverse is not true (the impact of spot prices on the futures returns is never significant when the oil equation is defined in terms of futures instead of spot prices). Moreover oil futures prices do not exert any effect in the stock conditional mean equation. The specification tests on the residuals confirm that the bivariate CCC GARCH-M is acceptable since the usual misspecification tests suggest that the standardized residual  $v_t$  are always well behaved. For each equation we find that  $E[v_t] = 0$ ,  $E[v_t^2] = 1$  and that  $v_t$  and  $v_t^2$  are serially uncorrelated.

### **2.3 Trivariate GARCH-M: oil price, US stock index and US dollar exchange rate**

In order to investigate the influence of the exchange rate on the pricing of oil we estimate a trivariate CCC GARCH(1,1)-M model (system (10)).

The ICAPM representations (7) are estimated for the rates of change of oil prices ( $\Delta s_t$ ) and of the Dow Jones stock index ( $\Delta j_t$ ). The lack of serial correlation suggests the use of an ICAPM with no feedback trading parameterization in the case of the rate of change of the US dollar effective exchange rate ( $\Delta z_t$ ).

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<sup>17</sup> There is negative feedback trading if  $-0.99h_{j,t} > 0.21 + 0.04h_{j,t}^2$  i.e. if the feedback trading coefficient is positive.



$$\Delta s_t = b_{0s} + b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + b_{8s}h_{sz,t} + (b_{4s}h_{s,t}^2 + b_{5s}h_{sj,t} + b_{9s}h_{sz,t})\Delta s_{t-1} + b_{6s}D_1 + b_{7s}\Delta f_{t-1} + u_{s,t}$$

$$\Delta j_t = b_{0j} + b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t} + (b_{3j} + b_{4j}h_{j,t}^2 + b_{5j}h_{sj,t})\Delta j_{t-1} + b_{6j}D_2 + u_{j,t}$$

$$\Delta z_t = b_{0z} + b_{1z}h_{z,t}^2 + b_{8z}h_{sz,t} + u_{z,t}$$

$$u_t = \begin{bmatrix} u_{s,t} \\ u_{j,t} \\ u_{z,t} \end{bmatrix}$$

$$u_t | \Omega_{t-1} \sim N(0, H_t) \quad (10)$$

$$H_t = \Delta R \Delta$$

$$R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix} \quad \Delta_t = \begin{bmatrix} h_{s,t} & 0 & 0 \\ 0 & h_{j,t} & 0 \\ 0 & 0 & h_{z,t} \end{bmatrix}$$

$$h_{s,t}^2 = \varpi_s + \alpha_s u_{s,t-1}^2 + \beta_s h_{s,t-1}^2; \quad h_{j,t}^2 = \varpi_j + \alpha_j u_{j,t-1}^2 + \beta_j h_{j,t-1}^2; \quad h_{z,t}^2 = \varpi_z + \alpha_z u_{z,t-1}^2 + \beta_z h_{z,t-1}^2$$

For the sake of notational simplicity let  $\lambda_{i,t}$  where  $i = s, j, z$ , be the CAPM coefficient - i.e.  $\lambda_{s,t} = b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + b_{8s}h_{sz,t}$   $\lambda_{j,t} = b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t}$  and  $\lambda_{z,t} = b_{1z}h_{z,t}^2 + b_{8z}h_{sz,t}$  - and  $\phi_{i,t}$  be the feedback trading coefficient - i.e.  $\phi_{s,t} = b_{4s}h_{s,t}^2 + b_{5s}h_{sj,t} + b_{9s}h_{sz,t}$  and  $\phi_{j,t} = b_{3j} + b_{4j}h_{j,t}^2 + b_{5j}h_{sj,t}$ . Parsimony suggests that the conditional mean determinants that are associated with the correlation between the stock price and the exchange rate changes  $h_{jz,t}$ , which is not significantly different from zero, be removed.<sup>18</sup> The diagnostic metrics on the standardized residuals suggest that the CCC GARCH(1,1)-M parameterization of the conditional variance is accurate. There is no evidence of residual heteroskedasticity and the mean and the conditional variance of the standardized residuals are very close to 0 and 1 respectively.

In the oil price conditional mean equation, we find that  $b_{1s}$  is significantly positive but smaller in size than its univariate estimate from equation (4'').

<sup>18</sup> We are thus estimating simultaneously, in order to improve efficiency, three ICAPMs. In the oil market ICAPM the exchange rate and the stock index rates of change are assumed to be the state variables that describe the behavior over time of the investment opportunity set. In both the stock and exchange rate models the state variable is the oil price rate of change.

The positive risk-return relationship, however, is strengthened by the algebraic sum of the impact of the two additional factors. The overall CAPM coefficient  $\lambda_{s,t}$  and the feedback trading coefficient  $\phi_{s,t}$ , computed with historical simulations which use the values of the conditional second moments, are found to be, respectively, positive and negative on average. (Their behavior over time is set out in Graph 1.) Both the dummy and the lagged futures changes coefficients are significant and of the expected sign. In the stock index return conditional mean equation an historical simulation shows that  $\lambda_{j,t}$  is, on average, positive. The coefficient  $\phi_{j,t}$  shifts from negative to positive values and reflects positive and negative feedback trading behavior (see Graph 2). The dummy  $D_2$  is significantly different from zero.

As for the US dollar effective exchange rate,  $b_{1z}$  is negative so that an increase in the volatility brings about a depreciation of the US effective exchange rate as traders sell dollars. The negativeness of the overall CAPM coefficient  $\lambda_{z,t}$  is mitigated by the impact of the covariance between the oil prices and the US dollar (see Graph 3).<sup>19</sup>

A visual inspection of the graphs, provides some useful insights of the reaction of speculators and informed traders to economic shocks. For example the period of stock market turmoil that followed 9/11 and the subsequent expansionary monetary policy is reflected in the huge negative spikes of  $\phi_{j,t}$  and in a large positive increase in the risk aversion on the US equity market. On the contrary the risk aversion on oil is sharply reduced, a possible evidence of the hedging role of oil in financial portfolios. An interesting finding is that no relevant shifts in risk aversion or feedback trading have accompanied the recent upswing in oil price. Our model's interpretation is that agents had no perception of increasing risks. Indeed, as

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<sup>19</sup> The unconditional means of the  $\lambda_{i,t}$  and  $\phi_{i,t}$  coefficients mentioned above are:  $\bar{\lambda}_s = 2.219$ ,  $\bar{\lambda}_j = 0.125$ ,  $\bar{\lambda}_z = -1.872$ ,  $\bar{\phi}_s = -0.350$  and  $\bar{\phi}_j = -0.050$ .

shown in the return panels of the graphs, conditional volatilities were not seriously affected.

Finally to check the soundness of our results we perform some specification tests of the asset pricing models within the parameterization of system (10). In our multilateral framework, two asset pricing models are examined (see Table 4). A distinction is drawn between the multifactor ICAPM and traditional CAPM – testing respectively the hypotheses  $H_{01}$  and  $H_{02}$ . These two sets of restrictions are always rejected and the empirical evidence strongly suggests that the feedback trading parameterization adopted in system (10) is correct.

### **3. Conclusions**

This paper investigates the relationship between oil prices, stock prices and US dollar exchange rate using a behavioral ICAPM approach, where noise traders are allowed to influence asset demands.

A non linear model of the rate of change of spot oil prices is developed in a univariate framework and then in a multivariate context, where the Dow Jones Industrial return and the rate of change in the US dollar nominal effective exchange rate are assumed to account for changes in the investment opportunity set. The empirical work reported here provides some insights on the recent oil price dynamics. First the higher the volatility the stronger the serial correlation of oil returns, consistently with a model where some traders follow feedback strategies. This result is reinforced when the impact of the futures oil prices on spot quotes is accounted for. As a matter of fact, futures oil markets are leading with respect to oil price changes while the reverse is never true. The enlargement of the investment opportunity set is coherent with the adoption of a multifactor behavioral ICAPM estimated with a multivariate CCC GARCH-M procedure.

We find strong evidence that the serial correlation of oil returns is influenced by the conditional covariances between factors (Dow Jones Industrial index

return and the US dollar percentage change). Moreover the conditional covariance between stock returns and oil returns is important for the feedback traders in the equity markets.

Overall these results suggest that traders hedge their portfolio considering oil as a component of their wealth allocation strategy and this may have some policy implications. Proving that speculation is affecting oil prices is, however, a slippery matter as it tends to occur against a background of changing fundamentals. Nonetheless, large daily upward and downward shifts in oil prices do not fit a fundamentals-driven market. Speculatively driven high prices can persist for a considerable time before fundamentals bring them down to fairer values. As a consequence, while measures of core inflation may remain quite well anchored, inflation expectations may edge higher. This complicates monetary policy decision making, as central banks move along a fine line between containing inflation and supporting demand. The rapid and unpredictable oil price movements raised global inflation, lowered incomes and deepened trade deficits, aggravating global financial instability and increasing the likelihood of a global recession. This is a clear indication that policy actions aimed at restricting speculators' activity should be welcomed. The CFTC already places limits on speculative energy trades, but speculators can avoid those limits if they move their holdings beyond the country borders.

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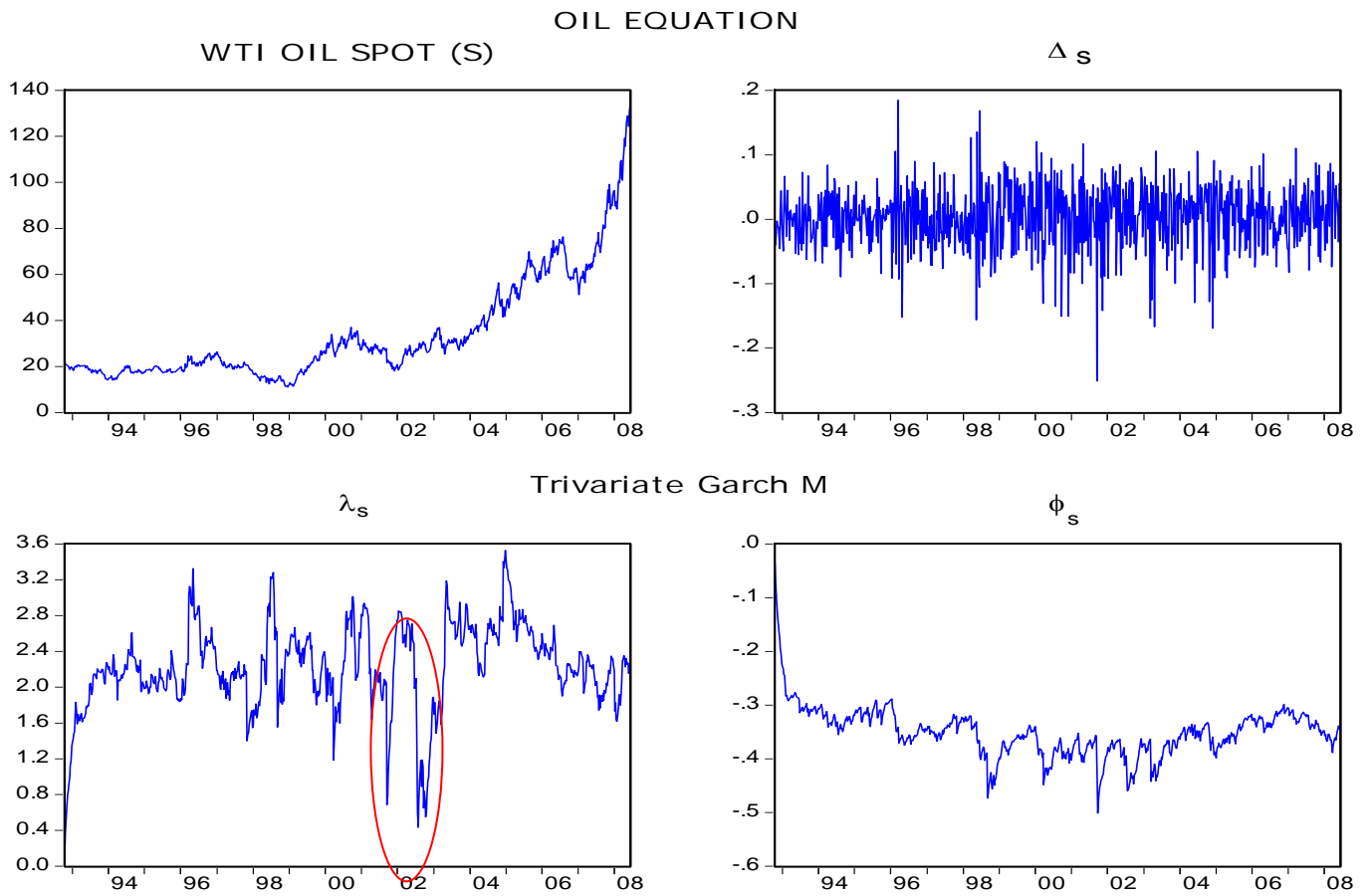
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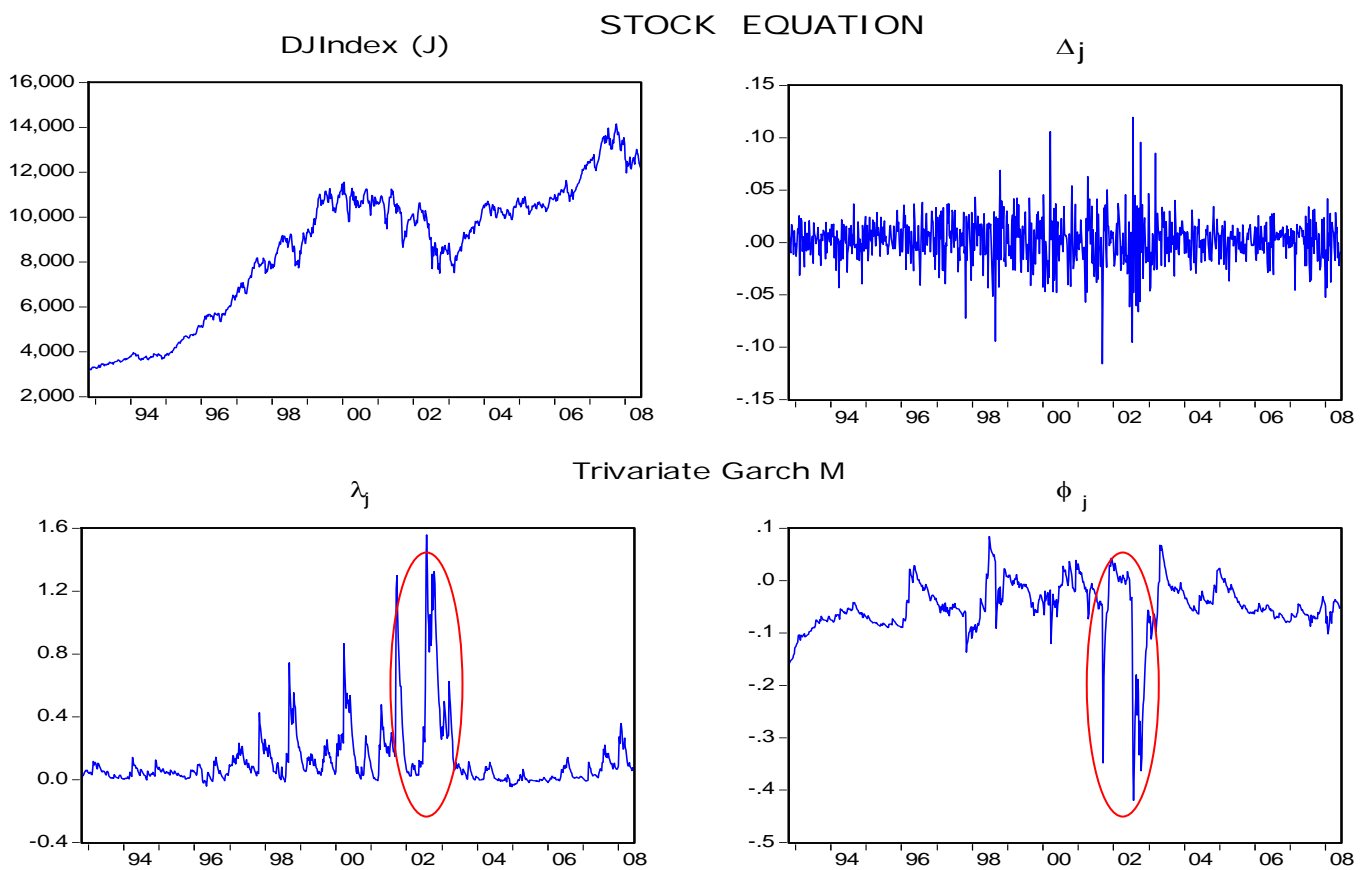
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### Graph 1

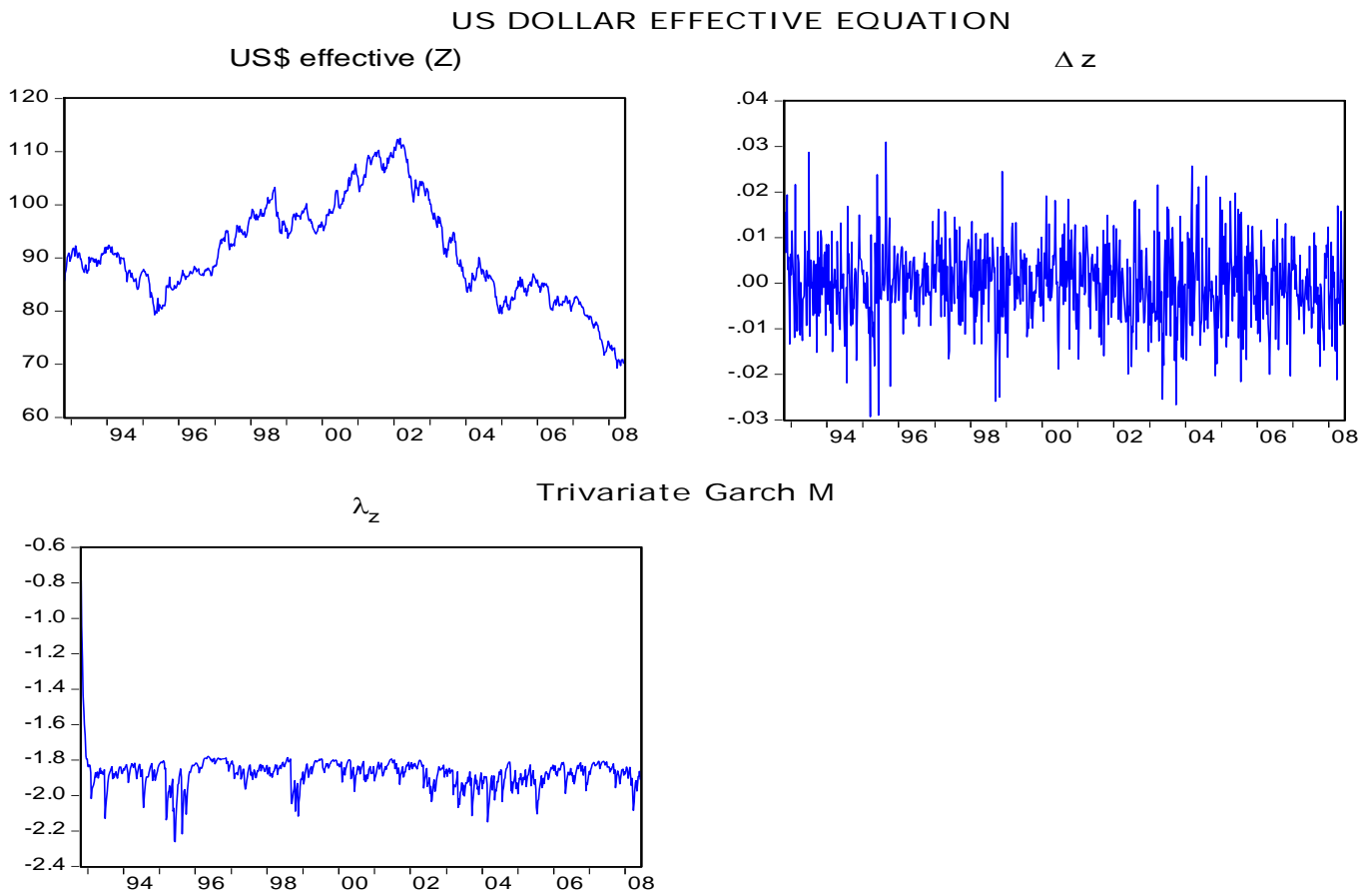


### Graph 2





### Graph 3



**Table 1. Descriptive statistics**

This table reports some basic descriptive statistics of the log first differences of the oil spot price, US dollar effective exchange rate, Dow Jones Industrial index and oil futures price at the shorted delivery date. ADF is the Augmented Dickey Fuller unit root test statistic;  $Q_x(k)$  is the Ljung Box Q-statistic for  $k^{\text{th}}$  order serial correlation of the x variable;  $Q_x^2(k)$  is the Ljung Box Q-statistic for  $k^{\text{th}}$  order serial correlation of the squared variable  $x^2$ . Data have a weekly frequency over the sample period 10/06/1992 - 6/30/2008. The sample includes 818 observations.

	Oil price rate of change	Stock price index rate of change	Effective exchange rate rate of change	Oil futures price rate of change
Mean	0.00225	0.00160	-0.00022	0.00223
Maximum	0.185	0.119	0.032	0.192
Minimum	-0.251	-0.116	-0.030	-0.239
Std. Dev.	0.048	0.022	0.009	0.048
Skewness	-0.447	-0.199	-0.008	-0.331
Kurtosis	4.595	6.984	3.574	4.535
Jarque-Bera	112.635	544.473	11.542	94.581
ADF	-32.04*	-31.53*	-28.30*	-32.14*
$Q_x(1)$	10.20*	8.26*	0.12	10.81*
$Q_x(12)$	38.02*	41.85*	11.610	34.83*
$Q_x^2(12)$	38.66*	203.20*	31.17*	37.94*

Note: \* significant at the 5 percent level.

**Table 2: Univariate CAPM with feedback trading - Oil price equation**

$$\Delta s_t = b_0 + (b_1 + b_3 \Delta s_{t-1}) h_t^2 + b_4 D_1 + b_5 \Delta f_{t-1} + b_6 \Delta SPC_{t-1} + u_t \quad (4'')$$

$$h_t^2 = \varpi + \alpha u_{t-1}^2 + \beta h_{t-1}^2 \quad (8)$$

Eq. (4'')				
$b_0$	-1.109 (-2.16)	-1.214 (-2.75)	-1.176 (-5.23)	-1.186 (-8.27)
$b_1$	0.059 (2.39)	0.058 (2.67)	0.058 (6.09)	0.058 (8.48)
$b_3$	-0.004 (-3.53)	-0.004 (-3.49)	-0.011 (-3.71)	-0.011 (-8.47)
$b_4$		1.501 (3.15)	1.411 (3.48)	1.407 (3.08)
$b_5$			0.177 (2.40)	0.177 (5.75)
$b_6$				-0.018 (-0.32)
Eq. (8)				
$\varpi$	0.606 (1.99)	0.587 (46.97)	0.571 (5.28)	0.571 (14.38)
$\alpha$	0.033 (1.56)	0.035 (2.47)	0.028 (10.26)	0.028 (15.06)
$\beta$	0.941 (28.76)	0.948 (70.34)	0.947 (317.50)	0.947 (517.86)
LLF	-2413.89	-2409.96	-2407.90	-2407.77
Residual diagnostics				
$E(v_t)$	0.003	0.002	0.0005	0.005
$E(v_t^2)$	0.999	0.999	0.999	0.999
Skew.	-0.42	-0.42	-0.40	-0.40
Kurtosis	1.51	1.50	1.47	1.48
LM(1)	0.25 [0.62]	0.21 [0.64]	0.03 [0.86]	0.03 [0.85]
LM(10)	13.31 [0.21]	13.12 [0.22]	12.67 [0.24]	12.57 [0.25]

Notes:  $v_t = u_t / \sqrt{h_t^2}$ ; Skew. : Skewness; LM(k) : Lagrange Multiplier test for k<sup>th</sup> order ARCH; t-statistics are in parentheses and probabilities in square brackets; the t-ratios are based on the robust standard errors computed with the Bollerslev and Wooldridge (1992) procedure. These notes apply also to table 3.

**Table 3: ICAPM with feedback trading - Multivariate CCC GARCH(1,1)-M**

*a. Bivariate setting: Oil price and Stock index*

System (9) Conditional mean equations															
	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$E(v_t)$	$E(v_t^2)$	Skew.	Kurtosis	LM(1)	LM(10)	LLF
$\Delta s_t$	-1.40 (-4.05)	0.12 (5.15)	3.39 (3.19)		-0.01 (-2.54)	0.06 (0.38)	1.44 (3.31)	0.18 (2.44)	0.04	1.00	-0.37	1.26	0.02 [0.88]	13.54 [0.19]	-4108.19
$\Delta j_t$	0.29 (3.42)	0.08 (2.15)	0.66 (2.04)	-0.21 (-4.08)	-0.04 (-6.88)	-0.99 (-8.48)	-0.22 (-2.11)		-0.05	1.00	-0.56	1.66	0.02 [0.88]	3.67 [0.96]	
Conditional variance equations															
	$\omega$	$\alpha$	$\beta$	$\rho_{12}$											
$h_{s,t}^2$	0.64 (4.84)	0.03 (3.89)	0.94 (99.20)	-0.04 (-6.51)											
$h_{j,t}^2$	0.16 (5.72)	0.12 (7.75)	0.85 (77.55)												

*b. Trivariate setting: Oil price, Stock index and US dollar*

System (10) Conditional mean equations																	
	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$	$b_9$	$E(v_t)$	$E(v_t^2)$	Skew.	Kurt.	LM(1)	LM(10)	LLF
$\Delta s_t$	-2.13 (-9.81)	0.02 (2.06)	3.25 (3.55)		0.01 (5.44)	0.13 (0.90)	1.33 (3.08)	0.25 (4.38)	-7.87 (-31.76)	1.27 (6.90)	0.01	1.00	-0.37	1.29	0.15 [0.70]	11.88 [0.29]	-5129.58
$\Delta j_t$	0.28 (3.26)	0.08 (4.21)	0.65 (1.52)	-0.16 (-3.83)	-0.04 (-4.82)	-0.75 (-8.38)	-0.21 (-1.87)				-0.05	1.00	-0.56	1.69	0.02 [0.89]	3.65 [0.96]	
$\Delta z_t$	1.86 (46.94)	-2.65 (-103.86)							-0.13 (-1.86)		-0.01	1.00	0.02	0.61	0.15 [0.69]	15.28 [0.12]	
Conditional variance equations																	
	$\omega$	$\alpha$	$\beta$	$\rho_{12}$	$\rho_{13}$	$\rho_{23}$											
$h_{s,t}^2$	0.61 (7.67)	0.04 (9.50)	0.93 (367.33)	-0.04 (-10.95)	-0.10 (-12.87)	0.02 (0.78)											
$h_{j,t}^2$	0.17 (4.55)	0.12 (8.42)	0.84 (150.07)														
$h_{z,t}^2$	0.17 (23.02)	0.01 (2.76)	0.75 (194.77)														

**Table 4: Likelihood Ratio tests of asset pricing restrictions within the multivariate GARCH(1,1)-M system (10)**

This table provides the results of testing two nested pricing models in the context of the trivariate GARCH-M system (10). The variance covariance matrix is estimated using the CCC GARCH(1,1) formulation.

$$\begin{aligned}\Delta s_t &= b_{0s} + b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + b_{8s}h_{sz,t} + (b_{4s}h_{s,t}^2 + b_{5s}h_{sj,t} + b_{9s}h_{sz,t})\Delta s_{t-1} + b_{6s}D_1 + b_{7s}\Delta f_{t-1} + u_{s,t} \\ \Delta j_t &= b_{0j} + b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t} + (b_{3j} + b_{4j}h_{j,t}^2 + b_{5j}h_{sj,t})\Delta j_{t-1} + b_{6j}D_2 + u_{j,t} \\ \Delta z_t &= b_{0z} + b_{1z}h_{z,t}^2 + b_{8z}h_{sz,t} + u_{z,t}\end{aligned}$$

$$u_t = \begin{bmatrix} u_{s,t} \\ u_{j,t} \\ u_{z,t} \end{bmatrix}$$

$$u_t | \Omega_{t-1} \sim N(0, H_t) \quad (10)$$

$$H_t = \Delta R \Delta$$

$$R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix} \quad \Delta_t = \begin{bmatrix} h_{s,t} & 0 & 0 \\ 0 & h_{j,t} & 0 \\ 0 & 0 & h_{z,t} \end{bmatrix}$$

$$h_{s,t}^2 = \omega_s + \alpha_s u_{s,t-1}^2 + \beta_s h_{s,t-1}^2; \quad h_{j,t}^2 = \omega_j + \alpha_j u_{j,t-1}^2 + \beta_j h_{j,t-1}^2; \quad h_{z,t}^2 = \omega_z + \alpha_z u_{z,t-1}^2 + \beta_z h_{z,t-1}^2$$

Null Model	
ICAPM	CAPM
$H_{01} :$ $b_{4s} = b_{5s} = b_{9s} = 0$ $b_{3j} = b_{4j} = b_{5j} = 0$	$H_{02} :$ $b_{2s} = b_{8s} = b_{4s} = b_{5s} = b_{9s} = 0$ $b_{2j} = b_{3j} = b_{4j} = b_{5j} = 0$ $b_{8z} = 0$
20.265* [0.002]	26.267* [0.003]

Notes: \* significant at the 5 percent level.