

NBER WORKING PAPER SERIES

OLIGOPOLISTIC PRICING AND THE EFFECTS OF AGGREGATE DEMAND ON ECONOMIC ACTIVITY

Julio J. Rotemberg

Michael Woodford

Working Paper No. 3206

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1989

We wish to thank Olivier Blanchard, Larry Christiano, Marty Eichenbaum, Bruce Greenwald, Ken Judd, Vicente Madrigal, Garth Saloner and Andrei Shleifer for comments, Robert Kollman for research assistance and the NSF and Sloan Foundations for research support. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #3206
December 1989

OLIGOPOLISTIC PRICING AND THE EFFECTS OF AGGREGATE DEMAND ON ECONOMIC ACTIVITY

ABSTRACT

We construct a dynamic general equilibrium model in which the typical industry colludes by threatening to punish deviations from an implicitly agreed upon pricing path. We argue that models of this type explain better than do competitive models the way in which the economy responds to aggregate demand shocks. When we calibrate a linearized version of the model using methods similar to those of Kydland and Prescott (1982), we obtain predictions concerning the economy's response to changes in military spending which are close to the response we estimate with postwar US data.

Julio J. Rotemberg
MIT
Sloan School of Management
Room E52-434
Cambridge, MA 02139

Michael Woodford
University of Chicago
Graduate School of Business
1101 East 58th Street
Chicago, IL 60637

In this paper we argue that the effects of aggregate demand shocks on economic activity are a consequence of imperfect competition. The aggregate demand shock which we model explicitly is a change in government purchases. For our empirical analysis we concentrate on the effect of military purchases because these are likely to be the most exogenous government purchases. In spite of this relatively narrow focus we believe that a similar analysis would be appropriate for other kinds of aggregate demand shocks.

We model the consequences of imperfect competition for aggregate fluctuations by constructing a completely specified intertemporal general equilibrium model. This model is identical to those studied in the real business cycle literature except that goods producers are modelled as oligopolistic producers using a variant of the Rotemberg and Saloner (1986) model of repeated oligopolistic interaction. We assign quantitative magnitudes to the parameters of this model on the basis of facts about the US economy that, for the most part, are unrelated to the economy's response to government purchases. We then solve the model and generate quantitative predictions regarding the effect of exogenous changes in government purchases. We then compare these predictions both to those of a similarly calibrated competitive model and to the estimated response of the economy to unpredictable changes in military purchases. We conclude that the fit between the theoretical model and the empirical observations improves when we assume that firms price oligopolistically.

Section 1 motivates our contention that a model of oligopolistic price setting is needed to understand the economy's response to changes in government purchases. Section 2 presents the

details of our model and section 3 discusses the linearization techniques used to generate quantitative predictions. Section 4 describes our choice of parameter values. Section 5 is devoted to an overview of the empirical effects of military spending in the US. Most of this discussion concerns the postwar era although we also touch on the economic effects of World War II. In Section 6 we compare the estimated responses of the economy to an unpredictable change in military purchases to those predicted by our model. Section 7 presents conclusions and directions for future research.

1. The Markup and the Transmission of Product Demand Shocks to the Labor Market

Perfectly competitive models have one major weakness when it comes to explain the effects of changes in government spending. They predict that changes of this type can increase employment *only* by increasing households' willingness to supply labor. Changes in government spending do not affect firm's demand for labor.

This weakness of the competitive model can be understood quite simply and has been noted before (Woodford (1988)).¹ Suppose that the aggregate production possibilities are described by a function $Y_t = F(K_t, H_t, z_t)$ where Y_t , K_t , H_t and z_t respectively represent output, capital, labor input and the state of technology at time t . F is a concave function of (K_t, H_t) . The capital stock K_t is predetermined at time t and z_t is exogenous with respect to firm's decisions. Then perfect competition in product and labor markets implies that the aggregate demand for labor is:

$$F_H(K_t, H_t, z_t) = w_t \quad (1.1)$$

where w_t is the real wage. For given K_t and z_t this demand curve slopes downward because F_{HH} is negative. The equilibrium level of employment (and hence output) as well as the real wage are the determined once we specify a labor supply function:

$$H_t = s(w_t, X_t) \quad (1.2)$$

representing the solution to households' optimization problem. The vector X_t includes variables such as the expected real return on savings, expectations of future real wages etc. which shift labor supply.

¹See also Lindbeck and Snower (1987).

Aggregate demand in period t can change for a variety of reasons. Demand rises if the government wants to increase its purchases. Demand for investment can rise in response to changed perceptions about future returns on investment. Demand for consumer goods can rise because consumers become more impatient and want to consume more now or because foreigners temporarily want to buy more of our output. None of these changes affect either K_t or z_t and so they cannot affect the demand for labor. Any effect they have on employment must come from an effect on the supply shifters X_t .

Several authors have proposed mechanisms through which increases in government spending shift the supply of labor. The principal such effects are wealth effects coming from the need to finance the additional government purchases and intertemporal substitution effects. The latter stem from the increase in real interest rates induced by increases in government purchases.² Other changes in aggregate demand could well have similar effects.

Insofar as demand shocks only operate through labor supply, they are unlikely to be a major factor in actual business fluctuations. The reason is that most of the evidence suggests that output fluctuations are accompanied by important changes in labor demand. First, real wages would be countercyclical if most output fluctuations were caused by changes in the variables X_t . However, a number of recent studies have found evidence of significant procyclical movements in real wages.³

Second, the evidence on quits and vacancies also suggests that business fluctuations are caused by changes in labor demand. This is apparent in Figure 1 where we show the well known relationships between unemployment, quits and vacancies.⁴ Periods of low unemployment (or high output) are also periods with large quits and large numbers of posted vacancies. This is easily rationalizable in a model such as Blanchard and Diamond (1989) as a response to changes in labor demand.

Suppose, as they do, that matching job seekers to jobs takes time so that the flow of new hires is an increasing function of both the number of job seekers and of the number of posted vacancies. An increase in labor demand would immediately increase the number of unfilled vacancies, raise the flow of new hires and reduce the unemployment rate (if the supply of available workers and the terms on which they are willing to work remain constant). The decrease in the average time spent by job-seekers in the unemployment state should also lower the threshold level of job dissatisfaction

² See Hall (1980), Barro (1981), Baxter and King (1988) and Aiyagari, Christiano and Eichenbaum (1989).

³ See Barsky and Solon (1989) and the references cited therein.

⁴ See Akerlof, Rose and Yellen (1988).

(due to mismatch between job and worker) at which workers are willing to quit and seek another job. Hence, quits should increase.

An increase in the supply of available workers or a rise in the range of offers a worker is willing to accept would also raise the number of new hires and the level of employment. But, this would deplete the pool of available vacancies and the average time spent in the unemployed state would increase. This would make quitting less attractive and the quit rate ought to decline. Variations in labor supply should result in countercyclical rather than procyclical movements in vacancies and quit rates.

In the context of the competitive model only changes in z_t and K_t shift labor demand. Therefore, if one wants to maintain the assumption of perfect competition, this evidence suggests that business fluctuations are caused by changes in technology z_t . This is an obvious reason for the popularity of real business cycle models with their attendant focus on changes in technological possibilities.

Suppose, however, that we knew some historical episodes in which aggregate demand was known to have changed. Insofar as these changes in aggregate demand raised employment, the competitive model would predict that real wages, vacancies and quits should all fall. Following Hall (1986) and (1988), we regard changes in military purchases as exogenous changes in government purchases, and hence in demand. In sections 5 and 6 we show that increases in military purchases tend to increase (and not reduce) real wages.

Figure 1 also supports the view that military purchases have an expansionary effect on labor demand. In the figure the logarithm of military purchases is plotted along with the labor market variables. One observes three main periods in which there were large increases in military purchases. These correspond to the Korean War, the Vietnam War and the Reagan military buildup. Each of these periods corresponds to a peak in both quit and vacancy rates⁵ although these events are not the only ones affecting quits and vacancies.

This evidence that increases in the demand for goods raise labor demand is evidence against the competitive model. However, it is consistent with equilibrium reasoning if the assumption of perfect competition is dropped. Suppose that producers have some market power and are able to

⁵Note that the quit data are unavailable after 1981.

set price above marginal cost. Then (1.1) must be replaced by:

$$F_H(K_t, H_t, z_t) = \mu_t w_t \quad (1.3)$$

where μ_t is the ratio of price to marginal cost (or markup) in period t . Variations in the markup now shift labor demand (the relationship between H_t and w_t) just as do technology shocks. Increases in demand, such as might be caused by increases in government purchases, can now raise output and employment even with constant labor supply as long as they lower markups.

Stiglitz (1984) also sees countercyclical markups as important in explaining macroeconomic fluctuations. We discuss some of his models as well as other models of countercyclical markups in the conclusion. The simple one we explore in depth is based on Rotemberg and Saloner (1986). The basic idea of their model is that a small number of firms within an oligopoly collude to keep their prices above marginal cost. This collusion is supported only by the threat to revert to lower prices in the future (to punish) if any member of the oligopoly deviates by cutting prices. An increase in the industry's current demand relative to the industry's future demand raises the gains from undercutting relative to the losses from the future punishment. To prevent an immediate breakdown, the optimal collusive agreement that is sustainable as an equilibrium then involves a smaller markup in these circumstances. In this way aggregate demand increases lower markups and increase labor demand.

We now proceed to describe this mechanism formally.

2. An Intertemporal General Equilibrium Model with Oligopolistic Price Setting

We embed the Rotemberg and Saloner (1986) model of oligopolistic pricing in a complete dynamic general equilibrium model. Apart from the endogenizing the demand curve and the production costs faced by each oligopoly, our presentation here extends that model in a number of respects. In particular, we allow for a much more general class of stochastic processes for industry demand,⁶ we allow for time-varying production costs, and we assume that the goods produced by the different firms in each industry are not perfect substitutes. The latter extension implies a non-trivial modification of the structure of the optimal symmetric collusive agreement. It no longer belongs to the class of equilibria in which a failure to comply with the collusive agreement results in reversion to Bertrand competition.

⁶ See also Haltiwanger and Harrington (1988), and Kandori (1988).

The economy consists of a large number of identical infinite-lived households. The representative household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \beta_s \right) N_t U \left(\frac{C_t}{N_t}, \frac{H_t}{N_t} \right) \right\} \quad (2.1)$$

where β_t denotes a (possibly stochastic) discount factor whose value is known in period t , N_t denotes the number of members per household in period t , C_t denotes total consumption by the household in period t , and H_t denotes total hours worked by members of the household in period t , including both hours supplied to the private sector and hours supplied to (or conscripted by) the government. By normalizing the number of households at one, we can use N_t also to represent the total population, C_t to denote aggregate consumption, and so on. We assume, as usual, that U is a concave function, increasing in its first argument, and decreasing in its second argument. (A more special functional form for preferences is introduced in the next section.) The representative household's budget constraint can be written

$$E_0 \left\{ \sum_{t=0}^{\infty} q_t (p_t C_t + \tau_t - H_t) \right\} \leq W_0 \quad (2.2)$$

where p_t denotes the price of the consumption good (the single final produced good) in period t in units of period t labor, τ_t denotes the period t lump sum tax liability per household, again in units of period t labor, and W_0 denotes initial wealth of the household, including both its per capita share of the initial capital stock and its per capita share of the present discounted value of profits to be obtained by operating the production technology, in units of period 0 labor. The stochastic process $\{q_t\}$ represents a stochastic pricing kernel for contingent claims; *i.e.*, a security whose payout in period $t+1$ in terms of period $t+1$ labor is a random variable $\{x_{t+1}\}$. This security has a period t value of

$$\frac{E_t q_{t+1} x_{t+1}}{q_t}$$

This process is normalized so that $q_0 = 1$.

The representative household optimizes by choosing the stochastic processes $\{C_t, H_t\}$ to maximize (2.1) subject to (2.2), given W_0 and the stochastic processes $\{p_t, q_t, \beta_t, N_t, \tau_t, H_t\}$. Sufficient conditions for this to be true are

$$\frac{U_1 \left(\frac{C_t}{N_t}, \frac{H_t}{N_t} \right)}{U_2 \left(\frac{C_t}{N_t}, \frac{H_t}{N_t} \right)} = p_t \quad (2.3)$$

$$\frac{q_{t+1}}{q_t} = \beta_t \frac{U_2\left(\frac{C_{t+1}}{N_{t+1}}, \frac{H_{t+1}}{N_{t+1}}\right)}{U_2\left(\frac{C_t}{N_t}, \frac{H_t}{N_t}\right)} \quad (2.4)$$

$$E_0 \left\{ \sum_{t=0}^{\infty} q_t (p_t C_t + \tau_t - H_t) \right\} = W_0 \quad (2.5)$$

$$E_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \beta_s \right) N_t U \left(\frac{C_t}{N_t}, \frac{H_t}{N_t} \right) \right\} < \infty \quad (2.6)$$

Under standard boundary conditions on the utility function, these conditions are also necessary for optimization, and so we assume that they hold in the equilibrium that we consider.

The single final produced good (which will be both the private consumption good, the good consumed by the government, and the capital good) is assumed to be produced using I composite intermediate goods, according to a technology

$$Y_t = f(x_t^1, \dots, x_t^I)$$

where Y_t denotes output of the final good in period t and x_t^i denotes the input used of the i th composite intermediate good. Each composite good is in turn produced using m lower-level intermediate goods, according to a technology

$$x_t^i = g(y_t^{i1}, \dots, y_t^{im})$$

where y_t^{ij} denotes the quantity used in period t of the good produced by the j th firm in the i th lower-level industry. Our intent is to model the economy as made up of large number (I) of oligopolistic industries, each of which consists of a small number (m) of firms, with collusion among the small number of firms in each industry, but monopolistic competition among industries. The producers of "composite intermediate goods" and "final goods" could be dispensed with by instead assuming that both the tastes of final consumers and the production technology used by each firm are defined over vectors of mI differentiated goods; we have chosen a more artificial route in order to simplify the description of the preferences of the representative household (2.1), and so to stress the fact that our characterization of household behavior in (2.3)-(2.6) is the same as in the RBC literature.

We make the following assumptions about the aggregation technologies:

(A1) The function f is homogeneous degree one, and a symmetric function of its arguments; *i.e.*,

$$f(x_1, \dots, x_I) = f(x_{\pi(1)}, \dots, x_{\pi(I)})$$

where π is any permutation of the indices $\{1, \dots, I\}$. In addition, $f(x, \dots, x) = x$, for all positive quantities x .

(A2) The function g is homogeneous degree one, and a symmetric function of its arguments; *i.e.*,

$$g(y_1, \dots, y_m) = g(y_{\pi(1)}, \dots, y_{\pi(m)})$$

where π is any permutation of the indices $\{1, \dots, m\}$. In addition, $g(y, \dots, y) = y$, for all positive quantities y .

The symmetry assumptions in (A1-A2) will allow us to characterize equilibrium in terms of the decision problem facing a single representative firm producing a lower-level intermediate good. The requirement that $f(x, \dots, x) = g(x, \dots, x) = x$, is a normalization that will allow us to identify quantities in the individual firm decision problem with the corresponding aggregates.

We also assume:

(A3) The isoquants of the function f (for any positive level of output Y) are entirely contained in the interior of the positive orthant.

(A4) The function g is such that the limit

$$\lim_{y^j \rightarrow 0} \frac{g_k(y, \dots, y^j, \dots, y)}{g_j(y, \dots, y^j, \dots, y)} = \tilde{\mu}$$

where the limit is well-defined and satisfies $0 < \tilde{\mu} < 1$, where y^j is the j 'th argument of g , and where k is any argument not equal to j , $\tilde{\mu}$

(A5) $\frac{m^2}{m-1}[-g_{jj}(1, \dots, 1)] < \frac{I^2}{I-1}[-f_{ii}(1, \dots, 1)]$, where i and j are arbitrary indices.

(A6) $-g_{jj}(1, \dots, 1) < \frac{(m-1)^2}{m^3}$, where j is an arbitrary index.

Assumption (A3) indicates that each industry's composite product is indispensable. Assumption (A4) implies just the opposite about the individual firms within each industry, since it implies that isoquants of g not only intersect the boundary of the positive orthant, but have a finite slope at the point of intersection. This is a reasonable assumption given that we want the goods produced by the different firms of a single "industry" to be relatively good substitutes.

If they were perfect substitutes, (A4) would hold, but with $\bar{\mu} = 1$; the assumption that $0 < \bar{\mu} < 1$ implies that they are not perfect substitutes, but nonetheless reasonably good substitutes. (Our reasons for disallowing the case of perfect substitutes are explained in the following section.) The contrary assumption regarding f indicates that the goods produced by distinct industries are less substitutable, and this helps to explain why we assume that collusion in pricing occurs between firms in the same industry and not between firms in distinct industries.

Assumption (A5) also indicates a sense in which the g technology involves greater factor substitutability than the f technology. (A5) says that the elasticity of substitution between factors, measured in the case at a symmetric production program in both cases, is greater in the case of the g technology. Finally, (A6) indicates a lower bound for factor substitutability in the g technology near a symmetrical production plan. Again, this is reasonable given our focus on industries where the various firms produce close substitutes.

We assume perfect competition in the production of final goods and the composite intermediate goods (or alternatively, optimization by the final consumers of the differentiated products of the lower-level firms), as a result of which we obtain a set of factor demand equations

$$y_i^{ij} = Y_i d^{ij}(p_i^{i1}, \dots, p_i^{im})$$

where for each ij , the function d^{ij} is homogeneous of degree zero. (This form for the demand equations follows from the homogeneity assumed for f and g .) The symmetry properties assumed in (A1-A2) imply corresponding symmetry properties for the factor demand functions. Specializing to the case where all firms outside industry i charge the same price p , we can write

$$d^{ij}(p, \dots, p^{i1}, \dots, p^{im}, \dots, p) \equiv D^j\left(\frac{p^{i1}}{p}, \dots, \frac{p^{im}}{p}\right)$$

where the demand functions D^j are the same for all industries. Also, each function D^j has the symmetry property

$$D^j(x_1, \dots, x_j, \dots, x_m) = D^j(x_{\pi(1)}, \dots, x_j, \dots, x_{\pi(m)})$$

where π is any permutation of the indices other than j . In addition, the demand functions D^j all have the same form, under appropriate permutation of the arguments, namely,

$$D^j(x_1, \dots, x_m) = D^{\pi(j)}(x_{\pi(1)}, \dots, x_{\pi(m)})$$

where π is any permutation of the indices $\{1, \dots, m\}$.

Each lower-level intermediate good is produced using capital and labor according to an increasing returns technology of the form

$$y_t^{ij} = F(K_t^{ij}, z_t H_t^{ij}) - \Phi z_t N_t$$

where K_t^{ij} denotes the capital and H_t^{ij} the hours of labor employed by firm ij , where F is increasing in both arguments and homogeneous degree one (so that marginal cost is independent of the scale of operation), and where the final term represents fixed costs. The stochastic process $\{z_t\}$ represents exogenous labor-augmenting technology shocks. Technological progress is assumed to take this form, as in King, Plosser, and Rebelo (1987), so as to allow for the existence of a stationary equilibrium without having to assume any special functional form for the production function F .

The fixed costs are included so as to make the postulated markup of prices over marginal costs (which we need in order to account for the observed procyclical response of the Solow productivity residual, as discussed the introduction) consistent with the observed moderate level of profits in the U.S. economy.⁷ The size of the fixed costs is assumed to grow over time, so that fixed costs don't eventually become unimportant as the economy grows; the particular form assumed here is convenient in allowing for the existence of a stationary equilibrium, discussed in the following section.

The m firms in each industry i take as given the stochastic process $\{p_t\}$ for the price charged by all other firms in all other industries, as well as the stochastic process $\{Y_t\}$ for final goods demand. However, they collude each period in choosing prices $(p_t^{i1}, \dots, p_t^{im})$ for their industry. In assuming that industry i does not regard its pricing decisions as having any effect upon $\{Y_t\}$, we are assuming that the large number of industries I are monopolistically competitive in the sense of authors such as Dixit and Stiglitz (1977).

The strategic interaction between the m firms in industry i can be described by the following repeated game. Each period t , each of the firms j chooses a price p_t^{ij} for its product. The single period net profits of firm j are then

$$P_t^{ij}(p_t^{i1}, \dots, p_t^{im}) = [p_t^{ij} - s_t] Y_t D^j \left(\frac{p_t^{i1}}{p_t}, \dots, \frac{p_t^{im}}{p_t} \right) - \Phi s_t z_t N_t$$

⁷ See Hall (1987).

Here s_t denotes the marginal cost of production in period t , given the price of capital services r_t and the state of technology z_t , i.e.,

$$s_t = \min_{K,H} r_t K + H \quad \text{s.t.} \quad F(K, z_t H) \geq 1 \quad (2.7)$$

The assumption of a competitive market for capital services (just as for labor services) is essential for the possibility of constant marginal costs, an assumption which (as in Rotemberg and Saloner (1986)) greatly simplifies the analysis of strategic interaction among the oligopolists.

This single-period game is repeated period after period (but with p_t , Y_t and s_t possibly time-varying); the overall objective of firm j is to maximize the discounted sum of profits

$$E_0 \left\{ \sum_{t=0}^{\infty} q_t P_t^{ij} \right\} \quad (2.8)$$

We assume that, because of antitrust laws, the several firms making up the oligopoly are unable to enforce any contractual penalties for breach of a collusive agreement. Such collusion as occurs must be enforced solely through the threat that the other firms will refuse to collude in the future if a given firm cheats on the (implicit) collusive agreement at any point in time. We also assume perfect information, i.e., each firm chooses its price p_t^{ij} with knowledge of all aggregate state variables realized in period t or earlier, and with knowledge of the complete history of prices charged by all firms in its industry in all periods prior to period t .

This sort of repeated game is well-known to admit of a very large set of Nash equilibria, and indeed of a large set of subgame perfect Nash equilibria, even if $\{p_t, Y_t, s_t\}$ are fixed over time. We further specify our equilibrium concept, as in Abreu (1986) and Rotemberg and Saloner (1986), and so are able to obtain a determinate equilibrium response to government purchases shocks in the cases considered below. We stipulate that the (implicit) collusive agreement between the firms in each industry i is the *optimal symmetric perfect equilibrium* of the oligopoly supergame. It is that subgame perfect equilibrium, among those that specify symmetric actions for all firms in the industry under all contingencies, which achieves the highest discounted present value of profits for each firm, taking as given the stochastic processes $\{p_t, Y_t, s_t\}$.

We show that, under our assumptions, the optimal symmetric collusive agreement can be given the following form (as discussed by Abreu (1986)). The agreement involves two phases, a "carrot" phase and a "stick" phase. It specifies a stochastic process $\{\bar{p}_t\}$ for the price to be charged by each

firm in the industry (which price may be contingent upon the realization of p_t, Y_t, s_t , or any other aggregate variables) if the "carrot" phase is in force in period t , and another process $\{\underline{p}_t\}$ (which may be similarly contingent) for the price to be charged if the "stick" phase is in force.

The agreement also specifies that the "carrot" phase is in force in period zero, and in all subsequent periods that follow a period in which all firms in the industry have complied with the collusive agreement, whereas the "stick" phase is in force in any period following a period in which one or more firms has failed to comply. The "carrot" phase represents successful collusion, in which all firms in the industry enjoy a high level of profits (at least, gross of fixed costs), while the "stick" phase represents a punishment phase in which profits are much lower (possibly even negative, net of fixed costs); it is the unattractiveness of the "stick" phase that sustains collusion in equilibrium.

In equilibrium, the "carrot" phase is always in force, although, as we shall see, the degree of collusion called for in the "carrot" phase may vary over time. The sort of agreement just described is a subgame perfect equilibrium if there is no incentive for any firm to deviate in either the "carrot" or the "stick" phase. Let us define

$$\pi(Y, s, p, \bar{p}) = [\bar{p} - s]Y D^j \left(\frac{\bar{p}}{p}, \dots, \frac{\bar{p}}{p} \right)$$

i.e., the level of profits gross of fixed costs obtained by each firm in the industry if all charge \bar{p} , while all firms in other industries charge p , final goods demand is Y , and marginal cost is s . Let us similarly define

$$\pi_M(Y, s, p, \bar{p}) = \max_{p'} [p' - s]Y D^j \left(\frac{\bar{p}}{p}, \dots, \frac{p'}{p}, \dots, \frac{\bar{p}}{p} \right)$$

where in this expression the $\frac{p'}{p}$ is the j th argument. This expression represents the maximum level of single period gross profits that any firm j can achieve by deviating from equilibrium play, if the equilibrium (*i.e.*, the collusive agreement) calls for each firm in the industry to charge \bar{p} . Then there is no incentive to deviate during the "carrot" phase only if

$$\pi_M(Y_t, s_t, p_t, \bar{p}_t) - \pi(Y_t, s_t, p_t, \bar{p}_t) \leq E_t \left\{ \frac{q_{t+1}}{q_t} \left[\pi(Y_{t+1}, s_{t+1}, p_{t+1}, \bar{p}_{t+1}) - \pi(Y_{t+1}, s_{t+1}, p_{t+1}, \underline{p}_{t+1}) \right] \right\} \quad (2.9)$$

at all times, and, correspondingly, there is no incentive to deviate during the "stick" phase only if

$$\pi_M(Y_t, s_t, p_t, \underline{p}_t) - \pi(Y_t, s_t, p_t, \underline{p}_t) \leq E_t \left\{ \frac{q_{t+1}}{q_t} \left[\pi(Y_{t+1}, s_{t+1}, p_{t+1}, \bar{p}_{t+1}) - \pi(Y_{t+1}, s_{t+1}, p_{t+1}, \underline{p}_{t+1}) \right] \right\} \quad (2.10)$$

at all times.

Conditions (2.9) and (2.10) guarantee that deviation in a single period, followed by compliance thereafter, is never worthwhile. They also guarantee that planning to deviate for any finite number of periods is never worthwhile. For there to be no gain from planning to deviate forever, it is also necessary that:

$$\begin{aligned} \pi_M(Y_t, s_t, p_t, \bar{p}_t) + E_t \left\{ \sum_{j=0}^{\infty} \frac{q_{t+j}}{q_t} \pi_M(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) - \Phi_{s_{t+j} z_{t+j}} N_{t+j} \right\} \leq \\ \pi(Y_t, s_t, p_t, \bar{p}_t) + E_t \left\{ \sum_{j=0}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \Phi_{s_{t+j} z_{t+j}} N_{t+j} \right\} < \infty \end{aligned}$$

where the second expression must be finite for the firm's objective (2.8) to be well-defined. An identical inequality must also hold with $\pi_M(Y_t, s_t, p_t, \bar{p}_t)$ and $\pi(Y_t, s_t, p_t, \bar{p}_t)$ replaced by $\pi_M(Y_t, s_t, p_t, \underline{p}_t)$ and $\pi(Y_t, s_t, p_t, \underline{p}_t)$ respectively. These inequalities follow from (2.9) and (2.10) if:

$$E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} [\pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) - \Phi_{s_{t+j} z_{t+j}} N_{t+j}] \right\} < \infty \quad (2.11)$$

and if

$$E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} [\pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) - \pi_M(Y_{t+j}, s_{t+j}, p_{t+j}, \underline{p}_{t+j})] \right\}.$$

is no smaller than either the left hand side of (2.9) or of (2.10). But, (2.9) and (2.10) already imply that:

$$E_t \left\{ \sum_{l=t+1}^T \frac{q_l}{q_t} [\pi(Y_l, s_l, p_l, \bar{p}_l) - \pi_M(Y_l, s_l, p_l, \underline{p}_l)] - \frac{q_T}{q_t} [\pi(Y_T, s_T, p_T, \bar{p}_T) - \pi_M(Y_T, s_T, p_T, \underline{p}_T)] \right\}$$

is no smaller than the left hand side of (2.9) or of (2.10) for all $T \geq t+1$. Hence a sufficient condition for the desired relations to hold is:

$$\lim_{T \rightarrow \infty} E_t \left\{ \frac{q_T}{q_t} [\pi_M(Y_T, s_T, p_T, \underline{p}_T) - \pi(Y_T, s_T, p_T, \underline{p}_T)] \right\} = 0. \quad (2.12)$$

Conditions (2.9)-(2.12) are thus sufficient for $\{\bar{p}_t, \underline{p}_t\}$ to characterize a subgame perfect Nash equilibrium of the oligopoly supergame.

As shown by Abreu, an agreement of this kind is the *optimal* symmetric collusive agreement only if the subgame perfect equilibrium in the “stick” phase gives all firms the *lowest possible* discounted present value of profits associated with any subgame perfect equilibrium. A sufficient condition for this to be true is

$$E_t \left\{ \pi(Y_t, s_t, p_t, \underline{p}_t) + \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \right\} = 0 \quad (2.13)$$

since a firm can always avail itself of zero profits by ceasing production. It is thus not possible to force any firm to have a discounted present value of gross profits less than zero. (We assume however that fixed costs must be paid in any event.) If (2.13) holds, (2.9) and (2.10) become

$$\pi_M(Y_t, s_t, p_t, \bar{p}_t) \leq E_t \left\{ \sum_{j=0}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \right\} \quad (2.14)$$

$$\pi_M(Y_t, s_t, p_t, \underline{p}_t) = 0 \quad (2.15)$$

where we use the fact that $\pi_M(Y, s, p, p') \geq 0$ for all values of the arguments to derive (2.15). Furthermore, (2.11) and (2.12) are both implied by the convergence of the infinite sum in (2.14) converges, if in addition:

$$E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} [s_{t+j} z_{t+j} N_{t+j}] \right\} < \infty \quad (2.16)$$

Thus conditions (2.13)-(2.16) suffice to describe a subgame perfect equilibrium in which the punishment in the “stick” phase is as bad as possible. Optimality of the collusive agreement also requires that the profits obtained by each firm in the “carrot” phase be as large as possible without violating (2.9), *i.e.*, that

$$\bar{p}_t = \arg \max_{p'} \pi(Y_t, s_t, p_t, p') \text{ s.t.}$$

$$\pi_M(Y_t, s_t, p_t, p') - \pi(Y_t, s_t, p_t, p') \leq E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \right\} \quad (2.17)$$

where we have used (2.13) to eliminate \underline{p}_{t+1} from (2.9). Note that (2.17) implies (2.14). Hence (assuming that the growth of fixed costs over time satisfies (2.16)) we have an optimal symmetric collusive agreement if the processes $\{\bar{p}_t, \underline{p}_t\}$ satisfy (2.13), (2.15), and (2.17).

Condition (2.17) is the central element in our model, since it is this condition that describes how the degree of collusion in equilibrium, and hence the equilibrium markup of prices over marginal costs, is affected by current and expected future market conditions.

We now further restrict our attention to equilibria in which the constraint in (2.17) is binding at all times. This is because we are ultimately interested only in a characterization of equilibria involving small fluctuations around a stationary deterministic growth path (in a sense to be made precise in the next section). We will assume that the constraint is strictly binding in that stationary growth path, and from this it will follow that it is binding at all times in any equilibria involving small enough fluctuations around that path. The only other case in which our linearization techniques would be applicable is that in which the constraint in (2.17) *never* binds; but in that case each oligopoly would behave like a monopolist at all times, and in that case (see footnote 7 below), no variations in the equilibrium markup are possible.

It is useful, for purposes of the characterization of small fluctuations by linear equilibrium conditions in the next section, to be able to replace (2.17) by a set of purely local conditions. In order to do this, we make the following assumptions regarding the form of the gross profit functions:

(A6) Given (Y, s, p) , $\pi(Y, s, p, p')$ is either non-decreasing for all $p' > s$, or is a unimodal function, i.e., non-decreasing for all p' below a certain critical value (the monopoly price), and non-increasing for all p' higher than that value.

(A7) There exists a unique Bertrand equilibrium price for the single-period game played by the firms in a given industry when they take as given (Y, s, p) . Also

$$\partial_{p'} \pi_M(Y, s, p, p') - \partial_{p'} \pi(Y, s, p, p') > 0$$

for all p' higher than this Bertrand price and lower than the critical value mentioned in (A6).

The behavior described in (A6) and (A7) is illustrated in Figure 2, where π and π_M are graphed as functions of p' , for given values of (Y, s, p) . Note that the inequality in (A7) might also hold for p' higher than the critical value, but cannot hold for p' slightly less than the Bertrand equilibrium price (indicated as p^B in the figure), since π and π_M must be tangent there. We do not derive these properties from assumptions regarding the underlying aggregation technology, though it is easy to show that the conditions are satisfied for well-behaved aggregation functions. In particular, (A7) holds if the products of the m firms in a given industry are sufficiently close substitutes, for in the case of perfect substitutes (treated by Rotemberg and Saloner (1986)) we have

$$\pi_M(Y, s, p, p') = m\pi(Y, s, p, p')$$

for all p' between the Bertrand price (s) and the critical value. (the monopoly price).

Given (A6) and (A7), the following conditions are then sufficient to imply (2.17), with the constraint in (2.17) always binding:

$$\pi_M(Y_t, s_t, p_t, \bar{p}_t) - \pi(Y_t, s_t, p_t, \bar{p}_t) \leq E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \right\} \quad (2.18)$$

$$\partial_{\bar{p}_t} \pi(Y_t, s_t, p_t, \bar{p}_t) > 0 \quad (2.19)$$

$$\partial_{p'} \left[(p' - s) D^j \left(\frac{\bar{p}_t}{p_t}, \dots, \frac{p'}{p}, \dots, \frac{\bar{p}_t}{p_t} \right) \right] < 0 \quad (2.20)$$

The situation described by conditions (2.18)-(2.20) is depicted in Figure 2. The figure shows how \bar{p}_t is determined, given the values of (Y_t, s_t, p_t) , and given an expectation regarding the value of

$$E_t \left\{ \sum_{j=1}^{\infty} \frac{q_{t+j}}{q_t} \pi(Y_{t+j}, s_{t+j}, p_{t+j}, \bar{p}_{t+j}) \right\}$$

We will consider only equilibria in which (2.18)-(2.20) hold. Thus the existence of an optimal symmetric collusive agreement is guaranteed if we assume that given the stochastic processes for $\{Y_t, s_t, p_t\}$, processes $\{\bar{p}_t, \underline{p}_t\}$ are specified satisfying (2.13), (2.15), and (2.18)-(2.20).

Condition (2.18) is equivalent to the pair of conditions

$$\pi_M(Y_t, s_t, p_t, \bar{p}_t) - \pi(Y_t, s_t, p_t, \bar{p}_t) = E_t \left\{ \frac{q_{t+1}}{q_t} \pi_M(Y_{t+1}, s_{t+1}, p_{t+1}, \bar{p}_{t+1}) \right\} \quad (2.21)$$

$$\lim_{T \rightarrow \infty} E_t \{ q_T \pi_M(Y_T, s_T, p_T, \bar{p}_T) \} = 0 \quad (2.22)$$

Finally, given (2.18), or alternatively, (2.21)-(2.22), (2.14) becomes

$$\pi(Y_t, s_t, p_t, \underline{p}_t) = \pi(Y_t, s_t, p_t, \bar{p}_t) - \pi_M(Y_t, s_t, p_t, \bar{p}_t) \quad (2.23)$$

It is clear that (2.23) has a solution $\underline{p}_t > 0$ for any specified values for $(Y_t, s_t, p_t, \bar{p}_t)$. For (A3) implies

$$\lim_{p \rightarrow 0} D^j(p, \dots, p) = +\infty$$

which in turn implies

$$\lim_{p' \rightarrow 0} \pi(Y, s, p, p') = -\infty$$

for all $Y, s, p > 0$. Since we also know that

$$\pi(Y, s, p, s) = 0$$

and that the right hand side of (2.23) must be non-positive, it follows from continuity that (2.23) has a solution in the interval $0 < \underline{p}_t \leq s$. Furthermore, (A4) implies that $\pi_M(Y, s, p, p') = 0$ for all $p' \leq \bar{\mu}s$. Hence a sufficient condition for (2.23) to have a solution that also satisfies (2.15) is

$$\pi(Y_t, s_t, p_t, \bar{\mu}s_t) \geq \pi(Y_t, s_t, p_t, \bar{p}_t) - \pi_M(Y_t, s_t, p_t, \bar{p}_t) \quad (2.24)$$

We will accordingly impose this condition upon the process $\{\bar{p}_t\}$ as well.

We will assume, then, that the process $\{\bar{p}_t\}$ satisfies (2.19)-(2.22) and (2.24). These conditions suffice to guarantee the existence of a process $\{\underline{p}_t\}$ such that (2.15) and (2.23) are satisfied as well, and hence that describes an optimal symmetric collusive agreement, given the processes $\{Y_t, s_t, p_t\}$. We will omit mention of the process $\{\underline{p}_t\}$ in our subsequent descriptions of equilibrium, since the "stick" phase is never in force in equilibrium, and so the process $\{\underline{p}_t\}$ neither enters any of the other equilibrium conditions of the model, nor needs to be characterized in order to predict the behavior of the observable variables.

Up to this point we have described a single industry's choice of $\{\bar{p}_t\}$ taking as given the pricing process $\{p_t\}$ for all other industries. But in the symmetrical equilibrium with which we are concerned (involving monopolistic competition among the I industries), it must turn out that $\bar{p}_t = p_t$ at all times. As a result the conditions (2.19)-(2.22) and (2.24) become

$$Y_t s_t [\sigma_M(\mu_t) - \sigma(\mu_t)] = E_t \left\{ \frac{q_{t+1}}{q_t} Y_{t+1} s_{t+1} \sigma_M(\mu_{t+1}) \right\} \quad (2.25)$$

$$\partial_{\mu'} \left[(\mu' - 1) D^j \left(\frac{\mu'}{\mu_t}, \dots, \frac{\mu'}{\mu_t} \right) \right]_{\mu' = \mu_t} > 0 \quad (2.26)$$

$$\partial_{\mu'} \left[(\mu' - 1) D^j (1, \dots, \frac{\mu'}{\mu_t}, \dots, 1) \right]_{\mu' = \mu_t} < 0 \quad (2.27)$$

$$\lim_{T \rightarrow \infty} E_t \{ q_T Y_T s_T \sigma_M(\mu_T) \} = 0 \quad (2.28)$$

$$\sigma(\bar{\mu}) \geq \sigma(\mu_t) - \sigma_M(\mu_t) \quad (2.29)$$

where we have used the fact that the gross profit functions are proportional to Y and homogeneous of degree one in (s, p, \bar{p}) to write

$$\pi(Y, s, p, p) \equiv Y s \sigma \left(\frac{p}{s} \right)$$

$$\pi_M(Y, s, p, p) \equiv Y s \sigma_M \left(\frac{p}{s} \right)$$

and where μ_t is the markup of price over marginal cost p_t/s_t . Therefore:

$$\sigma(\mu) = \mu - 1. \quad (2.30)$$

Because $D^j(1, \dots, 1) = 1$, conditions (2.26) and (2.27) are equivalent to:

$$\left[\sum_k D_k^j(1, \dots, 1) \right]^{-1} < \frac{1-\mu}{\mu} < \left[D_j^j(1, \dots, 1) \right]^{-1}$$

It follows from our definition of the function D^j that

$$\sum_k D_k^j(1, \dots, 1) = \frac{(I-1)^2}{I^3} [f_{ii}(1, \dots, 1)]^{-1} < 0$$

$$D_j^j(1, \dots, 1) = m^{-1} \frac{(I-1)^2}{I^3} [f_{ii}(1, \dots, 1)]^{-1} + \frac{(m-1)^2}{m^3} [g_{jj}(1, \dots, 1)]^{-1} < 0$$

where f_{ii} and g_{jj} denote arbitrary diagonal elements of the matrices of second derivatives in question. Both of these quantities are negative because of the concavity of the functions f and g , and (A.5) implies that

$$D_j^j(1, \dots, 1) < \sum_k D_k^j(1, \dots, 1) < 0$$

while (A.6) implies that

$$D_j^j(1, \dots, 1) < -1.$$

As a result, conditions (2.26) and (2.27) are equivalent to:

$$\underline{\mu} < \mu_t < \bar{\mu} \quad (2.31)$$

where

$$\underline{\mu} \equiv \frac{D_j^j(1, \dots, 1)}{1 + D_j^j(1, \dots, 1)}$$

$$\bar{\mu} \equiv \frac{\sum_k D_k^j(1, \dots, 1)}{\min(1 + \sum_k D_k^j(1, \dots, 1), 0)}$$

(We intend for $\bar{\mu}$ to be $+\infty$ if the denominator is zero). It follows from (A.5) and (A.6) that

$$1 < \underline{\mu} < \bar{\mu}$$

This makes it possible for solutions to exist to equation (2.31). (Note that it follows that assumptions similar to (A5) and (A6) are essential for the existence of equilibria of the kind considered here). The complete set of equilibrium conditions that must be satisfied by the markup process $\{\mu_t\}$ are therefore (2.25), (2.28), (2.29), and (2.31).⁸

Since the equilibrium is symmetric, each firm employs the same quantities of capital and labor so that we may write simply

$$Y_t = F(K_t, z_t H_t^P) - \Phi z_t N_t \quad (2.32)$$

where K_t denotes the aggregate capital stock in period t , and H_t^P denotes aggregate hours supplied to the private sector. In equilibrium the latter quantity is related to the representative household's supply of hours through the relation

$$H_t = H_t^P + H_t^g \quad (2.33)$$

where H_t^g denotes hours demanded by the government.⁹

Factor demands by the lower-level intermediate goods producing firms are determined by (2.32) together with the cost-minimization condition

$$r_t = \frac{F_K(K_t, z_t H_t^P)}{z_t F_H(K_t, z_t H_t^P)} \quad (2.34)$$

It follows that the marginal cost of production each period will be given by

$$s_t = \frac{1}{z_t F_H(K_t, z_t H_t^P)} \quad (2.35)$$

Furthermore, optimal capital accumulation requires

$$p_t = E_t \left\{ \frac{q_{t+1}}{q_t} [r_{t+1} + (1 - \delta)p_{t+1}] \right\} \quad (2.36)$$

⁸ It should be clear at this point that in an equilibrium where the constraint in (2.17) never binds, the markup of price over marginal cost cannot be time-varying. For in that case equations (2.25)-(2.26) would instead be replaced by

$$\partial_{\mu'} \left[(\mu' - 1) D^j \left(\frac{\mu'}{\mu_t}, \dots, \frac{\mu'}{\mu_t} \right) \right] = 0$$

and an inequality indicating that the constraint does not bind. And for the same reason that (2.26) is equivalent to $\mu_t < \beta$, the above equality is equivalent to $\mu_t = \beta$, so that the markup would be constant.

⁹ While we have written the household budget constraint (2.2) under the assumption that the market wage is paid for both government and private hours it does not matter for purposes of the derivation of the equilibrium dynamics of any quantities other than the lump sum tax variable whether we suppose that the government hours are purchased in a competitive labor market or directly levied through conscription.

where δ denotes the rate of depreciation of the capital stock.

Since the only non-zero profits in this economy must be earned by lower-level intermediate goods producers, the total initial wealth of the representative household is given by

$$W_0 = [r_0 + (1 - \delta)p_0]K_0 + E_0 \left\{ \sum_{t=0}^{\infty} q_t [\pi(Y_t, s_t, p_t, p_t) - \Phi s_t z_t N_t] \right\} \quad (2.37)$$

Finally, equilibrium requires market-clearing in the final goods market, *i. e.*,

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t + G_t \quad (2.38)$$

where G_t denotes period t government purchases of final goods (*i. e.*, the contribution of government purchases to *private sector value added*). The process for government purchases must be chosen to be consistent with the government budget constraint

$$E_0 \left\{ \sum_{t=0}^{\infty} q_t (p_t G_t + H_t^g - r_t) \right\} = 0 \quad (2.39)$$

An *equilibrium* is then a set of stochastic processes $\{p_t, r_t, q_t, C_t, H_t, K_t, H_t^p, Y_t, s_t, \mu_t\}$, and a quantity W_0 , that satisfy (2.3)-(2.6), (2.16), (2.25), and (2.28)-(2.38), taking as given K_0 , the exogenous processes $\{\beta_t, z_t, N_t\}$, and government policy choices $\{G_t, H_t^g, r_t\}$ that satisfy (2.39). In fact, (2.5) is implied by (2.32)-(2.39), so that it need not be listed as a separate equilibrium condition. We need only verify that the value for W_0 specified by (2.37) is indeed a well-defined quantity, and this is guaranteed by (2.16) and the convergence of the sum in (2.18), which in turn is implied by (2.25) and (2.28). Hence we can eliminate all of the equations involving W_0 from the definition of equilibrium. An equilibrium is then a set of stochastic processes $\{p_t, r_t, q_t, C_t, H_t, K_t, H_t^p, Y_t, s_t, \mu_t\}$ that satisfy (2.3), (2.4), (2.6), (2.16), (2.25), and (2.28)-(2.36), and (2.38), taking as given K_0 , and the exogenous processes $\{\beta_t, z_t, N_t, G_t, H_t^g\}$.

Despite the relatively involved derivation just presented, it is important to note that the final set of equilibrium conditions arrived at are only marginally more complex than those associated with a perfectly competitive stochastic growth model of the kind used to study the effects of government purchases by authors such as Baxter and King (1987), Aiyagari, Christiano, and Eichenbaum (1989), and Wynne (1988). In fact, all of the equilibrium conditions just listed are identical to those associated with a competitive representative consumer economy with a one-sector production

technology, with the exception of the block of conditions (2.25), (2.28), (2.29), and (2.31), describing determination of the equilibrium markup of price over marginal cost. The equilibrium conditions for the perfectly competitive model are obtained by supplementing (2.3), (2.4), (2.6), (2.16), (2.30), (2.32)-(2.36), and (2.38) with

$$\mu_t = 1 \quad (2.40)$$

From the perspective of computing solutions to the model the difference is even more minor. Assuming the various inequality and boundary conditions are met, solving the competitive model requires solving (2.3), (2.4), (2.30), (2.32)-(2.36) and (2.40). The oligopoly model is solved by replacing the static pricing equation (2.40) by the dynamic equation (2.25). We compare the two models predictions in section 6 below.

3. Small Fluctuations around a Stationary Growth Path

We restrict our attention further to economies in which the variance of the exogenous shock processes is small, and to equilibria in which the state variables fluctuate only a small amount around their values in a stationary growth path. Then, a log-linear approximation to the above equilibrium conditions yields a valid characterization of equilibrium fluctuations. The purpose of this is to allow us to characterize equilibrium fluctuations in terms of a linear approximation to the equilibrium conditions of the previous section, using the technique of Woodford (1986). This linear approximation allows a great saving in the computational difficulty of the simulations reported in section 6 as well as allowing a direct comparison of our simulation results with empirical evidence from linear vector autoregressions.¹⁰

We first consider a transformed set of state variables, in terms of which it is possible for a stationary equilibrium to exist. Let us define

$$\begin{array}{llll} n_t \equiv \frac{N_t}{N_{t-1}} & \lambda_t \equiv \frac{z_t}{z_{t-1}} & \eta_t \equiv \frac{q_t}{q_{t-1}} & \hat{s}_t \equiv z_t s_t \\ \hat{G}_t \equiv \frac{G_t}{z_t N_t} & \hat{Y}_t \equiv \frac{Y_t}{z_t N_t} & \hat{C}_t \equiv \frac{C_t}{z_t N_t} & \hat{p}_t \equiv z_t p_t \\ \hat{K}_{t+1} \equiv \frac{K_{t+1}}{\lambda n z_t N_t} & \hat{H}_t \equiv \frac{H_t}{N_t} & \hat{H}_t^p \equiv \frac{H_t^p}{N_t} & \hat{H}_t^q \equiv \frac{H_t^q}{N_t} \end{array}$$

¹⁰ Our method of linearisation is essentially equivalent to that of King, Plosser and Rebelo (1987). It is also closely related to the method of quadratic approximation of the representative household's objective function used by Kydland and Prescott (1982) in the case of the perfectly competitive model, although that method is not applicable to our model since the equilibrium does not solve a planning problem.

where λn is a constant to be explained below, and let us assume that the exogenous processes $\{\beta_t, n_t, \lambda_t, \hat{G}_t, \hat{H}_t^d\}$ are all stationary.¹¹ (Note that this allows for the possibility of either a unit root or a deterministic time trend in $\{N_t\}$ and/or in $\{z_t\}$, and for $\{G_t\}$ and $\{H_t^d\}$ to inherit unit roots or time trends from these variables.) We then wish to consider equilibria in which all of the variables $\{\hat{C}_t, \hat{Y}_t, \hat{K}_t, \hat{H}_t, \hat{H}_t^p, \hat{s}_t, \hat{p}_t, \mu_t, \eta_t\}$ follow stationary stochastic processes.

In order for such equilibria to exist, when both $\{N_t\}$ and $\{z_t\}$ possess either a time trend or a unit root, it is necessary that the utility function U have certain homogeneity properties. Specifically, it is necessary that

$$\frac{U_2(c, h)}{U_2(c', h')}$$

be homogeneous degree zero in (c, c') , and that

$$\frac{U_2(c, h)}{U_1(c, h)}$$

be homogeneous of degree one in c , because of (2.3) and (2.4). These requirements imply that we must restrict U (as in King, Plosser, and Rebelo (1987)) to be of the form

$$U(c, h) = \frac{c^{1-\rho} e^{(\rho-1)v(h)}}{1-\rho} \quad (3.1)$$

where $\rho > 0$, $\rho \neq 1$, and v is a monotonically increasing function, or of the form

$$U(c, h) = \log c - v(h)$$

where again v is an increasing function. The latter expression is simply a limiting case ($\rho = 1$) of the former one, and so (3.1) is assumed in the expressions written below. Concavity of U requires in addition that

$$(A8) \quad v''(h) + (\rho - 1)(v'(h))^2 > 0, \quad \rho v''(h) + (\rho - 1)(v'(h))^2 > 0, \quad \text{for all } h > 0.$$

With this change of variables, a *stationary equilibrium* is a set of jointly stationary stochastic processes $\{\hat{C}_t, \hat{Y}_t, \hat{K}_t, \hat{H}_t, \hat{H}_t^p, \hat{s}_t, \hat{p}_t, \mu_t, \eta_t\}$ that satisfy

$$\hat{Y}_t = F \left(\frac{\lambda n}{\lambda_t n_t} \hat{K}_t, \hat{H}_t^p \right) - \Phi \quad (3.2)$$

¹¹The variable \hat{K}_{t+1} is defined in the manner indicated so that \hat{K}_{t+1} , like K_{t+1} , is a variable that is determined at time t . The constant λn is included so that in the case of the stationary deterministic growth path described below, the constant ratio of $\frac{K_t}{Y_t}$ corresponds to the constant ratio of $\frac{K_t}{Y_t}$.

$$\hat{Y}_t = \hat{C}_t + \lambda n \hat{K}_{t+1} - (1 - \delta) \frac{\lambda n}{\lambda_t n_t} \hat{K}_t + \hat{G}_t \quad (3.3)$$

$$\hat{s}_t = \frac{1}{F_H \left(\frac{\lambda n}{\lambda_t n_t} \hat{K}_t, \hat{H}_t^p \right)} \quad (3.4)$$

$$\hat{p}_t = \frac{1}{\hat{C}_t v'(\hat{H}_t)} \quad (3.5)$$

$$\mu_t = \frac{\hat{p}_t}{\hat{s}_t} \quad (3.6)$$

$$\hat{H}_t = \hat{H}_t^p + \hat{H}_t^g \quad (3.7)$$

$$\eta_t = \beta_{t-1} \frac{v'(\hat{H}_t)}{v'(\hat{H}_{t-1})} \left(\frac{\hat{C}_t}{\hat{C}_{t-1}} g_t \right)^{1-\rho} e^{-(1-\rho)[v(\hat{H}_t) - v(\hat{H}_{t-1})]} \quad (3.8)$$

$$\hat{p}_t = E_t \left\{ \frac{\eta_{t+1}}{g_{t+1}} \left[\frac{F_K \left(\frac{\lambda n}{\lambda_{t+1} n_{t+1}} \hat{K}_{t+1}, \hat{H}_{t+1}^p \right)}{F_H \left(\frac{\lambda n}{\lambda_{t+1} n_{t+1}} \hat{K}_{t+1}, \hat{H}_{t+1}^p \right)} + (1 - \delta) \hat{p}_{t+1} \right] \right\} \quad (3.9)$$

$$\hat{Y}_t \hat{s}_t [\sigma_M(\mu_t) - \sigma(\mu_t)] = E_t \{ \eta_{t+1} \hat{Y}_{t+1} \hat{s}_{t+1} n_{t+1} \sigma_M(\mu_{t+1}) \} \quad (3.10)$$

$$\underline{\mu} < \mu_t < \bar{\mu} \quad (3.11)$$

$$\sigma(\bar{\mu}) \geq \sigma(\mu_t) - \sigma_M(\mu_t) \quad (3.12)$$

$$E_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \beta_s n_{s+1} \lambda_{s+1}^{1-\rho} \right) \hat{C}_t^{1-\rho} e^{-(1-\rho)v(\hat{H}_t)} \right\} < \infty \quad (3.13)$$

$$E_t \left\{ \sum_{j=1}^{\infty} \left(\prod_{s=t}^{t+j-1} \beta_s n_{s+1} \lambda_{s+1}^{1-\rho} \right) v'(\hat{H}_{t+j}) \hat{s}_{t+j} \hat{C}_{t+j}^{1-\rho} e^{-(1-\rho)v(\hat{H}_{t+j})} \right\} < \infty \quad (3.14)$$

$$\lim_{T \rightarrow \infty} E_t \left\{ \left(\prod_{s=1}^T \eta_s n_s \right) \hat{Y}_T \hat{s}_T \sigma_M(\mu_T) \right\} = 0 \quad (3.15)$$

given the exogenous processes $\{\beta_t, n_t, \lambda_t, \hat{G}_t, \hat{H}_t^g\}$.

Next we consider a stationary deterministic equilibrium growth path, under the assumption that the exogenous variables are all constant, *i.e.*, $\beta_t = \beta, \lambda_t = \lambda, n_t = n, \hat{G}_t = \hat{G}, \hat{H}_t^g = \hat{H}^g$. This is a solution to (3.2)-(3.15) in which the endogenous variables are also all constant, *i.e.*, $\hat{C}_t = \hat{C}, \hat{K}_t = \hat{K}$, etc. In other words, it is a set of constant values \hat{C}, \hat{K} , etc., that satisfy the equations

$$\hat{Y} = F(\hat{K}, \hat{H}^p) - \Phi \quad (3.16)$$

$$\hat{Y} = \hat{C} + [\lambda n - (1 - \delta)] \hat{K} + \hat{G} \quad (3.17)$$

$$\hat{s} = \frac{1}{F_H(\hat{K}, \hat{H}^p)} \quad (3.18)$$

$$\hat{p} = \frac{1}{\hat{C}v'(\hat{H})} \quad (3.19)$$

$$\mu = \frac{\hat{p}}{\hat{s}} \quad (3.20)$$

$$\hat{H} = \hat{H}^p + \hat{H}^g \quad (3.21)$$

$$\eta = \beta\lambda^{1-\rho} \quad (3.22)$$

$$\hat{p} = \beta\lambda^{-\rho} \left[\frac{F_K(\hat{K}, \hat{H}^p)}{F_H(\hat{K}, \hat{H}^p)} + (1 - \delta)\hat{p} \right] \quad (3.23)$$

$$\sigma_M(\mu) - \sigma(\mu) = \beta n\lambda^{1-\rho}\sigma_M(\mu) \quad (3.24)$$

$$\underline{\mu} < \mu < \bar{\mu} \quad (3.25)$$

$$\sigma_M(\mu) - \sigma(\mu) \geq -\sigma(\bar{\mu}) \quad (3.26)$$

$$\beta n\lambda^{1-\rho} < 1 \quad (3.27)$$

(Here condition (3.27) guarantees each of the conditions (3.13)-(3.15).)

We also imagine that, as in the Chamberlinian model, entry and exit are possible in the long run in the various intermediate goods industries. So, in the long run the number of firms per industry is determined by a zero profit condition. Since this entry can occur only very slowly, we ignore changes in the number of firms brought about by temporary disturbances. Rather, we assume that fluctuations occur around a steady state growth path along which there are zero profits (net of fixed costs), *i.e.*,

$$\hat{Y}\sigma(\mu) - \Phi = 0$$

Using (2.30), this becomes:

$$\mu = 1 + \frac{\Phi}{\hat{Y}} \quad (3.28)$$

We simply assume the existence of such a stationary equilibrium, for values of the exogenous variables $\beta, \lambda, n, \hat{G}$, and \hat{H}^g corresponding to their actual mean values, rather than showing that one must always exist under some general set of conditions. However, the existence question must be addressed at least briefly in order to indicate that the listed set of conditions are not internally contradictory.

In considering whether a solution to equations (3.16)-(3.28) exists, let us take as given the number of firms per industry, and the various production and utility functions, but solve for Φ as one of the endogenous variables.¹² The stationary markup μ must satisfy conditions (3.24)-(3.26), which involve μ alone. Let us suppose, first, that a solution exists. Then (3.23) implies that the stationary capital-labor ratio $\frac{\hat{K}}{\hat{H}^p}$ must satisfy

$$F_K \left(\frac{\hat{K}}{\hat{H}^p}, 1 \right) = \mu \left[\frac{\lambda^p}{\beta} - (1 - \delta) \right].$$

Thus existence of a solution requires that the exogenous parameters β, g , and δ happen to be such that

$$\frac{\lambda^p}{\beta} > 1 - \delta \quad (3.29)$$

But this is easily seen to be consistent with our other assumptions about these parameters, and with (3.27). If the condition is satisfied, then standard Inada conditions on the marginal product of capital will guarantee the existence of a solution $\frac{\hat{K}}{\hat{H}^p}$. Given $\frac{\hat{K}}{\hat{H}^p}$, \hat{s} is determined through (3.18), and this in turn determines \hat{p} through (3.20). Furthermore, (3.28) determines a positive solution for $\frac{\hat{\Phi}}{\hat{Y}}$. Then $\frac{\hat{Y}}{\hat{H}^p}$ is determined by

$$\frac{\hat{Y}}{\hat{H}^p} = \frac{1}{\mu} F \left(\frac{\hat{K}}{\hat{H}^p}, 1 \right)$$

because of (3.16). And (3.17) and (3.19) imply that \hat{H}^p must satisfy

$$\frac{\hat{Y}}{\hat{H}^p} = \frac{1}{\hat{p}\hat{H}^p v'(\hat{H}^p + \hat{H}^g)} + [\lambda n - (1 - \delta)] \frac{\hat{K}}{\hat{H}^p} + \frac{\hat{G}}{\hat{H}^p}$$

Given values for $\frac{\hat{Y}}{\hat{H}^p}, \hat{p}, \frac{\hat{K}}{\hat{H}^p}, \hat{G}, \hat{H}^g$, the right hand side of this equation is a monotonically decreasing function of \hat{H}^p , while the left hand side is a positive constant. Furthermore, the right hand side becomes an unboundedly large positive quantity for \hat{H}^p small enough, and approaches zero for \hat{H}^p very large. Hence by continuity there exists a value of \hat{H}^p satisfying the condition, and this in turn implies positive values for \hat{Y} and \hat{K} , as well as a positive value for \hat{C} from (3.19). Finally, the value of η is given by (3.22).

Thus if the exogenous parameters satisfy (3.27) and (3.29), and if there exists a value of μ satisfying (3.24)-(3.26), we can find a complete set of constant values for $\hat{Y}, \hat{K}, \hat{H}^p, \hat{C}, \hat{s}, \hat{p}, \mu, \eta$, and

¹²Given our story about entry as an equilibrating mechanism, it would be more appropriate to let Φ be exogenous and m endogenous. However, this would complicate the analysis: we would have to define functions such as g for variable m . Moreover the alternative procedure gives us equally valid conclusions regarding the consistency of equations (3.16)-(3.28) and the robustness of the case in which a solution exists.

Φ that satisfy all of conditions (3.16)-(3.28). It remains only to consider the internal consistency of conditions (3.24)-(3.26) for the determination of μ . The sort of situation in which these conditions are satisfied is illustrated in Figure 3.

Equation (2.30) implies that we can draw the curve $\sigma(\mu)$ as a straight line. We showed in the previous section that a point $\underline{\mu} > 1$, necessarily exists. Also, the definition of σ_M implies that $\sigma_M(\mu) > \sigma(\mu)$ for all $\mu \neq \underline{\mu}$, with tangency of the two curves at $\underline{\mu}$, as shown. Because of this, the curve $(1 - \beta n \lambda^{1-\rho})\sigma_M(\mu)$ necessarily lies below $\sigma(\mu)$ near $\underline{\mu}$. In order for a point $\mu^* > \underline{\mu}$ satisfying (3.24) to exist, it is only necessary that the ratio $\frac{\sigma_M(\mu)}{\sigma(\mu)}$ eventually become large enough as μ is made large.

Condition (3.26) is satisfied by μ^* if

$$\mu^* > 1 + (1 - \tilde{\mu}) \frac{1 - \beta n \lambda^{1-\rho}}{\beta n \lambda^{1-\rho}} \quad (3.30)$$

which holds if the curves cross in the way shown in the figure. If we suppose that the case of interest is that in which the "period" in our model (*i.e.*, the period of time before collusion breaks down in response to one firm's underpricing) is short, compared to consumers' rate of time preference and the rates of growth of both population and labor productivity), then we should assume that $\beta n \lambda^{1-\rho}$ is not much smaller than 1. And if we are most interested in the case in which the products of the several firms in a single industry are reasonably good substitutes, then we should assume that $\tilde{\mu}$ is not much smaller than 1. Either assumption makes it likely that (3.30) should obtain.

The upper bound inequality in (3.25) is clearly consistent with the other equations, since its tightness depends upon the size of $\sum_k D_k^j(1, \dots, 1)$ - and in particular, the inequality is necessarily satisfied if $\sum_k D_k^j(1, \dots, 1) > -1$ - whereas the condition defining μ^* depends only upon the form of the function $D^j(1, \dots, z, \dots, 1)$. The assumptions on the form of this function that are needed in order for a solution μ^* to exist to (3.24) place no restriction upon how small a negative number $\sum_k D_k^j(1, \dots, 1)$ might be, and hence do not contradict the requirement that $\mu^* \leq \tilde{\mu}$.

It is worth noting, however, that if the products of the firms within a single industry are perfect substitutes, a locally unique solution to (3.24)-(3.26), as shown in Figure 3, is not possible. In this case, $\tilde{\mu} = 1 = \underline{\mu}$, and

$$\sigma_M(\mu) = m \min(\sigma(\mu), \sigma(\tilde{\mu}))$$

This implies that there cannot be a solution to (3.24) that also satisfies (3.25), unless $\beta n \lambda^{1-\rho} = \frac{m-1}{m}$ exactly. In that case, the firms can support any outcome between the Bertrand and the monopoly point in the steady state. Our criterion that they pick the equilibrium that maximizes oligopoly profits then requires that they choose the monopoly point. But, then, variations in demand and cost conditions will make firms alternate between two regimes. In the first, the firms are able to support the fully collusive outcome. In the other regime they can only support lower prices than the ones that maximize industry profits. Because the local properties of the economy are not the same in the two regimes, our linearization technique is not valid in this case. For this reason we have assumed imperfect substitutability of goods and, as a result, our equilibrium is more complicated than the one in Rotemberg and Saloner (1986).

Given technology and preference specifications, and mean values for the exogenous shock variables $\{\beta_t, \lambda_t, n_t, \hat{G}_t, \hat{H}_t^p\}$ such that a stationary deterministic growth path of the kind just described would exist if the shock variables always took their mean values, let us now suppose that any or all of the shock variables are stochastic but confined to small neighborhoods of their mean values. Let us also consider solutions to (3.2)-(3.15) in which the endogenous state variables $\{\hat{C}_t, \hat{K}_t, \hat{Y}_t, \hat{H}_t, \hat{H}_t^p, \hat{s}_t, \hat{p}_t, \mu_t, \eta_t\}$ all remain forever within small neighborhoods of the constant values that satisfy (3.16)-(3.28). Conditions for the existence of such solutions, and a characterization of the co-movements of the various state variables in such solutions, can be given using the implicit function theorem techniques of Woodford (1986, especially Theorem 2).

The equilibria with which we are concerned involve "small fluctuations around a stationary deterministic growth path" because the transformed variables $\{\hat{C}_t, \hat{Y}_t, \dots\}$ have a small variance; but if $\{z_t\}$ and/or $\{N_t\}$ is a non-stationary process, the endogenous variables $\{C_t, Y_t, \dots\}$ will be non-stationary as well. In this case the latter variables will not have finite unconditional variances, and the equilibrium values will not remain forever near a deterministic trend path.

First, note that the state variables $\{\hat{Y}_t, \hat{H}_t, \hat{s}_t, \hat{p}_t, \mu_t, \eta_t\}$ can be eliminated using equations (3.2) and (3.4)-(3.8). What remain are a set of three stochastic difference equations of the form

$$F(X_t, Z_t, X_{t+1}) = 0$$

$$E_t\{G(X_t, Z_t, X_{t+1}, Z_{t+1})\} = 0$$

where

$$X_t = \begin{pmatrix} \hat{C}_t \\ \hat{H}_t^p \\ \hat{K}_t \end{pmatrix} \quad Z_t = \begin{pmatrix} \beta_t \\ \lambda_t \\ n_t \\ \hat{G}_t \\ \hat{H}_t^q \end{pmatrix}$$

F is a smooth scalar-valued function, and G is a smooth two-component vector-valued function, together with a set of inequalities. The inequalities are necessarily satisfied as long as the state variables all remain forever within small enough neighborhoods of their steady state values, and so the inequalities can be ignored. Necessary and sufficient conditions (given some rather weak regularity assumptions discussed further in Woodford (1986)) for the existence of stationary solutions in which the state variables remain forever within given neighborhoods of their steady state values, if the neighborhoods are chosen small enough, are (i) that the 3×3 matrix of derivatives

$$[D_3F \quad D_3G]$$

be non-singular, and (ii) that the 3×3 matrix

$$-[D_3F \quad D_3G]^{-1}[D_1F \quad D_1G]$$

have $q \geq 1$ eigenvalues with modulus less than one, and $3 - q$ with moduli greater than one. In this case, the stationary solutions involving small fluctuations to the exact nonlinear system can be approximated (with the validity of the approximation being greater the smaller the fluctuations in the state variables around their steady state values) by the stationary solutions to the linearized system of equations

$$[D_1F](X_t - X) + [D_2F](Z_t - Z) + [D_3F](X_{t+1} - X) = 0 \quad (3.31)$$

$$[D_1G](X_t - X) + [D_2G](Z_t - Z) + [D_3G]E_t(X_{t+1} - X) + [D_4G]E_t(Z_{t+1} - Z) = 0 \quad (3.32)$$

where X and Z are the vectors of steady-state values for the corresponding state variables, and where the derivatives of F and G are evaluated at the steady-state values of their arguments.

It is well known (see, *e.g.*, Blanchard and Kahn (1980)) that a system of linear equations of this sort has stationary solutions if and only if conditions (i) and (ii) are satisfied by the matrices of coefficients. Furthermore, there is a unique stationary solution if in addition $q = 1$ exactly (the case

of "saddle-point stability"). Woodford (1986) shows that this is also exactly the case in which the exact nonlinear equations have a locally unique stationary solution, near the deterministic steady state. For the calibrated parameter values discussed in the next section, and for all sufficiently nearby values, we find that $q = 1$, and so we are able to derive a unique equilibrium response to the type of shock to military spending with which we are concerned.¹³ We can approximate this unique response by calculating the solution to (3.31) and (3.32) using the formulae of Blanchard and Kahn (1980) and Hansen and Sargent (1980). Properties of this solution are reported below in the form of impulse response functions.

The actual elements of the matrices in (3.31) and (3.32) are provided in the Appendix as a function of our model's parameters. To facilitate calibration, we also write these elements as a function of the following variables:

$$k = \frac{\bar{K}}{\bar{Y}} = \text{capital-output ratio}$$

$$c = \frac{\bar{C}}{\bar{Y}} = \text{share of consumer expenditure in private sector value added}$$

$$g = \frac{\bar{G}}{\bar{Y}} = \text{share of government purchases in private sector value added}$$

$$s_K = \frac{F_K(\bar{K}, \bar{H}^p) \bar{K}}{F(\bar{K}, \bar{H}^p)} = \text{capital's share in national income}$$

$$s_L = \frac{F_H(\bar{K}, \bar{H}^p) \bar{H}}{F(\bar{K}, \bar{H}^p)} = \text{labor's share in national income}$$

$$\epsilon = -\frac{F_K(\bar{K}, \bar{H}^p) F_H(\bar{K}, \bar{H}^p)}{F_{KH}(\bar{K}, \bar{H}^p) F(\bar{K}, \bar{H}^p)} = \text{elasticity of substitution between capital and labor}$$

$$\theta^p = \frac{\bar{H}^p}{\bar{H}} = \text{share of total hours worked supplied to the private sector}$$

$$\theta^g = \frac{\bar{H}^g}{\bar{H}} = \text{share of hours worked supplied to the government}$$

$$v_1 = v'(\bar{H}) \bar{H}$$

$$v_2 = \frac{v''(\bar{H}) \bar{H}}{v'(\bar{H})}$$

$$\alpha = \frac{\sigma'_M(\mu) - \sigma'(\mu)}{\sigma_M(\mu) - \sigma(\mu)}$$

$$\gamma = \frac{\sigma'_M(\mu)}{\sigma_M(\mu)}$$

¹³There is no theoretical necessity for q to equal 1 in this model. In the case of the competitive models with lump-sum taxes considered by authors such as Aiyagari, Christiano, and Eichenbaum (1989), Baxter and King (1988), and Wynne (1989), an equilibrium necessarily maximises the welfare of the representative household given the quantity of resources consumed by the government, and as a result equilibrium must be unique. Since equilibrium in a model like ours does not correspond to the solution to any planning problem, it need not be unique. On the possible indeterminacy of equilibrium in the presence of distortions even when the economy can be modelled as having a representative consumer see Woodford (1986, 1988) and Kehoe, Levine, and Romer (1989).

4. Calibration of the Model's Parameters

The expressions given in the Appendix show that the dynamic properties of the linearized equilibrium depend upon the values assigned to the parameters $\beta, \lambda, n, \delta, k, c, g, s_K, s_L, \epsilon, \theta^p, \theta^g, \rho, v_1, v_2, \mu, \alpha, \gamma$. These parameters are not arbitrary. First of all, the parameters $\beta, \delta, c, G, s_K, s_L, \theta^p$, and θ^g must all take values between zero and one, and the parameters g, n, k , and ϵ must be positive. In addition, our model implies the following equalities relating parameter values:

$$s_K + s_L = 1 \quad (4.1)$$

$$\theta^p + \theta^g = 1 \quad (4.2)$$

$$c + g + [gn - (1 - \delta)]k = 1 \quad (4.3)$$

$$\lambda^\rho / \beta = (1 - \delta) + s_K / k \quad (4.4)$$

$$v_1 = s_L / c \quad (4.5)$$

Conditions (4.1)-(4.2) follow immediately from the definitions of the share variables in question. Condition (4.3) follows from (3.17), condition (4.4) follows from (3.18) and (3.23), and condition (4.5) follows from (3.16), (3.18), (3.19), and (3.28). Because of these relations, the linearized equations involve only 13 independent parameters.

The model also imposes a number of inequality constraints upon the parameters, for example:

$$\beta n \lambda^{1-\rho} < 1 \quad (4.6)$$

$$\lambda^\rho > \beta(1 - \delta) \quad (4.7)$$

$$\rho > 0 \quad (4.8)$$

$$v_2 + (\rho - 1)v_1 > 0 \quad (4.9)$$

$$\rho v_2 + (\rho - 1)v_1 > 0 \quad (4.10)$$

Conditions (4.6)-(4.7) simply repeat conditions (3.27) and (3.29), while (4.8)-(4.10) follow immediately from (A8).

Finally, the definitions of α and γ together with (2.30) and (3.24), imply that

$$\gamma(\mu - 1) - 1 = \beta n \lambda^{1-\rho} [\alpha(\mu - 1) - 1]. \quad (4.11)$$

If, in addition, there exists a unique stationary value μ^* , as shown in figure 3, then

$$\sigma'_M(\mu^*) > 0$$

so that

$$\gamma(\mu - 1) > 1. \quad (4.12)$$

These are the only general theoretical restrictions upon the parameters that determine the linearized dynamics. However, we can considerably tighten the predictions of our model if we recognize, as do Kydland and Prescott (1982), that empirically realistic values for many of the parameters can be selected on the basis of simple facts about the trend growth path of the U.S. economy.

The parameters of our model can be grouped in three groups. The first group consists of parameters describing the scale of various sectors and activities. In this group are such parameters as the economy's capital/output ratio k , the population growth rate n , the fraction of its output consumed by the government etc. In the second group are the parameters which describe the representative individuals' preferences. Finally, in the third group are the parameters which capture the character of the competitive interactions among firms.

Using the average quarterly growth rates for the period since World War II, we set the population growth parameter n equal to 1.0048. Since average GNP growth has been .008 per quarter, and the average rate of growth of the model equals λn , we set λ equal to 1.0032.

One important difference between our calibration and that of Kydland and Prescott (1982) is our treatment of consumer durables. In Kydland and Prescott (1982), these are treated as capital. This means that their concept of output exceeds the usual measurement of GNP by the services of these durables. This imputation of output raises some measurement difficulties. Even more severe difficulties attend the measurement of the labor input that goes into producing this output.

When developing their theory, Kydland and Prescott (1982) assume that output is produced via a Cobb-Douglas production function of capital and labor. On the other hand they ignore the time input of people who operate consumer durables whether these be lawnmowers or refrigerators. They treat the services of durables as being produced by the durables alone. An alternative would be to assume that some effort is spent in producing services from these durables. Unfortunately,

since this labor input is not measured, it is unclear how employment and thus productivity ought to be measured if one follows this approach.

We avoid these difficulties by treating consumer durables as consumed in the period in which they are purchased. We thus do not distinguish between durables and nondurables. We do this for simplicity and for two other reasons. First, Mankiw (1982) shows that the empirical behavior of consumer expenditure on durables closely approximates that of consumer expenditure on nondurables. Second, it is not reasonable to suppose that the services of durable goods are produced from durables and labor by imperfectly competitive firms. It is more reasonable to suppose that this production occurs in the home or, equivalently, by perfectly competitive firms.¹⁴ The construction of such a two sector model is left for future research.

As a result, our assumed capital output ratio k is 9.0¹⁵ The shares c and g are the average ratios of consumer expenditure and government purchases over private value added. They thus equal .697 and .117 respectively. Equation (3.17) then implies a quarterly depreciation δ of .013 rather than the quarterly depreciation of .025 assumed by Kydland and Prescott.

Combining federal, state and local governments, the ratio of total government employment to total employment has averaged .17 in the postwar era. We assume that hours per employee are the same in the government as in the rest of the economy so that θ^g equals .17 as well.

Our treatment of durables also affects our estimate of our share of labor. Viewing durables as consumption implies that the share of labor s_l is .75 (as in conventional measurements). Kydland and Prescott, who treat the entire services of durables as compensation to capital obtain instead a labor share equal to .64. Using (3.23), the real rate of interest is $1 - \delta + F_K/\mu$. Using (3.16) and (3.28) this equals $1 - d + (1 - st)/k$, which equals about .015 per quarter. Note that this is also the ratio of the net payments to capital over the capital stock itself.

The final parameter of technology is the elasticity of substitution ϵ . As is standard practice we assume that this equals one on the grounds that relative factor shares have remained roughly constant in spite of a secular increase in the cost of labor.

¹⁴A similar problem arises with respect to housing. Some housing services are sold by firms using labor services that are counted in the conventional measurements of employment. Other housing services are produced by their owner-occupants. Because a significant fraction of housing services are provided by firms and because much of the labor input that is employed here is measured as conventional employment we treat housing services as produced by imperfectly competitive firms.

¹⁵This is four times the ratio of private total tangible assets minus durables in 1987 over that year's private value added (i.e. GNP minus the value added by the federal, state and local governments).

We now turn to the preference parameters. As in standard real business cycle analysis there are two important parameters of this kind. The first, ρ gives the percent by which the ratio of consumption at $t + 1$ over consumption at t changes when the real return changes by one percent, holding hours worked in the two periods constant. The hypothesis of constant hours in this definition is important because, if ρ is different from 1, the utility function is not separable between employment and consumption. Raising ρ raises $\frac{\partial^2 U}{\partial C \partial H}$ so that they make consumption more complementary with employment.

The second parameter v_2 is related to the intertemporal substitutability of leisure. (The other preference parameter v_1 is given by (4.5)). The easiest way to describe this parameter is to focus on the elasticity of labor supply at t with respect to the wage at t holding constant the marginal utility of wealth (and thus of consumption). This elasticity can easily be computed to equal $\frac{\rho}{\rho v_2 + (\rho - 1)v_1}$.

Calibrations of real business cycle models assume that ρ equals one. This choice is based in part on the fact that several authors have obtained estimates of this parameter near one using "Euler equation" methods.¹⁶ Unfortunately the estimates from these methods are very sensitive to the instruments used as well as to the normalization adopted in the estimation procedure. Hall (1988) shows that the correlation between changes in consumption and expected real returns is small in practice and thus argues that the intertemporal substitutability of consumption is low (and ρ quite high, possibly infinite).

The real business cycle approach to estimation of the elasticity of labor supply deserves special comment. The idea is to assume a functional form of the utility function so special that it is possible to derive the elasticity of labor supply from knowledge of the ratio of hours worked to hours available for working. It is hard to see why there is any general connection between the average fraction of time spent in market activities and the elasticity of these hours with respect to temporary changes in the wage. Indeed, small modifications of the assumed utility function eliminate any such connection and no such connection exists for our class of utility functions.

This approach leads Aiyagari, Christiano and Eichenbaum (1989) to assume an elasticity of supply of labor of about 6 (which corresponds to a v_2 of about 0.17). This exceeds the vast majority of estimated labor supply elasticities whether the estimates are computed by carefully

¹⁶ See e.g. Hansen and Singleton (1982).

trying to keep the marginal utility of wealth constant or whether they incorporate the effect of varying participation rates. Typically, estimated labor supply elasticities for males are near zero.¹⁷ Estimates for female labor supply elasticities range more widely but, while many studies obtain estimates in the 0.5-1.5 range, hardly any obtain estimates above 2.¹⁸

In spite of these reservations we use these parameter values when simulating the competitive version of our model. We do this both to preserve comparison with their work and because these parameters make the competitive model predict accurately the output response to military spending.

The baseline simulations of our model are computed using substantially lower elasticities for both consumption and labor supply. Since we do not know the appropriate values for these elasticities, we have picked them to some extent with the objective of making the model perform well. In our baseline simulation ρ is 10. The parameter v_2 is 0.07 which implies an elasticity of labor (holding marginal utility of wealth constant) of 0.97. While the model seems to fit better with such relatively low elasticities, the qualitative features of the model's response is not sensitive to their precise values.

Given these parameters, condition (4.4), which equates the steady state marginal rate of substitution between current and future consumption to the real return, implies that β equals 1.017. That this coefficient exceeds one may be worrisome to some. It certainly implies that people are not as impatient as is commonly thought. Beyond this, the fact that it exceeds one does not affect the analysis. Since (3.27) is satisfied the sums in (3.13) and (3.14) remain bounded.

There are three parameters that capture the competitive structure in which the firms operate. The first is the markup μ . We assume μ equals two. This is on the low end of Hall's (1988) estimates which are based on the extent to which total factor productivity rises when various indicators of demand go up.

The only *a priori* restrictions on α and γ are conditions (4.11) and (4.12). This leaves us with one free parameter between the two. To obtain a baseline case we focus on the implications of an easily understood specific functional form for demand. We consider a stationary equilibrium with

¹⁷See Pencavel (1986) and Killingsworth (1983).

¹⁸See Killingsworth (1983) and Killingsworth and Heckman (1986).

price \hat{p} . Suppose that the demand curve facing an individual firm is

$$D^j(1, \dots, X, \dots, 1) = X^{-\epsilon} \quad \text{for } X \leq 1$$

when all other firms charge \hat{p} .¹⁹ As a result a deviating firm charges $e/(e-1)$ times marginal cost. Thus, while $\sigma(\mu)$ equals $(\mu-1)$, $\sigma_M(\mu)$ equals $\left[\frac{(\mu-1)\mu}{e}\right]^\epsilon / (e-1)$.

Plugging these formulas in (3.24) we can solve for the elasticity of demand e given values for μ , β , g and n . One can show that the resulting equation has a unique solution in the range $e > \mu/(\mu-1)$ for each $\mu > 1$.²⁰ It can also be shown that e is monotonically decreasing as a function of μ for fixed $\beta n \lambda^{(1-\rho)}$ which is itself the product of the real discount factor $\beta \lambda^{-\rho}$ and the growth factor $n \lambda$. Moreover:

$$\begin{aligned} \lim_{\mu \rightarrow 1} e(\mu, \beta n \lambda^{(1-\rho)}) &= \infty \\ \lim_{\mu \rightarrow \infty} e(\mu, \beta n \lambda^{(1-\rho)}) &= 1 \end{aligned}$$

We obtain γ from e using the relation $\gamma = e/\mu$. Using this value of γ we use (4.11) to compute α . In particular, with μ equal to 2, and the other parameters as above, e must equal about 11.8. This elasticity is substantially higher than the typical elasticity computed in marketing studies (Tellis (1988)). However, many of these studies are not focused on single firms, do not control for the endogeneity of price and quantity, etc. Baker and Bresnahan (1984) deal with these issues in their study of three beer producers. For Anheuser-Busch, Coors and Pabst, their estimated elasticities are 1.3, 3.2 and 17.2 respectively so that our estimate of 11.8 appears high but not unreasonable.

Using this estimate of e , α equals 6.054 and γ equals 6.016. We thus use these parameters for our base case. Note that, with this method of deriving α and γ the only additional free parameter beyond those of standard real business cycle models is the markup μ .

5. Estimation of Impulse Responses

In this section we describe how we estimate the economy's response to changes in military spending. We compare these empirical responses to those predicted by the oligopolistic and competitive models in the next section. The macroeconomic variables we focus on are the national

¹⁹This is not meant as a complete description of demand. In particular, we are not specifying how demand varies when all the firms in industry i 's charge a price different from that charged by the firms operating in other industries. More importantly, we are not specifying demand when, as is typical of deviations in the stick phase, firm i charges a price above that of the other members of its industry. Large increases of this type can still lead the firm to make zero sales as is required by (A4).

²⁰We need $e > \mu/(\mu-1)$ for the solution to satisfy (3.25) with $\mu > \bar{\mu} = e/(e-1)$.

defense purchases of goods and services on the one hand and the value added produced by the private sector (which equals GNP minus the value added produced by the federal, state and local governments), total hours worked in the private sector, aggregate consumption, aggregate investment, the real wage and the level of real profits on the other.²¹

The real wage is defined by dividing our measure of the nominal wage by the deflator for private value added. Our basic measure of the nominal wage is the level of hourly earnings in manufacturing.²² This private value added deflator is constructed by dividing nominal value added produced in the private sector by constant dollar value added in the private sector.

The level of real profits is the ratio of the difference between national income and the compensation of employees over the deflator for private value added. The level of profits is included because its ratio to the stock of capital is a measure of the *ex post* real rate of return. While we do not have quarterly measurements of the capital stock, this variable evolves smoothly through time. We therefore expect that our measurement of the changes in the real rate of return is not too severely distorted by the fact that the capital stock is not a deterministic trend even though we only include such a trend in our regressions.

Our total private hours variable is computed by multiplying the number of people on private nonagricultural payrolls by the average hours worked by all nonagricultural employees. It thus is proportional to the variable that enters our theoretical model only if average hours per employee are the same in the government as in the private sector and if farm hours and employment behave like other private hours and employment.

In common with the other empirical studies of the connection between military spending and aggregate activity, we treat military spending as exogenous. Figure 1 shows the evolution of military spending after World War II. It is apparent from this figure that the three big military buildups correspond to the Korean War, the Vietnam War and the Reagan buildup. All three of these episodes appear to be responses to the perceived threat of communist regimes and thus can

²¹ We are hardly the first to have focused on these relationships. Among recent efforts we note Garcia-Mila's (1987) study of the relationship between military spending and GNP, Hall's (1986) of the relationship between military spending and consumption, and Hall's (1988) of that between military spending and productivity.

²² We have also experimented by using two alternative measurements of the nominal wage. The first is more comprehensive, it equals total compensation of non-government employees divided by total hours worked in the private sector. One possible problem with our basic measure as well as with this first alternative is that the measured wage is affected by changes in either the sectoral composition of output or by changes in the extent to which firms use overtime. We thus also experimented with the measure of hourly earnings in manufacturing that is corrected for overtime and interindustry shifts. The results using these alternative measures of wages are broadly similar so that we only report those using hourly earnings in manufacturing.

plausibly be treated as exogenous to the state of US economic activity. Assuming that military spending is accurately measured we thus treat the disturbances in the relationship between military spending and other variables as independent of military spending.

Given these assumptions we model the logarithm of military spending as a univariate stochastic process. Because the Dickey-Fuller test rejects the presence of a unit root in this series, we estimate its stochastic process as:

$$\begin{aligned} \text{mil}_t = & 0.145 + 1.655\text{mil}_{t-1} - 0.683\text{mil}_{t-2} + 2.89e^{-5}\text{trend} \\ & (0.04) \quad (0.06) \quad (0.05) \quad (5.7e^{-5}) \end{aligned} \quad (5.1)$$

$$\text{Period : 1947 : IV - 1988 : I; } R^2 = 0.989; \quad \text{D.W.} = 2.06$$

Note that an increase in military spending is followed by further increases and only later does military spending return to its normal value.

To compute the responses of the other variables to changes in military spending we also estimate bivariate relationships between the logarithm of military spending and the logarithm of our other variables. In these regressions the logarithm of each particular variable is explained by three lagged values of the variable, a linear trend, the current and three lagged values of military spending. We ran these regressions using quarterly seasonally adjusted data from 1947:IV to 1988:I. We then computed the response of a particular variable to a unit impulse in military spending by combining the estimated coefficients of (5.1) with those of the regression which explains the behavior of this variable.

This is only one of many possible procedures for computing impulse responses. One alternative is to compute responses from a vector autoregression which includes the logarithms of all our variables. As long as the series are covariance stationary, both methods produce consistent estimates of the dynamic response of all endogenous variables to an innovation in military spending²³. In practice both methods give very similar estimates for the effect of military spending on other variables. We choose to focus on the bivariate relationships because they involve fewer parameters.

We also experimented with specifications in which the variables of interest are included as first differences instead of as levels.²⁴ The result is that, because fewer parameters are needed to

²³ See Zellner and Palm (1974).

²⁴ This would be more efficient if the series are integrated and there is no cointegrating vector. On the other hand, it is less robust in that, the estimates cease being consistent if these integration assumptions are false.

capture the effect of military spending, the statistical significance of the military spending regressors is enhanced. The implied short run responses from estimation in first differences are essentially identical to those from estimation in levels though the long run responses differ. Since we are more confident of the validity of the short term responses, our discussion of impulse responses is limited to what happens within ten quarters of a given shock and places most emphasis on the first few quarters.

6. Estimated and Simulated Impulse Responses

In this section we compare the predictions of the oligopolistic and competitive models to the US experience since World War II. As discussed above our version of the competitive model differs from the oligopolistic one not only in its markup but also in its preference parameters. To compute the predicted responses to a unit impulse in military spending we assumed that agents forecast future military spending by using the actual stochastic process for this variable (5.1).

To compute these theoretical predictions we must take into account that military expenditure has two components. The military both hires people and buys goods. For simplicity we assume that the percent deviation from trend in both people hired and goods purchased is equal to the percent deviation from trend of military expenditure. Let ΔM_t be the percent deviation of military spending. Then, given that military employment averaged only 18% of government employment and that military purchases of goods accounted for about 35% of total government purchases of goods $\Delta \hat{g}_t$ equals $0.35\Delta M_t$ while $\Delta \hat{H}_t^g$ equals $0.18\Delta M_t$.

We then compare these theoretical responses to those generated by the bivariate regressions discussed in the section 5. To get a sense of the performance of the two models we also plotted the two standard error confidence intervals around our empirical impulse responses.²⁵

Figure 5 presents the response of private value added to a unit shock in military spending. The positive association between military purchases and output has been noted before. Both Hall (1986) and Garcia-Mila show that in post war US data increases in military purchases raise GNP though slightly less than one for one. Since government value added rises much less than one for one with military purchases, private output must rise as well.

Our choice of parameters ensures that our noncompetitive model tracks this response reasonable well. The competitive version predicts the output response about as well. This is a certain measure of success for the preference parameters proposed by Aiyagari, Christiano and Eichenbaum (1989) since they did not choose these parameters to make this particular simulation fit these particular facts. In particular, their simulations assume rather different values for the capital output ratio, for the parameters of the stochastic process describing military spending etc.

Figure 6 presents the response of hours worked. Here the competitive model encounters well

²⁵ These intervals are computed using the asymptotic method applied by Poterba, Rotemberg and Summers (1986).

known difficulties. The actual percent increase in hours worked is smaller than the percent increase in output. Given diminishing returns the competitive model predicts that the percent increase in hours worked should be larger than the percent increase in output.²⁶ Models with increasing returns such as ours can account for this discrepancy. Indeed Hall (1987) uses the extent to which productivity increases in response to demand variations such as changes in military purchases to measure the degree of increasing returns.²⁷

Note that we underpredict the extent to which output per man hour rises with military spending. Similarly, we underpredict the extent to which the Solow residual rises with military spending. We could have predicted these more accurately if we had chosen a larger markup. A μ of two should be regarded as a conservative measurement of the markup.

Figure 7 presents the response of the real hourly earnings in manufacturing. The observed positive effect on real wages is the principal difference between the predictions of our model and those of the competitive model.²⁸

Here the prediction of our competitive model fall decidedly outside our two standard error confidence band while our predicted response of the real wage is almost entirely contained within it. It is worth emphasizing that this statistical failure of the competitive model is not really sensitive to the choice of preference parameters. It depends almost entirely on technology. As we explained in Section 1, competitive firms are simply unwilling to hire the extra workers needed to account for the big increase in output unless the real wage falls substantially.

Figure 8 plots the effect of military purchases on consumption. In the initial quarter consumption actually rises (though not significantly so) and only later declines. As a result, consumption initially falls significantly less than is predicted by the competitive model. A similar rejection of the competitive model is reported in Hall though for different preference parameters. Our choice of parameters, by contrast leads to an overprediction (though not a statistically significant one) of

²⁶Eichenbaum and Christiano (1988) point out that actual productivity is less procyclical than is implied by models where technology is the only random influence on output. They seek to remedy this by letting government purchases be random as well. This remedy is not altogether convincing given that productivity and output rise together in response to unexpected changes in military spending. What is more, productivity actually rises more for a given percent increase in output when that increase in output is correlated with current and lagged values of military spending than when it is not.

²⁷Similarly, Hall (1988) uses the degree to which the Solow residual increases with military spending to measure the size of the markup. In practice, measured economic profits are small so that his two measurements are related as in equation (3.28) of our model.

²⁸It also differentiates the predictions of our model from textbook Keynesian models with rigid nominal wages. In those models output expands only when real wages fall. It is also worth pointing out that the increase in real wages is even stronger when earnings in manufacturing are deflated by the overall GNP deflator instead of being deflated by the deflator for private value added.

consumption in later quarters.

One interesting feature of the measured consumption response is that consumption is predicted to decline over time after the initial shock. Since the coefficient of lagged military spending growth in the regression explaining consumption is extremely significant, the random walk model of Hall(1978) is rejected.²⁹ The significant negative effect of lagged military spending is also at odds with the standard competitive model. That model implies that the increase in real rates leads consumption to rise over time. Our oligopolistic model, by contrast, predicts that consumption should decline though starting only in the third quarter after the shock. The model makes this prediction because our preference parameters imply that the marginal utility of consumption rises with the number of hours worked.

Figure 9 presents the empirical consequences of military purchases for the *ex post* average profitability of capital. As both the competitive and the oligopolistic model require, *ex post* real rates in the form of the profitability of capital rise.³⁰ Such increases in real returns are an important component of the both the competitive and the oligopolistic economies' reactions to increases in military spending. Indeed, markups fall in the oligopolistic economy in part because the rise in real rates of interest makes each firm less concerned about future punishments. Similarly, in the competitive model the increases in real rates promote intertemporal substitution in labor supply and thus are largely necessary for output to increase.³¹

The largest increase in profitability occurs on impact. However, the estimates are very imprecise and the confidence intervals are rather large. They are so large that, on this scale, the predicted responses are not visibly different from zero. Figure 10 presents the predicted responses which are quite small and rather similar for both models.³² The small size of the response in the competitive model could be guessed from the predicted response of consumption whose predicted change over time is negligible.

Finally, figure 11 plots the response of the capital stock. This response is computed by applying

²⁹ Tests of that model usually focus on nondurables and services only while we are using total consumption expenditure. However, we have obtained similar negative and significant coefficients when studying the behavior of nondurables and services.

³⁰ We also experimented with other measures of the rate of return. Interestingly, current military spending does exert a strong and statistically significant positive effect on later values of the *ex post* real return from holding Treasury bills.

³¹ Mankiw (1987) showed that it is possible for real rates not to rise in competitive economies. In his model real rates can fall because consumers respond to the increase in military purchases by selling their durable goods to firms who then employ them as productive capital.

³² Both these responses are much smaller and statistically significantly different from the estimated responses of the *ex post* real return on treasury bills.

our depreciation rate to the empirical response of investment. As can be seen also in table 2 investment (and hence the capital stock) fall after an increase in military purchases. By contrast both models predict an initial buildup of the capital stock. The increase in capital predicted by both models for the first few quarters is within the two standard error band. After that, the noncompetitive model predicts that capital declines. By contrast the competitive model predicts a longer period of capital accumulation so that it tracks the actual response of capital somewhat less well.

In conclusion the competitive model performs poorly with regards to employment, productivity, the real wage and the composition of output between consumption and capital accumulation. Our noncompetitive model, which in some sense has only one additional free parameter performs somewhat better, particularly in terms of its explanation for changes in real wages.

Before closing this section we report some sensitivity analysis on the parameters of our model. Figure 12 shows how the response of the initial level of output depends on ρ and on "els", the (Frisch) elasticity of labor supply. Figure 13 presents the initial responses of initial real wages. These simulations are conducted leaving all other parameters at the values described in Section 4.

The output response is increasing in ρ and in "els". It is particularly sensitive to the elasticity of labor supply when this elasticity is between zero and one. This means that it would be difficult, unless one were willing to change some other parameter, to reproduce the actual response of output with elasticities of labor supply much below one.

The real wage response is also slightly increasing in ρ but is falling in "els". Thus relatively small elasticities of labor supply are necessary for real wages to increase. Larger elasticities make the intertemporal substitution effect so large that the real wage falls even in the presence of imperfect competition.

The competitive model behaves in similar fashion with respect to changes in parameters. Higher elasticities of labor supply mean that labor supply rises more in response to increases in military purchases. So output rises more and the real wage falls more (just as it falls less in the oligopoly case). This similarity of behavior means that the competitive model can only produce the observed output responses if labor is quite elastic. However, such high elasticities then imply large real wage declines.

A different type of sensitivity analysis is presented in Figure 14. There we plot the initial response of output and the real wage as we vary the elasticity of demand faced by an individual firm ϵ . As we vary ϵ we keep all the other parameters constant except the markup μ whose value is computed using the procedure outlined at the end of Section 4 and the fraction of fixed costs in total costs which varies so as to keep aggregate profits equal to zero. Higher values of ϵ correspond to lower values of μ and to more competitive conditions. Increasing ϵ in this manner lowers the responses of both output and the real wage to changes in military spending. For the reasons discussed above, sufficiently high values of ϵ imply that real wages actually decline in the aftermath of the increase in military spending.

7. World War II

In this subsection we discuss some of the salient features of the US economy around World War II. The main purpose of this section is to show that several of our post-war results are quite consistent with the experience of the 1930's and 40's. Consider in particular Figure 4. It exhibits the change in the logarithm of the real wage (defined as the wages and salaries of full time equivalent employees over the deflator for private value added) on the left scale. The right scale measures the change in federal purchases (no separate series is available for national defense expenditures) and of private value added. Each of these two series is measured in 1972 dollars.

The figure shows a striking positive relationship between changes in federal purchases and changes in real wages. There is also a weaker positive relationship between the federal government's purchases and the level of private value added.

One potentially troubling feature of World War II is the large extent of price controls. It is possible that the increased real wages of 1942-43 are attributable to price controls. Similarly, the reduction in real wages in 1946 might be due to the lifting of price controls.

In this context the year 1941 is worth discussing in detail because price controls were enacted only after passage of the Emergency Price Control Act of 1942. Military purchases were much higher in 1941 than in 1940. Indeed, this is the first year in which the change in federal purchases is substantially higher than in the early 1930's. At the same time, the real wage rose substantially. Moreover, private value added also rose although less than the government purchases themselves. This reflects both the fact that the government purchased labor services and that the multiplier is

relatively small.

Thus the picture conveys the same basic message as the impulse responses of the previous subsection. Changes in military purchases (which undoubtedly account for the bulk of the changes in federal government's purchases over the period 1939-1947) move private value added and the real wage in the same direction.

8. Concluding Remarks

In this conclusion we tentatively discuss some alternative interpretations for the facts put forth in this paper. In future work we intend to pursue the comparison between these ideas and those of our model more systematically. The first possibility is that the competitive model is accurate and that we have misspecified the model in important respects.

One possibility along these lines is that the one sector structure of our model is misleading. Increases in government purchases may actually lower product wages in the sectors which are led to produce more while raising them only in those sectors where output falls. Our measurement of increased real wages may be due only to inappropriate aggregation of the two sectors. While this possibility cannot be dismissed, we note that it implies that the prices of the sectors that expand ought to rise relative to other goods' prices. In particular, the prices of military goods ought to rise relative to other prices when military spending rises. Yet, regressions using postwar data show that the deflator for military purchases falls relative to the GNP deflator when military spending goes up. Similarly, the disaggregated post 1972 data show that both the ammunition deflator and the military equipment deflator fall more relative to that for private value added the more real military purchases rise.

A second competitive explanation of our findings is that increases in the demand for goods lead firms to use their capital more intensely. A simple model of variations in the "workweek of capital" such as that of Lucas (1970) will not suffice, however. In that model, capital can be used more intensely by employing it for additional shifts. But one must ask why capital should not be fully employed. In Lucas' explanation, overtime hours are more costly than regular hours and hours of both kinds are employed to the point where the marginal product of each kind of hour equals its wage. But, then, it is again impossible for firms to be induced to hire more hours of either kind in response to an increase in demand, unless one or both real wages fall due to a labor supply shift.

The variable workweek of capital can explain our real wage puzzle only if the real wage increase is an artifact of aggregation; both regular and overtime wages might fall while the composition of hours shifts towards overtime hours to such an extent that average hourly earnings increase. But, increases in military purchases raise real wages even when we use a real wage measure corrected for shifts between regular and overtime hours (see footnote 22).

Another possible explanation for variable utilization of capital would be that more intense utilization results in faster depreciation of the capital stock, as in the model of Greenwood, Hercowitz and Huffman (1988). But, again, this does not lead to an easy resolution of the puzzle. Following Greenwood *et al*, consider a production function $F(K, H, u; z)$ and a depreciation rate $\delta(u)$ that both depend on the intensity of capital utilization u . Then, in equilibrium, the utilization u maximizes $F(K, H, u; z) - \delta(u)K$. If we define a modified production function

$$\tilde{F}(K, H; z) = \max_u [F(K, H, u; z) - \delta(u)K]$$

the first order conditions for the employment decisions of competitive firms are

$$\tilde{F}_H(K_t, H_t; z_t) = w_t.$$

This is exactly analogous to (1.1) and the puzzle remains. For labor demand to shift when the demand for output increases, it is necessary not only that depreciation vary with utilization but also that new output and leftover capital be imperfect substitutes. Then, a demand shock that changes their relative values affects the utilization of capital and also labor demand.

The second class of possibilities is that aggregate demand does indeed affect the economy by changing the markup but that implicit collusion plays no role. We therefore briefly survey alternative reasons for countercyclical markups. The oldest proposal is due to Robinson (1932).³³ Robinson's (1932) view, which has recently been revived by Bills (1987) and Lindbeck and Snower (1987), is that decreases in aggregate demand also reduce the elasticity of demand faced by the typical monopolistically competitive firm. This leads the firms, (which are regarded as maximizing profits independently in each period) to increase the markup. One difficulty with this view is a

³³ Similar ideas are expressed by Kalecki (1938) and Keynes (1939), with Kalecki developing more fully the role of markup variation in the generation of business cycles. Another early hypothesis is that of Hall and Hitch (1939), who explained countercyclical markups in terms of strategic interactions among oligopolists and decreasing average costs. In these respects their view is a precursor of our own although their model is entirely static. Both these factors also play a role in Kalecki (1938).

lack of persuasive reasons for the elasticity of demand to vary with the strength of demand. What is more, it is not enough that the elasticity depend on the level of sales (or output) alone. Suppose that the markup μ is only a function of Y so that (1.1) can be replaced by:

$$F_H(K_t, H_t, z_t) = \mu(K_t, H_t, z_t)w_t.$$

As Woodford (1988) points out, this still describes a relationship between H_t and w_t which depends only on K_t and z_t so that it cannot be affected by aggregate demand. Aggregate demand can still only affect employment by shifting labor supply. Increases in real wages following increases in aggregate demand could still be consistent with this story if the derivative of μ with respect to Y were so large that the labor demand curve sloped upwards. But, such a story would be harder to square with the other evidence for increases in labor demand presented in section 1.

For labor demand to shift with aggregate demand in this type of model the shifts in aggregate demand must be accompanied by changes in the elasticity of demand even at unchanged output. This is easiest to rationalize if the shift in aggregate demand also changes the composition of demand and if the components differ in their demand elasticity. Indeed Lindbeck and Snower (1987) propose that government demand is more elastic than private sector demand while Bills (1987) proposes that demand of the young is more elastic than that of high income consumers.

In such a model it would be sheer coincidence if all the shocks that increase aggregate demand (changes in the perceived profitability of investment, changes in the tastes of foreigners, etc.) also shifted demand towards more elastic customers. One would expect instead that some increases in demand would increase while others would decrease the markup. This story thus tends to be inconsistent with the long tradition of macroeconomics which treats all changes in aggregate demand (*i.e.* all shifts from future to current spending at given interest rates) as having roughly similar effects.

Stiglitz (1984) proposes several other models of countercyclical markups. Some of these are also static. Others are dynamic but have rather different implications from ours. In one model an incumbent monopolist faces the threat of future entry and responds by limit pricing. As real interest rates go up, the incumbent is less worried about future entry so that he charges a higher price (which is closer to the price he would charge in the absence of potential entry). This theory predicts that markups should rise when the real interest rate rises whereas ours predicts that

markups should fall in this case.

A final theory of countercyclical markups is based on the model of Phelps and Winter (1970). Suppose that customers are divided into two groups. There are some who have already experienced and liked the good. These are willing to pay a great deal. There are also customers who have yet to try the good and so are willing to pay less. Low prices are then an investment in new customers. When firms are discounting the future highly they will invest little and exploit their existing customer base by charging high prices. In this simple version of the theory, markups will also be low when interest rates are low. At least in the case of military spending the increases in interest rates appear to be matched by falls in markups though the effect of other variables is still worth investigating.

Gottfries (1986) and Greenwald and Stiglitz (1988) propose a more complicated version of this theory where informational barriers impede financial flows between firms and outside investors. Thus, the observed real interest rate is not the rate used by firms when they select investment projects. In particular, falls in demand exacerbate the firm's liquidity problems and thus raise the rate at which firms discount the future. This reduces investment in physical assets as well as investment and customers so that prices rise. According to this version of the theory, increases in demand ought to raise output only if they also raise physical investment. For the case of military spending this is not true. The regressions of sections 5 and 6 show that military spending reduces physical investment while lowering markups. Still, the general relationship between physical investment and markups deserves to be explored further.

An alternative to all these approaches is the view that countercyclical markups are a result of some kind of sluggishness on the part of firms in changing the prices at which they sell their output, even if marginal costs have changed.³⁴ According to this view, it is posted money prices that are slow to adjust. So, a prediction that differentiates this view from all the "real" theories just discussed would be the claim that markups should decline most when costs are increasing most rapidly in money terms. However, a detailed comparison between this kind of model and the others is difficult because choice-based dynamic models of this type where real interest rates and changes in the expectation of future demand play a role remain to be developed. Still, the role of nominal

³⁴ See e.g. Woodford (1988). For a survey of evidence for and theoretical models of nominal price rigidity, see Rotemberg (1988). Rotemberg and Summers (1989) show that price rigidity can also rationalize the increase in measured productivity that accompanies increases in military spending.

rigidities in explaining markup variation is an important problem for future research.

9. References

- Abreu, Dilip: "Extremal Equilibria of Oligopolistic Supergames," *Journal of Economic Theory*, 39, 1986, 191-225.
- Aiyagari, S.Rao, Lawrence Christiano and Martin Eichenbaum: "The Output and Employment Effects of Government Purchases," mimeo, February 1989.
- Akerlof, George, Andrew K. Rose and Janet Yellen: "Job Switching and Job Satisfaction in the US Labor Market," *Brookings Papers on Economic Activity*, 1988:2, 485-582.
- Baker, Jonathan B. and Timothy F. Bresnahan: "Estimating the Elasticity of Demand Facing a Single Firm: Evidence on Three Brewing Firms," Stanford Working Paper, June 1984
- Barsky, Robert and Gary Solon: "Real Wages over the Business Cycle," *NBER Working Paper 2888*, March 1989.
- Barro, Robert J.: "Output Effects of Government Purchases," *Journal of Political Economy*, 89, 1981.
- Baxter, Marianne and Robert G. King: "Multipliers in Equilibrium Business Cycle Models," mimeo, November 1988.
- Bils, Mark: "Pricing in a Customer Market," mimeo, 1987.
- Blanchard, Olivier Jean and Charles M. Kahn: "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48, 1305-13, July 1980.
- and Peter Diamond: "Beveridge and Phillips Curves," mimeo, 1988.
- Christiano, Lawrence J. and Martin Eichenbaum: "Is Theory Really ahead of Measurement? Current Business Cycle Theories and Aggregate Labor Market Fluctuations" *NBER Working paper 2700*, September 1988.
- Dixit, Avinash and Joseph Stiglitz: "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 297-308, 1977.
- Greenwald, Bruce and Joseph Stiglitz: "Financial Market Imperfections and Business Cycles," *NBER Working Paper 2494*, 1988.
- Garcia-Mila, Teresa: "Government Purchases and Real Output: An Empirical Analysis and Equilibrium Model with Public Capital," W.P. 93.88, Institut d'Anàlisi Econòmica, CSIC, Universidad Autònoma de Barcelona, August 1987.

- Gottfries, Nils: "A Permanent Demand Theory of Pricing," Institute for International Economic Studies, Seminar Paper 345, January 1986.
- Greenwood, Jeremy, Zvi Hercowitz and Gregory W. Huffman: "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78, 402-18, 1988.
- Hall, Robert E. "Stochastic Implications of The Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, 971-87, December 1978.
- : "Labor Supply and Aggregate Fluctuations" in *On the State of Macroeconomics, Carnegie-Rochester Conference on Public Policy* North Holland, Amsterdam, 1980.
- : "The Role of Consumption in Economic Fluctuations" in R.J.Gordon ed. *The American Business Cycle*, University of Chicago, Chicago, 1986.
- : "Investment under Uncertainty: Theory and Test with Industry Data," *NBER Working Paper 2264*, May 1987.
- : "The Relation between Price and Marginal Cost in US Industry," *Journal of Political Economy*, 96, October 1988, 921-48.
- Hall, R.L. and C.J.Hitch: "Price Theory and Business Behavior," *Oxford Economic Papers*, May 1939, 13-45.
- Haltiwanger, John and Joseph Harrington Jr.: "The Impact of Cyclical Demand Movements on Collusive Behavior," mimeo, April 1988.
- Hansen, Lars Peter and Thomas Sargent: "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 2, 7-46, February 1980.
- and Kenneth J. Singleton: "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269-86, 1982.
- Kalecki, Michael: "The Determinants of the Distribution of National Income," *Econometrica*, 6, 97-112, 1938.
- Kandori, Michihiro: "Price Wars during Booms—a Generalization," mimeo, June 1988.
- Kehoe, Timothy J., David K. Levine and Paul M. Romer: "Characterizing Equilibria of Models with Externalities and Taxes as Solutions to Optimization Problems," unpublished, Federal Reserve Bank of Minneapolis, April 1989.
- Keynes, John Maynard: "Relative Movements of Real Wages and Output," *Economic Journal*, 49, 34-51, 1939.

- Killingsworth, Mark R.: *Labor Supply*, Cambridge University Press, Cambridge, 1983.
- and James J. Heckman: "Female Labor Supply: A Survey," in *Handbook of Labor Economics*, Ashenfelter and Layard eds., North Holland, Amsterdam, 1986.
- King, Robert G., Charles I. Plosser and Sergio Rebelo: "Production, Growth and Business cycles: Technical Appendix," unpublished, University of Rochester, August 1987.
- Kydland, Finn E. and Edward C. Prescott: "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, November 1982, 1345-70.
- Lindbeck, Assar and Dennis J. Snower: "Transmission Mechanisms from the Product to the Labor Market," Seminar Paper 403, Institute for International Economic Studies, Stockholm, December 1987.
- Lucas, Robert E. Jr.: "Capacity, Overtime and Empirical Production Functions," *American Economic Review*, 60, 23-27, 1970.
- Mankiw, N. Gregory: "Hall's Consumption Hypothesis and Durable Goods," *Journal of Monetary Economics*, 10, 417-25, 1982.
- : "Government Purchases and Real Interest Rates," *Journal of Political Economy*, 95, April 1987, 407-19.
- Pencavel, John: "Labor Supply of Men: A Survey," in *Handbook of Labor Economics*, Ashenfelter and Layard eds., North Holland, Amsterdam, 1986.
- Phelps, Edmund S. and S.G. Winter: "Optimal Price Policy under Atomistic Competition," in *Microeconomic Foundations of Employment and Inflation Theory*, E.S. Phelps ed., W.W. Norton Co., NY, 1970.
- Poterba, James M., Julio J. Rotemberg and Lawrence H. Summers: "A Tax-Based Test for Nominal Rigidities," *American Economic Review*, 76, September 1986, 659-75.
- Robinson, Joan: "The Economics of Imperfect Competition," Macmillan, London, 1932.
- Rotemberg, Julio J.: "The New Keynesian Microfoundations," *NBER Macroeconomics Annual* 1987.
- and Garth Saloner: "A Supergame-Theoretic Model of Price Wars during Booms," *American Economic Review*, 76, June 1986, 390-407.
- and Lawrence Summers: "Inflexible Prices and Labor Hoarding's Role in Explaining Procyclical Productivity," mimeo, 1989.
- Stiglitz, Joseph E.: "Price Rigidities and Market Structure," *American Economic Review*, 74,

May 1984, 350-355.

Tellis, Gerard J.: "The Price Elasticity of Selective Demand: A Meta-Analysis of Econometric Models of Sales," *Journal of Marketing Research*, 25, November 1988, 331-41.

Woodford, Michael: "Stationary Sunspot Equilibria: The case of Small Fluctuations around a Deterministic Steady State," Unpublished, University of Chicago, 1986.

—: "Self-Fulfilling Expectations and Fluctuations in Agregate Demand," unpublished, University of Chicago, 1988.

Wynne, Mark: "The Aggregate Effects of Temporary Government Purchases," mimeo 1989.

Zellner, Arnold and F. Palm: "Time Series Analysis and Simultaneous Equation Econometric Models," *Journal of Econometrics*, 2, 17-54, 1974.

10. APPENDIX

In the case of our model, the linearized equilibrium conditions (3.31) and (3.32) can be rearranged in the form:

$$A \begin{pmatrix} E_t \Delta \hat{C}_{t+1} \\ E_t \Delta \hat{H}_{t+1}^p \\ \Delta \hat{K}_{t+1} \end{pmatrix} = B \begin{pmatrix} \Delta \hat{C}_t \\ \Delta \hat{H}_t^p \\ \Delta \hat{K}_t \end{pmatrix} + C \begin{pmatrix} \Delta \hat{\beta}_t \\ \Delta \hat{g}_t \\ \Delta \hat{n}_t \\ \Delta \hat{G}_t \\ \Delta \hat{H}_t^g \end{pmatrix} + D \begin{pmatrix} E_t \Delta \hat{\beta}_{t+1} \\ E_t \Delta \hat{g}_{t+1} \\ E_t \Delta \hat{n}_{t+1} \\ E_t \Delta \hat{G}_{t+1} \\ E_t \Delta \hat{H}_{t+1}^g \end{pmatrix}$$

and the elements of the matrices A, B, C, D are

$$a_{11} = a_{12} = 0, \quad a_{13} = \lambda n k, \quad a_{21} = [1 - \beta(1 - \delta)\lambda^{-\rho}] - \rho,$$

$$a_{22} = [1 - \beta(1 - \delta)\lambda^{-\rho}](\epsilon^{-1} + v_2 \theta^p) - (1 - \rho)v_1, \quad a_{23} = -\frac{1 - \beta(1 - \delta)\lambda^{-\rho}}{\epsilon}, \quad a_{31} = 1 - \rho - \gamma\mu,$$

$$a_{32} = \mu s_L + \frac{sK}{\epsilon}(1 - \gamma\mu) + v_2 \theta^p(1 - \gamma\mu) - (1 - \rho)v_1 \theta^p, \quad a_{33} = \mu s_K - \frac{sK}{\epsilon}(1 - \gamma\mu)$$

$$b_{11} = -c, \quad b_{12} = \mu s_L, \quad b_{13} = \mu s_K + (1 - \delta)k, \quad b_{21} = -\rho, \quad b_{22} = -(1 - \rho)v_1 \theta^p, \quad b_{23} = 0,$$

$$b_{31} = 1 - \rho - \alpha\mu, \quad b_{32} = \mu s_L + \frac{sK}{\epsilon}(1 - \alpha\mu) + v_2 \theta^p(1 - \alpha\mu) - (1 - \rho)v_1 \theta^p,$$

$$b_{33} = \mu s_K - \frac{sK}{\epsilon}(1 - \alpha\mu)$$

$$c_{11} = 0, \quad c_{12} = -(\mu s_K + (1 - \delta)k), \quad c_{13} = -(\mu s_K + (1 - \delta)k)$$

$$c_{14} = -g, \quad c_{15} = 0, \quad c_{21} = -1, \quad c_{22} = c_{23} = c_{24} = 0, \quad c_{25} = -(1 - \rho)v_1 \theta^g,$$

$$c_{31} = -1, \quad c_{32} = \frac{sK}{\epsilon}(1 - \alpha\mu) - \mu s_K, \quad c_{33} = \frac{sK}{\epsilon}(1 - \alpha\mu) - \mu s_K,$$

$$c_{34} = 0, \quad c_{35} = v_2 \theta^g(1 - \alpha\mu) - (1 - \rho)v_1 \theta^g$$

$$d_{11} = d_{12} = d_{13} = d_{14} = d_{15} = 0, \quad d_{21} = 0, \quad d_{22} = \rho - \frac{1 - \beta(1 - \delta)\lambda^{-\rho}}{\epsilon},$$

$$d_{23} = -\frac{1 - \beta(1 - \delta)\lambda^{-\rho}}{\epsilon}, \quad d_{24} = 0, \quad d_{25} = (1 - \rho)v_1\theta^g - [1 - \beta(1 - \delta)\lambda^{-\rho}]v_2\theta^g,$$

$$d_{31} = 0, \quad d_{32} = \mu s_K - \frac{s_K}{\epsilon}(1 - \gamma\mu) - (1 - \rho), \quad d_{33} = \mu s_K - \frac{s_K}{\epsilon}(1 - \gamma\mu) - 1,$$

$$d_{34} = 0, \quad d_{35} = (1 - \rho)v_1\theta^g - v_2(1 - \gamma\mu)\theta^g$$

Figure 1. The Labor Market and Military Spending.

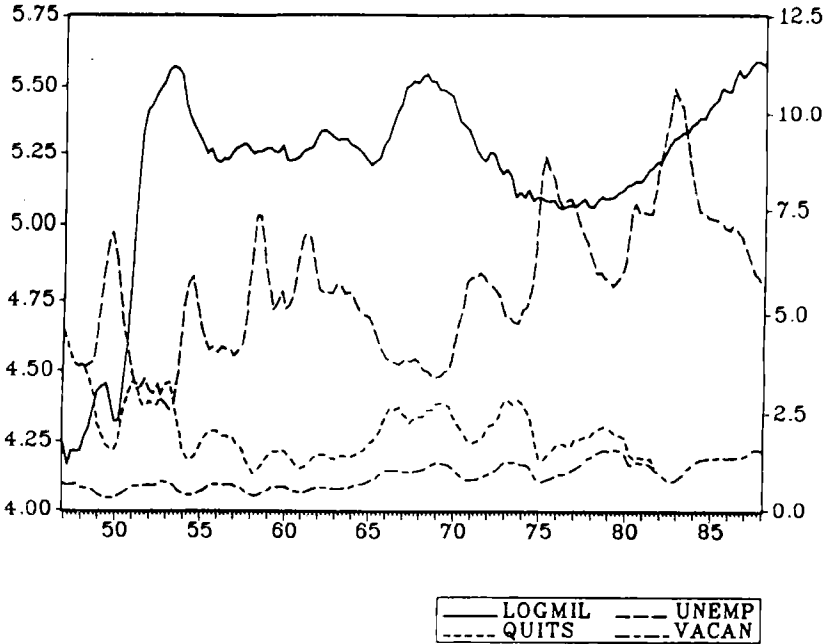


Figure 2. Properties of Profit Functions.

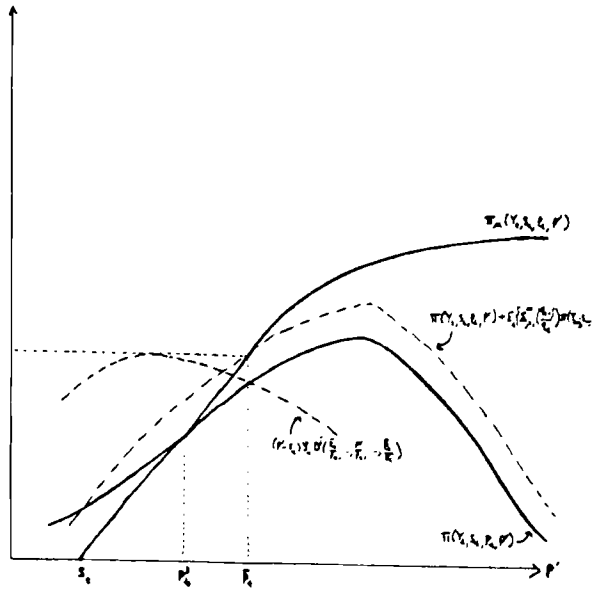


Figure 3. Sustainable Profits in the Steady State.

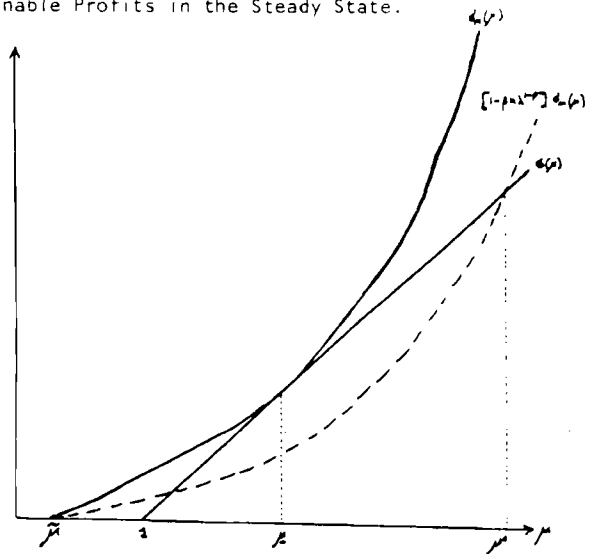


Figure 4. World War II.

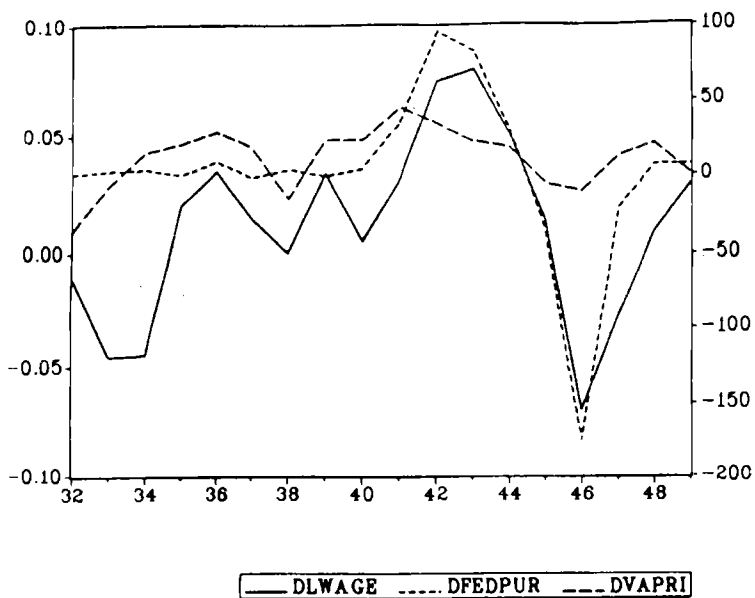


Figure 5. Response of Private Value Added.

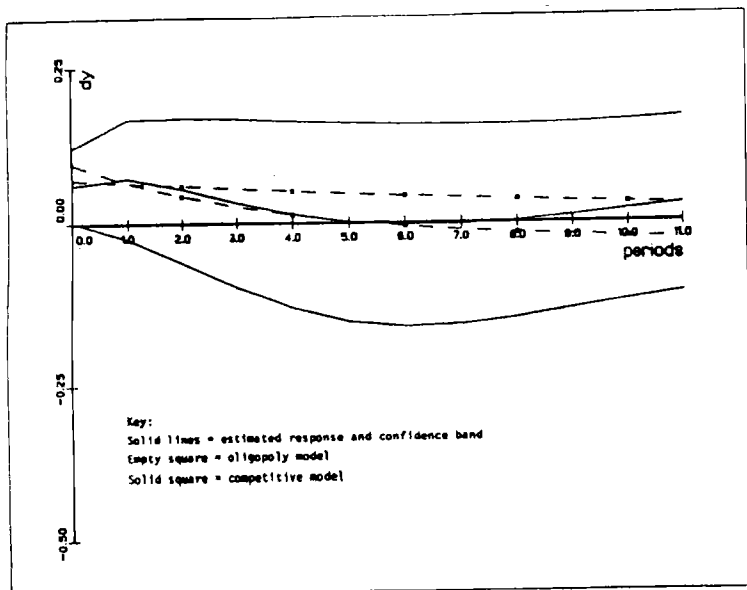


Figure 6. Response of Hours Worked.

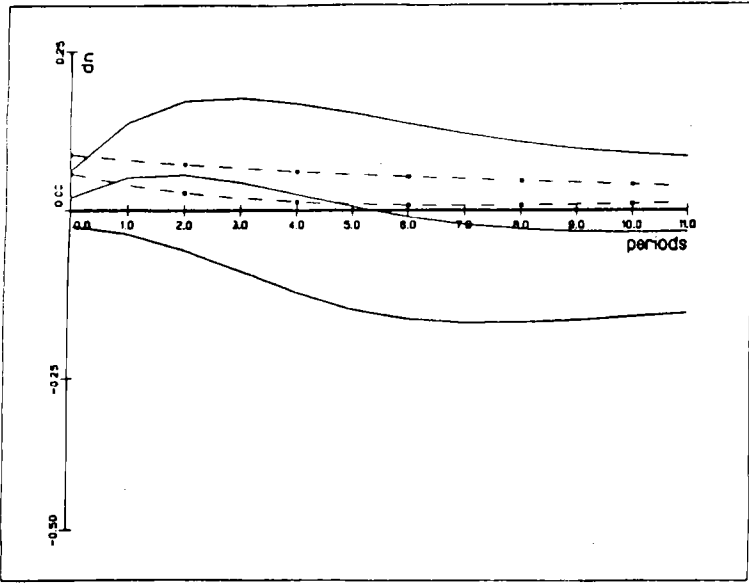


Figure 7. Response of the Real Wage.

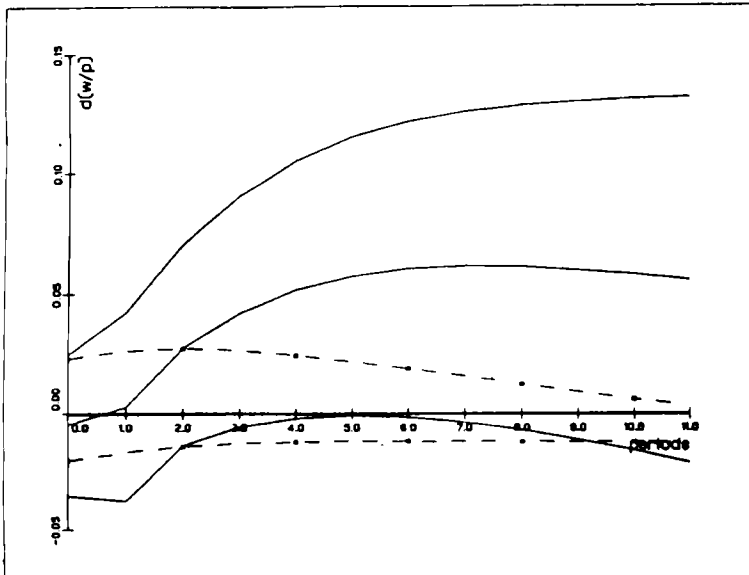


Figure 8. Response of Private Consumption.

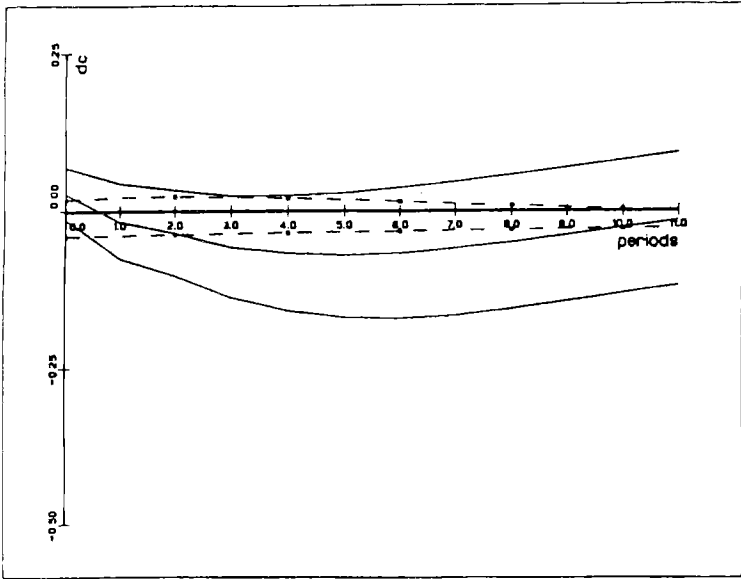


Figure 9. Estimated Response of Profits Relative to Trend.

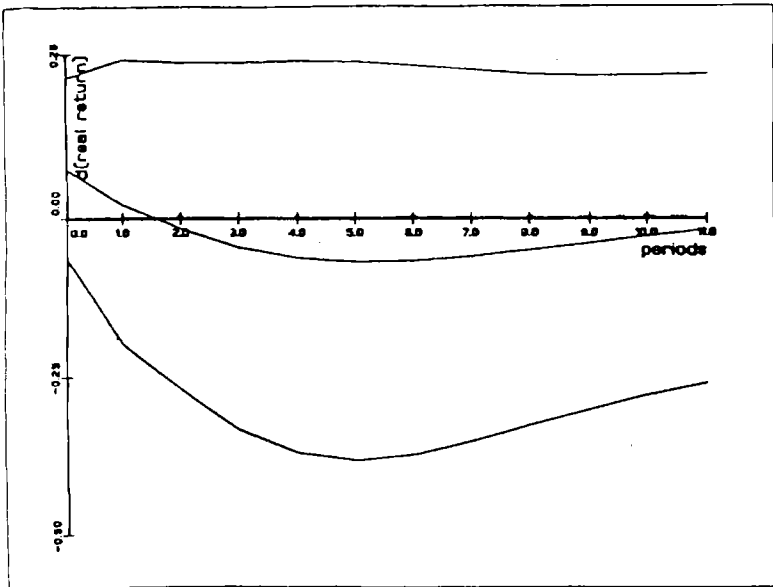


Figure 10. Simulated Response on the Real Return.

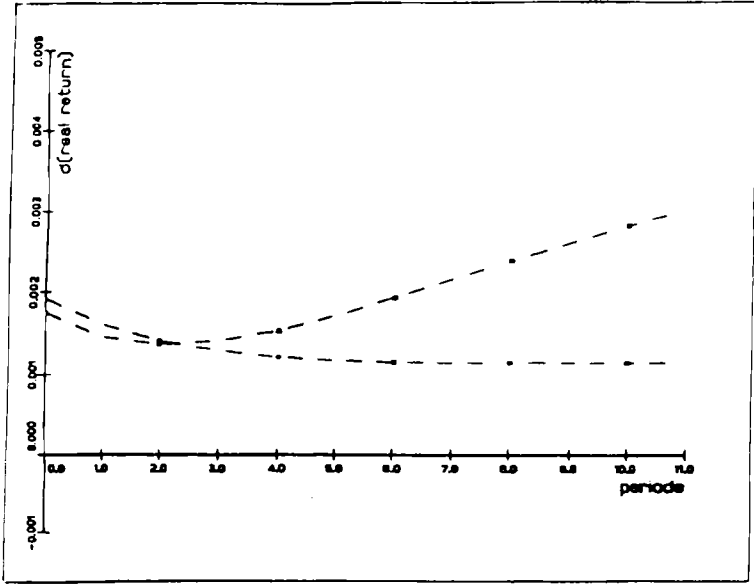


Figure 11. Simulated Response of the Capital Stock.

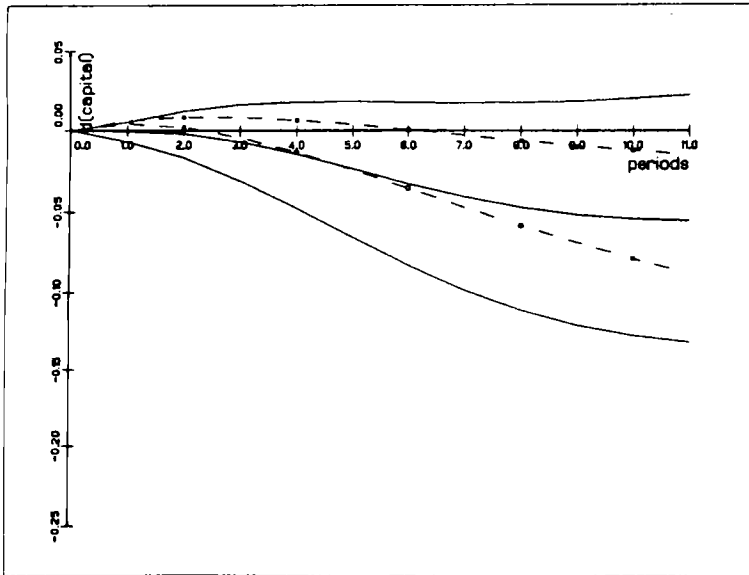


Figure 12. Sensitivity of Output Response to Elasticities of Consumption and Labor Supply.

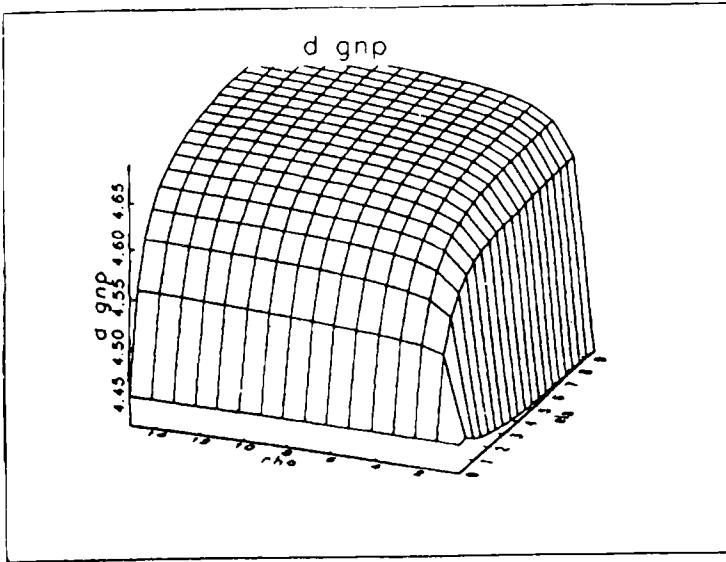


Figure 13. Sensitivity of Real Wage Response to Elasticities of Consumption and Labor Supply.

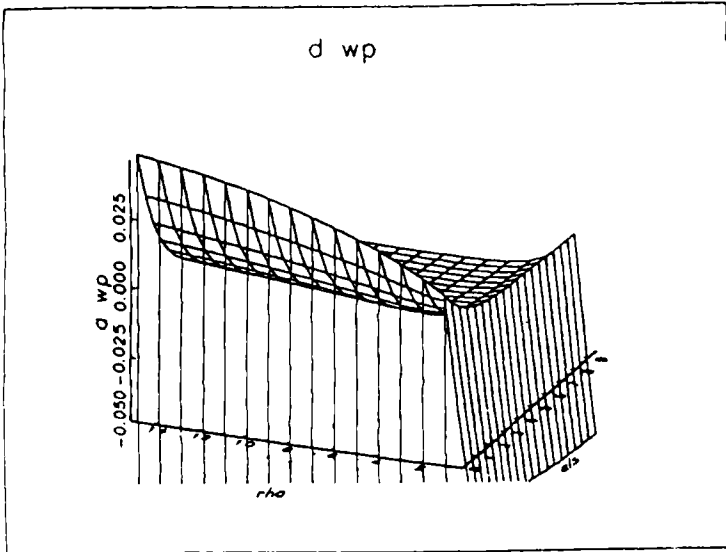


Figure 14. Sensitivity with Respect to Elasticity of Demand.

