Economics
The Open-Access, Open-Assessment E-Journal
Vol. 13, 2019-52 | December 11, 2019 | http://dx.doi.org/10.5018/economics-ejournal.ja.2019-52

Oligopoly price discrimination, competitive pressure and total output

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Abstract
This paper extends the traditional analysis of the output effect under monopoly (third-degree) price discrimination to a multimarket oligopoly. The author shows that under oligopoly price discrimination, differences in competitive pressure, measured by the number of firms, across markets are more important than the relative demand curvature when determining the effect on total output.

JEL D42 L12 L13
Keywords Third-degree price discrimination; total output; oligopoly; welfare

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Citation Iñaki Aguirre (2019). Oligopoly price discrimination, competitive pressure and total output. *Economics: The Open-Access, Open-Assessment E-Journal*, 13 (2019-52): 1–16. http://dx.doi.org/10.5018/economics-ejournal.ja.2019-52

Received September 12, 2019 **Published as Economics Discussion Paper** September 26, 2019
Revised November 29, 2019 **Accepted** December 3, 2019 **Published** December 11, 2019

1 Introduction

With respect to uniform pricing, third-degree price discrimination generates two effects: first, price discrimination causes a misallocation of goods from high to low value users and, second, price discrimination affects total output.[1] Therefore, a necessary condition for third-degree price discrimination to increase social welfare is an increase in total output.[2] As a result, a focal point has been the analysis of the effects of price discrimination on output.[3] It is known from Pigou (1920) that under linear demands price discrimination does not change output. In the general non-linear case, however, the effect of price discrimination on output may be either positive or negative. It is also well known (see, for example, Robinson, 1933, or Schmalensee, 1981) that when all the strong markets (markets where the optimal discriminatory price exceeds the optimal single price) have concave demands and the weak markets (where the optimal discriminatory prices are lower than the single price) have convex demands (with at least one market with strict concavity or convexity), then third-degree price discrimination increases output. When strong markets have convex demands and weak markets concave demands price discrimination reduces output. In the case in which all the demand curves have similar curvature the answer is more complicated. Shih et al. (1988) and Cheung and Wang (1994) obtain more general results and Aguirre (2009), Aguirre et al. (2010) and Cowan (2016) show that the effect of third-degree price discrimination on total output is intrinsically related to the shape of demands and inverse demands in strong markets as compared to the shape of direct and inverse demands in weak markets.

Over the last few decades much research has analyzed price discrimination in oligopolistic markets both under price competition and quantity competition.[4] Here we mainly focus on price discrimination under quantity competition following Stole's (2007) insight: "Perhaps the simplest model of imperfect competition and price discrimination is the immediate extension of Cournot's quantity-setting, homogeneous-good game to firms competing in distinct market segments."[5] The Cournot model has been widely used to analyze price discrimination in many different contexts.[6] This paper extends the traditional analysis of the output effect under

[1] See, for example, Schmalensee (1981) and Aguirre et al. (2010) for an explicit decomposition of the change in social welfare into these two effects: the misallocation effect and the output effect.

[2] See Schmalensee (1981), Varian (1985) and Schwartz (1990).

[3] It is assumed throughout the paper that all markets are served under both pricing policies, uniform pricing and price discrimination. The possibility that price discrimination opens up new markets (and that may even yield to Pareto improvements) has been analyzed for instance by Hausman and Mackie-Mason (1988).

[4] Many papers have analyzed oligopolistic price discrimination under price competition, including Holmes (1989), Corts (1998), Dastidar (2006), Adachi and Matsushima (2014), Adachi and Fabinger (2019) and Chen et al. (2019).

[5] Various empirical studies provide support for the assumption that Cournot competition prevails in some markets as, for instance, in the airline market, a market where price discrimination is quite common (see, for example, Brander and Zhang, 1990, and Oum et al., 1993).

[6] Neven and Phlips (1985), Howell (1991), Cheung and Wang (1997), Aguirre (2000), Galera and Zaratiegui (2006), Hazledine (2006), Stole (2007), Hazledine (2010), Kutlu (2012), Bakó and Kálecz-Simon (2012), Czerny and Zhang (2014), Kumar and Kutlu (2016) and Aguirre (2016) consider price discrimination under quantity competition in the final good market.

monopoly third-degree price discrimination to a multimarket oligopoly. We show that under symmetric Cournot oligopoly (all firms selling in all markets) similar results to those under monopoly are obtained:[7] in order for total output to increase with price discrimination the demand of the strong market (the high price market) should be, as conjecture by Robinson (1933), more concave than the demand of the weak market (the low price one). When competitive pressure (measured by the number of firms) varies across markets the effect of price discrimination on total output crucially depends on which market, the strong or the weak, is more competitive. Importantly, some new unexpected results are obtained, even with linear demand. First, we show that price discrimination in favor of the more competitive market is quite generally output reducing, therefore leading to a welfare deterioration. This result maintains unambiguously under linear demand, and also when the strong market exhibits convex demand and the weak market concave demand. Our results are in line with those of Holmes (1989) and Weyl and Fabinger (2013) who suggest that price discrimination against the more competitive markets (measured by the number of firms) might reduce social welfare through decreasing total output. Second, when the competitive pressure is higher in the strong market we obtain an important result: independently of the shape of demands and inverse demands, price discrimination tends to increase total output. In particular, we show that even with linear demand price discrimination increases total output.

The paper is organized as follows. Section 2 analyzes the output effect of price discrimination for a Cournot oligopoly. It shows that the results crucially depend on whether competitive pressure varies across markets and on which market, the strong or the weak, is more competitive. In Section 3, we consider an oligopoly model of price discrimination considering price competition and we show that our main result maintains under linear demand. That is, when the competitive pressure is higher in the strong market then price discrimination increases total output. Section 4 presents some concluding remarks.

2 Analysis

Consider a Cournot oligopoly selling a homogeneous product in two perfectly separated markets. Market 1 is served by n_1 firms and market 2 by n_2. The inverse demand function in market i is given by $p_i(q_i)$, where q_i is the total quantity sold and $p_i'(q_i) < 0$. Unit cost, c, is assumed constant and common for all firms. The profit function of firm j in market $i = 1,2$ is given by: $\pi_{ji}\big(q_{ji}, q_{-ji}\big) = [p_i(q_i) - c]q_{ji}$, where q_{ji} is the quantity sold by firm j in market i and $q_{-ji} = q_i - q_{ji}$, which is assumed to be strictly concave. We shall obtain the change in total output due to a move from third-degree price discrimination to uniform pricing.

[7] Our model of multimarket Cournot oligopoly can be seen as a particular case of multiproduct Cournot oligopoly (see, for instance, Johnson and Myatt, 2006).

Under price discrimination firms present in both markets choose their production in each market independently. By adding first order conditions we obtain that the equilibrium total output in market *i* satisfies:[8]

$$n_i\left[p_i\left(q_i^d\right)-c\right]+q_i^d p_i'\left(q_i^d\right)=0 \qquad i=1,2, \tag{1}$$

where q_i^d denotes the Cournot total output in market i.[9] From condition (1) we obtain that the equilibrium price can be written as:

$$p_i\left(q_i^d\right)=\frac{c}{1-\dfrac{1}{n_i\varepsilon_i\left(q_i^d\right)}} \qquad i=1,2, \tag{2}$$

where $\varepsilon_i(q_i)=-\frac{1}{p_i'(q_i)}\frac{p_i(q_i)}{q_i}$ is the elasticity of demand in market $i=1,2$. Therefore, we obtain a generalization of the monopolistic price discrimination rule to a Cournot oligopoly: $p_1\left(q_1^d\right)>p_2\left(q_2^d\right)$ iff $n_1\varepsilon_1\left(q_1^d\right)<n_2\varepsilon_2\left(q_2^d\right)$. From now on, we assume that market 1 is the strong market, $p_1\left(q_1^d\right)>p_2\left(q_2^d\right)$. Consequently, if the number of firms does not vary across markets the Cournot price will be higher in the market with the lower elasticity. The total output under price discrimination is $Q^d=\sum_{i=1}^{2} q_i^d$, which, given (1), can be expressed as:

$$Q^d=\sum_{i=1}^{2} q_i^d=-\sum_{i=1}^{2} n_i\frac{\left[p_i\left(q_i^d\right)-c\right]}{p_i'\left(q_i^d\right)}. \tag{3}$$

In order to solve the problem under uniform pricing, we distinguish between firms that sell in both markets and firms that only sell in one market.[10]Assume that there are $n_B>0$ firms selling in both markets, $n_1-n_B>0$ firms selling only in market 1 and $n_2-n_B>0$ firms selling only in market 2. If we aggregate first order conditions for firms that are only in market *i* we get:

$$(n_i-n_B)\left[p_i\left(q_i^0\right)-c\right]+q_{ii}^0 p_i'\left(q_i^0\right)=0 \qquad i=1,2, \tag{4}$$

where q_{ii}^0 is the total output produced by firms only set in market $i=1,2$ and q_i^0 is the total output sold in market $i=1,2$. Under uniform pricing a firm that sells in both markets has to adjust production in order to maintain the same price in both markets.[11] From the first order conditions and by adding over firms set in both markets we get:

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[8] We assume that conditions for existence and uniqueness of the Cournot equilibrium are satisfied. See, for instance, Kolstad and Mathiesen (1987). Dastidar (2000) shows that, in a case of symmetric costs like ours, a unique Cournot equilibrium is always stable.

[9] Following Kreps and Sheinkman (1983) Cournot competition can be interpreted as the reduced form of a two-stage game where firms first choose capacities and then set prices. Dana and Williams (2019) develop an oligopoly model of sequential quantity-price games where firms compete in multiple advance-purchase markets and explore the conditions required for intertemporal price discrimination to arise in an oligopoly setting.

[10] Uniform pricing is an empirically plausible price policy. In fact, DellaVigna and Gentzkow (2019) show that most U.S. food, drugstore, and mass-merchandise chains charge nearly uniform prices across stores, despite wide variation in consumer demographics and competition.

[11] Note that price discrimination might be illegal or impracticable due to regulation or arbitrage and the multimarket firms could be forced to adjust output across markets in order to satisfy price uniformity. Or, equivalently,

$$n_B[p_1(q_1^0)-c]p_2'(q_2^0)+n_B[p_2(q_2^0)-c]p_1'(q_1^0)+(q_{B1}^0+q_{B2}^0)p_1'(q_1^0)p_2'(q_2^0)=0, \tag{5}$$

where q_{Bi}^0 is the total output sold in market $i=1,2$ by the firms selling in both markets.[12] Condition (5) can be written as:

$$q_{B1}^0+q_{B2}^0=-\frac{n_B[p_1(q_1^0)-c]}{p_1'(q_1^0)}-n_B\frac{[p_2(q_2^0)-c]}{p_2'(q_2^0)}. \tag{6}$$

It is satisfied that $p_1(q_1^d)>p_1(q_1^0)=p_2(q_2^0)>p_2(q_2^d)$ and the total output, $Q^0=\sum_{i=1}^2 q_i^0$, can be expressed, given (4) and (6), as:

$$Q^0=\sum_{i=1}^{2}q_i^0=-\sum_{i=1}^{2}n_i\frac{[p_i(q_i^0)-c]}{p_i'(q_i^0)}. \tag{7}$$

Given (3) and (7), the change in total output is given by:

$$\Delta Q=Q^d-Q^0=-\sum_{i=1}^{2}n_i\frac{[p_i(q_i^d)-c]}{p_i'(q_i^d)}+\sum_{i=1}^{2}n_i\frac{[p_i(q_i^0)-c]}{p_i'(q_i^0)}, \tag{8}$$

which can be written as: [13]

$$\Delta Q=-\sum_{i=1}^{2}\left\{\int_{q_i^0}^{q_i^d}d\left[n_i\frac{[p_i(q_i)-c]}{p_i'(q_i)}\right]\right\}. \tag{9}$$

Therefore, the change in total output becomes:

$$\Delta Q=-\sum_{i=1}^{2}n_i\left\{\int_{q_i^0}^{q_i^d}\frac{[p_i'(q_i)]^2-[p_i(q_i)-c]p_i''(q_i)}{[p_i'(q_i)]^2}dq_i\right\}$$

$$\Delta Q=-\sum_{i=1}^{2}n_i\Delta q_i+\sum_{i=1}^{2}n_i\left\{\int_{q_i^0}^{q_i^d}\frac{[p_i(q_i)-c]p_i''(q_i)}{[p_i'(q_i)]^2}dq_i\right\}, \tag{10}$$

which can be rewritten as:

$$\Delta Q=-\sum_{i=1}^{2}n_i\Delta q_i+\sum_{i=1}^{2}n_i\left\{\int_{q_i^0}^{q_i^d}L_i(q_i)\varepsilon_i(q_i)C_i^I(q_i)dq_i\right\}, \tag{11}$$

multimarket firms might sign most-favored-customer (MFC) clauses with their clients committing to price uniformly (see, for instance, Aguirre, 2000). The argument is as follows. Assume that multimarket firm *j* adopts an MFC policy and that market 1 is strong. If firm *j* chooses its outputs q_{j1} and q_{j2} so that $p_1(q_{j1}+q_{-j1})>p_2(q_{j2}+q_{-j2})$ then it must rebate $[p_1(q_{j1}+q_{-j1})-p_2(q_{j2}+q_{-j2})]q_{j1}$ to market 1 customers and therefore its profits will be $[p_1(q_{j1}+q_{-j1})-c]q_{j1}-[p_2(q_{j2}+q_{-j2})-c]q_{j2}-[p_1(q_{j1}+q_{-j1})-p_2(q_{j2}+q_{-j2})]q_{j1}=[p_2(q_{j2}+q_{-j2})-c](q_{j1}+q_{j2})$. Firm *j* can increase its profits by choosing an output $q_{j1}'>q_{j1}$, such that given q_{-j1}, $p_2(q_{j2}+q_{-j2})$ is the price of both markets obtaining $[p_2(q_{j2}+q_{-j2})-c](q_{j1}'+q_{j2})$.

[12] We assume that the bordered Hessian is negative definite.

[13] We follow closely the analysis by Cheung and Wang (1994), (1997), Aguirre (2009) and Cowan (2016).

where $L_i(q_i) = \frac{p_i(q_i)-c}{p_i(q_i)}$ is the Lerner index of market i, $\varepsilon_i(q_i) = -\frac{1}{p_i'(q_i)}\frac{p_i(q_i)}{q_i}$ is the elasticity of demand of market i and $C_i^I(q_i) = q_i\frac{p_i''(q_i)}{p_i'(q_i)}$ is the adjusted concavity of the inverse demand in market i (this is analogous to relative risk aversion for a utility function). The adjusted concavity of the direct demand, $C_i^D(q_i) = -p_i(q_i)\frac{p_i''(q_i)}{[p_i'(q_i)]^2}$, is given by $C_i^D(q_i) = \varepsilon_i(q_i)C_i^I(q_i)$. Therefore, we can express the change in total output as:

$$\Delta Q = -\sum_{i=1}^{2} n_i \Delta q_i + \sum_{i=1}^{2} n_i \left\{\int_{q_i^0}^{q_i^d} L_i(q_i)C_i^D(q_i)dq_i\right\}. \tag{12}$$

We next show that the change of total output crucially depends on whether all firms are present in all markets.

(i) *Symmetric multimarket Cournot oligopoly*

First, consider a symmetric multimarket Cournot oligopoly with all firms selling in all markets and, therefore, $n_1 = n_2 = n$. The change in total output is (see Cheung and Wang, 1997):

$$\Delta Q = \frac{n}{1+n}\left\{\int_{q_1^0}^{q_1^d} \frac{[p_1(q_1)-c]p_1''(q_1)}{[p_1'(q_1)]^2}dq_1 + \int_{q_2^0}^{q_2^d} \frac{[p_2(q_2)-c]p_2''(q_2)}{[p_2'(q_2)]^2}dq_2\right\}. \tag{13}$$

One advantage of the Cournot oligopoly is that it converges to the monopoly case when $n = 1$. Under monopoly, a move from uniform pricing to third-degree price discrimination leads to (see, Cheung and Wang, 1994, Aguirre, 2009, or Cowan, 2016):

$$\Delta Q = \frac{1}{2}\left\{\int_{q_1^0}^{q_1^d} \frac{[p_1(q_1)-c]p_1''(q_1)}{[p_1'(q_1)]^2}dq_1 + \int_{q_2^0}^{q_2^d} \frac{[p_2(q_2)-c]p_2''(q_2)}{[p_2'(q_2)]^2}dq_2\right\}. \tag{14}$$

Therefore, we can immediately extend the results under monopoly obtained by Pigou (1920), Robinson (1933), Schmalense (1981), and, more recently, by Shih et al. (1988), Cheung and Wang (1994), Aguirre, 2009, Aguirre et al. (2010) and Cowan (2016) to a symmetric Cournot oligopoly. For instance, under linear demand total output changes neither in the monopoly case nor in the case of symmetric Cournot oligopoly independently of the number of firms (see Neven and Phlips, 1985, and Howell, 1991; Stole, 2007, provides a more elegant proof). On the other hand, third-degree price discrimination decreases output when strong markets exhibit strictly convex demands and weak markets have concave demands (see, for example, Robinson, 1933, Schmalensee, 1981 or Shih et al., 1988).

The next remark summarizes the effect of third-degree price discrimination on total output under monopoly and under a symmetric Cournot oligopoly.[14]

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[14] Weyl and Fabinger (2013) suggest that the results under monopoly might be extended to symmetric imperfect competition. Here, we prove it for a symmetric Cournot oligopoly.

Remark 1. *Effect of third-degree price discrimination on total output under monopoly and symmetric Cournot oligopoly.*

(i) If direct demand curves and inverse demand curves are both more concave in strong markets than in weak markets, then third-degree price discrimination increases total output.
(ii) If direct demand curves and inverse demand curves are both less (or equally) concave in strong markets than in weak markets, then third-degree price discrimination does not increase total output.

With respect to the results under monopoly, see the proof of Remark 1 in Aguirre (2009) for the *n*-market case. In Aguirre et al. (2010), this result appears as a corollary of their Proposition 4, for the two-market case. Cowan (2016) presents an elegant proof of case (i) for the *n*-market case.[15] Cheung and Wang (1994) for the case of monopoly and Cheung and Wang (1997) for the case of Cournot oligopoly provide a weaker version of this general result.

(ii) *Asymmetric multimarket Cournot oligopoly*

Consider an asymmetric multimarket Cournot oligopoly with $n_1 \neq n_2$. The effect of price discrimination on total output crucially depends on which market, the strong or the weak, exhibits more competitive pressure as measured by the number of firms.

a) $n_1 < n_2$

When the weak market has more firms we can rewrite (10), by adding and subtracting $n_1 \Delta q_2$, as:

$$\Delta Q = -\frac{n_2 - n_1}{1 + n_1}\Delta q_2 + \frac{1}{1 + n_1}\sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}. \tag{15}$$

As the next proposition and corollary show, when the number of firms is higher in the weak market the potential negative effects of price discrimination are aggravated.

Proposition 1. *When the weak market exhibits more competitive pressure, total output decreases with price discrimination under linear demand.*

Proof. Note that second terms in (15) are zero under linear demand, and that the first term is negative given that market 2 is the weak market, $\Delta q_2 > 0$, and it exhibits higher competitive pressure, $n_1 < n_2$.■

Corollary 1. *Effect of third-degree price discrimination on total output when the weak market is more competitive.*
If competitive pressure, measured by the number of firms, is higher in the weak market, then ceteris paribus it is more likely that price discrimination will reduce total output and therefore social welfare.

[15] See Weyl and Fabinger (2013) for a nice interpretation in terms of pass-through.

Proof. Note that the first term in (15) is negative given that market 2 is the weak market, $\Delta q_2 > 0$, and it exhibits higher competitive pressure, $n_1 < n_2$. Even with inverse and direct demands more concave in the strong market, price discrimination may reduce total output.■

Given that the first term in (15) is negative, we obtain the result that total output (and, therefore, social welfare) may decrease regardless of the shape of inverse and direct demands when firms price discriminate in favor of the market with more competitive pressure. This result is in line with the results of Holmes (1989) and Weyl and Fabinger (2013) who suggest that when discrimination is in favor of individuals for whom competition is more intense, discrimination is more likely to be harmful.

b) $n_1 > n_2$

When the strong market has more firms we can rewrite (10), by adding and subtracting $n_2 \Delta q_1$, as:

$$\Delta Q = -\frac{n_1 - n_2}{1 + n_2}\Delta q_1 + \frac{1}{1 + n_2}\sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}. \quad (16)$$

Note that, regardless of the shape of demands and inverse demands, there is a tendency for price discrimination to increase total output given that the first term in (16) is positive. The next proposition and corollary present some perhaps unexpected results that stress the importance of the differences in competitive pressure across markets. In particular, the next proposition shows that what is perhaps the most cited result in Pigou (1920) and Robinson (1933) does not hold.

Proposition 2. *When the strong market exhibits more competitive pressure total output increases with price discrimination under linear demand.*

Proof. Note that second terms in (16) are zero under linear demand and the first term is positive given that market 1 is the strong market, $\Delta q_1 < 0$, and it exhibits higher competitive pressure, $n_1 > n_2$.■

Corollary 2. *Effect of third-degree price discrimination on total output when the strong market is more competitive.*
If competitive pressure measured by the number of firms is higher in the strong market, then, regardless of the shape of direct demands and inverse demands, total output can increase with price discrimination.

Proof. Note that the first term in (16) is positive given that market 2 is the strong market, $\Delta q_1 > 0$, and it exhibits higher competitive pressure, $n_1 > n_2$. Even when inverse and direct demands are more concave in the weak market price discrimination might increase total output.■

Note that when the strong market exhibits more competitive pressure, $n_1 > n_2$, the general result of part (i) in Remark 1 maintains: if both direct demand curves are more concave in strong markets than in weak markets, then third-degree price discrimination increases total output. However, it is now possible that price discrimination leads to an increase in total output when the inverse and direct demands are not more concave in the strong markets.

These results are in sharp contrast with the well-known results under monopoly (see, for instance, Pigou, 1920) and under symmetric Cournot oligopoly (see, for instance, Stole, 2007) that price discrimination keeps total output unchanged with linear demand. To stress the importance of the above result, consider the case where the strong market has a strictly convex inverse demand and the weak market a strictly concave one (a situation in which under monopoly output unambiguously decreases as graphically proved by Robinson, 1933). Note that given that under price discrimination output decreases in the strong market and increases in the weak market, the second term in condition (16) is strictly negative. However, as the first term is strictly positive, if $|p_i''(q_i)|$ is small enough we can find easily examples where price discrimination increases total output.

We next show that similar results maintain under Bertrand competition in the case of linear demand.

3 Bertrand oligopoly price discrimination and linear demand

We consider a simple model due to Shubik and Levitan (1980) (see Motta, 2004, for a discussion of the advantages of this model in a context of oligopoly) to analyze the output effect of price discrimination under price competition.

Consider a Bertrand oligopoly selling differentiated goods in two perfectly separated markets. Market 1 is served by n_1 firms and market 2 by n_2.[16] The demand function for good *j* in market *i* is given by

$$q_{ji}\left(p_{ji}, p_{-ji}\right) = \frac{1}{n_i}\left[a_i - p_{ji}(1+\mu) + \frac{\mu}{n_i}\sum_{j=1}^{n_i} p_{ji}\right],$$

where $a_i > c$ and $\mu \in [0, \infty)$ represents the degree of substitutability between the n_i products that we assume constant across markets. The profit function of firm *j* in market $i = 1,2$ is given by: $\pi_{ji}\left(p_{ji}, p_{-ji}\right) = \left(p_{ji} - c\right) q_{ji}\left(p_{ji}, p_{-ji}\right)$.

Under price discrimination firms present in both markets price their product in each market independently. From first order conditions and imposing symmetry we obtain the Bertrand equilibrium price in market $i = 1,2$ as:

$$p_i^d = \frac{n_i a_i + c(n_i + n_i\mu - \mu)}{2n_i + n_i\mu - \mu}.$$

In equilibrium, the quantity sold by firm *j* in market $i = 1,2$ is given by:

[16] Adachi and Fabinger (2019) and Chen et al. (2019) analyze the welfare effects of price discrimination considering a Bertrand duopoly with differentiated product allowing marginal cost to vary across markets. They also consider, following Holmes (1989), symmetric models with all firms selling in all markets.

$$q_{ji}^* = \frac{(a_i - c)(n_i + n_i\mu - \mu)}{n_i(2n_i + n_i\mu - \mu)}.$$

And the total output sold in market $i = 1,2$ is:

$$q_i^d = \frac{(a_i - c)(n_i + n_i\mu - \mu)}{(2n_i + n_i\mu - \mu)}.$$

Total output under price discrimination is given by

$$Q^d = \sum_{i=1}^{2} q_i^d = \sum_{i=1}^{2}(a_i - p_i^d) = \sum_{i=1}^{2} \frac{(a_i - c)(n_i + n_i\mu - \mu)}{(2n_i + n_i\mu - \mu)}. \quad (17)$$

In order to solve the problem under uniform pricing, we distinguish between firms that sell in both markets and firms that only sell in one market. Assume that there are $n_B > 0$ firms selling in both markets, $n_1 - n_B > 0$ firms selling only in market 1 and $n_2 - n_B > 0$ firms selling only in market 2. Denote as p_1^0, p_2^0 and p^0 equilibrium prices for firms that sell only in market 1, only in market 2 and in both markets, respectively, when firms that sell in both markets are restricted to price uniformly. The first order condition for a firm that sells only in market i is:

$$\frac{1}{n_i}\left[a_i - p_i^0(1+\mu) + \frac{\mu}{n_i}\left[(n_i - n_B)p_i^0 + n_B p^0\right]\right] - \frac{\left(p_i^0 - c\right)}{n_i^2}[n_i(1+\mu) - \mu] = 0. \quad (18)$$

From condition (17) we obtain:

$$p_i^0 = \frac{n_i a_i + c[n_i(1+\mu) - \mu] + \mu n_B p^0}{2n_i + n_i\mu - \mu + \mu n_B}, \qquad i = 1,2 \quad (19)$$

The first order condition for a firm that sells in both markets is:

$$\frac{1}{n_1}\left[a_1 - p^0(1+\mu) + \frac{\mu}{n_1}[(n_1 - n_B)p_1^0 + n_B p^0]\right] - \frac{(p^0 - c)}{n_1^2}[n_1(1+\mu) - \mu]$$

$$+\frac{1}{n_2}\left[a_2 - p^0(1+\mu) + \frac{\mu}{n_2}[(n_2 - n_B)p_2^0 + n_B p^0]\right] - \frac{(p^0 - c)}{n_2^2}[n_2(1+\mu) - \mu] = 0. \quad (20)$$

We first show that under symmetric competition (all firms selling in all markets) a move from uniform pricing to price discrimination does not change total output. When it is satisfied that $n_1 = n_2 = n_B = n$, then from (19) and imposing symmetry we obtain:

$$p^0 = \frac{(a_1 + a_2)n + 2c[n(1+\mu) - \mu]}{4n + 2n\mu - 2\mu} = \frac{p_1^d + p_2^d}{2}. \quad (21)$$

Therefore, total output does not change:[17]

[17] Dastidar (2006) shows, for the duopoly case, that total output can increase, decrease or remain constant, with linear demand, depending on the relationship between the "own effect" ($\partial q_{ji}/\partial p_{ji}$) and the "cross effect" ($\partial q_{ji}/\partial p_{ki}$) across markets, when we move from uniform pricing to price discrimination. When "the own effect" and "the

$$Q^d = \sum_{i=1}^{2} q_i^d = a_1 - p_i^d + a_2 - p_2^d = a_1 - \frac{p_1^d + p_2^d}{2} + a_2 - \frac{p_1^d + p_2^d}{2} = \sum_{i=1}^{2} q_i^0 = Q^0.$$

The next proposition states that the results under Cournot competition can be extended to a product differentiation Bertrand oligopoly under linear demand.

***Proposition 3**. Effect of third-degree price discrimination on total output for a product differentiation Bertrand oligopoly under linear demand.*
When the strong market exhibits more (equal/less) competitive pressure, total output increases (does not change/decreases) with price discrimination under linear demand.

Proof. See Appendix.

Therefore, differences in competitive pressure across markets are also crucial to determine the effect of total output under price competition and product differentiation.

4 Concluding remarks

The analysis of the effects of third-degree price discrimination on total output and social welfare has been the focus of much theoretical research beginning at least from the pioneering work by Pigou (1920) and Robinson (1933). This paper contributes to the literature by showing that the basic results under monopoly can be directly extended to a Cournot oligopoly with homogeneous product in the case in which all firms are established in all markets. When competitive pressure measured by the number of firms varies across markets we find some perhaps unexpected results. We have obtained that when competitive pressure is higher in the strong market, there is a strong tendency for price discrimination to increase total output. On the other hand, price discrimination tends to reduce total output when competitive pressure is higher in the weak market.[18]

We have also found that for the two oligopoly canonical models in the literature (Bertrand with differentiated products and Cournot with homogeneous product), under linear demand price discrimination increases (does not change) (decreases) total output if the competitive pressure, measured by the number of firms, is greater (equal) (lower) in the strong market.

It is well known that the effect of monopoly third-degree price discrimination on total output is intrinsically related to the shape of both the demands and inverse demands in strong markets as compared to the shape of direct and inverse demands in weak markets. We have found that, however, differences in competitive pressure across markets may be even more important than the relative demand curvatures to determine the effect on total output in the oligopoly case.

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cross effect" remain constant across markets, in his model and in ours (with the same number of firms in both markets) total output does not change with discrimination.

[18] In a recent paper, Miklós-Thal and Shaffer (2019) obtain a related result in a context of intermediate price discrimination. In particular, they show that total output and social welfare are more likely to increase for more intense competition in the market where price rises with price discrimination (the strong market) and less intense competition in the market where price falls (the weak market).

Acknowledgements Financial support from the Ministerio de Economía y Competitividad (ECO2015-66027-P), and from the Departamento de Educación, Política Lingüística y Cultura del Gobierno Vasco (IT-1367-19) is gratefully acknowledged. A previous version of the paper circulated under the title "Cournot Oligopoly, Price Discrimination and Total Output." I would like to thank Takanori Adachi, Simon Cowan, Achim Czerny, Rajnish Kumar, Ignacio Palacios-Huerta and Glen Weyl for helpful comments to a previous version of the paper. I also appreciate the comments and suggestions of two anonymous referees and an invited reader.

Appendix

Proof of Proposition 3

In order to simplify the analysis, we assume that marginal cost is zero, $n_1 > n_B$ and $n_2 = n_B$. When firms that sell in both markets are restricted to price uniformly, the equilibrium price for firms selling in both markets and the equilibrium price for firms that sell only in market 1 are, respectively:

$$p^0 = \frac{(a_1 n_B + a_2 n_1) n_1 n_B (2n_1 + n_1\mu - \mu + \mu n_B) + a_1 \mu (n_1 - n_B) n_1 n_B^2}{4(n_B + n_1) n_1^2 n_B + \mu B + \mu^2 C}$$

and

$$p_1^0 = \frac{1}{(2n_1 + n_1\mu - \mu + \mu n_B)[4(n_B + n_1) n_1^2 n_B + \mu B + \mu^2 C]} \{a_1 n_1 [4(n_B + n_1) n_1^2 n_B + \mu B + \mu^2 C] + \mu (a_1 n_B + a_2 n_1) n_1 n_B^2 (2n_1 + n_1\mu - \mu + \mu n_B) + a_1 \mu^2 (n_1 - n_B) n_1 n_B^3\},$$

where

$$B = 8n_1^2 n_B^2 + 4n_1^3 n_B - 4n_1 n_B^2 - 2n_1^3 - 2n_1^2 n_B$$

$$C = 3n_1^2 n_B^2 + n_1^3 n_B - 3n_1 n_B^2 - n_1^3 - 2n_1^2 n_B + n_B^2 + n_1^2.$$

Total output in market 1 and market 2, under uniform pricing, are, respectively:

$$Q_1^0 = a_1 - p_1^0 \frac{(n_1 - n_B)}{n_1} - p^0 \frac{n_B}{n_1}$$

and

$$Q_2^0 = a_2 - p^0.$$

The changes in prices due to a movement from uniform pricing to price discrimination are: $\Delta p_{11} = p_1^d - p_1^0$ (the change in the price for firms that only sell in market 1), $\Delta p_{B1} = p_1^d - p^0$ (the change in their market 1's price for firms that sell in both markets), and $\Delta p_2 = p_2^d - p^0$ (the change in their market 2's price for firms that sell in both markets).

The change in total output due to a move from uniform pricing to third-degree price discrimination is:

$$\Delta Q = -\frac{(n_1 - n_B)}{n_1} \Delta p_{11} - \frac{n_B}{n_1} \Delta p_{B1} - \Delta p_2.$$

If the market with more firms, market 1, is the strong market, $p_1^d > p_2^d$, then price discrimination increases total output. If the market with more firms is the weak market, $p_1^d < p_2^d$, then price discrimination reduces total output.

Therefore, when the strong market exhibits more (equal/less) competitive pressure, total output increases (does not change/decreases) with price discrimination under linear demand. ■

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