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Research Article

On 3-Dimensional Contact Metric Generalized (k, μ)-Space Forms

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The present paper deals with a study of 3-dimensional contact metric generalized (k, μ)-space forms. We obtained necessary and sufficient condition for a 3-dimensional contact metric generalized (k, μ)-space form with $Q\phi = \phi Q$ to be of constant curvature. We also obtained some conditions of such space forms to be pseudosymmetric and ξ-projectively flat, respectively.

1. Introduction

In 1995, Blair et al. [1] introduced the notion of contact metric manifolds with characteristic vector field ξ belonging to the (k, μ)-nullity distribution and such types of manifolds are called (k, μ)-contact metric manifolds. They obtained several results and examples of such a manifold. A full classification of this manifold has been given by Boeckx [2]. A contact metric manifold (M, ϕ, ξ, η, g) is said to be a generalized (k, μ)-space if its curvature tensor satisfies the condition

$$R(X,Y)\xi = k\{\eta(Y)X - \eta(X)Y\} + \mu\{\eta(Y)hX - \eta(X)hY\} \tag{1}$$

for some smooth functions k and μ on M independent choice of vector fields X and Y. If k and μ are constant, the manifold is called a (k, μ)-space. If a (k, μ)-space M has a constant ϕ-sectional curvature c and a dimension greater than 3, the curvature tensor of this (k, μ)-space form is given by [3]

$$R = \frac{c+3}{4}R_1 + \frac{c-1}{4}R_2 + \left(\frac{c+3}{4} - k\right)R_3 + R_4 + \frac{1}{2}R_5 + (1-\mu)R_6, \tag{2}$$

where R_1, R_2, R_3, R_4, R_5, and R_6 are the tensors defined by

$$\begin{aligned}
R_1(X,Y)Z &= g(Y,Z)X - g(X,Z)Y,\\
R_2(X,Y)Z &= g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z,\\
R_3(X,Y)Z &= \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi,\\
R_4(X,Y)Z &= g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y,\\
R_5(X,Y)Z &= g(hY,Z)hX - g(hX,Z)hY + g(\phi hX,Z)\phi hY - g(\phi hY,Z)\phi hX,\\
R_6(X,Y)Z &= \eta(X)\eta(Z)hY - \eta(Y)\eta(Z)hX + g(hX,Z)\eta(Y)\xi - g(hY,Z)\eta(X)\xi
\end{aligned} \tag{3}$$

for all vector fields X, Y, Z on M, where $2h = L_\xi\phi$ and L is the usual Lie derivative.

The notion of generalized Sasakian-space-forms was introduced and studied by Alegre et al. [4] with several examples. A generalized Sasakian-space-form is an almost contact metric manifold (M, ϕ, ξ, η, g) whose curvature tensor is given by

$$R(X,Y)Z = f_1 R_1 + f_2 R_2 + R_3 f_3, \tag{4}$$

where R_1, R_2, and R_3 are the tensors defined above and f_1, f_2, f_3 are differentiable functions on M. In such case we will write the manifold as $M(f_1, f_2, f_3)$. Generalized Sasakian-space-forms have been studied by several authors, namely, [5–11].

By motivating the works on generalized Sasakian-space-forms and (k,μ)-space forms, Carriazo et al. [12] introduced the concept of generalized (k,μ)-space forms. A generalized (k,μ)-space form is an almost contact metric manifold (M, ϕ, ξ, η, g) whose curvature tensor R is given by

$$R = f_1 R_1 + f_2 R_2 + f_3 R_3 + f_4 R_4 + f_5 R_5 + f_6 R_6, \tag{5}$$

where $R_1, R_2, R_3, R_4, R_5, R_6$ are the tensors defined above and f_1, f_2, f_3, f_4, f_5, f_6 are differentiable functions on M.

The object of the paper is to study 3-dimensional contact metric generalized (k,μ)-space forms. The paper is organized as follows. Section 2 deals with some preliminaries on contact metric manifolds and contact metric generalized (k,μ)-space forms. Section 3 is concerned with 3-dimensional contact metric generalized (k,μ)-space forms. Here it is proved that a 3-dimensional contact metric generalized (k,μ)-space form with $Q\phi = \phi Q$ is of constant curvature if and only if $3f_2 + f_3 = 0$. In Section 4, it is proved that a 3-dimensional contact metric generalized (k,μ)-space form is Ricci-semisymmetric; then either $f_1 = f_3$ or $3f_2 + f_3 = 0$. In Sections 5 and 6, we obtained that some equivalent conditions for a 3-dimensional contact metric generalized (k,μ)-space form are pseudosymmetric and ξ-projectively flat, respectively.

2. Contact Metric Generalized (k,μ)-Space Forms

A contact manifold is a C^∞-$(2n+1)$ manifold M^{2n+1} equipped with a global 1-form η such that $\eta \wedge (d\eta)^n \neq 0$ everywhere on M^{2n+1}. Given a contact form η it is well known that there exists a unique vector field ξ called the characteristic vector field of η such that $\eta(\xi) = 1$ and $d\eta(X, \xi) = 0$ for every vector field X on M^{2n+1}. A Riemannian metric is said to be associated metric if there exists a tensor field ϕ of type (1, 1) such that $d\eta(X,Y) = g(X, \phi Y)$, $\eta(X) = g(X, \xi)$, $\phi^2 X = -X + \eta(X)\xi$, $\phi\xi = 0$, $\eta(\phi X) = 0$, and $g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$ for all vector fields X, Y on M^{2n+1}. Then the structure (ϕ, ξ, η, g) on M^{2n+1} is called a contact metric structure, and then manifold M^{2n+1} equipped with such a structure is called a contact metric manifold [13].

Given a contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ we define a (1, 1) tensor field h by $2h = L_\xi \phi$. Then h is symmetric and satisfies the following relations:

$$h\xi = 0, \qquad h\phi = -\phi h, \qquad \operatorname{tr}(h) = \operatorname{tr}(\phi h) = 0, \qquad \eta \cdot h = 0. \tag{6}$$

Moreover, if ∇ denotes the Riemannian connection of g, then the following relation holds:

$$\nabla_X \xi = -\phi X - \phi h X. \tag{7}$$

The vector field ξ is a Killing vector with respect to g if and only if $h = 0$. A contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ for which ξ is a Killing vector is said to be a K-contact manifold. Therefore, a generalized (k,μ)-space form with such a structure is actually a generalized Sasakian-space-form.

A generalized (k,μ)-space-form is an almost contact metric manifold (M, ϕ, ξ, η, g) whose curvature tensor R is given by

$$\begin{aligned} R(X,Y)Z &= f_1 \{g(Y,Z)X - g(X,Z)Y\} \\ &\quad + f_2 \{g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z\} \\ &\quad + f_3 \{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi \\ &\qquad - g(Y,Z)\eta(X)\xi\} \\ &\quad + f_4 \{g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X \\ &\qquad - g(hX,Z)Y\} \\ &\quad + f_5 \{g(hY,Z)hX - g(hX,Z)hY + g(\phi hX, Z)\phi hY \\ &\qquad - g(\phi hY, Z)\phi hX\} \\ &\quad + f_6 \{\eta(X)\eta(Z)hY - \eta(Y)\eta(Z)hX + g(hX,Z)\eta(Y)\xi \\ &\qquad - g(hY,Z)\eta(X)\xi\} \end{aligned} \tag{8}$$

for all vector fields X, Y, Z on M, where f_1, f_2, f_3, f_4, f_5, f_6 are differentiable functions on M, $2h = L_\xi \phi$, and L is the usual Lie derivative. In such case we will denote the manifold as $M(f_1, \dots, f_6)$. This kind of manifold appears as natural generalization of the (k,μ)-space forms by taking

$$f_1 = \frac{c+3}{4}, \qquad f_2 = \frac{c-1}{4}, \qquad f_3 = \frac{c+3}{4} - k, \qquad f_4 = 1, \qquad f_5 = \frac{1}{2}, \qquad f_6 = 1 - \mu \tag{9}$$

as constant. Here c denotes constant ϕ-sectional curvature. The ϕ-sectional curvature of generalized (k,μ)-space form $M(f_1, \dots, f_6)$ is $f_1 + 3f_2 + f_4 g((h - \phi h \phi)X, X)$. Generalized Sasakian-space-forms [4] are also examples with $f_4 = f_5 = f_6 = 0$ and f_1, f_2, f_3 not necessarily constant.

We inferred the following result from [12].

Theorem 1. *Let $M(f_1, \dots, f_6)$ be a generalized (k,μ)-space form. If M is a contact metric manifold with $f_3 = f_1 - 1$, then it is a Sasakian manifold.*

Next, by using the definitions of the tensors $R_1, R_2, R_3, R_4, R_5, R_6$ and properties (6) of the tensor h in the formula (8), we obtain that the curvature tensor of a generalized (k,μ)-space form satisfies

$$R(X,Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\} \tag{10}$$

for every X, Y. Thus we have the following Theorem.

Theorem 2 (see [12]). *If $M(f_1,\ldots,f_6)$ is a contact metric generalized (k,μ)-space form, then it is a generalized (k,μ)-space, with $k = f_1 - f_3$ and $\mu = f_4 - f_6$.*

We know from Theorem 1 that $M(f_1,\ldots,f_6)$ is Sasakian if $k = f_1 - f_3 = 1$. Under the same hypothesis, we also know that $f_2 = f_3 = f_1 - 1$ and $h = 0$, so we may take $f_4 = f_5 = f_6 = 0$. Therefore, in this paper we will consider non-Sasakian generalized (k,μ)-space forms $M(f_1,\ldots,f_6)$, that is, those with $k = f_1 - f_3 \neq 1$.

A contact metric manifold is said to be η-Einstein manifold [13] if it satisfies

$$S = ag + b\eta \otimes \eta \tag{11}$$

for some smooth functions a and b.

3. On 3-Dimensional Contact Metric Generalized (k,μ)-Space Forms

If $M^3(f_1,\ldots,f_6)$ is a contact metric generalized (k,μ)-space form, then its Ricci tensor S and the scalar curvature r can be written as [12]

$$S(X,Y) = (2f_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + f_3)\eta(X)\eta(Y) + (f_4 - f_6)g(hX,Y), \tag{12}$$

$$r = 2(3f_1 + 3f_2 - 2f_3). \tag{13}$$

It is known that in a 3-dimensional Riemannian manifold (M^3, g), the curvature tensor R is given by

$$R(X,Y)Z = S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y] \tag{14}$$

for any vector fields X, Y, Z on M. By substituting (12) and (13) in (14) we have

$$\begin{aligned} R(X,Y)Z &= (f_1 + 3f_2)\{g(Y,Z)X - g(X,Z)Y\} \\ &\quad + (3f_2 + f_3)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \\ &\quad + (f_4 - f_6)\{g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y\}. \end{aligned} \tag{15}$$

From (12), we have

$$S(X,\xi) = 2(f_1 - f_3)\eta(X). \tag{16}$$

Also, from (15) we obtain

$$R(\xi,Y)Z = (f_1 - f_3)\{g(Y,Z)\xi - \eta(Z)Y\} + (f_4 - f_6)\{g(hY,Z)\xi - \eta(Z)hY\}. \tag{17}$$

If $M^3(f_1,\ldots,f_6)$ is a contact metric generalized (k,μ)-space form, then $Q\phi = \phi Q$ is true if and only if $f_4 - f_6 = 0$. In this connection, the following result appears in [12].

Lemma 3. *If $M^3(f_1,\ldots,f_6)$ is a contact metric generalized (k,μ)-space form, then the following conditions are equivalent:*

(i) *M^3 is an η-Einstein,*

(ii) *$Q\phi = \phi Q$, where Q denotes the Ricci tensor,*

(iii) *M^3 is a $(f_1 - f_3, 0)$-space,*

(iv) $f_4 - f_6 = 0$.

Now we begin with the following.

Lemma 4. *A 3-dimensional contact metric generalized (k,μ)-space form with $Q\phi = \phi Q$ is of constant curvature if and only if $3f_2 + f_3 = 0$.*

Proof. Let a 3-dimensional contact metric generalized (k,μ)-space form $M^3(f_1,\ldots,f_6)$ with $Q\phi = \phi Q$ be a space of constant curvature. Then

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}, \tag{18}$$

where c is constant curvature of the manifold. By using the definition of Ricci curvature and (18) we have

$$S(X,Y) = 2cg(X,Y). \tag{19}$$

If we use (19) in the definition of the scalar curvature we get

$$r = 6c. \tag{20}$$

From (19) and (20) one can easily see that

$$S(X,Y) = \frac{r}{3}g(X,Y). \tag{21}$$

By putting $X = Y = \xi$ in (12) and using (21) we obtain $r = 6(f_1 - f_3)$. By comparing this value of r with (13), we have $3f_2 + f_3 = 0$.

Conversely, if $3f_2 + f_3 = 0$, then from (15) we can easily see that

$$R(X,Y)Z = (f_1 - f_3)\{g(Y,Z)X - g(X,Z)Y\} + (f_4 - f_6)\{g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y\}. \tag{22}$$

If $M^3(f_1, \ldots, f_6)$ satisfies $Q\phi = \phi Q$, then in view of Lemma 3 we have $f_4 - f_6 = 0$. Thus from (22), we have

$$R(X,Y)Z = (f_1 - f_3)\{g(Y,Z)X - g(X,Z)Y\}. \quad (23)$$

If $M^3(f_1, \ldots, f_6)$ satisfies $f_4 - f_6 = 0$, then $f_1 - f_3$ is constant. Hence, from (23), we conclude that the M^3 is of constant curvature. This completes the proof. □

Theorem 5. *Every 3-dimensional contact metric generalized (k,μ)-space form with $Q\phi = \phi Q$ is an $N(f_1 - f_3)$-η-Einstein manifold.*

Proof. The proof follows from (12), (10), and Lemma 3. □

4. Ricci-Semisymmetric 3-Dimensional Contact Metric Generalized (k,μ)-Space Forms

A contact metric generalized (k,μ)-space form is said to be Ricci-semisymmetric if its Ricci tensor S satisfies the condition

$$R(X,Y)\cdot S = 0, \quad X, Y \in \chi(M), \quad (24)$$

where $R(X,Y)$ acts as a derivation on S. This notion was introduced by Mirzoyan [14] for Riemannian spaces.

From (24) we have

$$S(R(X,Y)U,V) + S(U,R(X,Y)V) = 0. \quad (25)$$

If we substitute $X = \xi$ in (25) and then using (17), we get

$$\begin{aligned}(f_1 - f_3)&[g(Y,U)S(\xi,V) - \eta(U)S(Y,V) \\ &+ g(Y,V)S(\xi,U) - \eta(V)S(Y,U)] \\ &+ (f_4 - f_6)[g(hY,U)S(\xi,V) - \eta(U)S(hY,V) \\ &+ g(hY,V)S(\xi,U) - \eta(V)S(hY,U)] = 0.\end{aligned} \quad (26)$$

By using (16) in (26) we obtain

$$\begin{aligned}2(f_1 - f_3)&[(f_1 - f_3)\{g(Y,U)\eta(V) + g(Y,V)\eta(U)\} \\ &+ (f_4 - f_6)\{g(hY,U)\eta(V) + g(hY,V)\eta(U)\}] \\ &- (f_1 - f_3)[S(Y,V)\eta(U) + S(Y,U)\eta(V)] \\ &- (f_4 - f_6)[S(hY,V)\eta(U) + S(hY,U)\eta(V)] = 0.\end{aligned} \quad (27)$$

Consider that $\{e_1, e_2, e_3\}$ is an orthonormal basis of the T_pM, $p \in M$. Then by putting $Y = U = e_i$ in (27) and taking the summation for $1 \le i \le 3$, we have

$$(f_1 - f_3)[8(f_1 - f_3)\eta(V) - S(V,\xi) - r\eta(V)] = 0. \quad (28)$$

Again by using (16) in (28), we get

$$(f_1 - f_3)[6(f_1 - f_3) - r]\eta(V) = 0, \quad (29)$$

which gives either $f_1 = f_3$ or $r = 6(f_1 - f_3)$. For the second case, that is, $r = 6(f_1 - f_3)$, we have $3f_2 + f_3 = 0$. Thus we can state the following.

Theorem 6. *If a 3-dimensional contact metric generalized (k,μ)-space form is Ricci-semisymmetric then either $f_1 = f_3$ or $3f_2 + f_3 = 0$.*

5. Pseudosymmetric 3-Dimensional Contact Metric Generalized (k,μ)-Space Forms

Let (M,g) be a Riemannian manifold with the Riemannian metric ∇. A tensor field $F : \chi(M)\times\chi(M)\times\chi(M) \to \chi(M)$ of type $(1,3)$ is said to be curvature-like if it has the properties of R. For example, the tensor R given by

$$\begin{aligned}R(X,Y)Z = (X\wedge Y)Z = g(Y,Z)X - g(X,Z)Y, \\ X, Y, Z \in \chi(M)\end{aligned} \quad (30)$$

defines a curvature-like tensor field on M. For example, if $R(X,Y)Z = k((X\wedge Y)Z) = k(g(Y,Z)X - g(X,Z)Y)$, then the manifold is of constant curvature k.

It is well known that every curvature-like tensor field F acts on the algebra $\mathfrak{T}^1_s(M)$ of all tensor fields on M of type $(1,s)$ as a derivation [15]

$$\begin{aligned}(F\cdot P)&(X_1,\ldots,X_s;Y,X) \\ &= F(X,Y)\{P(X_1,\ldots,X_s)\} \\ &- \sum_{j=1}^{s} P(X_1,\ldots,F(X,Y)X_j,\ldots,X_s)\end{aligned} \quad (31)$$

for all $X_1,\ldots,X_s \in \chi(M)$ and $P \in \mathfrak{T}^1_s(M)$. The derivation $F\cdot P$ of P by F is a tensor field of type $(1,s+2)$. For a tensor field P of type $(1,s)$, we define the derivative of P with respect to the curvature-like tensor defined by (30) as

$$\begin{aligned}Q(g,P)&(X_1,\ldots,X_s;Y,X) \\ &= (X\wedge Y)P(X_1,\ldots,X_s) \\ &- \sum_{j=1}^{s} P(X_1,\ldots,(X\wedge Y)X_j,\ldots,X_s).\end{aligned} \quad (32)$$

As a generalization of locally symmetric ($\nabla R = 0$) manifolds, the notion of a semisymmetric ($R\cdot R = 0$) manifold is introduced by Szabó [16]. As a proper generalization of a semisymmetric manifold, the notion of pseudosymmetric manifold is introduced by Deszcz [17]. A Riemannian manifold (M,g) is said to be pseudosymmetric if there exists a real valued smooth function $L : M \to \mathbb{R}$ such that $R\cdot R = L(g,R)$. In particular, if L is constant, then M is said to be a pseudosymmetric manifold of constant type [17]. A pseudosymmetric manifold is said to be proper if it is not semisymmetric.

For 3-dimensional Riemannian spaces, the following characterization of pseudosymmetry is known (cf. [18, 19]).

Theorem 7. *A 3-dimensional Riemannian space is pseudosymmetric if and only if it is η-Einstein.*

We also know that a 3-dimensional contact metric generalized (k,μ)-space form is η-Einstein if and only if $f_4 - f_6 = 0$. Hence, in view of Theorem 7, we can state the following.

Theorem 8. *A 3-dimensional contact metric generalized (k,μ)-space form is pseudosymmetric if and only if $f_4 - f_6 = 0$.*

From Lemma 3 and Theorem 8, we have following.

Lemma 9. *A 3-dimensional contact metric generalized (k,μ)-space form with $Q\phi = \phi Q$ is pseudosymmetric.*

6. ξ-Projectively Flat 3-Dimensional Contact Metric Generalized (k,μ)-Space Forms

In a 3-dimensional contact metric generalized (k,μ)-space form $M^3(f_1,\ldots,f_6)$ the projective curvature tensor $\mathscr{P}$ is defined as

$$\mathscr{P}(X,Y)Z = R(X,Y)Z - \frac{1}{2}[S(Y,Z)X - S(X,Z)Y] \quad (33)$$

for all $X, Y, Z \in TM$.

By plugging $Z = \xi$ in (33) and using (16) and (10), we get

$$\mathscr{P}(X,Y)\xi = (f_4 - f_6)[\eta(Y)hX - \eta(X)hY]. \quad (34)$$

If $f_4 - f_6 = 0$, then from (34) one can get

$$\mathscr{P}(X,Y)\xi = 0; \quad (35)$$

that is, $M^3(f_1,\ldots,f_6)$ is ξ-projectively flat.

Conversely, suppose that $M^3(f_1,\ldots,f_6)$ is ξ-projectively flat. Then from (34) we obtain

$$(f_4 - f_6)[\eta(Y)hX - \eta(X)hY] = 0. \quad (36)$$

This implies that $f_4 - f_6 = 0$. Thus we can state the following.

Theorem 10. *A 3-dimensional contact metric generalized (k,μ)-space form is ξ-projectively flat if and only if $f_4 - f_6 = 0$.*

From Lemma 3 and Theorem 10, we obtain the following.

Lemma 11. *A 3-dimensional contact metric generalized (k,μ)-space form with $Q\phi = \phi Q$ is ξ-projectively flat.*

By combining Lemma 3, Theorem 5, Theorem 8, and Theorem 10 the following result is obtained.

Theorem 12. *If $M^3(f_1,\ldots,f_6)$ is a contact metric generalized (k,μ)-space form, then the following conditions are equivalent to each other:*

(i) *M^3 is $N(f_1 - f_3)$-η-Einstein,*

(ii) *$Q\phi = \phi Q$, where Q denotes the Ricci operator,*

(iii) *$f_4 - f_6 = 0$,*

(iv) *M^3 is pseudosymmetric,*

(v) *M^3 is ξ-projectively flat.*

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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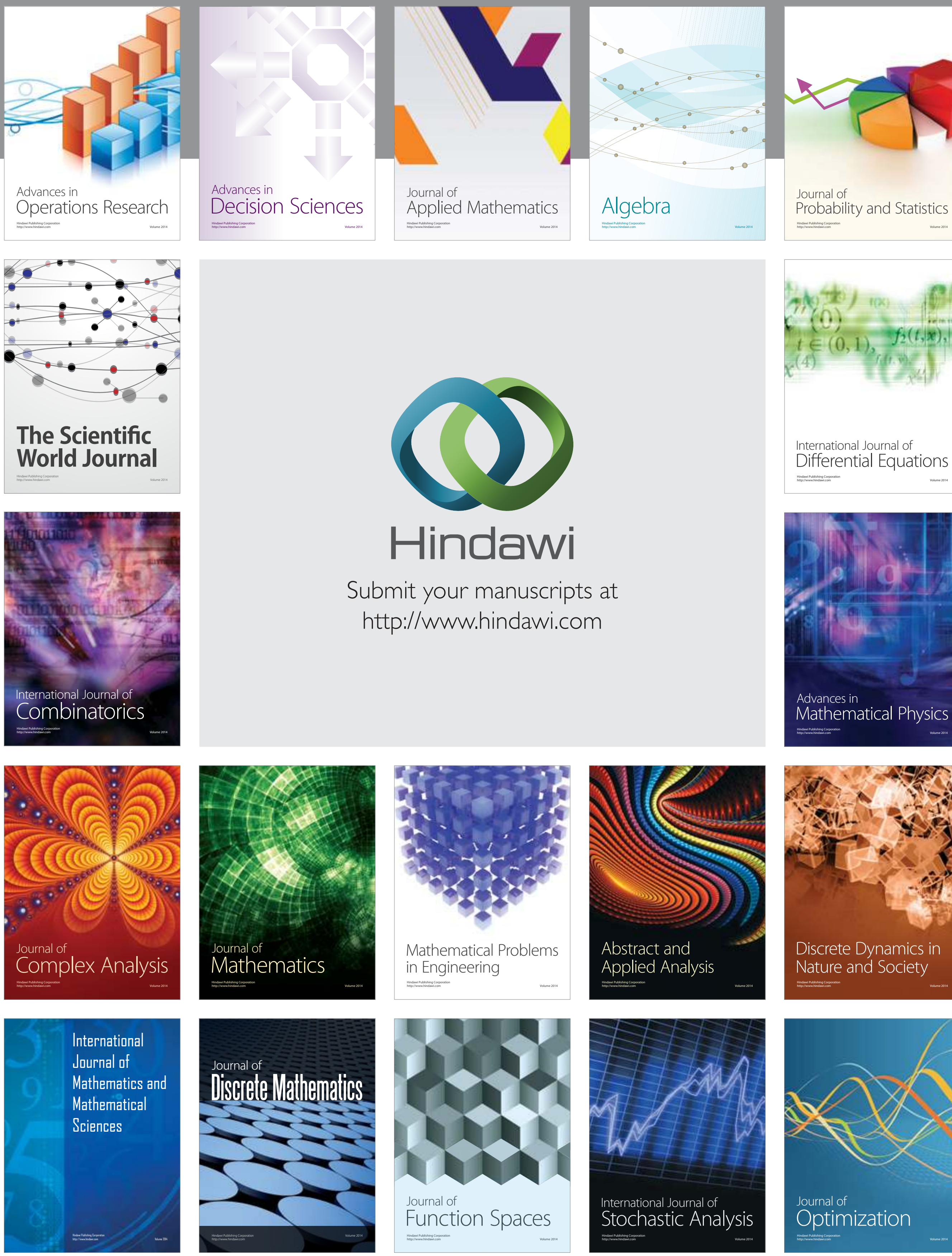
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