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On a new iteration scheme for numerical reckoning fixed points of Berinde mappings with convergence analysis

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Abstract

The aim of this work is to introduce a new three step iteration scheme for approximating fixed points of the nonlinear self mappings on a normed linear spaces satisfying Berinde contractive condition. We also study the sufficient condition to prove that our iteration process is faster than the iteration processes of Mann, Ishikawa and Agarwal, et al. Furthermore, we give two numerical examples which fixed points are approximated by using MATLAB.

Keywords: Picard iteration process, Mann iteration process, Ishikawa iteration process, rate of convergence, mean valued theorem.
2010 MSC: 47H09, 47H10.

1. Introduction and preliminaries

It is well-known that several mathematics problems are naturally formulated as fixed point problem,

$$Tx = x, \tag{1.1}$$

where T is some suitable mapping, may be nonlinear.

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For example, for a given mappings $\phi : [a,b] \subseteq \mathbb{R} \to \mathbb{R}$ and $K : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$, the solution of the following nonlinear integral equation:

$$x(c) = \phi(c) + \int_a^b K(c, r, x(r))dr, \tag{1.2}$$

where $x \in C[a,b]$ (the set of all continuous real-valued functions defined on $[a,b] \subseteq \mathbb{R}$), is equivalently with fixed point problem for a mapping $T : C[a,b] \to C[a,b]$ which is defined by

$$(Tx)(c) = \phi(c) + \int_a^b K(c, r, x(r))dr$$

for all $x \in C[a,b]$.

A solution x^* of the problem (1.1) is called a fixed point of the mapping T. Throughout this paper, we denote by $Dom(T)$ and $Fix(T)$ the domain of a mapping T and the set of all fixed points of a mapping T, respectively.

Consider *the fixed point iteration*, which is given by

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, ..., \tag{P_n}$$

where x_0 is arbitrary point but fixed in $Dom(T)$. Sometime the iterative method (P_n) is also called *the Picard iteration*, or *the Richardson iteration*, or *the method of successive substitution*. The standard result for a fixed point iteration is the Banach contraction mapping principle as follows:

Theorem 1.1 ([3])**.** *Let (X, d) be a complete metric space and $T : X \to X$ be a contraction mapping, i.e., a mapping for which there exists a constant $k \in [0, 1)$ such that*

$$d(Tx, Ty) \leq kd(x, y) \tag{1.3}$$

for all $x, y \in X$. Then T has a unique fixed point $x^* \in X$ and the iterates (P_n) converge to the fixed point x^*. Moreover, the error estimation is given by:

$$d(T^n x, x^*) \leq \frac{k^n}{1-k} d(x, Tx)$$

for each $x \in X$.

If constant k in condition (1.3) is equal to 1, then T is called a nonexpansive mapping. In fact, the Picard iteration (P_n) has been successfully employed for approximating the fixed point of contraction mappings and its variants. This success, however, has not extended to some nonlinear mapping such as nonexpansive mappings whenever the existence of a fixed point of such mappings is known.

Consider the mapping $T : [0,1] \to [0,1]$ which is defined by $Tx = 1 - x$ for all $x \in [0,1]$. Then T is a nonexpansive mapping on a usual metric with a unique fixed point $x^* = \frac{1}{2}$. We observe that the Picard iteration (P_n) of T with the starting value $x_0 \in [0,1]$ such that $x_0 \neq \frac{1}{2}$ yield the sequence $\{1-x_0, x_0, 1-x_0, ...\}$ for which does not converge to a fixed point x^* of T. Therefore, when a fixed point of nonexpansive mappings exists, other approximation techniques are needed to approximate it.

Iteration schemes for numerical reckoning fixed points of various classes of nonlinear operators have been introduced and studied by many mathematicians. For instance, the class of nonexpansive mappings via iteration methods is extensively studied in results of Tan and Xu [17] and Thakur et al. [20]. Also, the class of pseudocontractive mappings in their relation with iteration procedures has been studied by several researchers under suitable conditions (see main results of Yao et al. [21, 22], Thakur et al. [18, 19], Dewangan et al. [8, 9]).

Throughout this paper, unless otherwise specified, let E be a normed linear space and $T : E \to E$ be a given mapping. Here, we give some concepts of other approximation techniques.

The Mann iteration process [13] is defined by the following sequence $\{x_n\}$:

$$\begin{cases} x_0 \in E, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad n = 0,1,2,..., \end{cases} \tag{M_n}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is real control sequence in the interval $[0,1]$.

Remark 1.2. For $\alpha_n = \alpha \in [0,1]$ (constant), the iteration (M_n) reduces to the *the Krasnoselskij iteration*, while for $\alpha_n = 1$ the iteration (M_n) becomes the Picard iteration (P_n).

In 1974, Ishikawa [12] introduced an iteration process $\{x_n\}$ defined iteratively by

$$\begin{cases} x_0 \in E, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \quad n = 0,1,2,..., \end{cases} \tag{I_n}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are real control sequences in the interval $[0,1]$.

Remark 1.3. The Ishikawa iteration (I_n) reduces to the Mann iteration process (M_n) when take $\beta_n = 0$ for all $n = 0,1,2,\ldots$.

In 2007, Agarwal et al. [2] introduced an iteration process $\{s_n\}$ defined iteratively by

$$\begin{cases} s_0 \in E, \\ t_n = (1-\beta_n)s_n + \beta_n T s_n, \\ s_{n+1} = (1-\alpha_n)T s_n + \alpha_n T t_n, \quad n = 0,1,2,..., \end{cases} \tag{ARS_n}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are real control sequences in the interval $[0,1]$.

Now we come back to the contractive condition (1.3). We can easily see that this condition forces T to be continuous on X. It is then natural to ask that there exist contractive conditions which do not imply the continuity of T. In 1968, Kannan give answer for this question by considering instead of (1.3) the next condition of mappings that need not be continuous: there exists $k \in [0, 1/2)$ such that

$$d(Tx,Ty) \le k[d(x,Tx) + d(y,Ty)] \tag{1.4}$$

for all $x, y \in E$.

Example 1.4. Let $X = \mathbb{R}$ be a usual metric space and $T : X \to X$ be defined by

$$Tx = \begin{cases} 0, & x \in (-\infty, 2] \\ -\frac{1}{2}, & x \in (2, \infty). \end{cases}$$

Then T is not continuous on $\mathbb{R}$ but it satisfies condition (1.4) with $k = \frac{1}{5}$.

In 1975, Subrahmanyam [16] proved that Kannan contractive condition (1.4) characterizes the metric completeness, that is, a metric space E is complete if and only if every Kannan contraction mapping on E has a fixed point. Especially, the similar contractive condition of (1.4) has been introduced by Chatterjea [7] as follows: there exists $k \in [0, 1/2)$ such that

$$d(Tx,Ty) \le k[d(x,Ty) + d(y,Tx)] \tag{1.5}$$

for all $x, y \in E$.

Remark 1.5. Note that conditions (1.3), (1.4) and (1.5) are independent contractive conditions (see in [15]).

In 1972, Zamfirescu [23] obtained a very interesting fixed point theorem, by combining (1.3), (1.4) and (1.5) as follows:

Theorem 1.6. *Let (X,d) be a complete metric space and $T : X \to X$ be a Zamfirescu mapping, i.e., there exist the real numbers a, b and c satisfying $a \in [0,1)$ and $b, c \in [0,1/2)$ such that for each $x, y \in X$, at least one of the following is true:*

(Z_1) $d(Tx,Ty) \le ad(x,y)$;

(Z_2) $d(Tx,Ty) \le b[d(x,Tx) + d(y,Ty)]$;

(Z_3) $d(Tx,Ty) \le c[d(x,Ty) + d(y,Tx)]$.

Then T has a unique fixed point x^* and the Picard iteration $\{x_n\}$ defined as (P_n) converges to x^* for arbitrary but fixed $x_0 \in X$.

In 2004, Berinde [4] introduced a new class of mappings on a metric space (X,d) satisfying

$$d(Tx,Ty) \le \delta d(x,y) + Ld(x,Tx) \text{ for all } x,y \in X, \tag{1.6}$$

where $0 \le \delta < 1$ and $L \ge 0$.

Remark 1.7. It follows from the symmetry of the metric that the weak contractive condition (1.6) implies the following condition:

$$d(Tx,Ty) \le \delta d(x,y) + Ld(y,Ty) \text{ for all } x,y \in X, \tag{1.7}$$

Therefore, in order to check the weak contractiveness of T, it is necessary to check both (1.6) and (1.7).

He also showed that the class of nonlinear mapping satisfying the condition (1.6) is wider than the class of Zamfirescu mappings. In next year, Berinde [6] used the Ishikawa iteration process (I_n) to approximate fixed points of this class in a normed linear space.

By using the iteration process (ARS_n), Hussain et al. [11] proved a general theorem to approximate fixed points for nonlinear self mappings T on a nonempty closed convex subset C of a normed linear space E satisfying the condition (1.6) as follows:

Theorem 1.8 ([11])**.** *Let C be a nonempty closed convex subset of a Banach space E and $T : C \to C$ be a mapping satisfying the condition* (1.6)*. Suppose that the sequence $\{s_n\}$ is defined through the iterative process (ARS_n) and $s_0 \in C$, where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are sequences in the interval* $[0,1]$ *satisfying $\sum\limits_{n=0}^{\infty} \alpha_n = \infty$. If $Fix(T) \neq \emptyset$, then the sequence $\{s_n\}$ converges strongly to the fixed point of T.*

They also give some example to show that iteration process (ARS_n) is faster than the iteration processes (M_n) and (I_n) in the sense of Berinde [5] (see in Definition 1.9).

Definition 1.9 ([5])**.** Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers that converge to a and b, respectively, and assume that there exists

$$l := \lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|}.$$

$(\mathcal{R}_1)$ If $l = 0$, then it can be said that $\{a_n\}$ converges faster to a than $\{b_n\}$ to b.

$(\mathcal{R}_2)$ If $0 < l < \infty$, then it can be said that $\{a_n\}$ and $\{b_n\}$ have the same rate of convergence.

Next, we give the useful concept about rate of convergence due to Abbas and Nazir [1].

Definition 1.10 ([1])**.** Let $(X, \|\cdot\|)$ be a normed linear space and $\{u_n\}, \{v_n\}$ be two sequences in X. Suppose that $\{u_n\}$ and $\{v_n\}$ converging to the same point $p \in X$ and the following error estimates

$$||u_n - p|| \le a_n, \text{ for all } n \in \mathbb{N};$$
$$||v_n - p|| \le b_n, \text{ for all } n \in \mathbb{N};$$

are available, where $\{a_n\}$ and $\{b_n\}$ are two sequences of positive numbers (converging to zero). If $\{a_n\}$ converges faster than $\{b_n\}$, then $\{u_n\}$ *converges faster than* $\{v_n\}$ to p.

In this work, the authors deal with the iterates of Berinde mappings, in normed linear spaces, under a new iteration process (S_n) (see this process in Section 2), with convergence analysis. We also support analytic proof by numerical examples in Section 3.

2. Approximation results

In this section, we prove the new theorem to approximate fixed points for nonlinear self mappings T on a nonempty closed convex subset C of normed linear space E satisfying the condition (1.6) through the new iteration process as follows:

$$\begin{cases} x_0 \in C, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ z_n = (1-\gamma_n)x_n + \gamma_n y_n, \\ x_{n+1} = (1-\alpha_n)Tz_n + \alpha_n T y_n, \quad n = 0,1,2,..., \end{cases} \tag{S_n}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ are real control sequences in the interval $[0,1]$.

Theorem 2.1. *Let C be a nonempty closed convex subset of a Banach space $(E, \|\cdot\|)$ and $T\colon C \to C$ be a mapping satisfying the contractive condition (1.6), with the fixed point w. Suppose that the sequence $\{x_n\}$ is defined by the iteration process (S_n) and the sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, and $\{\gamma_n\}_{n=0}^{\infty}$ are in $[\alpha, 1-\alpha]$, $[\beta, 1-\beta]$, and $[\gamma, 1-\gamma]$ respectively, with $\alpha, \beta, \gamma \in (0, \frac{1}{2})$. If $\alpha(2-\gamma) < \gamma$, then the iteration process (S_n) converges strongly to the fixed point w of T faster than (ARS_n).*

Proof. For each $n \in \{0,1,2,...\}$, by using (S_n), we get

$$\begin{aligned} \|x_{n+1} - w\| &= \|(1-\alpha_n)Tz_n + \alpha_n T y_n - w\| \\ &= \|(1-\alpha_n)(Tz_n - w) + \alpha_n(Ty_n - w)\| \\ &\le (1-\alpha_n)\|Tz_n - w\| + \alpha_n\|Ty_n - w\| \\ &\le (1-\alpha_n)\delta\|z_n - w\| + \alpha_n\delta\|y_n - w\| \\ &\le (1-\alpha_n)\|z_n - w\| + \alpha_n\|y_n - w\|. \end{aligned} \tag{2.1}$$

Using (S_n) again, for each $n \in \{0,1,2,...\}$, we have

$$\begin{aligned} \|y_n - w\| &= \|(1-\beta_n)x_n + \beta_n T x_n - w\| \\ &= \|(1-\beta_n)(x_n - w) + \beta_n(Tx_n - w)\| \\ &\le (1-\beta_n)\|x_n - w\| + \beta_n\|Tx_n - w\| \\ &\le (1-\beta_n)\|x_n - w\| + \beta_n\delta\|x_n - w\| \\ &= (1-(1-\delta)\beta_n)\|x_n - w\| \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} \|z_n - w\| &= \|(1-\gamma_n)x_n + \gamma_n y_n - w\| \\ &= \|(1-\gamma_n)(x_n - w) + \gamma_n(y_n - w)\| \\ &\le (1-\gamma_n)\|x_n - w\| + \gamma_n\|y_n - w\| \\ &\le (1-\gamma_n)\|x_n - w\| + \gamma_n(1-(1-\delta)\beta_n)\|x_n - w\| \\ &= (1-(1-\delta)\beta_n\gamma_n)\|x_n - w\|. \end{aligned} \tag{2.3}$$

From (2.1), (2.2) and (2.3), we have

$$\begin{aligned} \|x_{n+1} - w\| &\le \{(1-\alpha_n)[1-(1-\delta)\beta_n\gamma_n] + \alpha_n[1-(1-\delta)\beta_n]\}\|x_n - w\| \\ &= \{1-(1-\delta)\beta_n\gamma_n - (1-\delta)\alpha_n\beta_n(1-\gamma_n)\}\|x_n - w\| \\ &= \{1-(1-\delta)\beta_n[\gamma_n + \alpha_n(1-\gamma_n)]\}\|x_n - w\| \\ &\le \{1-(1-\delta)\beta[\gamma - \alpha + \alpha\gamma]\}\|x_n - w\| \end{aligned}$$

for all $n \in \{0, 1, 2, ...\}$. Therefore,

$$\|x_n - w\| \leq \{1 - (1 - \delta)\beta[\gamma - \alpha + \alpha\gamma]\}^n \|x_0 - w\|$$

for all $n \in \{1, 2, ...\}$. Let

$$a_n := \{1 - (1 - \delta)\beta[\gamma - \alpha + \alpha\gamma]\}^n \|x_0 - w\|, \quad n = 1, 2, \dots$$

Now, let us refer to the (ARS_n) iteration. We have

$$\begin{aligned} \|s_{n+1} - w\| &= \|(1 - \alpha_n)Ts_n + \alpha_n Tt_n - w\| \\ &= \|(1 - \alpha_n)(Ts_n - w) + \alpha_n(Tt_n - w)\| \\ &\leq (1 - \alpha_n)\|Ts_n - w\| + \alpha_n\|Tt_n - w\| \\ &\leq \delta[(1 - \alpha_n)\|s_n - w\| + \alpha_n\|t_n - w\|] \\ &\leq (1 - \alpha_n)\|s_n - w\| + \alpha_n\|t_n - w\|. \end{aligned} \tag{2.4}$$

On the other hand,

$$\begin{aligned} \|t_n - w\| &= \|(1 - \beta_n)s_n + \beta_n Ts_n - w\| \\ &= \|(1 - \beta_n)(s_n - w) + \beta_n(Ts_n - w)\| \\ &\leq (1 - \beta_n)\|s_n - w\| + \beta_n\|Ts_n - w\| \\ &\leq (1 - \beta_n)\|s_n - w\| + \delta\beta_n\|Ts_n - w\| \\ &= [1 - (1 - \delta)\beta_n]\|s_n - w\|. \end{aligned} \tag{2.5}$$

Using (2.4), and (2.5), we obtain

$$\begin{aligned} \|s_{n+1} - w\| &\leq [1 - (1 - \delta)\alpha_n\beta_n]\|s_n - w\| \\ &\leq [1 - (1 - \delta)\alpha\beta]\|s_n - w\|. \end{aligned}$$

It follows that

$$\|s_n - w\| \leq [1 - (1 - \delta)\alpha\beta]^n\|s_0 - w\|, \; n = 0, 1, 2, \dots \; .$$

Let

$$b_n := [1 - (1 - \delta)\alpha\beta]^n\|s_0 - w\|, \quad n = 1, 2, \dots \; .$$

Since $\alpha(2 - \gamma) < \gamma$, we get that

$$1 - (1 - \delta)\beta(\gamma - \alpha + \alpha\gamma) < 1 - (1 - \delta)\alpha\beta < 1.$$

In this respect,

$$\lim_{n\to\infty} \|x_n - w\| \leq \lim_{n\to\infty} \{1 - (1 - \delta)\beta(\gamma - \alpha + \alpha\gamma)\}^n\|x_0 - w\| = 0,$$

$$\lim_{n\to\infty} \|s_n - w\| \leq \lim_{n\to\infty} 1 - (1 - \delta)\alpha\beta^n\|s_0 - w\| = 0,$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left\{\frac{1 - (1 - \delta)\beta(\gamma - \alpha + \alpha\gamma)}{1 - (1 - \delta)\alpha\beta}\right\}^n \frac{\|x_0 - w\|}{\|s_0 - w\|} = 0,$$

and the conclusion follows. □

3. Numerical results

In this section, we consider the following examples to illustrate the theoretical results that our iteration process (S_n) is faster than the iteration process (ARS_n) for mapping satisfying condition (1.6) and then it is also faster than the iteration processes (M_n) and (I_n).

Example 3.1. Let $C = [1, 100]$ be a subset of a usual normed space $E = \mathbb{R}$ and $T : C \to C$ be a mapping which is defined by

$$Tx = \sqrt{x^2 - 8x + 40}$$

for all $x \in C$. Choose $\alpha = \beta = 0.1, \gamma = 0.2$ and $\alpha_n = \beta_n = \gamma_n = \frac{1}{2}$ for all $n = 0, 1, 2, \ldots$. By mean valued theorem, we can prove that T satisfies the condition (1.6). It is easy to see that T has a unique fixed point $w := 5$. Also, it clear that sequences $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and parameters α, β, γ satisfy all the conditions of Theorem 2.1. So our corresponding iteration process (S_n) is faster than the Agarwal et al. iteration process (ARS_n) and then it is also faster than the Mann iteration process (M_n), the Ishikawa iteration process (I_n).

For the initial point $x_0 = 100$, our corresponding iteration process (S_n), the Agarwal et al. iteration process (ARS_n), the Ishikawa iteration process (I_n), the Mann iteration process (M_n) are, respectively, given in Table 1.

Table 1: Comparative results of Example 3.1

Step	**Iteration** (M_n)	**Iteration** (I_n)	**Iteration** (ARS_n)	**Iteration** (S_n)
1	98.062459362791700	97.094973886224900	95.157433249016600	94.673683974767200
2	96.126203567620800	94.192948098135200	90.323288827253500	89.357673090634100
3	94.191285640815600	91.294114752765900	85.498500618548100	84.053241902263000
4	92.257761918007700	88.398684766890500	80.684164558474000	78.761913211678600
5	90.325692322426100	85.506890363085900	75.881577927300300	73.485526099539800
6	88.395140672799500	82.618988003211200	71.092291170561100	68.226328658877100
7	86.466175024695300	79.735261838952700	66.318177306360700	62.987106889142000
8	84.538868049712100	76.856027791591100	61.561526502810500	57.771367836679900
9	82.613297457629000	73.981638402287800	56.825177462530800	52.583606400377400
10	80.689546467432200	71.112488632202200	52.112703944630700	47.429705306909100
⋮	⋮	⋮	⋮	⋮
36	31.864913492268900	6.570632482402260	5.000000000005160	5.000000000000020
37	30.078599364633200	5.940916407416960	5.000000000000830	5.000000000000000
38	28.306678330914200	5.548084108494060	5.000000000000130	5.000000000000000
39	26.551066895595100	5.313896276989430	5.000000000000020	5.000000000000000
40	24.814062579901100	5.178022277672790	5.000000000000000	5.000000000000000

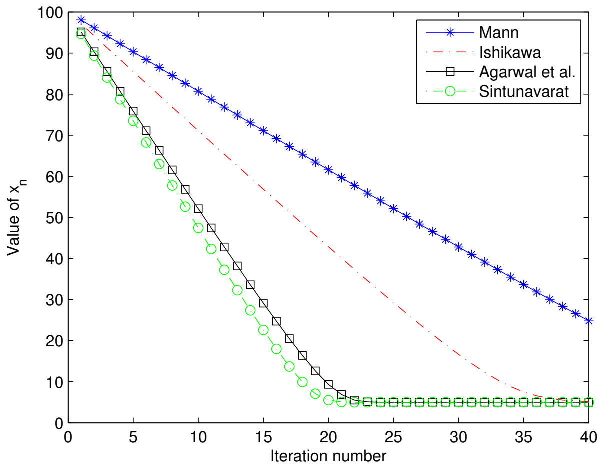

Figure 1: Behavior of of the Mann iteration process (M_n), the Ishikawa iteration process (I_n), the Agarwal et al. iteration process (ARS_n), and the Sintunavarat iteration process (S_n) for the given function in Example 3.1.

Example 3.2. Let $C = [0, 20]$ be a subset of a usual normed space $E = \mathbb{R}$ and $T : C \to C$ be a mapping which is defined by

$$Tx = \cos(\cos x)$$

for all $x \in C$. Choose $\alpha = \beta = \gamma = 0.25$ and $\alpha_n = \beta_n = \gamma_n = \frac{1}{2} + \frac{1}{2\sqrt{n+3}}$ for all $n = 0, 1, 2, \ldots$. By mean valued theorem, we can show that T satisfies the condition (1.6). It is easy to see that T has a unique fixed point $w \approx 0.739085133215161$. Also, it is clear that sequences $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and parameters α, β, γ satisfy all the conditions of Theorem 2.1. So our corresponding iteration process (S_n) is faster than the Agarwal et al. iteration process (ARS_n) and then it is also faster than the Mann iteration process (M_n), the Ishikawa iteration process (I_n).

For the initial point $x_0 = 5$, our corresponding iteration process (S_n), the Agarwal et al. iteration process (ARS_n), the Ishikawa iteration process (I_n), the Mann iteration process (M_n) are, respectively, given in Table 2.

Table 2: Comparative results of Example 3.2

Step	Iteration (M_n)	Iteration (I_n)	Iteration (ARS_n)	Iteration (S_n)
1	1.970027697721000	1.978396159559040	0.968405392132710	0.903759405132275
2	1.214127002979290	1.220146760824190	0.813134967469598	0.786108010872498
3	1.020869628246380	0.970455207657142	0.763601708031390	0.752788371219904
4	0.914519972971912	0.856215366672090	0.747297336769758	0.743142048582790
5	0.850246556143590	0.799726878828884	0.741866585940653	0.740303764756083
6	0.810159605749810	0.770977864281416	0.740036673484997	0.739455781629162
7	0.784808478488821	0.756079417424699	0.739413479402521	0.739199057746805
8	0.768647126128122	0.748244740823917	0.739199273870616	0.739120463031230
⋮	⋮	⋮	⋮	⋮
29	0.739091111576946	0.739085212233260	0.739085133215219	0.739085133215162
30	0.739089194508728	0.739085180129703	0.739085133215182	0.739085133215161
31	0.739087895043010	0.739085161120275	0.739085133215168	0.739085133215161
32	0.739087013220185	0.739085149842359	0.739085133215164	0.739085133215161
33	0.739086414168057	0.739085143138995	0.739085133215162	0.739085133215161
34	0.739086006793563	0.739085139147605	0.739085133215161	0.739085133215161
35	0.739085729493707	0.739085136766993	0.739085133215161	0.739085133215161

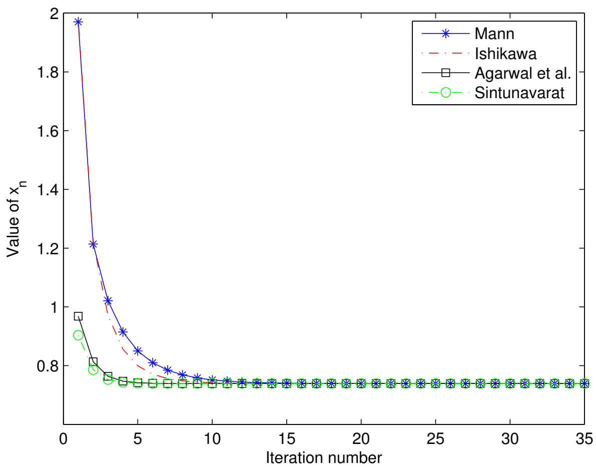

Figure 2: Behavior of of the Mann iteration process (M_n), the Ishikawa iteration process (I_n), the Agarwal et al. iteration process (ARS_n), and the Sintunavarat iteration process (S_n) for the given function in Example 3.2.

4. Conclusion and open problem

Convergence behavior of the sequence $\{x_n\}$ generated by the fixed point iteration process (S_n) was investigated under general assumptions on the parameter. The rate of convergence of this iteration process was studied. Finally, some illustrative numerical results are furnished which demonstrate the validity of the hypotheses and degree of utility of our results. It shows the behavior of iteration (S_n) with respect to the Mann iteration process (M_n), the Ishikawa iteration process (I_n) and the Agarwal et al. iteration process (ARS_n).

On the other hand, stability results established in metric spaces and normed linear spaces have been studied by several mathematicians such as Haghi et al. [10], Olatinwo and Postolache [14]. Therefore, the stability of iteration scheme (S_n) still open for interested mathematicians.

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