On a Null Method of Testing Vibration Galvanometers

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XXVIII. *On a Null Method of Testing Vibration Galvanometers.*

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1. In a recent Paper A. Campbell* has given the constants of a number of vibration galvanometers. In obtaining these constants the measurements made were the alternating-current and voltage sensitivities, the direct-current sensitivity and the resonating frequency. Of these, the alternating-current and voltage sensitivities are the most difficult to measure, since, because of the sharpness of resonance, there is a considerable fluctuation in the amplitude of vibration for very small variations in the frequency of the source, while the presence of any harmonics in the source will introduce a definite error in the results.

In the course of an investigation on electrically-maintained vibrations, the author has succeeded in developing a null method whereby the vibration constants of a galvanometer may be determined and in which the harmonics in the source have no effect. Measurements by this method, together with a knowledge of the direct-current sensitivity, are sufficient to determine the galvanometer constants.

The principle of the method depends on an extension of the theory of the vibration galvanometer. It is shown that, as far as the electrical behaviour of the instrument is concerned, we may regard it as built up of a parallel combination of a capacity, an inductance and a resistance, in series with a resistance.

At resonance, the capacity and inductance neutralise each other and the galvanometer then behaves as a pure resistance. The value of this resistance is the effective resistance as measured by Campbell.

A direct measurement of the effective resistance at resonance by a null method, is, however, practically impossible, since fluctuations in the frequency of the source have their maximum effect on the impedance of the galvanometer in this state.

It will be seen, however, that by a suitable arrangement of inductances and capacities a complete balance can be obtained

* "Proc." Phys. Soc., Feb., 1914.

for any form of current, and that the method will be applicable, with certain limitations, to all the galvanometers tested by Campbell.

The units employed will be those of the C.G.S. electromagnetic system, unless where otherwise stated. For the physical interpretation of the symbols reference should be made to Campbell's Paper. The present notation as compared with his is as follows :—

α	β	γ	A	r_v	r	p	p_0	k
mk^2	b	c	$10g$	R′—R	R	ω	ω_1	$\frac{2}{h}\times 10^{-4}$

The symbols in the second line are those used by Campbell.

2. Using the notation employed in a previous Paper,* the equation of motion of the coil is

$$\alpha\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}+\gamma y=\mathrm{A}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where y and i are the instantaneous values of the deflection and current respectively. If the galvanometer has ohmic resistance r and inductance l, the E.M.F. across the galvanometer terminals is

$$e=ri+l\frac{di}{dt}+\mathrm{A}\frac{dy}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the last term being the back E.M.F. due to the motion of the coil.

When the current is alternating with a frequency $p/2\pi$, equations (1) and (2) transform into the vector equations

$$\left.\begin{array}{l}(\gamma-\alpha p^2+jp\beta)\mathrm{Y}=\mathrm{AI}\\ \mathrm{E}=(r+jpl)\mathrm{I}+jp\mathrm{AY}\end{array}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Y, I, E are the vectors representing the vibration current and E.M.F. respectively. Eliminating Y

$$\mathrm{E}=\left(r+jpl+\frac{jp\mathrm{A}^2}{\gamma-\alpha p^2+jp\beta}\right)\mathrm{I}. \quad . \quad . \quad . \quad . \quad (4)$$

The quantity in ( ) is the vector impedance of the vibration

* Butterworth, "Proc." Phys. Soc., Vol. 24, p. 75, 1912.

galvanometer, the last term being that contributed by the vibration. If we put

$$\beta/A^2=S_v=\frac{1}{r_v},\quad \alpha/A^2=C_v,\quad A^2/\gamma=L_v, \quad . \quad . \quad . \quad (5)$$

then the vector impedance becomes

$$G=r+jpl+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}. \quad . \quad . \quad . \quad (6)$$

The inductance (l) of the galvanometer can be neglected in practice, so that with this assumption we see that the galvanometer is equivalent to a parallel combination of a conductance (S_v), a capacity (C_v) and an inductance (L_v) in series with the ohmic resistance (r).

If $$p^2=\frac{1}{L_vC_v}\equiv\frac{\gamma}{\alpha}=p_0^2 \text{ (say)}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

then $$G=r+\frac{1}{S_v}=r+r_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6A)$$

This holds when the galvanometer is in resonance with the source, so that (6A) gives the effective resistance at resonance.

If $p\neq p_0$, then for good galvanometers the damping (β) may be neglected, so that, putting $S_v=0$, we have

$$p>p_0 \qquad G=r-j/pC_v\left(1-\frac{1}{n^2}\right), \quad . \quad . \quad . \quad . \quad (6B)$$

$$p<p_0 \qquad G=r+jpL_v/(1-n^2), \quad . \quad . \quad . \quad . \quad . \quad (6C)$$

where $n=p/p_0$.

Hence, for frequencies of the source above resonance the galvanometer may be treated as a condenser with a series resistance, and for frequencies of the source below resonance as an inductive resistance.

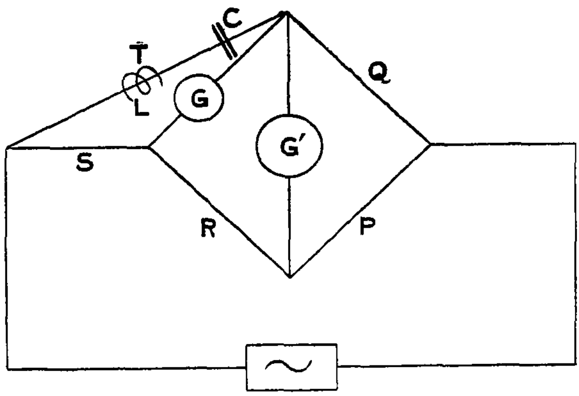
The quantities r_v, C_v, L_v will be referred to as the *vibration resistance, vibration capacity* and *vibration inductance* respectively, and they will be called generally the *vibration constants* to distinguish them from the intrinsic constants α, β, γ, A.

3. *Determination of the Vibration Constants.*

In order to obtain approximate values of L_v and C_v all that is necessary is to measure the apparent inductance or capacity of the galvanometer at two known source frequencies. Then, by means of equations (7), (6B), (6C), the values of L_v, C_v and p_0 can be found. The quantities thus measured depend on the frequency, so that the source should be free from harmonics and the fluctuations of frequency should be small. If the source frequencies chosen are well removed from the resonating frequency, however, the effect of fluctuations may be neglected.

In order to obtain the vibration resistance, it will be necessary, however, to work with the galvanometer almost in resonance with the source. The effect of the fluctuations in frequency is now very large, so that although equation (6A) holds at resonance no balance will be possible on a direct resistance bridge unless the source is almost perfectly steady. By using the bridge below, however, a balance can be got even with a very impure form of alternating current.

In the figure G is the galvanometer under test, G′ is the detector, P, Q, R, S are non-inductive resistances, and the arm



T is made up of a non-inductive resistance, ρ, an inductive resistance (U, L), and a condenser (C) all in series.

If T, G are the vector impedances of the corresponding arms, the condition of balance is

$$T(GP-QR)=Q\{G(R+S)+RS\}. \quad . \quad . \quad . \qquad (8)$$

In this equation T is given by

$$T=\rho+U+j(pL-1/pC)$$

and

$$G=r+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}.$$

Now, let the two following balances be made with *direct* currents :—

(*a*) With the arm T open, adjust P or Q for balance. Then, since G=r,

$$rP=RQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

(*b*) With the condenser (C) short-circuited, $\rho=0$, and the arm G open, adjust S for balance. Then

$$UP=(R+S)Q. \qquad (10)$$

When these conditions are satisfied equation (8) reduces to

$$(T-U)(G-r)=r(U+S), \qquad (11)$$

so that, putting in the values of T and G, the further conditions of balance *for all frequencies* are

$$\frac{\rho}{S_v}=\frac{L}{C_v}=\frac{L_v}{C}=r(U+S). \qquad (12)$$

In attaining these conditions in practice the values of L_v and C_v are already known approximately, so that the values of L and C are thus roughly fixed. Also L_v depends on γ (*see* equation 5), which in turn is altered by the tuning of the galvanometer. Hence, by using a variable self-induction for L, all the adjustments may be made without disturbing the direct-current balances (9) and (10).

Hence, complete balance for alternating currents can be got by successive adjustments of L_v (by tuning), of L, and of ρ.

4. *Limitations of the Method.*

The most important limitation of the method lies in the fact that the time constant (L/U) of the inductive coil in the arm T cannot readily be made greater than about 0·01. Hence, by (12), putting S=0, the maximum vibration capacity measurable is given by $0{\cdot}01/r$. An examination of the table of vibration constants for Campbell's galvanometers shows that, with the exception of galvanometers H, J_1, J_2, all the vibration capacities are greater than this, so that before the method will apply to these galvanometers the vibration capacities must be reduced.

Since the vibration capacities are inversely proportional to A^2 (*see* equation 5), and A is proportional to the flux density (B) of the air-gap of the galvanometer magnet, the required reduction may be brought about by increasing this flux density. The flux density necessary is given in the last column of the table. It will be seen that these values may be readily obtained by an electromagnet.

5. *Experimental Illustration of Method.*

As a suitable vibration galvanometer was not available, the following arrangement was employed in order to test the method :—

A loop of No. 32 phosphor bronze wire (diameter=0·27 mm.) was stretched between two ivory stops at a distance of 10 cm. apart. Its tension could be adjusted by means of a spring attached to an ivory pulley over which the wire passed. The arrangement was placed between the poles of an electromagnet, the width of the air-gap being about 3 mm.

As source, a small alternator, whose normal frequency was 100 ∾ per second, was employed. An oscillogram of the wave-form of this machine showed that it possessed pronounced third and fifth harmonics.

The detector was a Duddell vibration galvanometer. This instrument could not be brought to resonance with the fundamental frequency of the machine, so that in all measurements the galvanometer was tuned to the third harmonic.

The frequency of the fundamental of the source was measured by a reed frequency meter.

Tests were carried out as follows :—

(*a*) With the fundamental frequency of the machine at 100 ∾ per second, a current of about $\frac{1}{3}$ ampere was passed along the wire, and its tension adjusted until it was in resonance with the third harmonic. This adjustment could be made with some precision, as the wire emitted a distinct note at resonance.

(*b*) The source frequency was lowered first to 90 ∾ per second and then to 80 ∾ per second, and the apparent inductance of the wire was measured in each case by Anderson's method.

In this test it was found necessary to bring the third harmonic to resonance and to eliminate the fifth harmonic by means of a Campbell wave-sifter.*

The results obtained were :—

At a frequency of 270 ∾ per second, inductance=0·260 mh.
,, ,, 240 ∾ ,, ,, =0·140 mh.

Hence, by (6c) and (7),

Resonating frequency$\doteqdot$300, L_v=0·05 mh., C_v=6 millifarads.

* A. Campbell, "Proc." Phys. Soc., Vol. 24, p. 107, 1912.

(*c*) Using the approximate values of L_v and C_v as a guide, the bridge of section 3 was built up as follows :—

The arm T consisted of an Ayrton and Perry variable standard of self-induction of range 4 to 33 millihenries, in series with a condenser of capacity 12·6 mfd. and an adjustable resistance ranging from 0·1 to 10 ohms.

The resistances P and R were fixed at 2·00 and 0·100 ohms respectively. The values of Q and S were obtained by the direct-current adjustments of equations (9) and (10), the fractions of an ohm being obtained by using parallel combinations of resistance boxes. These gave $Q=8{\cdot}75$ ohms, $S=2{\cdot}083$ ohms. Hence, from (9) and (10),

$$r=0{\cdot}438 \text{ ohm}, \quad U=9{\cdot}55 \text{ ohms}.$$

The alternating-current balance was obtained by successive adjustments of L, the tension of the wire, and ρ ; these gave

$$L=30{\cdot}05 \text{ millihenries}, \quad \rho=0{\cdot}90 \text{ ohm}.$$

Hence, from equation (12),

$$r_v=5{\cdot}7 \text{ ohms}, \quad L_v=0{\cdot}0641 \text{ mh.}, \quad C_v=5{\cdot}90 \text{ millifarads}.$$

The resonating frequency of the wire follows from equation (7) as 259 ∿ per second. In order to test whether fluctuations of frequency disturbed the balance, the frequency of the source was varied from 70 ∿ per second to 90 ∿ per second. It was found that, unless the frequency was varied abruptly (thus causing unsteady motion of the wire), the balance remained perfect.

Further, it was found to be quite unnecessary to eliminate either the fundamental or the fifth harmonic of the source.

6. *Values of the Vibration Constants.*

In order to show the magnitude of the vibration constants for actual galvanometers the following table has been prepared from Campbell's data. The resonating frequency is taken in every case as 100 ∿ per second. For any other frequency only the vibration inductance will be affected. Its value at any frequency (f) may readily be obtained from the relation $L_v f^2=$constant. Also a variation of the flux density (B) of the air-gap will cause a variation of all the vibration constants, the variation being such that r_v/B^2, L_v/B^2 and $C_v B^2$ are constant.

Galvanometer A of Campbell's table is omitted. The vibration resistance of this galvanometer is less than its ohmic resistance, and this condition serves no useful purpose, since, for the most sensitive conditions of the working of a vibration galvanometer, the vibration resistance of the instrument must be at least equal to the ohmic resistance of the working circuit.*

Table of Vibration Constants.

Galvanometer.	r ohms.	r_v ohms.	C_v millifarads.	L_v millihenries.	B	B′.
B_1	6·0	14·1	21·2	0·118	2,500	8,900
B_2	4·6	11·5	16·9	0·148	2,500	7,100
C	10·5	37·5	5·02	0·500	2,500	5,800
D_1	5·7	8·4	43·0	0·058	1,650	8,300
D_2	5·7	25·3	15·6	0·160	2,500	7,500
E_1	6·0	68	3·75	0·667	2,500	3,800
E_2	6·0	94	2·70	0·926	2,500	3,200
F	7·0	168	2·08	1·20	2,500	3,100
G	8·8	342	1·21	2·07	2,500	2,600
H	14·2	2,290	0·251	10·0	2,500	1,500
J_1	14·5	765	0·921	2·72	(1,650)	2,000
J_2	14·5	1,525	0·349	7·16	(2,700)	2,000

r = ohmic resistance of galvanometer.
r_v = vibration resistance.
C_v = vibration capacity.
L_v = vibration inductance.
B = actual flux density.
B′ = flux density required for measurement.

7. *Determination of the Intrinsic Constants.*

By equation (5) a knowledge of r_v, L_v, C_v enables us to determine A^2/β, A^2/γ and α/A^2. In order to obtain α, β, γ, A separately we require another relation. This is supplied by the steady current constant k (defined as the current to produce unit deflection of the coil), which is equivalent to γ/A.

We then have

$$A=L_v k,\quad \alpha=C_v A^2,\quad \beta=S_v A^2,\quad \gamma=kA.$$

Further, the steady current constant will generally give the most rapid method of comparing the flux densities in different air-gaps by means of the relation kB=constant, so that if it is necessary to test a vibration galvanometer suspension in an air-gap other than the one in which it is to be used, this relation, together with the relations in section 6, will be sufficient to determine the vibration constants for any air-gap.

* Butterworth, *loc. cit.*

8. *Conclusion.*

The method given in section 3 has a number of possible uses. The theory, with slight modifications, applies to the motion of any electrically-maintained vibrations—*e.g.*, the motion of the diaphragm of a telephone receiver. Hence the method may be employed to determine the energy losses in such systems. Again, if the galvanometer under test is replaced by a leaky condenser with a series resistance and the condenser in the arm T is removed, the balance obtained in this case will give the values of the leakage, capacity and series resistance.

Finally, the fact that the vibration constants depend on the square of the flux density suggests a method of comparing intense magnetic fields, such as exist in the air-gaps of dynamos, by a null method.

I wish to thank Prof. Gee, Mr. J. Hollingworth, and Mr. A. Campbell for valuable criticism and advice, and Mr. White for the construction of the vibrating system.

ABSTRACT.

The methods usually employed in the determination of the constants of a vibration galvanometer involve the measurement of a deflection under three different conditions. Two of these deflections can only be obtained very approximately.

By extending the theory of the vibration galvanometer it is shown how the constants may be determined by methods which involve only the measurement of one deflection. The remaining measurements are carried out on an alternating-current bridge, and the results obtained are practically independent of the wave-form of the source.

The principle of the method depends on the fact that a vibration galvanometer behaves as a parallel combination of a conductance, a capacity and an inductance, in series with a resistance. It is shown how to balance such a combination, and the method is illustrated experimentally. The constants of various galvanometers are quoted in order to show the applicability of the method. Other uses of the bridge are suggested.

DISCUSSION.

Mr. A. Campbell remarked that it was most interesting to find that the electrical behaviour of a circuit capable of dynamical resonance could be imitated exactly by putting in parallel a resistance, a condenser and an inductance. It was a pity that this combination could not be realised in practice since the inductance must have zero resistance. However, Mr. Butterworth got over the difficulty by his special form of bridge. The limitations of this bridge somewhat lessened the range of application to practical cases, and he hoped that the author would be able to modify the bridge so as to remove these limitations.

Mr. D Owen stated that he had found no difficulty in maintaining the frequency of the source sufficiently steady to maintain the voltage sensitivity of a vibration galvanometer constant within one or two per cent. for a considerable time. The author's analysis of the vibrating coil was very ingenious, but one would like to know whether sensitivities calculated by his rather complex bridge agreed with those obtained by the usual direct method.

Dr. R. S. WILLOWS asked whether the method could be used to find the dielectric constant of a slightly conducting liquid; if so, was it sensitive enough to be practically useful and did the largeness of the required inductance again limit its applicability.

Mr. BUTTERWORTH, in reply, stated that the method would apply to any vibrating system provided that the conditions mentioned in the Paper could be satisfied. It was true, as Mr. Owen had pointed out, that the *voltage* sensitivity of certain galvanometers could be determined very precisely. This held in the case of instruments capable of developing a high back E.M.F. The reduction of current at resonance would then prevent any considerable rise in the vibration in spite of the large increase in *current* sensitivity. The application of the method to the measurement of small capacities required investigation.