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ON A PARAMETER ADAPTIVE SELF-ORGANIZING SYSTEM WITH THE MINIMUM VARIANCE CONTROL LAW IN THE PRESENCE OF LARGE OUTLIERS IN OBSERVATIONS

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Abstract. The aim of the given paper is development of a minimum variance control (MVC) approach for a closed-loop discrete-time linear time-invariant (LTI) system when the parameters of a dynamic system as well as that of a controller are not known and ought to be estimated. The parametric identification of the open-loop LTI system and the determination of the coefficients of the MV controller are performed in each current operation by processing observations in the case of additive noise on the output with contaminating outliers uniformly spread in it. The robust recursive technique, based on the S-algorithm, with a version of Shweppe's GM-estimator is applied here in the calculation of estimates of the parameters of a LTI system with one time-varying coefficient in the numerator of the system transfer function. Then, the recursive parameter estimates are used in each current iteration to determine unknown parameters of the MV controller. Afterwards, the current value of the control signal is found in each operation, and it is used to generate the output of the system, too. The results of numerical simulation by computer are presented and discussed.

Key words: Adaptive systems, closed-loop, self-tuning controller, the minimum variance control law, parametric identification, observations, outliers.

1. Introduction

To provide self-tuning control of a real plant several ordinary control approaches, such as, minimum-variance control MVC, generalized MVC (GMVC), incremental GMVC, and so on, are frequently used that take into consideration random disturbances affecting the process. The MVC and GMVC algorithms, as noted in [17], were the first that were designed specially for self-tuning applications and are now considered 'classical' formulations. The algorithms described there can be implemented as self-tuning controllers that underpin the design and development of a modern model based predictive control approach. On the other hand, it has been emphasized in [10] that in designing a robust control system, one ought to determine the type of uncertainties appearing in the system to be controlled. There are many types of uncertainties in system description models. One of the main ones of them is the uncertainty arising in the output disturbance description of a plant model to be used. It is frequently assumed that system's output is affected by Gaussian disturbance. However, nonnormal noise, and particularly the presence of outliers, degrades the performance of a system acting in a closed-loop. In such a case ordinary recursive techniques used for a parametric identification of any control systems, are inefficient, as a rule. It is known [1, 4, 9, 11–16] that, for parametric identification of open-loop, as well as of closed-loop systems, robust recursive techniques ought to be applied that are efficient in the case of nonnormal noise. To implement the self-tuning MV controller, it is necessary, firstly, to estimate LTI system's model parameters in such a noisy environment and, secondly, to determine the controller coefficients in each current operation (see Fig. 1,[2]). that are recalculated using the values of abovementioned estimates. Then, the current value of the control signal, based on the values of the reference signal and that of input-noisy output of a system, multiplied by the respective weighting coefficients, is obtained according to Fig. 1. However, in such a case, the transfer of meanings of large outliers proceeds in random noise appearing in output observations. Therefore, in each current operation before calculating the value of the control signal it is important to find the values of the output that have not been harmed by outliers. To this end, we propose here to generate an auxiliary output signal that will be without outliers.

In Section 2, a statement of the problem is presented. In Section 3, the method for a design of a self-organizing system is given. In Section 4, an ordinary direct approach is described for a parametric identification of the system transfer function. We analyze a recursive parametric identification, based on GM- estimators in the presence of outliers in output observations, in Section 5. Section 6 presents the simulation and parametric identification results. Section 7 contains conclusions.

2. Statement of the Problem

Assume that a system to be observed is a causal and LTI system with one output $\{y(k)\}$ and one input $\{u(k)\}$, expressed by the equation

$$y(k) = q^{-\tau} G_0(q^{-1};\ \theta\,) u(k) + \underbrace{H_0(q^{-1}; \varphi)\xi(k)}_{v(k)}, \tag{1}$$

that consists of two parts (Fig. 2): a system model $G_0(q^{-1}; \theta)$ and a noise model $H_0(q^{-1}; \varphi)$. Here k is the current number of observations of a respective signal, τ is an known time delay, θ, φ are unknown parameter vectors to be estimated, q^{-1} is the backward time-shift operator such that $q^{-1}u(k) = u(k-1)$, and $H_0(q^{-1}, \varphi)$ is an inversely stable monic filter [3]. Given the model (1) and measured data

$$\mathbf{Z}^N = \{u(1), u(2), \ldots, u(N), y(1), y(2), \ldots, y(N)\} \tag{2}$$

and assuming that a noise $\{\xi(k)\}, k = 1, 2, ...$ is really a sequence of independent identically distributed variables with an ϵ-contaminated distribution of the form

$$p(\xi(k)) = (1-\epsilon)N(0, \sigma_\mu^2) + \epsilon N(0, \sigma_\varsigma^2), \tag{3}$$

and the variance

$$\sigma_\xi^2 = (1-\epsilon)\sigma_\mu^2 + \epsilon\sigma_\varsigma^2, \tag{4}$$

let us suppose that $\{\xi(k)\}$ is used to generate unmeasurable noise $\{v(k)\}$. Here $p\{\xi(k)\}$ is the probability density distribution of the sequence $\{\xi(k)\}, k = 1, 2, \ldots$;

$$\xi(k) = (1-\gamma_k)\mu_k + \gamma_k\varsigma_k \tag{5}$$

is the value of the sequence $\{\xi(k)\}, k = 1, 2, \ldots$ at a time moment k; γ is a random variable, taking values 0 or 1 with probabilities $p(\gamma_k = 0) = 1-\epsilon, p(\gamma_k = 1) = \epsilon$; μ_k, ς_k are sequences of independent Gaussian variables with zero means and variances $\sigma_\mu^2, \sigma_\varsigma^2$, respectively; besides, $\sigma_\mu < \sigma_\varsigma$; $0\epsilon 1$ is the unknown fraction of contamination;

The aim of the given paper is to design a parameter adaptive self-organizing system with the MV control law, shown in Fig. 1, in the case of additive noise $\{v(k)\}$, that contains large outliers and corrupts the output $\{y(k)\}$ of the LTI system (see Fig. 2).

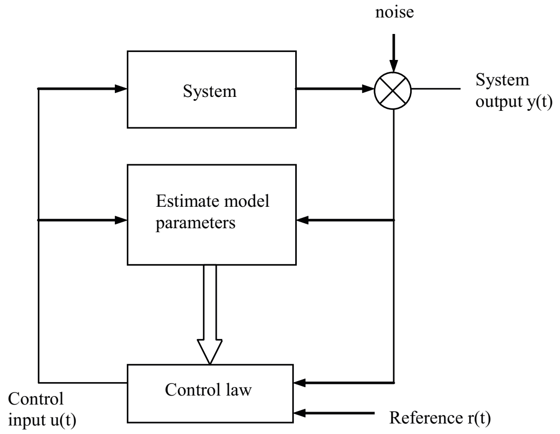

Figure 1. Self-organizing system [2]

3. Design of a self-organizing system

The MVC controller seeks to design the required control signal

$$u(k) = \frac{1}{b_0}\{\sum_{l=1}^{n_a} a_l y(k+\tau-l) - \sum_{i=1}^{n_b} b_i u(k-i) + r(k)\} \tag{6}$$

by minimizing with respect to $\{u(k)\}$ the quadratic performance function

$$J_{MV} = \lim_{N\to\infty} E\{\frac{1}{N}\sum_{k=0}^{N-1}[r(k) - y(k+\tau)]^2\}, \tag{7}$$

that refers to the variance of the error between setpoint $r(k)$ and the controlled output τ-time steps in the future, $y(k+\tau)$ [17]. Here n_a, n_b are general numbers of respective parameter sets of difference equation (6).

To implement the self-tuning MVC controller, it is necessary, firstly, to estimate LTI system's model unknown parameters in such a noisy environment using robust techniques [1, 4, 5, 8, 9, 11 – 14, 16]

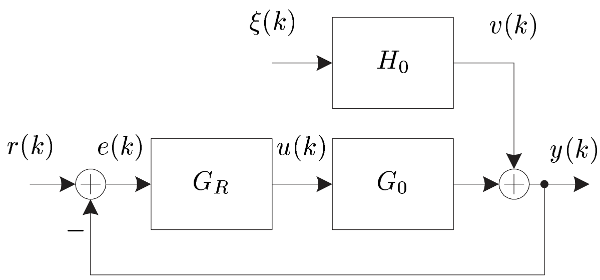

Figure 2. The closed-loop system to be observed. Here $G_R \equiv G_R(q^{-1}; \alpha)$, $G_0 \equiv G_0(q^{-1}; \theta)$, and $H_0 \equiv H_0(q^{-1}; \varphi)$

and, secondly, to determine the value of control signal $u(k)$ in each current operation by substituting in (6) the values of abovementioned estimates $\hat{\mathbf{b}}^T = (\hat{b}_0, \hat{b}_1, \ldots, \hat{b}_{n_b}), \hat{\mathbf{a}}^T = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{n_a})$. However, in such a case, the transfer of meanings of large outliers proceeds in random noise appearing in output observations. Therefore, in each current operation before calculating the value of the control signal $\{u(k)\}$ it is important to find the values of the output that have not been harmed by outliers. In such a case, we propose here to generate an auxiliary output signal $\hat{y}(k+\tau)$ that will be without outliers.

A self-organizing MVC strategy is achieved when estimation and control are carried out every current instant k simultaneously.

4. The direct approach

The direct approach ignores the feedback and identifies the system $G_0(q^{-1};\theta)$ using the measurements of the input $u(k)$ and output $y(k)$ $\forall k = 1, 2, ...$ [3] assuming that the noise $\{\xi(k)\}, k = 1, 2, ...$ is statistically independent and stationary with the following characteristics:

$$E\{\xi(k)\} = 0,\ E\{\xi(k)\xi(k+\tau)\} = \sigma_\xi^2\delta(\tau), \quad (8)$$

where $E\{\xi(k)\}$ is the mean value, σ_ξ^2 is the variance, $\delta(\tau)$ is the Kronecker delta function. Using the direct parameric identification method one has to estimate the prediction error value $\hat{\theta}_N$ of the vector of parameters θ by

$$\hat{\theta}_N = arg \min_{\theta \in D_M} V_N(\theta, \mathbf{Z}^N). \quad (9)$$

Here D_M is the set of allowable parameter values, which is assumed compact and connected [3],

$$V_N(\theta, \mathbf{Z}^N) = \frac{1}{N}\sum_{k=1}^{N} e_F^T(k,\theta)\mathbf{\Lambda}^{-1}e_F^T(k,\theta), \quad (10)$$

with

$$e_F(k,\theta) = L(q^{-1},\theta)\epsilon(k,\theta), \quad (11)$$

$\mathbf{\Lambda}$ is a symmetric, positive definite weighting matrix, and $L(q^{-1};\theta)$ is a monic prefilter that can be used to enhance certain frequency regions [3]. The prediction error is calculated by

$$\epsilon(k,\theta) = y(k) - \hat{y}(k,\hat{\theta}) = \quad (12)$$
$$H^{-1}(q^{-1};\,\hat{\varphi})[y(k) - G(q^{-1};\,\hat{\theta})u(k)].$$

Here the output $y(k)$ of the general model of the LTI system $G(q^{-1};\,\theta)$ and noise filter $H(q^{-1};\,\varphi)$, respectively, are of the form

$$y(k) = G(q^{-1};\,\theta)u(k) + H(q^{-1};\varphi)\xi(k) \quad (13)$$

where $G(q^{-1};\,\theta)$ corresponds to the first part of equation (1) and $H(q^{-1};\varphi)$ to the second one. Then, the one-step-ahead predictor for the model structure (13) is

$$\hat{y}(k,\hat{\theta}) = H^{-1}(q^{-1};\,\hat{\varphi})G(q^{-1};\,\hat{\theta})u(k) + \quad (14)$$
$$[1 - H^{-1}(q^{-1};\,\hat{\varphi})]y(k).$$

Here $\hat{\varphi}$ is the estimate of the parameter vector φ. The parameter vector θ can be determined by an ordinary prediction error method, based on the recursive LS (RLS) of the form

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\mathbf{\Gamma}(k-1)\mathbf{z}(k)}{1 + \mathbf{z}^T(k)\mathbf{\Gamma}(k-1)\mathbf{z}(k)}\hat{\varepsilon}(k), \quad (15)$$
$$\mathbf{\Gamma}(k) = \mathbf{\Gamma}(k-1) - \frac{\mathbf{\Gamma}(k-1)\mathbf{z}(k)\mathbf{z}^T(k)\mathbf{\Gamma}(k-1)}{1 + \mathbf{z}^T(k)\mathbf{\Gamma}(k-1)\mathbf{z}(k)}$$

with the vector of observations $\mathbf{z}^T(k) = [-y(k-1), \ldots, -y(k-m), u(k-1), \ldots, u(k-m)]$, and some initial values of the vector $\hat{\theta}(0)$ and matrix $\mathbf{\Gamma}(0)$. Here

$$\hat{\theta}^T(k) = [\hat{\mathbf{a}}^T(k), \hat{\mathbf{b}}^T(k)] = \quad (16)$$
$$[\hat{a}_1(k), \ldots, \hat{a}_m(k), \hat{b}_0(k), \hat{b}_1(k), \ldots, \hat{b}_m(k)],$$

is the current estimate of the vector $\theta^T = (\mathbf{a}^T, \mathbf{b}^T) = (a_1, \ldots, a_m, b_0, b_1, \ldots, b_m)$, and

$$\hat{\varepsilon}(k) = y(k) - \mathbf{z}^T(k)\hat{\theta}(k-1) \quad (17)$$

is the prediction error on the current k-th iteration, respectively, where $G_0(q^{-1};\theta)$ is the system transfer function of the form

$$G_0(q^{-1};\theta) = \frac{B(q^{-1};\mathbf{b})}{A(q^{-1};\mathbf{a})} = \quad (18)$$
$$\frac{b_0 + b_1q^{-1} + b_2q^{-2} + \ldots + b_mq^{-m}}{1 + a_1q^{-1} + \ldots + a_mq^{-m}}.$$

Here $\mathbf{b}^T = (b_0, b_1, \ldots, b_m)$, and $\mathbf{a}^T = (a_1, \ldots, a_m)$ are vectors of the parameters to be estimated, $m = n_b = n_a$.

It is known that RLS is efficient only in the case where

$$H_0(q^{-1};\varphi)=\frac{1}{1+A(q^{-1};\mathbf{a})}= \qquad (19)$$
$$\frac{1}{1+a_1q^{-1}+\ldots+a_mq^{-m}},\varphi\equiv\mathbf{a}.$$

It could be emphasized that, before the closed-loop direct parametric identification, the respective identifiability conditions should be satisfied according to [7].

5. Parametric identification in the presence of large outliers

In what follows, we introduce the robust recursive generalized maximum likelihood (GM) for calculating robust estimates of the parameters of LTI dynamic systems, acting in a closed-loop (Fig. 2) in the case of correlated noise of special form (19) with outliers in it. A class of GM-estimators is defined implicitly by the first order condition [6]

$$\sum_{t=1}^{N}\mathbf{x}(t)\zeta\{\mathbf{x}(t),[y(t)-\mathbf{x}^T(t)\theta]/\sigma\}=0. \qquad (20)$$

Here $\mathbf{x}(t)$ is a set of regressors, σ denotes the scale of residuals $\mathbf{n}(t)$ of the linear regression model $y(t)=\mathbf{x}^T(t)\theta+\delta(t), t=1,...,N$, where θ is a vector of unknown parameters. The function $\zeta\{\cdot,\cdot\}$ in (20) depends on both the set of regressors $\mathbf{x}(t)$ and the standardized residual $\delta(t)/\sigma$. The conditions that ought to be satisfied by $\zeta\{\cdot,\cdot\}$ in order that the GM-estimator have nice asymptotic properties are known in advance [8]. The ordinary least-squares estimator could be obtained as a special case of (20) by setting in it the function $\tau(\mathbf{x}(t),r)=r^2/2$ with $\partial\tau(\mathbf{x}(t),r)/\partial r=\zeta\{\mathbf{x}(t),r\}$, where r is a short form of the standardized residual. Thus, one can determine the prediction error estimate $\hat{\theta}_N$ of the parameter vector $\theta^T=(\mathbf{a}^T,\mathbf{b}^T)=(a_1,\ldots,a_m,b_0,b_1,\ldots,b_m)$ by minimizing

$$\hat{\theta}_N=arg\min_{\theta\in D_M}\{V_N(\theta,\mathbf{Z}^N). \qquad (21)$$

with

$$\{V_N(\theta,\mathbf{Z}^N)=\frac{1}{N}\sum_{k=1}^{N}\rho(e_F(k,\theta/s)), \qquad (22)$$

or by solving the equation

$$\sum_{t=1}^{N}\mathbf{z}(t)\{\psi[y(t)-\mathbf{z}^T(t)\theta]\}=0, \qquad (23)$$

in the vector form. Here $\hat{\theta}_N$ is the robust estimate of the parameter vector θ, established by processing N pairs of input-output samples; s is the scale of residuals (examples of the scale are the standard deviation, the median, absolute deviation from the median, etc.,); $\rho(\cdot)$ is a real-valued function that is even and nondecreasing for positive residuals, and $\rho(0)=0,\psi=\rho^{'}$.
For the Huber M-estimator, the ρ-function is given by [6]

$$\rho(x)=\begin{cases}c_H\mid x\mid -c_H^2/2 & \text{if}\mid x\mid>c_H,\\ x^2/2 & \text{otherwise},\end{cases} \qquad (24)$$

where c_H is a cutoff value. The most often used function ψ is:

$$\psi(x)=\begin{cases}c_H sign(x) & \text{if}\mid x\mid>c_H,\\ x & \text{otherwise}\end{cases} \qquad (25)$$

with given $c_H>0$. To get a better performance of $\hat{\theta}_N$ in the case of very long-tailed distributions, the function (25), satisfying $\psi(x)=0$, if $\mid x\mid>c_H$, for some $c_H>0$ could be selected. It is known [9] that, in both such cases, i.e., $\epsilon\neq 0$ and $H_0(q^{-1};\varphi)$ of the form (19), the current M- estimates of an unknown vector of the parameters θ of LTI system (1) with $G(q^{-1},\theta)$ of form (18) can be calculated using three techniques: the S-algorithm, the H-algorithm, and the W-one. All the three of them could be written in the general form:

$$\hat{\theta}(k)=\hat{\theta}(k-1)+\frac{\mathbf{\Gamma}(k-1)\mathbf{z}(k)}{\lambda(k)+\mathbf{z}^T(k)\mathbf{\Gamma}(k-1)\mathbf{z}(k)}\beta(k) \qquad (26)$$
$$\mathbf{\Gamma}(k)=\mathbf{\Gamma}(k-1)-\frac{\mathbf{\Gamma}(k-1)\mathbf{z}(k)\mathbf{z}^T(k)\mathbf{\Gamma}(k-1)}{\lambda(k)+\mathbf{z}^T(k)\mathbf{\Gamma}(k-1)\mathbf{z}(k)}$$

Here

$$\beta(k)=\hat{s}\psi[\alpha(k)] \qquad (27)$$

with

$$\alpha(k)=\hat{\varepsilon}(k)/\hat{s} \qquad (28)$$

for the S- and H-algorithms, and

$$\beta(k)=\hat{s}\hat{\varepsilon}(k) \qquad (29)$$

for the W-algorithm;

$$\hat{\varepsilon}(k)/\hat{s}=\{y(k)-\mathbf{z}^T(k)\hat{\theta}(k-1)\}/\hat{s} \qquad (30)$$

is the same for all the three algorithms, while

$$\lambda(k)=1 \qquad (31)$$

for the H-algorithm,

$$\lambda(k) = \begin{cases} \{\hat{s}\psi[\alpha(k)]/\hat{\varepsilon}(k)\}^{-1} & \text{for } \hat{\varepsilon}(k) \neq 0, \\ 1 & \text{for } \hat{\varepsilon}(k) = 0, \end{cases} \tag{32}$$

for the W-algorithm, and

$$\lambda(k) = \psi^{'}[\alpha(k)]^{-1} \tag{33}$$

for the S-algorithm. Here $\hat{s}$ is the robust estimate of the scale s of residuals.

In [4] it has been proposed to use

$$\beta(k) = \hat{s}\phi_{z1}\psi[\alpha(k)/\phi_{z2}], \tag{34}$$

and

$$\lambda(k) = \begin{cases} \phi_{z1}\psi[\alpha(k)/\phi_{z2}]/[\alpha(k)/\phi_{z2}] & \text{for } \alpha(k) \neq 0, \\ \phi_{z1} & \text{for } \alpha(k) = 0, \end{cases} \tag{35}$$

respectively, instead of (27) and (32). Here

$$\phi_{z1} = \phi_{z2} = 1 \tag{36}$$

for Huber's M-estimator;

$$\phi_{z1} = \phi_z[h(k)], \phi_{z2} = 1 \tag{37}$$

for Mallow's, and

$$\phi_{z1} = \phi_{z2} = \phi_z[h(k)], \tag{38}$$

for Shweppe's GM-estimators [8], respectively, where

$$\phi_z[h(k)] = \sqrt{1 - h(k)} \tag{39}$$

with

$$h(k) = \mathbf{z}^T(k)\mathbf{\Gamma}(k)\mathbf{z}(k). \tag{40}$$

The S-algorithm represents a version of the algorithm proposed by [9] for an on-line robust identification of parameters of a linear dynamic model of an LTI system. The ordinary RLS (15) is modified by substituting the "winsorization" step of the residuals in the first equation and changing the second equation in the system of equations (15). The recursive H-algorithm is obtained only by inserting the "winsorization" step into the first equation of (15). The W- algorithm is worked out by inserting different weights in respect to the function $\psi\{\cdot\}$ into the already existing ordinary RLS.

6. Simulation example

A closed-loop system to be simulated is shown in Fig. 2 and described by a linear difference equation of the form

$$(1 + a_1q^{-1} + a_2q^{-2})y(k) = q^{-1}(b_0 + b_1q^{-1})u(k) + (1 + a_1q^{-1} + a_2q^{-2})^{-1}\xi(k), \tag{41}$$

while the GMV controller design equation is [2, 17]

$$u(k) = \frac{P(a_1y(k) + a_2y(k-1) - b_1u(k-1) + Rr(k))}{Pb_0 + Q}. \tag{42}$$

Here $a_1 = -1.5, a_2 = 0.7, b_0 = 1$ and the value of coefficient b_1 varries from 0.5 to 0.6 over 400 observations, P, Q, and R are tuning parameters. Thus,

$$G_0 = \frac{q^{-1} - (0.5 + 0.1k/400)q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}}, \tag{43}$$

$$H_0 = \frac{1}{1 - 1.5q^{-1} + 0.7q^{-2}} \tag{44}$$

in Fig. 2. If P=R=1 and Q=0 the controller becomes a MV controller that will be used in our paper. The value of control signal $u(k)$ in each current operation k has been determined by substituting in (42) the values of estimates $\hat{b}_0(k), \hat{b}_1(k), \hat{a}_1(k), \hat{a}_2(k)$ of the true parameters, respectively. The output $\{y(k)\}, k = 0, 1, 2, \ldots, 400$ of the closed-loop system will be observed under the additive noise $\{v(k)\}$ in the presence of large outliers according to (3)–(5) (see Fig. 3a – 3c). Note that all the three noise realizations given there are the same except that their amplitudes are artificially increased from one realization to the other by ten times. In such a case, the meanings of rare outliers have especially grown in any realization of $\{v(k)\}$. The reference signal $\{r(k)\}, k = 0, 1, 2, \ldots, 400$ is given in Fig. 3d.

The parameter adaptive self-organizing system has been implemented here according to the structure shown in Fig. 1. Firstly, the initial values of estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of the true parameters a_1, a_2, b_0, b_1 of equation (41) were calculated by the ordinary LS with Mallow's estimator using 23 pairs of observations of $u(k), y(k)$. Secondly, we recursively calculate the estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of the same parameters a_1, a_2, b_0, b_1 by processing $k = 24, 25, \ldots, 400$ observations of the control signal $\{u(k)\}$ and the output $\{y(k)\}$ in each current iteration, using two S-algorithms (26) with a version of Shweppe's GM-estimator (38)–(40) (see Fig.'s 4 – 6). The output signals $\{y(k)\}$ of the same system (41) to be processed by both algorithms were different and generated in two ways:

$$y(k) = y_*(k) + (1 + a_1q^{-1} + a_2q^{-2})^{-1}\xi(k), \quad (45)$$

$$y_*(k) = q^{-1}(b_0+b_1q^{-1})u(k)-(a_1q^{-1}+a_2q^{-2})y_*(k), \quad (46)$$

with

$$u(k) = \frac{\hat{a}_1y(k) + \hat{a}_2y(k-1) - \hat{b}_1u(k-1) + r(k)}{\hat{b}_0}, \quad (47)$$

and with

$$u(k) = \frac{\hat{a}_1\hat{y}(k) + \hat{a}_2\hat{y}(k-1) - \hat{b}_1u(k-1) + r(k)}{\hat{b}_0}, \quad (48)$$

where

$$\hat{y}(k) = q^{-1}(\hat{b}_0+\hat{b}_1q^{-1})u(k)-(\hat{a}_1q^{-1}+\hat{a}_2q^{-2})\hat{y}(k), \quad (49)$$

respectively, because in each recursive iteration $k = 24, 25, \ldots, 400$ the current value of the control signal $\{u(k)\}$ is generated according to (47) (here the observed noisy values of $\{y(k)\}$ are substituted), and according to (48) (here the values of the noiseless auxiliary signal $\{\hat{y}(k)\}$ are applied). In both cases the current estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ are used. Afterwards, two different current values of the output signal $\{y(k)\}$ are calculated by formulas (45), where different current values of $\{u(k)\}$ are used. Then, different values of $u(k), y(k)$ are processed separately, in calculating the estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of true values of the parameters a_1, a_2, b_0, b_1, respectively, using two recursive procedures (26) with the same versions of Shweppe's GM-estimator (38)–(40) (see Fig.'s 4 – 6).

It follows that the accuracy of estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of the parameters a_1, a_2, b_0, b_1, obtained by two separate acting recursive procedures (26) with the version of Shweppe's GM-estimator (38)–(40) (see Fig.'s 4c, d – 6 c, d), decreases when the amplitudes of values of the additive noise $\{v(k)\}$ with outliers in it are increasing (see Fig. 3a – 3c). In such a case, the true output signal $\{y_*(k)\}$ (46) does not track the reference one (Fig. 3d), if the control signal $\{u(k)\}$ is calculated according to (47) (see Fig.'s 4e – 6 e). Therefore it is important for calculating current values of the control signal $\{u(k)\}$ to use formulas (48)–(49) because, in such a case, the output signal $\{y_*(k)\}$ of form (46) tracks the reference one (Fig.'s 4f – 6f).

7. Conclusions

Despite that the MV approach has been worked out for a random disturbance generated from the statistically independent and stationary sequence with (8), it appears to be also applicable in the presence of large, but rare outliers in output observations (see Fig.'s 3a – 3c) in case the robust recursive parametric identification algorithms are used. If the amplitude values of outliers are increasing, then the recursive estimates, obtained by the S-algorithm (26) with the version of Shweppe's GM-estimator (38)–(40), and the auxiliary signal of form (49) used to calculate the current values of the control signal $\{u(k)\}$, approach the respective true values of parameters more rapidly (Fig.'s 4d – 6d) than that calculated by the same procedure without determining such a signal (Fig.'s 4c – 6c). In such a case, the true output $\{y_*(k)\}$ of the LTI system tracks the reference signal (Fig. 3d) much more accurately (Fig.'s 4f – 6f) for relatively large, in the sense of amplitudes, outliers in the additive noise $\{v(k)\}$ (see Fig.'s 3a – 3c) in comparison with (Fig.'s 4e – 6e), respectively. Thus, one can state that the use of auxiliary signal $\{\hat{y}(k)\}$ (49) allowed us to increase the efficiency of an parameter adaptive LTI system with a self-tuning MV controller.

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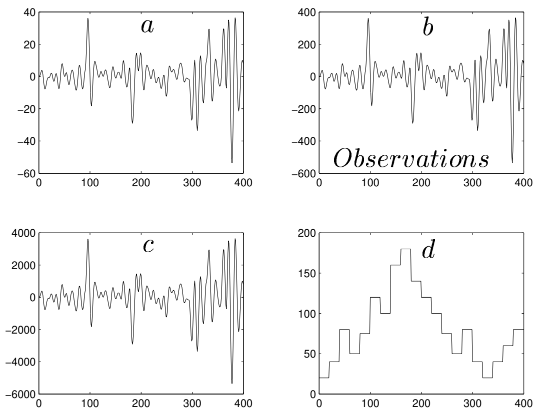

Figure 3. Three realizations of noise $v(k)\}$ with outliers (a, b, c) and the reference signal (d) for a simulated closed-loop system. The fraction of contamination $\epsilon = 0.1$.

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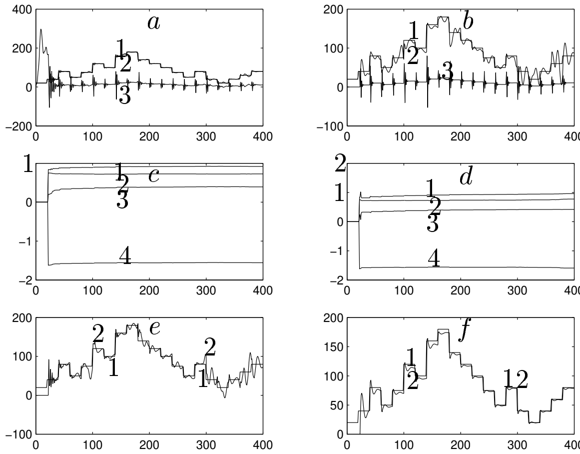

Figure 4. The signals and parametric identification results in the presence of additive noise given in Fig.3a depending on the number of recursive iterations: x-axis – numbers of iterations, y-axis – meanings of the signals (a, b, e, f) and estimates (c, d); a, b, e, f are signals: the reference signal $\{r(k)\}$ - 1, output signals: $\{y(k)\}$ - 2 (a, b), $\{y_*(k)\}$ - 2 (e, f), the control signal $\{u(k)\}$ - 3, respectively, the current values of which have been determined as follows: by substituting the observed noisy values of $\{y(k)\}$ in formula (47) (a, c, e), and by substituting noiseless values of the auxiliary signal $\{\hat{y}(k)\}$ in formula (48) (b, d, f); in c, d: $\hat{b}_0, \hat{b}_1, \hat{a}_1, \hat{a}_2$ - 1, 3, 4, 2.

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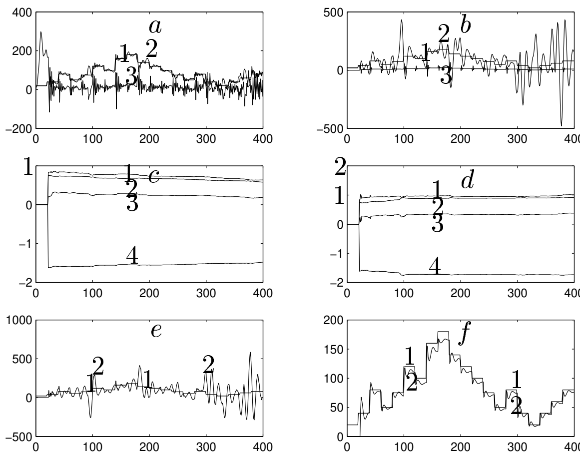

Figure 5. The signals and parametric identification results in the presence of additive noise (see, Fig. 3b). Other values and notation are the same as in Fig.4.

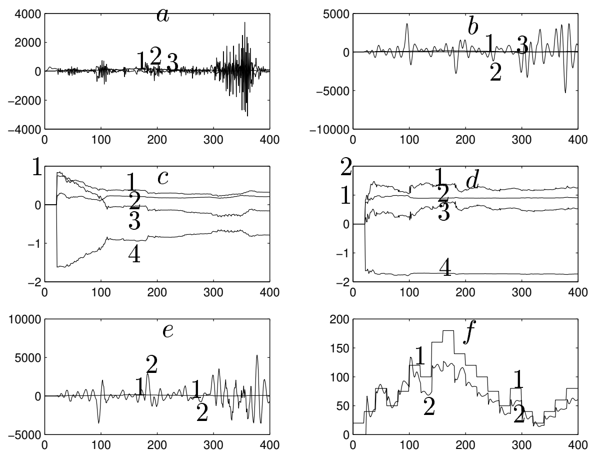

Figure 6. The signals and parametric identification results in the presence of additive noise (see Fig. 3c). Other values and notation are the same as in Fig.4.