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V. G. KADYSHEVSKY
AND
M. D. MATEEV

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V.G. Kadyshevsky **

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** On leave of absence from Joint Institute for Nuclear Research, Dubna, USSR.

*** On leave of absence from University of Sofia, Bulgaria.

ABSTRACT

Quasipotential equations are constructed for the relativistic scattering amplitude and the wave function of two interacting particles with spin $\frac{1}{2}$. This is done with the help of specific covariant extrapolation of the scattering amplitude off-the-energy-momentum shell. Suitable diagram techniques are developed. The quasipotential is defined as a sum of "irreducible" diagrams. The free part of the wave equation is spin independent and all features connected with spin appear in the interaction (quasipotential). The spin structure of the quasipotential is investigated.

ON A RELATIVISTIC QUASIPOTENTIAL EQUATION IN THE CASE OF PARTICLES WITH SPIN

I. INTRODUCTION

In quantum field theory the system of two interacting particles can be described in the framework of the relativistic invariant Bethe-Salpeter formalism [1]. However, as is well known, the Bethe-Salpeter wave function has two time variables and hence cannot be interpreted in the usual quantum mechanical sense as the probability amplitude for finding the system in a definite state. For this reason the question of its normalization is not at all trivial; and, up to now, a complete understanding of this feature has not been achieved.

Several years ago Logunov and Tavkhelidze developed the quasipotential approach to the problem of the interaction of two relativistic particles [2]. The wave function which they introduced is a natural generalization of the non-relativistic one since it is a function of only one time variable and satisfies a Schrödinger type equation. For interacting scalar particles with equal masses in the centre-of-mass system, the quasipotential equation in momentum space can be written in the form:

$$(\vec{p}^2-\vec{q}^2)\Psi_q(\vec{p}) = \frac{1}{4(2\pi)^3}\frac{1}{\sqrt{\vec{p}^2+M^2}}\int V(\vec{p},\vec{k};E_q)\Psi_q(\vec{k})\,d\vec{k} \qquad (1.1)$$

The "quasipotential" $V(\vec{p},\vec{k};E_q)$ (in general a complex quantity) can be built according to [2] with the help of perturbation theory. This can be done in two different ways either by using the two-time Green function of the system or with the help of the scattering amplitude on the mass shell. In the formalism of Logunov and Tavkhelidze there is an equation of the Lippman-Schwinger type for the off-the-energy-shell scattering amplitude:

$$T(\vec{p},\vec{q}) = V(\vec{p},\vec{q};E_q) + \frac{1}{4(2\pi)^3}\int V(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+M^2}}\,\frac{T(\vec{k},\vec{q})}{k^2-q^2-i\varepsilon} \qquad (1.2)$$

It is important to stress that the integration on the right-hand sides of eqs. (1.1) and (1.2) is performed over a <u>three-dimensional</u> manifold, while in the corresponding Bethe-Salpeter equations there is integration over the four-dimensional momentum of the virtual particle.

The quasipotential method can be successfully applied to the solution of the problem of scattering and bound states for any particles independently of their concrete nature [3], [4]. However, in the case of particles with spin, certain specific complications arise in the construction of the quasipotential. They are connected with the fact that the two-time, bare, free Green function for a system of two spinor particles is a singular matrix. In Ref. [4] this difficulty is avoided by projecting the quasipotential equation onto the subspace of the spinors corresponding to positive energies, the quasipotential being built with the help of the on-mass-shell scattering amplitude. The problem of constructing the quasipotential in the spinor case using the two-time Green function has been discussed in detail in Ref. [5].

The purpose of this work is the construction of quasipotential equations of the type (1.1) and (1.2) for the case of particles with spin using a method which is not connected with the usual Green function apparatus and the Bethe-Salpeter formalism. In this method [6], which is a covariant formulation of "old-fashioned" perturbation theory, all the particles in the initial, intermediate and final states are on the mass shell and have positive energies; but, due to the presence of specific spurions (we will call them "quasiparticles"), the total four-momentum of the system is not conserved. It can be said that this approach is an alternative to the usual Feynman perturbation theory. Indeed, in the first case, we always have $p^2 = m^2$, $p_o > 0$; and, in general, the four-momentum is not conserved;

$$\sum p_i - \sum q_k \neq 0 .$$

In the second case the four-momentum is conserved;

$$\sum p_i - \sum q_k = 0 \;;$$

but, in general, $p^2 \neq m^2$ and $p_0 \gtrless 0$.

In [7], with the help of the diagram technique from [6], equations of the same type as (1.1) and (1.2) have been obtained:

$$E_p(E_p - E_q)\Psi_q(\vec{p}) = \frac{1}{(4\pi)^3}\int V(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\,\Psi_q(\vec{k}), \tag{1.3}$$

and

$$T(\vec{p},q) = V(\vec{p},\vec{q};E_q) + \frac{1}{(4\pi)^3}\int V(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{k^2+m^2}}\,\frac{T(\vec{p},\vec{q})}{E_k(E_k-E_q-i\varepsilon)} \tag{1.4}$$

$$\left(E_q = \sqrt{\vec{q}^2+M^2}\,,\ E_k = \sqrt{\vec{k}^2+m^2}\right).$$

The quasipotential $V(\vec{p},\vec{q};E_q)$ has been built with the help of special kinds of irreducible diagrams (compare with the Bethe-Salpeter formalism [1]).

In the following we shall show that a similar procedure can be directly applied in the spinor case and we shall find the corresponding analogues of eqs. (1.3) and (1.4).

Special attention will be paid to the question of the spin structure of the quasipotential (Sec. IV). For simplicity and to avoid superfluous references to the scalar case, in Sec. III we shall formulate in detail the diagram technique [6,7]. As a concrete model of the interaction the pseudoscalar coupling is chosen:

$$H_{int} = g : \bar{\Psi}(x)\gamma_5 \Psi(x)\, \varphi(x): \tag{1.5}$$

where one may consider that the spinor field $\Psi(x)$ describes nucleons and antinucleons with mass M, while $\varphi(x)$ corresponds to pseudoscalar mesons with mass m.

II. DIAGRAM TECHNIQUE

Let

$$\tilde{H}(p) = \int e^{-ipx} H(x)\, d^4x \tag{2.1}$$

be the four-dimensional Fourier-transform of the interaction Hamiltonian (1.5), and

$$S = I + i R \tag{2.2}$$

be the scattering matrix corresponding to this interaction. Then according to [6,7],

$$R \equiv R(0\,,\,0) = R(\lambda\varkappa, \lambda\varkappa')\Big|_{\varkappa = \varkappa' = 0}\,, \tag{2.3}$$

where the operator $R(\lambda\varkappa\,,\,\lambda\varkappa')$ is determined from the equation *)

$$R(\lambda\varkappa,\lambda\varkappa') = -\tilde{H}(\lambda\varkappa-\lambda\varkappa') - \frac{1}{2\pi}\int \tilde{H}(\lambda\varkappa-\lambda\varkappa_1)\frac{d\varkappa_1}{\varkappa_1 - i\varepsilon}\, R(\lambda\varkappa_1, \lambda\varkappa'). \tag{2.4}$$

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*) It is evident that from the very beginning, we could put $\varkappa' = 0$ and use, instead of (2.4), the equation

$$R(\lambda\varkappa, 0) = -\tilde{H}(\lambda\varkappa) - \frac{1}{2\pi}\int \tilde{H}(\lambda\varkappa-\lambda\varkappa_1)\frac{d\varkappa_1}{\varkappa_1 - i\varepsilon}\, R(\lambda\varkappa_1, 0). \tag{2.4'}$$

However we will need the matrix $R(\lambda\varkappa, \lambda\varkappa')$ for the analysis of the T-invariance of the scattering amplitude (Sec. IV), and this is why we shall make all further considerations on the basis of eq. (2.4).

The quantity λ, appearing in these formulae, is a four-vector with the property that

$$\lambda^2 = 1\,, \quad \lambda_0 > 0 \tag{2.5}$$

and $\varkappa$, $\varkappa_1$ and $\varkappa'$ are one-dimensional invariant parameters.

From (2.3) it follows that on the surface

$$\varkappa = \varkappa' = 0\,, \tag{2.6}$$

which we shall call the energy-momentum shell, the matrix elements of the operator $R(\lambda\varkappa, \lambda\varkappa')$ must be independent of the direction of λ *). Therefore the vector λ, with the properties (2.5), can be chosen in a completely arbitrary way, for example, as

$$\lambda \sim P\,, \tag{2.7}$$

where P is the total four-momentum of the system.

To first order in the coupling constant we have

$$R_1(\lambda\varkappa, \lambda\varkappa') = -\widetilde{H}(\lambda\varkappa - \lambda\varkappa') =$$

$$= -\int e^{-i\lambda(\varkappa-\varkappa')x}\, g : \overline{\Psi}(x)\Psi(x)\varphi(x) : dx = \tag{2.8}$$

$$= -\frac{g}{\sqrt{2\pi}} \int \delta(\lambda\varkappa - \lambda\varkappa' - p - q - k) : \overline{\Psi}(p)\Psi(q)\varphi(k) : dp\, dq\, dk$$

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*) This fact is a corollary from the locality of the operator H(x) [6]:

$$[H(x), H(y)] = 0 \quad \text{for} \quad (x-y)^2 < 0.$$

(the Fourier decompositions of the operators $\overline{\psi}(x)$, $\psi(x)$ and $\varphi(x)$ are given in the Appendix - eqs. (A.1), (A.5) and (A.6)). Eq. (2.8) is the sum of eight normal products:

$$R_1(\lambda\varkappa-\lambda\varkappa') =$$

$$= -g\int e^{-\lambda(\varkappa-\varkappa')x} dx \Big\{ :\overline{\psi}^{(+)}(x)\gamma_5\psi^{(+)}(x)\varphi^{(+)}(x): + :\overline{\psi}^{(-)}(x)\gamma_5\psi^{(-)}(x)\varphi^{(+)}(x): +$$

$$+ :\overline{\psi}^{(+)}(x)\gamma_5\psi^{(-)}(x)\varphi^{(+)}(x): + :\overline{\psi}^{(-)}(x)\gamma_5\psi^{(+)}(x)\varphi^{(+)}(x): + \tag{2.9}$$

$$+ :\overline{\psi}^{(+)}(x)\gamma_5\psi^{(+)}(x)\varphi^{(-)}(x): + :\overline{\psi}^{(-)}(x)\gamma_5\psi^{(-)}(x)\varphi^{(-)}(x): +$$

$$+ :\overline{\psi}^{(+)}(x)\gamma_5\psi^{(-)}(x)\varphi^{(-)}(x): + :\overline{\psi}^{(-)}(x)\gamma_5\psi^{(+)}(x)\varphi^{(-)}(x): \Big\} \equiv$$

$$\equiv \int e^{-i\lambda(\varkappa-\varkappa')x} dx \sum_{a=1}^{8} H_a(x)$$

By calculating matrix elements of (2.9) with normalized states (for normalization of states see Appendix - eqs. (A.9)), it can be seen that, for example, the operator

$$g\int e^{-\lambda(\varkappa-\varkappa')x} H_1(x)\,dx = \int e^{-i\lambda(\varkappa-\varkappa')x} dx :\overline{\psi}^{(+)}(x)\gamma_5\psi^{(+)}(x)\varphi^{(+)}(x):$$

has a non-zero matrix element for the transition vacuum $\longrightarrow$ nucleon + antinucleon + meson:

$$\langle p_1, \mu_1; p_2, \mu_2; k | \int e^{-i\lambda(\varkappa - \varkappa')x} H_1(x)\, dx | 0 \rangle =$$

$$= (2\pi)^{9/2} \frac{g}{\sqrt{2\pi}} \delta(\lambda\varkappa - \lambda\varkappa' - p_1 - p_2 - k) \frac{1}{\sqrt{2p_{10} 2p_{20} 2k_0}} \cdot \tag{2.10}$$

$$\cdot \; \bar{u}^{\mu_1}(\vec{p}_1) \gamma_5 v^{\mu_2}(-\vec{p}_2)$$

and the operator

$$\int e^{-\lambda(\varkappa - \varkappa')x} H_7(x)\, dx = g \int e^{-i\lambda(\varkappa - \varkappa')x} : \bar{\psi}^{(+)}(x) \gamma_5 \psi^{(-)}(x) \varphi^{(-)}(x) : dx$$

gives a non-vanishing contribution to the amplitude for the process nucleon + meson $\longrightarrow$ nucleon:

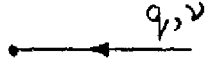
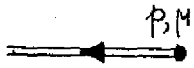
$$\langle p, \mu | \int e^{-i\lambda(\varkappa - \varkappa')x} H_7(x)\, dx | q, \nu; k \rangle =$$

$$= (2\pi)^{9/2} \frac{g}{\sqrt{2\pi}} \delta(\lambda\varkappa - \lambda\varkappa' - p + q + k) \frac{1}{\sqrt{2p_0 2q_0 2k_0}} \cdot \tag{2.11}$$

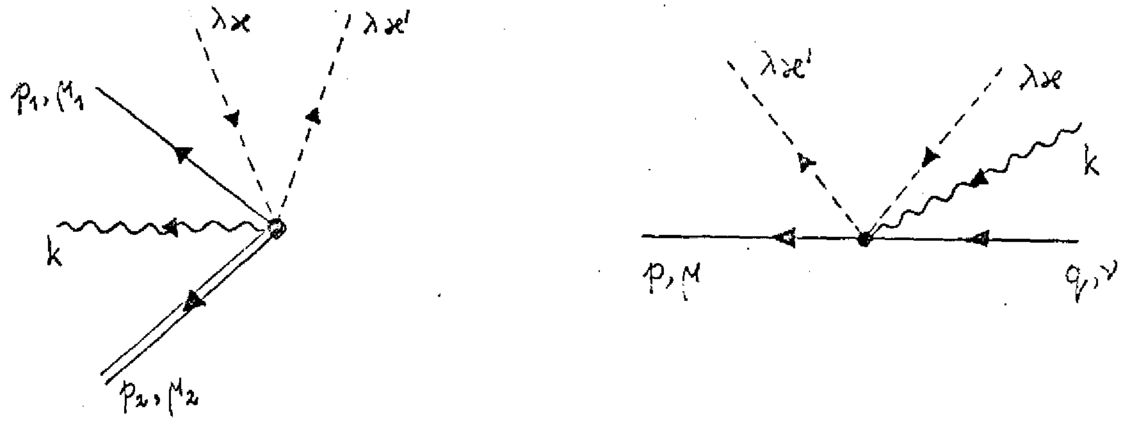
$$\cdot \; \bar{u}^{\mu}(\vec{p}) \gamma_5 u^{\nu}(\vec{q})$$

Let us accept the following rules for the graphical description of the particles in the initial and final states.

<u>TABLE I</u>

Line	Particle	State	Factor in the matrix element
q, ν 	nucleon	in	$\frac{(2\pi)^{3/2}}{\sqrt{2q_0}}\, u^\nu(\vec{q})$
q, ν 	antinucleon	in	$\frac{(2\pi)^{3/2}}{\sqrt{2q_0}}\, \bar{v}^\nu(\vec{q})$
k 	meson	in	$\frac{(2\pi)^{3/2}}{\sqrt{2k_0}}$
p, μ 	nucleon	out	$\frac{(2\pi)^{3/2}}{\sqrt{2p_0}}\, \bar{u}^\mu(\vec{p})$
p, μ 	antinucleon	out	$\frac{(2\pi)^{3/2}}{\sqrt{2p_0}}\, v^\mu(\vec{p})$
k 	meson	out	$\frac{(2\pi)^{3/2}}{\sqrt{2k_0}}$

Now the process (2.10) can be described by diagram a) in Fig. 1 where the spurion dotted lines, which carry the four-momenta $\lambda\varkappa$ and $\lambda\varkappa'$, are introduced so that the conservation law, $\lambda\varkappa - \lambda\varkappa' - p_1 - p_2 - k = 0$, can be satisfied at the vertex.



a) b)

Fig. 1

In a similar manner we construct the diagram of the process (2.11) (see b) in Fig. 1).

In the x-representation as it follows from (2.10) and (2.11), the dotted lines (we will call them quasiparticles), correspond to plane waves of the form $e^{-i\lambda\varkappa x}$. Therefore the operator $\widetilde{H}(\lambda\varkappa - \lambda\varkappa')$ can be interpreted also as an interaction of the fields ψ and φ with the plane wave.

When we iterate eq. (2.4), operator terms of the form

$$R_n(\lambda\varkappa, \lambda\varkappa') =$$

$$= \frac{(-1)^n}{(2\pi)^{n-1}} \int \widetilde{H}(\lambda\varkappa - \lambda\varkappa_1) \frac{d\varkappa_1}{\varkappa_1 - i\varepsilon} \cdots \frac{d\varkappa_{j-1}}{\varkappa_{j-1} - i\varepsilon} \widetilde{H}(\lambda\varkappa_{j-1} - \lambda\varkappa_j) \frac{d\varkappa_j}{\varkappa_j - i\varepsilon} \cdots \tag{2.12}$$

$$\cdots \frac{d\varkappa_{n-1}}{\varkappa_{n-1} - i\varepsilon} \widetilde{H}(\lambda\varkappa_{n-1} - \lambda\varkappa'),$$

appear. This must be reduced to normal form. We shall assume that the Hamiltonians in (2.8) are numbered, with number <u>one</u> assigned to $\tilde{H}(\lambda\varkappa - \lambda\varkappa_1)$, number <u>two</u> to $\tilde{H}(\lambda\varkappa_1 - \lambda\varkappa_2)$, etc., so that the number of the last operator $\tilde{H}(\lambda\varkappa_{n-1} - \lambda\varkappa')$ is n. Further, to each of the operators $\bar{\psi}$, ψ and φ is assigned the number of the Hamiltonian to which this operator belongs. Then, recognizing that the Hamiltonian $\tilde{H}$ is already specified in normal form, we can state that when $R_n(\lambda\varkappa, \lambda\varkappa')$ is reduced to normal form it is necessary to pair only the operators with different numbers. In the case considered here, due to the absence of chronological ordering in $R_n(\lambda\varkappa, \lambda\varkappa')$, the pairings have the form:

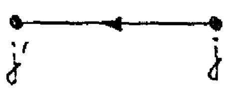
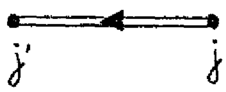
$$\underbrace{\psi_\beta(q_\nu)\,\bar{\psi}_\alpha}(p) = \delta(q_\nu + p)\,\theta(p_0)\,(\hat{p}+M)_{\beta\alpha}\,\delta(p^2 - M^2) \equiv \\ \equiv \delta(q_\nu + p)\, S^{(+)}_{\beta\alpha}(p, M) \tag{2.13}$$

$$\underbrace{\bar{\psi}_\alpha(p)\,\psi_\beta}(q_\nu) = \delta(p + q_\nu)\,\theta(q_{\nu 0})\,(\hat{q}_\nu - M)_{\beta\alpha}\,\delta(q_\nu^2 - M^2) \equiv \\ \equiv \delta(p + q_\nu)\, S^{(+)}_{\beta\alpha}(q_\nu, -M) \tag{2.14}$$

$$\underbrace{\varphi(k)\,\varphi}(k') = \delta(k + k')\,\theta(k'_0)\,\delta(k'^2 - m^2) \equiv \\ \equiv \delta(k + k')\,\Delta^{(+)}(k') \tag{2.15}$$

It is easy to see that the argument of the functions $S^{(+)}$ and $\Delta^{(+)}$ is the argument of those operators ψ and φ which are at the right side of the pairing, i.e., those which have a larger index number. The last circumstance determines the rule for the orientation of the lines in the graphical description of the pairings (2.13)-(2.15) (Table II).

TABLE II

Line	Particle	Pairing	Factor in the matrix element
	nucleon	$\underbrace{\psi_\beta(q_{j'})\,\overline{\psi}_\alpha}(p_j)\,;\quad j'<j$	$S^{(+)}_{\beta\alpha}(p_j, M)$
	antinucleon	$\underbrace{\overline{\psi}_\alpha(p_j)\,\psi_\beta}(q_{v_{j'}})\,;\quad j'<j$	$S^{(+)}_{\beta\alpha}(q_j, -M)$
	meson	$\underbrace{\varphi(k_{j'})\,\varphi}(k_j)\,;\quad j'<j$	$\Delta^{(+)}(k_j)$

The reason why the first pairing in Table II is made to correspond to the nucleon and the second to the antinucleon is, for example, the exact correspondence of these pairings to the contribution of the intermediate nucleon and antinucleon one-particle states, in the unitarity condition.

Beginning with the second-order terms in g in the matrix element, it can be seen from (2.12) that there also appear factors of the form:

$$g_0(\varkappa_j) = \frac{1}{2\pi}\,\frac{1}{\varkappa_j - i\varepsilon} \quad (j = 1, \dots, n-1) \qquad (2.16)$$

which corresponds to a "virtual" quasiparticle with four-momentum $\lambda \varkappa_j$ going out from the vertex with number j and coming into a vertex with number j + 1. Graphically we shall describe such a quasiparticle in the following way:

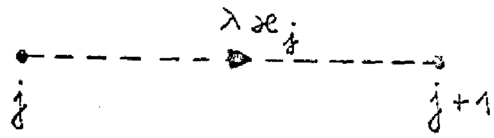


Fig. 2

Now, taking into account Tables I and II, we can formulate in a general form the rules for constructing the matrix elements in this formalism. These rules are as follows:

1) Draw the Feynman diagram (or a set of diagrams) corresponding to the process considered and describe the free nucleon and antinucleon states in accordance with Table I. Number arbitrarily all the vertices and orient every internal line in a direction from the larger to the smaller number. Then, without changing the orientation, change some of the single (nucleon) internal lines to double (antinucleon) lines in such a way that the nucleon charge is conserved in every vertex. Assign to every internal line some momentum p.

2) Connect the first vertex to the second, the second to the third, the third to the fourth, etc., with dotted lines oriented in the direction of increasing numbers; and to every such line assign a four-momentum $\lambda \varkappa_j$, where j = 1,2,..., n-1 is the number of the vertex which the given dotted line leaves. Then join an incoming external dotted line with momentum $\lambda \varkappa$ to the first vertex and an outgoing external line with momentum $\lambda \varkappa'$ to the last vertex (with number n).

3) To each internal dotted line with four-momentum $\lambda \varkappa_j$ assign a propagator (2.16) and to every internal line of a physical particle with momentum p assign one of the functions $S^{(+)}(p,M)$, $S^{(+)}(p,-M)$ or $\Delta^{(+)}(p)$ in accordance with Table II.

4) To each vertex of the diagram assign a factor $-\frac{g\gamma_5}{\sqrt{2\pi}}$ *) and a four-dimensional δ-function that gives the conservation of the total four-momentum of the incoming and outgoing particles and quasiparticles.

5) Integrate between infinite limits over all variables $\varkappa_j$ and over all the independent momenta among the vectors p.

6) Repeat the operations 1) to 5) for all n! numberings of the vertices of the given diagram; add together the expressions obtained and multiply the result by $\frac{\delta_p}{h}$, where h is the number of permutations of the external vertices appearing in the diagram in a symmetrical way and δ_p is the well-known sign factor connected with the parity of the permutations of the external nucleon and antinucleon lines (see, for instance [8] **).

Let us illustrate this procedure with several examples.

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*) All the matrices acting on spinor indices have to be ordered in a sequence from left to right, in the order in which they are met, if one moves along the spinor line passing the antinucleon lines in the direction of their orientation and the nucleon in the direction opposite to their orientation.

**) The sign factor connected with closed spinor loops does not appear in the present diagram technique.

1) Scattering of nucleons and antinucleons in second order.

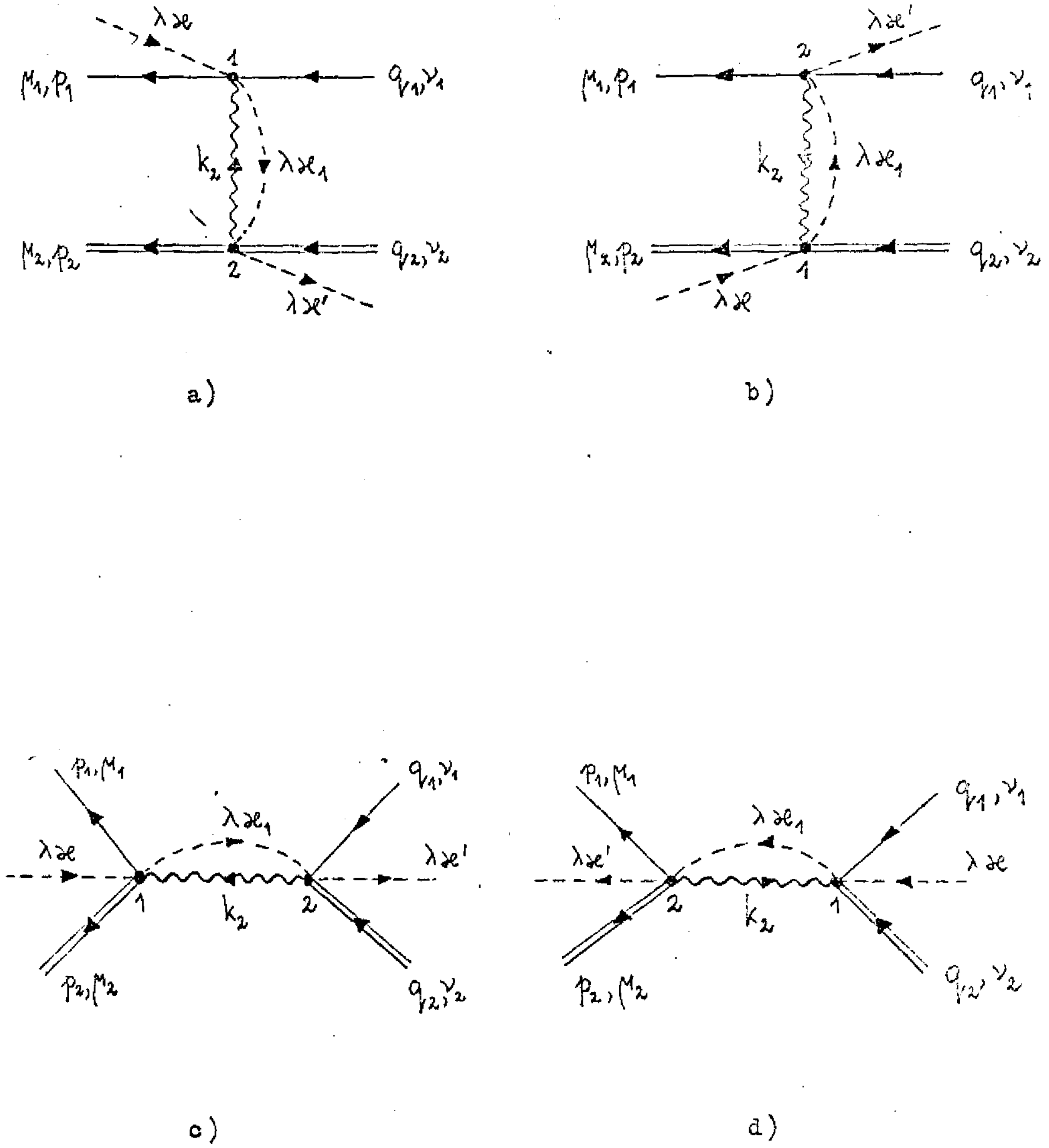


Fig. 3

The matrix element corresponding to the diagrams a), b), c) and d) in Fig. 3 has the form:

$$T_2 =$$

$$= \frac{(-1)(2\pi)^{3/2\cdot 4}\left(-\frac{g}{\sqrt{2\pi}}\right)^2}{\sqrt{2p_{10}2p_{20}2q_{10}2q_{20}}} \bar{u}^{\mu_1}(\vec{p}_1)\gamma_5 u^{\nu_1}(\vec{q}_1)\cdot\bar{v}^{\nu_2}(\vec{q}_2)\gamma_5 v^{\mu_2}(\vec{p}_2)\cdot$$

$$\cdot\frac{1}{2\pi}\int d^4k_2 \frac{d\varkappa_1}{\varkappa_1 - i\varepsilon}\Delta^{(+)}(k_2)\Big[\delta(\lambda\varkappa + q_1 + k_2 - p_1 - \lambda\varkappa_1)\delta(\lambda\varkappa_1 + q_2 - k_2 - p_2 - \lambda\varkappa') +$$

$$+\delta(\lambda\varkappa + q_2 + k_2 - p_2 - \lambda\varkappa_1)\delta(\lambda\varkappa_1 + q_1 - k_2 - p_1 - \lambda\varkappa')\Big] +$$

$$+\frac{(2\pi)^{3/2\cdot 4}\left(-\frac{g}{\sqrt{2\pi}}\right)^2}{\sqrt{2p_{10}2p_{20}2q_{10}2q_{20}}} \bar{u}^{\mu_1}(\vec{p}_1)\gamma_5 v^{\mu_2}(\vec{p}_2)\cdot\bar{v}^{\nu_2}(\vec{q}_2)\gamma_5 u^{\nu_1}(\vec{q}_1)\cdot$$

$$\cdot\frac{1}{2\pi}\int d^4k_2 \frac{d\varkappa_1}{\varkappa_1 - i\varepsilon}\Delta^{(+)}(k_2)\Big[\delta(\lambda\varkappa + k_2 - p_1 - p_2 - \lambda\varkappa_1)\delta(\lambda\varkappa_1 + q_1 + q_2 - k_2 - \lambda\varkappa') +$$

$$+\delta(\lambda\varkappa + q_1 + q_2 + k_2 - \lambda\varkappa_1)\delta(\lambda\varkappa_1 - p_1 - p_2 - k_2 - \lambda\varkappa')\Big] \equiv$$

$$\equiv (2\pi)^4 \frac{\delta(\lambda\varkappa' + p_1 + p_2 - q_1 - q_2 - \lambda\varkappa)}{\sqrt{2p_{10}2p_{20}2q_{10}2q_{20}}} \left(T_2\right)^{\mu_1,\nu_1}_{\mu_2\nu_2}(\lambda\varkappa, p_1, p_2 | q_1, q_2, \lambda\varkappa')\cdot$$

If we choose the vector λ, in accordance with (2.7),*) to be:

$$\lambda = \frac{p_1+p_2}{\sqrt{(p_1+p_2)^2}} = \frac{q_1+q_2}{\sqrt{(q_1+q_2)^2}} \tag{2.17}$$

after some simple calculations we obtain:

$$\left(T_2\right)^{\mu_1 \nu_1}_{\mu_2 \nu_2} (\varkappa', p_1, p_2 | q_1, q_2, \varkappa) =$$

$$(2.18)$$

$$= - \frac{g^2}{\sqrt{m^2 - t + \frac{1}{4}(\varkappa - \varkappa')^2}} \quad \frac{1}{\frac{1}{2}(\varkappa + \varkappa') + \sqrt{m^2 - t + \frac{1}{4}(\varkappa - \varkappa')^2} - i\varepsilon} \, \cdot$$

$$\cdot \, \bar{u}^{\mu_1}(\vec{p}_1) \gamma_5 u^{\nu_1}(\vec{q}_1) \cdot \bar{v}^{\nu_2}(\vec{q}_2) \gamma_5 v^{\mu_2}(\vec{p}_2) +$$

$$+ \frac{g^2}{2m} \left(\frac{1}{\varkappa' + \sqrt{s_p} + m - i\varepsilon} + \frac{1}{\varkappa - \sqrt{s_p} + m - i\varepsilon} \right) \cdot$$

$$\bar{u}^{\mu_1}(\vec{p}_1) \gamma_5 v^{\mu_2}(\vec{p}_2) \cdot \bar{v}^{\nu_2}(\vec{q}_2) \gamma_5 u^{\nu_1}(\vec{q}_1) \quad ,$$

---

*) It is easy to see that, due to the conservation law of the four-momentum, the collinearity of the vectors λ and $\frac{p_1+p_2}{\sqrt{(p_1+p_2)^2}}$ automatically leads to a collinearity between $\frac{p_1+p_2}{\sqrt{(p_1+p_2)^2}}$ and $\frac{q_1+q_2}{\sqrt{(q_1+q_2)^2}}$. In other words, with our choice of λ, the four-velocity of the system is conserved even off the shell, (2.6).

where

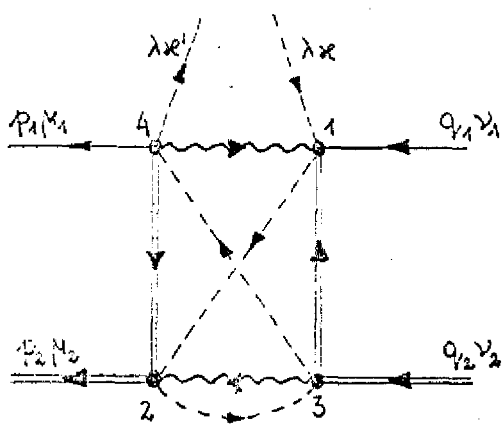
$$
\begin{aligned}
t &= (p_1 - q_1)^2 \\
s_p &= (p_1 + p_2)^2 \\
s_q &= (q_1 + q_2)^2
\end{aligned}
\tag{2.19}
$$

and

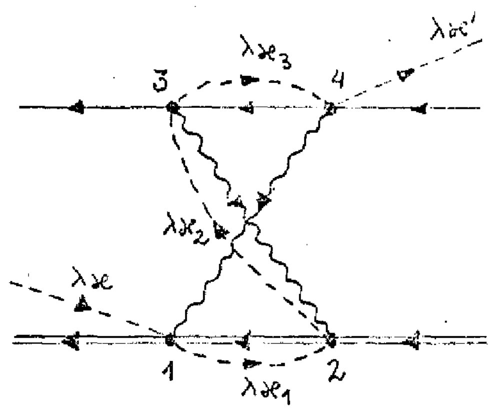
$$\varkappa' + \sqrt{s_p} = \varkappa + \sqrt{s_q} \quad . \tag{2.20}$$

It is evident that on the energy-momentum shell, (2.6), the formula (2.18) gives the same result as the Feynman technique.

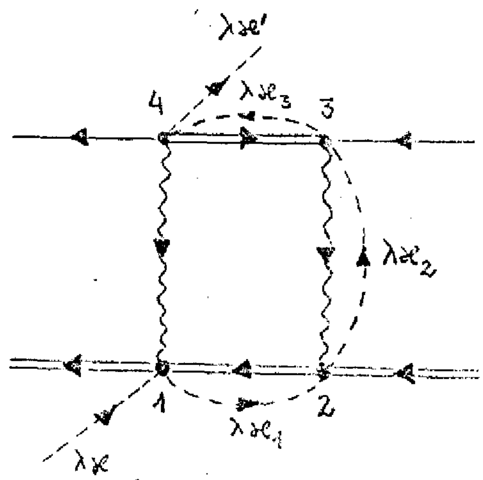
2) Some higher order graphs



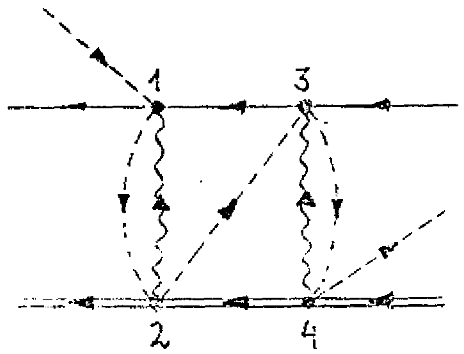
a)



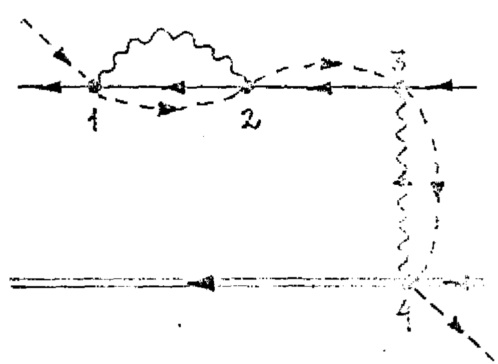
b)



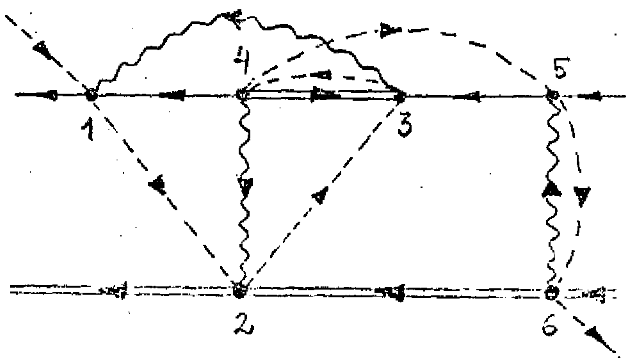
c)



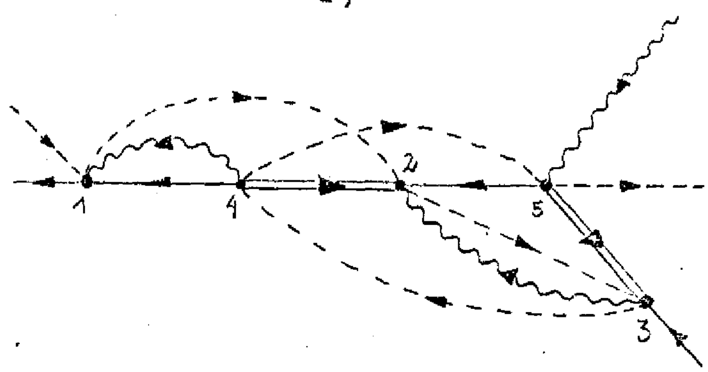
d)



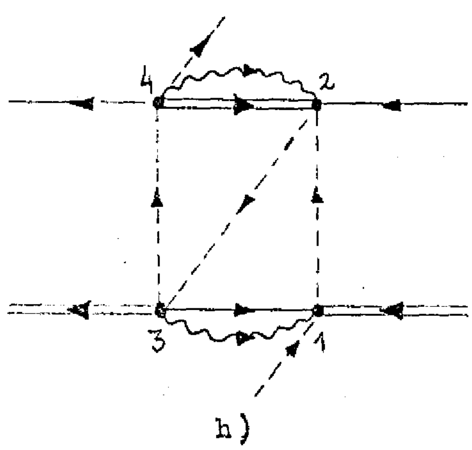
e)



f)



g)



h)

It is necessary to emphasize that since the theory developed here is equivalent to the usual one, the traditional "ultra-violet" divergences appear as they should. But an essential feature of this approach is that these divergences occur only in one-dimensional "dispersion-like" integrals over $\varkappa_j$, which correspond to dotted lines. All the other integrals obtained from products of $S^{(+)}$ and $\Delta^{(+)}$ functions are always convergent [9]. Because of this the removal of the divergences in the proposed scheme is connected with a subtraction procedure in the integrals over $\varkappa_j$ [7]. Further, we shall always suppose that it has been done in this way.

III. THE QUASIPOTENTIAL EQUATION

In this section we shall obtain, with the help of the preceding diagram technique, an analogue of eq. (1.4) for the amplitude of the elastic nucleon-antinucleon scattering

$$N + \bar{N} \longrightarrow N + \bar{N} \tag{3.1}$$

and an analogue of eq. (1.3) for the wave function of the nucleon-antinucleon system.

First let us introduce some definitions.

1) We shall call a diagram <u>unconnected</u> if it can be split into parts which are not connected by <u>physical particle</u> lines. For the opposite case we will call the diagram <u>connected</u> (for example, the diagram h) in Fig. 4 is unconnected and all the others in the same figure are connected).

2) Let us choose a definite time-direction, for instance from right to left, and let us correspondingly orient the free ends of the diagram describing the process (3.1) off the energy-momentum shell, (2.6). A connected diagram belonging to this class will be called <u>irreducible</u> if it is impossible to separate it into two connected subdiagrams, which are linked to each other by two spinor lines (a nucleon and an antinucleon), oriented from right to left, and one dotted line oriented in the opposite direction. If such

a separation can be found, we will call the diagram reducible. For examplo, all the diagrams in Fig. 1 and the diagrams a), b), c), and e) in Fig. 4 are irreducible, but the diagrams d) and f) are reducible. It is evident that all the connected diagrams can be built from irreducible components. We will use this fact in writing the equation for the scattering amplitude (compare this with the corresponding procedure in the B-S formalism [1]).

Let

$$V = \frac{(2\pi)^4\,\delta(\lambda\varkappa' + p_1 + p_2 - q_1 - q_2 - \lambda\varkappa)}{\sqrt{2p_{10}\,2p_{20}\,2q_{10}\,2q_{20}}}\, V^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa, p_1, p_2 | \lambda\varkappa_1, q_1, q_2) = \tag{3.2}$$

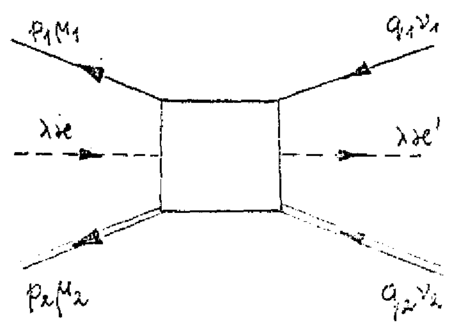
$$= \frac{(2\pi)^4\,\delta(\lambda\varkappa' + p_1 + p_2 - q_1 - q_2 - \lambda\varkappa)}{\sqrt{2p_{10}\,2p_{20}\,2q_{10}\,2q_{20}}}\,.$$

$$\cdot\, \bar{u}^{\mu_1}_{\alpha_1}(\vec{p}_1)\, \bar{v}^{\nu_2}_{\alpha_2}(\vec{q}_2)\, V_{\alpha_1\beta_1;\alpha_2\beta_2}(\lambda\varkappa, p_1, p_2 | \lambda\varkappa', q_1, q_2)\, u^{\nu_1}_{\beta_1}(\vec{q}_1)\, v^{\mu_2}_{\beta_2}(\vec{p}_2)$$

be the matrix élement corresponding to the set of all irreducible diagrams describing the process (3.1). (All the variables and indices have the same meaning here as in the second-order matrix element (2.18).)

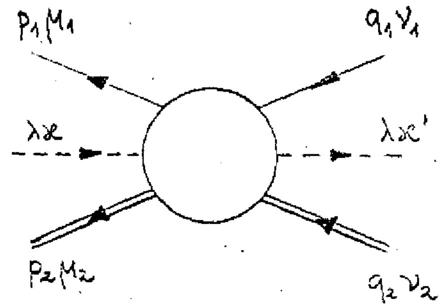
Let us agree that we will draw V graphically as:

$V =$



(3.3)

Further, let

$T =$  $=$

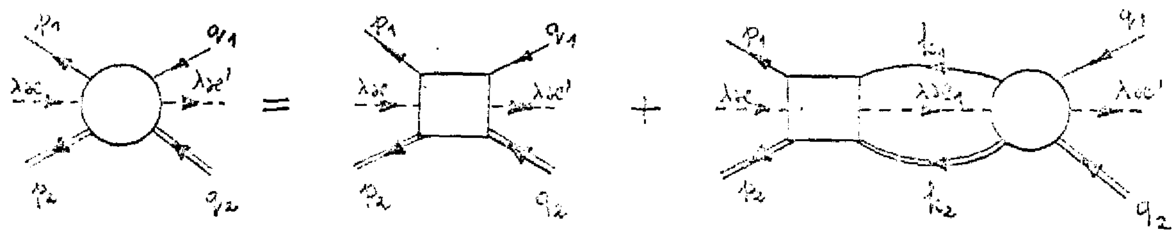
$$= \frac{(2\pi)^4\,\delta(-\lambda\varkappa + p_1 + p_2 - q_1 - q_2 + \lambda\varkappa')}{\sqrt{2p_{10}\,2p_{20}\,2q_{10}\,2q_{20}}}\; T^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa, p_1, p_2|\lambda\varkappa', q_1, q_2) =$$

$$= \frac{(2\pi)^4\,\delta(-\lambda\varkappa + p_1 + p_2 - q_1 - q_2 + \lambda\varkappa')}{\sqrt{2p_{10}\,2p_{20}\,2q_{10}\,2q_{20}}}\;\cdot$$

$$\cdot\; \bar{u}^{\mu_1}_{\alpha_1}(\vec{p}_1)\,\bar{v}^{\mu_2}_{\alpha_2}(\vec{q}_2)\; T_{\alpha_1\beta_1;\alpha_2\beta_2}(\lambda\varkappa, p_1, p_2|\lambda\varkappa', q_1, q_2)\, u^{\nu_1}_{\beta}(\vec{q}_1)\, v^{\mu_2}_{\beta_2}(\vec{p}_2)$$

be the amplitude of the $N\bar{N}$ scattering off the shell, (2.6); i.e., the set of all connected diagrams corresponding to the process (3.1).

Then, taking into account the definition of irreducible diagrams, we can write the following graphical equation:



which, according to the rules of our diagram technique, is equivalent to the integral equation

$$\delta(-\lambda\varkappa+p_1+p_2-q_1-q_2+\lambda\varkappa')\, T^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa',q_1,q_2)=$$

$$=\delta(-\lambda\varkappa+p_1+p_2-q_1-q_2+\lambda\varkappa')\, V^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa',q_1,q_2)\;+$$

$$+\frac{1}{(2\pi)^2}\int\delta(-\lambda\varkappa+p_1+p_2-k_1-k_2+\lambda\varkappa_1)\,\bar{u}^{\mu_1}_{\alpha_1}(\vec{p}_1)\,\bar{v}^{\nu_2}_{\alpha_2}(\vec{q}_2)\;\cdot \tag{3.5}$$

$$\cdot\, V_{\alpha_1\gamma_1;\delta_2\beta_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa_1,k_1,k_2)\,d^4k_1\,d^4k_2\quad S^{(+)}_{\gamma_1\delta_1}(k_1,M)\,\cdot$$

$$\cdot\, S^{(+)}_{\gamma_2\delta_2}(k_2,-M)\,\frac{d\varkappa_1}{2\pi(\varkappa_1-i\varepsilon)}\,\delta(-\lambda\varkappa_1+k_1+k_2-q_1-q_2+\lambda\varkappa')\,\cdot$$

$$\cdot\, T_{\delta_1\beta_1;\alpha_2\gamma_2}(\lambda\varkappa_1,k_1,k_2|\lambda\varkappa',q_1,q_2)\,u^{\nu_1}_{\beta_1}(\vec{q}_1)\quad v^{\mu_2}_{\beta_2}(\vec{p}_2)$$

If we use the completeness condition (eq. (A.10) and drop the common δ-function, expressing the conservation law $p_1+p_2-\lambda\varkappa=q_1+q_2-\lambda\varkappa'$, then this equation can be written in the form:

$$T^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa',q_1,q_2)=V^{\mu_1\nu_1}_{\mu_2\nu_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa',q_1,q_2)+$$

$$+\sum_{\varsigma_1\varsigma_2}\int V^{\mu_1\varsigma_1}_{\mu_2\varsigma_2}(\lambda\varkappa,p_1,p_2|\lambda\varkappa_1,k_1,k_2)\,\theta(k_1^0)\,\delta(k_1^2-M^2)\,d^4k_1\;\cdot \tag{3.6}$$

$$\cdot\left\{\frac{1}{(2\pi)^3}\,\frac{\theta(k_2^0)\,\delta(k_2^2-M^2)}{\varkappa_1-i\varepsilon}\,\delta(-\lambda\varkappa_1+k_1+k_2-q_1-q_2+\lambda\varkappa')\right\}d^4k_2\,d\varkappa_1\,\cdot$$

$$\cdot\, T^{\varsigma_1\nu_1}_{\varsigma_2\nu_2}(\lambda\varkappa_1,k_1,k_2|\lambda\varkappa',q_1,q_2)\,.$$

Under the condition

$$\lambda = \frac{p_1 + p_2}{\sqrt{(p_1+p_2)^2}} = \frac{k_1 + k_2}{\sqrt{(k_1+k_2)^2}} = \frac{q_1+q_2}{\sqrt{(q_1+q_2)^2}} \tag{3.7}$$

(compare with 2.17) we can introduce the standard invariant variables

$$\begin{aligned} s_p &= (p_1 + p_2)^2 \\ s_k &= (k_1 + k_2)^2 \\ s_q &= (q_1 + q_2)^2 \\ t_{pq} &= (p_1 - q_1)^2 \\ u_{pq} &= (p_2 - q_1)^2 \\ t_{pk} &= (p_1 - k_1)^2 \end{aligned} \tag{3.8}$$

etc. It is easily seen that the quantities (3.7) are connected by relations of the form

$$\sqrt{s_\varkappa s_q} + t_{\varkappa q} + u_{\varkappa q} = 4M^2 , \tag{3.9}$$

which, on the energy-momentum shell, reduce to the well-known equality:

$$s + t + u = 4M^2 .$$

If we now integrate over k_2 and $\varkappa_1$ in (3.6) the expression in the curly brackets takes the form:

$$\frac{1}{(2\pi)^3}\frac{1}{\sqrt{s_k}\left(\varkappa' + \sqrt{s_k} - \sqrt{s_q} - i\varepsilon\right)} \tag{3.10}$$

As mentioned above, the parameter $\varkappa'$ is an auxiliary quantity facilitating the T-invariance analysis of the theory. For practical calculations it is sufficient to consider only the case $\varkappa' = 0$. In this case (3.10) is equal to

$$\frac{1}{(2\pi)^3}\frac{1}{\sqrt{s_k}\left(\sqrt{s_k} - \sqrt{s_q} - i\varepsilon\right)} \equiv G_q^0(k) , \tag{3.11}$$

or, using (3.8),

$$G_q^0(k) = \frac{1}{(2\pi)^3}\frac{1}{s_k + t_{kq} + u_{kq} - 4M^2 - i\varepsilon} . \tag{3.12}$$

It is clear that the quantity $G_q^0(k)\,\delta^{\rho_1\sigma_1}\,\delta_{\rho_2\sigma_2}$ in eq. (3.6) plays the role of a free Green function of the two-particle system considered, and V is the quasipotential. Let us mention here that, in the scalar case, the free Green function is also given by eq. (3.12) [7]. The complete analogy between eq. (3.6), when $\varkappa' = 0$, and the scalar equation (1.4) becomes obvious after going to the centre-of-mass system in (3.6). Introducing the notation

$$\begin{aligned} \vec{p}_1 &= -\vec{p}_2 = \vec{p} \\ \vec{q}_1 &= -\vec{q}_2 = \vec{q} \\ \vec{k}_1 &= -\vec{k}_2 = \vec{k} \end{aligned} \tag{3.13}$$

$$E_p = \sqrt{\vec{p}^{\,2} + M^2} , \quad E_k = \sqrt{\vec{k}^{\,2} + M^2} , \quad E_q = \sqrt{\vec{q}^{\,2} + M^2}$$

and taking into account the equalities

$$
\begin{aligned}
2E_q + \varkappa &= 2E_p \\
2E_k + \varkappa &= 2E_p + \varkappa_1 ,
\end{aligned}
\tag{3.14}
$$

we will have, instead of (3.6) *),

$$
T^{\mu_1\nu_1}_{\mu_2\nu_2}(\vec{p},\vec{q}) = V^{\mu_1\nu_1}_{\mu_2\nu_2}(\vec{p},\vec{q};E_q) +
+ \frac{1}{(4\pi)^3}\sum_{\rho_1\rho_2}\int V^{\mu_1\rho_1}_{\mu_2\rho_2}(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+M^2}}\,\frac{T^{\rho_1\nu_1}_{\rho_2\nu_2}(\vec{k},\vec{q})}{E_k(E_k-E_q-i\varepsilon)} \tag{3.15}
$$

Thus the quasipotential equation for the scattering amplitude for the case of spinor particles differs from the equation in the scalar case only by the appearance of a trivial summation over the intermediate spinor indices. The free Green function is the same in both cases.

By repeating literally the reasoning from [7], it is easy to show that, in the case of real quasipotential, the $N\bar{N}$-scattering amplitude satisfies the relativistic two-particle unitarity condition. This fact reflects one of the basic ideas of the quasipotential approach [2].

It is convenient in the following to substitute the spinor amplitudes v and $\bar{v}$ appearing in eqs. (3.2) and (3.4) with their charge-conjugated spinors u^c and $\bar{u}^c$, which correspond to antinucleons

*) In terms of the variables of (3.13), we see directly that the Green function $G^0(k)$ has the correct non-relativistic limit:

$$
G^0_q(k) = \frac{1}{(2\pi)^3\, 4E_k(E_k - E_q - i\varepsilon)} \longrightarrow \frac{1}{2(2\pi)^3(\vec{k}^2 - \vec{q}^2 - i\varepsilon)}
$$

and, afterwards, to pass to the two-component spinors φ, φ^*, χ, χ^* whose components correspond to states with a given helicity (see Appendix). Finally we obtain (the polarization indices are omitted in the left-hand side)

$$\bar{u}_{\alpha_1}\bar{v}_{\alpha_2}\, T_{\alpha_1\beta_1;\alpha_2\beta_2}\, u_{\beta_1} v_{\beta_2} = \varphi^+_{i_1}\chi^+_{i_2}\; t_{i_1,k_1;i_2,k_2}\, \varphi_{k_1}\chi_{k_2}$$

$$\bar{u}_{\alpha_1}\bar{v}_{\alpha_2}\, V_{\alpha_1\beta_1;\alpha_2\beta_2}\, u_{\beta_1} v_{\beta_2} = \varphi^+_{i_1}\chi^+_{i_2}\; V_{i_1,k_1;i_2,k_2}\, \varphi_{k_1}\chi_{k_2} \tag{3.16}$$

$$(\alpha,\beta = 1,2,3,4\;;\; i,k = 1,2).$$

It is easy to see that, taking into account (3.16) and also the completeness of the system of functions φ and χ, eq. (3.15) can be rewritten in the form

$$t_{i_1k_1;i_2k_2}(\vec{p},\vec{q})\,\varphi_{k_1}\chi_{k_2} = V_{i_1k_1;i_2k_2}(\vec{p},\vec{q};E_q)\,\varphi_{k_1}\chi_{k_2} +$$

$$+\frac{1}{(4\pi)^3}\int V_{i_1j_1;i_2j_2}(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\,\frac{t_{j_1k_1;j_2k_2}(\vec{k},\vec{q})\,\varphi_{k_1}\chi_{k_2}}{E_k(E_k-E_q-i\varepsilon)}\,. \tag{3.17}$$

Let us define now the wave function of the $N\bar{N}$-system, corresponding to the continuous spectrum, as

$$\Psi_q(\vec{p})_{i_1i_2} = \frac{(2\pi)^3}{M}\,\delta(\vec{p}-\vec{q})\sqrt{\vec{p}^2+M^2}\;\varphi_{i_1}\chi_{i_2} +$$

$$+\frac{1}{8ME_p(E_p-E_q-i\varepsilon)}\; t_{i_1k_1;i_2k_2}(\vec{p},\vec{q})\,\varphi_{k_1}\chi_{k_2}\,. \tag{3.18}$$

Substituting (3.18) into (3.17) will give

$$\Psi_q(\vec{p})_{i_1 i_2} = \frac{(2\pi)^3}{M}\,\delta(\vec{p}-\vec{q})\sqrt{\vec{p}^2+M^2}\;\varphi_{i_1}\chi_{i_2} + \tag{3.19}$$

$$+\frac{1}{E_p(E_p-E_q-i\varepsilon)}\,\frac{1}{(4\pi)^3}\int\frac{d\vec{k}}{\sqrt{k^2+M^2}}\,V_{i_1 j_1;\, i_2 j_2}(\vec{p},\vec{q};E_q)\,\Psi_q(\vec{k})_{j_1 j_2}$$

or

$$E_p(E_p-E_q)\Psi_q(\vec{p})_{i_1 i_2} = \tag{3.20}$$

$$=\frac{1}{(4\pi)^3}\int\frac{d\vec{k}}{\sqrt{k^2+M^2}}\,V_{i_1 j_1;\, i_2 j_2}(\vec{p},\vec{k};E_q)\,\Psi_q(\vec{k})_{j_1 j_2}$$

The equation obtained here, (3.20), is the analogue of the Schrödinger equation for the $N\bar{N}$ system *) and can be used to find the wave function corresponding to the continuous spectrum, as well as for the bound state problems. The quasipotential $V_{i_1 j_1;\, i_2 j_2}(\vec{p},\vec{k};E_q)$, as stated before, is defined as the set of all irreducible diagrams corresponding to the $N\bar{N}$-scattering off the energy-momentum shell, (2.6).

IV THE SPIN STRUCTURES OF THE QUASIPOTENTIAL

In the present section we will consider the problem of spin structures which the quasipotential for the $N\bar{N}$-system can have in the general case.

When $\varkappa' \neq 0$, in the centre-of-mass system, the quasipotential can be written in the form of a $4 \otimes 4$ matrix (compare with (3.20))

---

*) We wish to point out the complete analogy between (3.20) and (1.3).

$$V_{i_1k_1;i_2k_2}(\varkappa,\vec{p}\,|\,\varkappa',\vec{q}) \tag{4.1}$$

$$(i,k = 1,2)$$

The matrix (4.1) must be invariant with respect to space rotations, space reflections, charge conjugation and time reversal. Let us introduce in the three-dimensional $\vec{p}$-space a system of orthogonal unit vectors, *)

$$\begin{aligned}
\vec{l} &= \frac{1}{N_l}(\vec{p}+\vec{q}) \\
\vec{n} &= \frac{1}{N_n}(\vec{p}\times\vec{q}) \\
\vec{m} &= \frac{1}{N_m}\left[(\vec{p}+\vec{q})\times(\vec{p}\times\vec{q})\right]
\end{aligned} \tag{4.2}$$

The system (4.2) on the energy shell ($E_p = E_q$) is transformed into the familiar basis:

$$\begin{aligned}
\vec{l}' &= \frac{1}{N_{l'}}(\vec{p}+\vec{q}) \\
\vec{n}' &= \frac{1}{N_{n'}}(\vec{p}\times\vec{q}) \\
\vec{m}' &= \frac{1}{N_{m'}}(\vec{p}-\vec{q})
\end{aligned} \tag{4.3}$$

Let us write, with the help of the Pauli matrices, a basis in the space of the $4\otimes 4$ matrices to which the operator (4.1) belongs:

$$I^{(1)}\otimes I^{(2)}\;,\quad I^{(1)}\otimes\sigma_r^{(2)}\;,\quad \sigma_r^{(1)}\otimes I^{(2)}\;,\quad \sigma_r^{(1)}\otimes\sigma_s^{(2)} \tag{4.4}$$

$$(r,s = 1,2,3)$$

*) The quantities N_l, N_n and N_m are normalization factors, i.e., for example, $N_l = |\vec{p}+\vec{q}|$, etc.

Here the index (1) corresponds to the spin space of the nucleon and (2) to the spin space of the antinucleon.

Reasoning in the usual manner, we conclude that the requirement of invariance, with respect to space rotations and reflections, allows only quasipotential with the following structure:

$$\begin{aligned} V = {} & V_1\, I^{(1)} \otimes I^{(2)} + V_2\, I^{(1)} \otimes \vec{n}\cdot\vec{\sigma}^{(2)} + V_3\, \vec{n}\cdot\vec{\sigma}^{(1)} \times \vec{n}\cdot\vec{\sigma}^{(2)} + \\ & + V_4\, \vec{l}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} + V_5\, \vec{m}\cdot\vec{\sigma}^{(1)} \otimes \vec{m}\cdot\vec{\sigma}^{(2)} + \\ & + V_6\, \vec{l}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} + V_7\, \vec{m}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} + \\ & + V_8\, \vec{n}\cdot\vec{\sigma}^{(1)} \otimes I^{(2)} , \end{aligned} \tag{4.5}$$

where $V_1, \ldots, V_8$ are, in general, complex scalar functions of the vectors $\vec{p}$, $\vec{q}$, and the parameters $\varkappa$ and $\varkappa'$. Taking into account the conservation law,

$$2E_p + \varkappa' = 2E_q + \varkappa ,$$

we can write these functions in the form

$$V_i(\vec{p}\cdot\vec{q},\ E_p + E_q ,\ \varkappa + \varkappa' ,\ E_p - E_q) \tag{4.6}$$

$$i = 1, 2, \ldots, 8 .$$

The invariance of the theory with respect to charge conjugation in the case of the $N\bar{N}$-system means simply a symmetry of the potential (4.4) with respect to permutation of the indices (1) and (2). From this it follows that

$$V_2 = V_8$$
$$V_6 = V_7 \qquad (4.7)$$

and therefore (4.5) takes the form

$$V = V_1\, I^{(1)} \otimes I^{(2)} + V_2 \left(I^{(1)} \otimes \vec{n}\cdot\vec{\sigma}^{(2)} + \vec{n}\cdot\vec{\sigma}^{(1)} \otimes I^{(2)} \right) +$$

$$+ V_3\, \vec{n}\cdot\vec{\sigma}^{(1)} \otimes \vec{n}\cdot\vec{\sigma}^{(2)} + V_4\, \vec{l}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} + \qquad (4.8)$$

$$+ V_5\, \vec{m}\cdot\vec{\sigma}^{(1)} \otimes \vec{m}\cdot\vec{\sigma}^{(2)} + V_6 \left(\vec{l}\cdot\vec{\sigma}^{(1)} \otimes \vec{m}\cdot\vec{\sigma}^{(2)} + \vec{m}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} \right).$$

Let us consider now the condition implied by the T-invariance. It is easy to show that the weak (Wigner) time reversal T_W leads to the following transformation of the matrix elements of the operator $R(\lambda\varkappa, \lambda\varkappa')$:

$$\langle \vec{p}_1, \sigma_1; \vec{p}_2, \sigma_2; \dots | R(\lambda\varkappa, \lambda\varkappa') | \vec{q}_1, \sigma_1; \vec{q}_2, \sigma_2; \dots \rangle \xrightarrow{T_W}$$
$$(4.9)$$
$$\longrightarrow \langle -\vec{q}_1, -\sigma_1; -\vec{q}_2, -\sigma_2; \dots | R(\hat{\lambda}\varkappa', \hat{\lambda}\varkappa) | -\vec{p}_1, -\sigma_1; -\vec{p}_2, -\sigma_2; \dots \rangle ,$$

where $\sigma_1, \sigma_2, \dots$ are the values of the helicities of the nucleons and antinucleons and *)

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*) Taking into account (4.9) and (4.10), it can be said that, under time reversal, the quasiparticle in the "initial state" turns into a quasiparticle in the "final state" with a change in sign of its three-momentum.

Let us also note that, if the vector λ is chosen in accordance with (3.7), the transformation $\lambda \longrightarrow \hat{\lambda}$ in (4.9) is automatically performed.

$$\hat{\lambda} = (\lambda_0, -\vec{\lambda}). \tag{4.10}$$

The quasipotential transforms similarly to (4.9); and this is the reason why, in the case of time-reversal invariance of the theory, a new restriction appears: the quasipotential must not be changed under the simultaneous transformations

$$\begin{aligned} &\vec{p} \to -\vec{q} \quad , \quad \vec{q} \to -\vec{p} \\ &\sigma_1 \to -\sigma_2 \quad , \quad \sigma_2 \to -\sigma_1 \\ &\varkappa \to \varkappa' \quad ; \quad \varkappa' \to \varkappa \end{aligned} \tag{4.11}$$

With the help of (4.2) it is easy to verify that, under the transformation (4.11),

$$\begin{aligned} &\vec{l} \to -\vec{l} \\ &\vec{n} \to -\vec{n} \\ &\vec{m} \to \vec{m} \end{aligned} \tag{4.12}$$

From (4.12) and (4.11) we find that the term

$$\vec{l}\cdot\vec{\sigma}^{(1)} \otimes \vec{m}\cdot\vec{\sigma}^{(2)} + \vec{m}\cdot\vec{\sigma}^{(1)} \otimes \vec{l}\cdot\vec{\sigma}^{(2)} , \tag{4.13}$$

is odd with respect to time-reversal, while the other structures in (4.8) do not change under this transformation. Hence the quantity $V_6(\vec{p}\cdot\vec{q}, E_p + E_q, \varkappa + \varkappa', E_p - E_q)$ must be an odd function of its last argument and V_1, V_2, V_3, V_4 and V_5, correspondingly, must be even functions of $E_p - E_q$. Evidently, on the shell, $E_p = E_q$,

$$V_6 = -V_6 = 0$$

and the potential contains only five independent spin-structures.

These results are in complete agreement with those of the authors of [10] and [11], where the structure of the nucleon-nucleon potential in the non-relativistic case has been analysed (see also [12]).

For illustrative purposes we now calculate the quasipotential to second order in g. It is evident that here we can use formula (2.18), as the only irreducible diagrams of second order are the diagrams a), b), c) and d) in Fig. 3. After some simple calculations, we find the following expressions for the quantities V_i from (4.8). (The index (2) shows that quantities of second order are considered.),

$$V_1^{(2)} = V_3^{(2)} = - g^2 2 p_0 q_0 \frac{1 + \frac{\varkappa+\varkappa'}{2m}}{\left(\frac{\varkappa+\varkappa'}{2} + m\right)^2 - (p_0+q_0)^2}$$

$$V_2^{(2)} = 0$$

$$V_4^{(2)} = g^2 \left\{ \frac{N_e^2 (p_0-q_0)^2 \left[(p_0+M)(q_0+M) - \vec{p}\vec{q}\right]}{\sqrt{m^2 - t_{pq} + \frac{1}{4}(\varkappa-\varkappa')^2}\left(\frac{\varkappa+\varkappa'}{2} + \sqrt{m^2 - t_{pq} + \frac{(\varkappa-\varkappa')^2}{4}}\right)} + V_1^{(2)} \right\}$$

$$(4.14)$$

$$V_5^{(2)} = g^2 \left\{ \frac{N_m^2 \left[\vec{p}^2\vec{q}^2 - (\vec{p}\vec{q})^2\right]^2 (p_0+M)(q_0+M)\left(\frac{1}{p_0+M} + \frac{1}{q_0+M}\right)^2}{\sqrt{m^2 - t_{pq} + \frac{1}{4}(\varkappa-\varkappa')^2}\left(\frac{\varkappa+\varkappa'}{2} + \sqrt{m^2 - t_{pq} + \frac{(\varkappa-\varkappa')^2}{4}}\right)} + V_1^{(2)} \right\}$$

$$V_6^{(2)} = g^2 \frac{N_e N_m (p_0+q_0+2M)\left[(p_0+M)(q_0+M) - \vec{p}\vec{q}\right]\left[\vec{p}^2\vec{q}^2 - (\vec{p}\vec{q})^2\right](p_0-q_0)}{(p_0+M)(q_0+M)\sqrt{m^2 - t_{pq} + \frac{1}{4}(\varkappa-\varkappa')^2}\left(\frac{\varkappa+\varkappa'}{2} + \sqrt{m^2 - t_{pq} + \frac{1}{4}(\varkappa-\varkappa')^2}\right)}$$

The function $V_6^{(2)}$ is obviously antisymmetric with respect to the transformation $p_0 \longrightarrow q_0$, while $V_1^{(2)}$, $V_2^{(2)}$, $V_3^{(2)}$, $V_4^{(2)}$ and $V_5^{(2)}$ are invariant under this transformation.

In the higher order terms of the perturbation series the T-odd structure appears as well. For example, the irreducible diagram shown in Fig. 4a) gives the following contribution in $V_6^{(4)}$:

$$- \frac{1}{(2\pi)^5} 4 g^4 p_0 q_0 \int \frac{d\varkappa_1}{\varkappa_1 - i\varepsilon} \frac{d\varkappa_2}{\varkappa_2 - i\varepsilon} \frac{d\varkappa_3}{\varkappa_3 - i\varepsilon} \Delta^{(+)}(k) .$$

$$\Delta^{(+)}(\lambda\varkappa - \lambda\varkappa_1 - \lambda\varkappa_3 - \lambda\varkappa_3 + q_1 + q_2 + k) \, \Delta_M^{(+)}(\lambda\varkappa_1 - \lambda\varkappa - q_1 - k) . \tag{4.15}$$

$$\cdot \Delta_M^{(+)}(\lambda\varkappa_3 - \lambda\varkappa' - p_1 - k) \cdot \left[(\vec{a}\cdot\vec{\ell})(\vec{b}\cdot\vec{m}) + (\vec{b}\cdot\vec{\ell})(\vec{a}\cdot\vec{m})\right] ,$$

where λ is determined by eq. (3.7), $\Delta_M^{(+)}(p) = \theta(p_0) \; \delta(p^2 - M^2)$, and the vectors $\vec{a}$ and $\vec{b}$ are equal to

$$\vec{a} = \frac{\vec{k}\cdot\vec{q}}{q_0(q_0+M)} \vec{q} - \vec{k} \tag{4.16}$$

$$\vec{b} = \frac{\vec{k}\cdot\vec{p}}{p_0(p_0+M)} \vec{p} - \vec{k} .$$

It is easy to verify that, under the transformation (4.11)-(4.12), the expression (4.15) changes sign.

To conclude this section we write the spin structures of the quasipotential (3.2) in a Lorentz covariant form:

$$\begin{aligned} V = {} & F_1 \, I^{(1)} \otimes I^{(2)} + F_2 \left(\not{L}^{(1)} \otimes I^{(2)} + I^{(1)} \otimes \not{M}^{(2)} \right) + \\ & + F_3 \left(\not{M}^{(1)} \otimes \not{L}^{(2)} \right) + F_4 \left[(\gamma_5 \not{M}^{(1)}) \otimes (\gamma_5 \otimes \not{L}^{(2)}) \right] + \\ & + F_5 \, \gamma_5^{(1)} \otimes \gamma_5^{(2)} + F_6 \left[\gamma_5^{(1)} \otimes (\gamma_5 \not{L}^{(2)}) + (\gamma_5 \not{M}^{(1)}) \otimes \gamma_5^{(2)} \right] , \end{aligned} \tag{4.17}$$

where

$$L = \tfrac{1}{2}(p_1 + q_1)$$
$$M = \tfrac{1}{2}(p_2 + q_2) \tag{4.18}$$

The quantities F_i $(i = 1,2, \dots, 6)$ are invariant functions of the type

$$F_i = F_i(t_{pq}, s_p + s_q, \varkappa + \varkappa', s_p - s_q). \tag{4.19}$$

The first five spin-structures in (4.7) are T-even, and so their coefficients F must be even functions of the last argument. The function F_6 is multiplied by a T-odd structure and hence it must be an odd function of $s_p - s_q$. Obviously $F_6 = 0$ when $s_p = s_q$.

The decomposition (4.17) can be obtained using the well-known reasoning described, for instance, in [13]. In our case it is convenient to choose the vector basis in the four-dimensional p-space in the form:

$$L_\mu = \tfrac{1}{2}(p_{1\mu} + q_{1\mu})$$
$$M_\mu = \tfrac{1}{2}(p_{2\mu} + q_{2\mu})$$
$$N_\mu = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma} L_\nu M_\rho (q_{1\sigma} - q_{2\sigma} + p_{2\sigma} - p_{1\sigma}) \tag{4.20}$$
$$K_\mu = \varepsilon_{\mu\nu\rho\sigma} L_\nu M_\rho N_\sigma .$$

V. CONCLUSION

As already mentioned, the quasipotential equations we have obtained for the $N\bar{N}$ system are completely analogous to the corresponding scalar equations (1.3) and (1.4). In both cases the free Green function is the same and all the specific features introduced by the spin appear only in the structure of the quasipotential. Therefore, the situation here is the same as in the non-relativistic case where the free Hamiltonian is a scalar in the spin space and only the interaction terms are spin dependent.

In connection with this fact, it is tempting to try to apply our approach to a relativistic formulation of the higher symmetries, for instance SU(6), where the invariance of the free equations with respect to proper spin transformations is highly desirable. Then, one of the interesting questions which can arise is: what must be the form of the initial interaction Hamiltonian $H(x)$ in order that the quasi-potential built up from $H(x)$ by using our procedure would be approximately SU(6) invariant?

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APPENDIX

In this part of the paper, information about the symbols, notations and normalizations used above. is collected.

I. Metric

$$pq = p_o q_o - \vec{p}\cdot\vec{q} \quad .$$

II. Field equations, Fourier decompositions and quantization

1) Pseudoscalar field

a) $\left(\Box - m^2\right)\varphi(x) = 0 \quad ; \quad \varphi = \varphi^{+}$

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \varphi(k)\, e^{ikx}\, d^4k =$$

$$= \frac{1}{(2\pi)^{3/2}} \int e^{ikx} \frac{d\vec{k}}{\sqrt{2k_o}}\, \alpha^{+}(\vec{k}) + \frac{1}{(2\pi)^{3/2}} \int e^{-ikx} \frac{d\vec{k}}{\sqrt{2k_o}}\, \alpha(\vec{k}) = \tag{A.1}$$

$$= \varphi^{(+)}(x) + \varphi^{(-)}(x) \quad ,$$

where

$$\left[\,\alpha(\vec{k}), \alpha^{+}(\vec{k}')\right] = \delta(\vec{k}-\vec{k}') \quad . \tag{A.2}$$

b) Pairing of two φ-operators

$$\underbrace{\varphi(k)\,\varphi}(k') = \delta(k+k')\,\Delta^{(+)}(k') =$$

$$= \delta(k+k')\,\Theta(k'_o)\,\delta(k'^2 - m^2) \, . \tag{A.3}$$

2) <u>Spinor field</u>

a) $$\left(i\frac{\partial\!\!\!/}{\partial\!\!\!/ x} - M\right)\Psi(x) = 0,$$

where $$\frac{\partial\!\!\!/}{\partial\!\!\!/ x} \equiv \gamma^0\frac{\partial}{\partial x^0} - \vec{\gamma}\cdot\frac{\partial}{\partial\vec{x}}\,, \tag{A.4}$$

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}; \quad \gamma_5 = -i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix};$$

$$\begin{aligned}\Psi(x) &= \frac{1}{(2\pi)^{3/2}}\int e^{iqx}\Psi(q)\,d^4q = \\ &= \frac{1}{(2\pi)^{3/2}}\sum_{\nu=1,2}\int\frac{d\vec{q}}{\sqrt{2q_0}}\,e^{iqx}\,b^+_\nu(\vec{q})\,v^\nu(\vec{q}) + \\ &+ \frac{1}{(2\pi)^{3/2}}\sum_{\nu=1,2}\int\frac{d\vec{q}}{\sqrt{2q_0}}\,e^{-iqx}\,a_\nu(\vec{q})\,u^\nu(\vec{q}) \equiv \\ &\equiv \Psi^{(+)}(x) + \Psi^{(-)}(x)\end{aligned} \tag{A.5}$$

$$\begin{aligned}\overline{\Psi}(x) = \Psi^+(x)\gamma^0 &= \frac{1}{(2\pi)^{3/2}}\int e^{ipx}\,\Psi^+(-p)\,\gamma^0\,dp = \\ &= \frac{1}{(2\pi)^{3/2}}\sum_{\nu=1,2}\int\frac{d\vec{p}}{\sqrt{2p_0}}\,e^{ipx}\,a^+_\nu(\vec{p})\,\overline{u}^\nu(\vec{p}) + \\ &+ \frac{1}{(2\pi)^{3/2}}\sum_{\nu=1,2}\int\frac{d\vec{p}}{\sqrt{2p_0}}\,e^{-ipx}\,b_\nu(\vec{p})\,\overline{v}^\nu(\vec{p}) \equiv \\ &\equiv \overline{\Psi}^{(+)}(x) + \overline{\Psi}^{(-)}(x)\,,\end{aligned} \tag{A.6}$$

where

$$\left\{ a_\mu(\vec{q}),\ a_\nu^+(\vec{q}\,') \right\}_+ = \delta_{\mu\nu}\,\delta(\vec{q}-\vec{q}\,')$$

$$\left\{ b_\mu(\vec{q}),\ b_\nu^+(\vec{q}\,') \right\}_+ = \delta_{\mu\nu}\,\delta(\vec{q}-\vec{q}\,')\ .$$

b) <u>Pairings of two spinor operators</u>

$$\underbrace{\psi_\beta(q)\overline{\psi}_\alpha(p)} = \delta(q+p)\ S^{(+)}_{\beta\alpha}(p,M) =$$

$$= \delta(q+p)\,\theta(p^0)(\not p+M)_{\beta\alpha}\,\delta(p^2-M^2) \tag{A.7}$$

$$\underbrace{\overline{\psi}_\alpha(p)\,\psi_\beta(q)} = \delta(p+q)\ S^{(+)}_{\beta\alpha}(q,-M) = \tag{A.8}$$

$$= \delta(p+q)\ \theta(q^0)(\not q-M)_{\beta\alpha}\,\delta(q^2-M^2)\ ,$$

where

$$\overline{\psi}(p) \equiv \psi^+(-p)\,\gamma^0$$

c) <u>Normalization and orthogonality relations and completeness conditions for the spinors u and v</u>

$$\bar u^\mu_\alpha(\vec{q})\,u^\nu_\alpha(\vec{q}) = 2M\,\delta^{\mu\nu} \tag{A.9}$$

$$\bar v^\mu_\alpha(\vec{q})\,v^\nu_\alpha(\vec{q}) = -2M\,\delta^{\mu\nu}$$

$$\sum_{\mu=1,2} \bar{u}^{\mu}_{\alpha}(\vec{q})\, u^{\mu}_{\beta}(\vec{q}) = (\not{q} + M)_{\beta\alpha}$$

$$\sum_{\mu=1,2} v^{\mu}_{\alpha}(\vec{q})\, \bar{v}^{\mu}_{\beta}(\vec{q}) = (\not{q} - M)_{\beta\alpha}\,, \qquad \text{(A.10)}$$

where

$$\bar{v}(\vec{q}) = v^{\dagger}(\vec{q})\gamma^{0}\,; \quad \bar{u}(\vec{q}) = u^{\dagger}(\vec{q})\gamma^{0}.$$

d) Explicit form of u and v in terms of two-component quantities

$$u(\vec{q}) = \sqrt{q_0+M}\begin{pmatrix} \varphi \\ \dfrac{\vec{\sigma}.\vec{q}}{q_0+M}\,\varphi \end{pmatrix} \qquad \text{(A.11)}$$

$$v(\vec{q}) = \sqrt{q_0+M}\begin{pmatrix} \dfrac{\vec{\sigma}.\vec{q}}{q_0+M}\,\xi \\ \xi \end{pmatrix}, \qquad \text{(A.12)}$$

where $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ and $\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$ are the spin wave functions normalized to unity.

e) Charge-conjugated spinors

$$u^{c} = C\,\bar{v}^{T} \qquad \text{(A.13)}$$

$$\bar{u}^{c} = v^{T}(C^{T})^{-1}$$

$$C = \gamma_0\gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

The wave functions $u^c(\vec{q})$ can also be written in the form:

$$u^c(\vec{q}) = \sqrt{q^0+M} \begin{pmatrix} \chi \\ \dfrac{\vec{\sigma}\cdot\vec{q}}{q_0+M}\chi \end{pmatrix}, \tag{A.14}$$

where

$$\chi = \left(\xi^+ \sigma_2\right)^T. \tag{A.15}$$

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