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Abstract

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On a Research Programme about the Teaching and Learning of Linear Algebra in First Year of French Science University

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Abstract: In this paper, we present the current teaching of linear algebra in first year of French science university and the main difficulties of the students. Then we give a brief account of our first steps towards the design of a teaching experiment: from a joint didactical and historical survey we draw out our first hypotheses : epistemological specificity, use of 'meta-lever', use of changes of settings and points of view, importance of the concept of rank. We present briefly the main aspects and objectives of the teaching design experimented over a whole teaching semester with around 200 students during five years. Finally, we explain the type of evaluations that have set up and the difficulties we encountered. Our conclusion deals with issues on the teaching and learning of linear algebra as well as issues on methodological and theoretical points in relation with the original didactical framework we have elaborated.

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I. General Presentation of the Research

During the period of modern mathematics, in France, the theory of vector spaces was taught in secondary schools. Today vector spaces have completely disappeared from teaching in secondary schools. In the teaching of geometry, although the fundamentals of Cartesian and vectorial geometry are still taught, the approach is essentially synthetic. For instance, transformations are introduced through the study of their action on figures, rather than through their properties as transformations on a set of points with structural features. On the

other hand, the teaching in secondary school has become less formal in general: there is no study of algebraic structures as such, the formal definition of a limit in analysis is replaced by theorems of comparison with elementary functions. Students have therefore no practice of elementary logic or set theory in their mathematical curriculum in French secondary schools. Finally, the study of systems of linear equations has also changed; determinants and Cramer's rule have disappeared, and Gaussian elimination is now taught as the method for solving such systems.

The theory of vector spaces is taught in the first year of all French science universities, and it represents more or less a third of the mathematical content. Traditionally, this teaching starts with the axiomatic definition of a vector space and finishes with the diagonalisation of linear operators. In a survey, Robert and Robinet (see [1-2]) showed that the main criticisms made by the students toward linear algebra concern the use of formalism, the overwhelming amount of new definitions and the lack of connection with what they already know in mathematics. It is quite clear that many students have the feeling of landing on a new planet and are not able to find their way in this new world. On the other hand, teachers usually complain of their students' erratic use of basic tools of logic or set theory. They also complain that students have no skills in elementary Cartesian geometry and consequently cannot use intuition to build geometrical representation of the basic concepts of the theory of vector spaces. In several universities, the teachers decided to set up prior teaching in Cartesian geometry or/and logic and set theory, but these attempts at remediation did not improve the situation substantially. The general attitude of teachers consists more often of a compromise: there is less and less emphasis on the most formal part of the teaching (especially at the beginning) and most of the evaluation deals with the algorithmic tasks connected with the reduction of matrices of linear operators. However, this leads to a contradiction which cannot satisfy us. Indeed, the students may be able to find the Jordan reduced form of an operator, but, on the other hand, suffer from severe misunderstanding on elementary notions such as linear dependence, generators, or complementary subspaces.

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In response to this situation, we have developed a research programme on the learning and teaching of linear algebra in the first year of French science universities. This work[1], which started some ten years ago, includes the elaboration and evaluation of experimental teaching based on a substantial historical study and a theoretical approach within the French context of 'didactique des mathématiques', and in collaboration with other teams working on the teaching of linear algebra, in France and abroad (see [4-15]).

At first, we made several analyses of students' works in ordinary teaching; we also created tests and interviewed students and teachers in order to better understand the meaning of what we have called the 'obstacle of formalism' (see ([3] part II-1) and [16]). We also investigated the history of linear algebra through the epistemological analysis of original texts from the 17th century up to recent developments (see ([3] part I) and [17]). From this first stage of our research, we have drawn a first conclusion, which we will call the *fundamental epistemological hypothesis* :

- The theory of vector spaces is a very recent theory that emerged in the late 19th century but only spread in the 1930s. At this stage, it became widely used not so much because it allowed new problems to be solved (for instance linear functional equations were solved with use of the theory of determinants generalized to infinite dimensions) but mostly because it was a way of unifying different methods and tools used in different contexts and generalizing them (this was necessary because of the rapid increase in the number of new mathematical results). Therefore linear algebra is a unifying and generalizing theory. As a consequence, it is also a formal theory; it simplifies the solving of many problems, but the simplification is only visible to the specialist who can anticipate the advantage of generalization because they already know many contexts in which the new theory can be used. For a beginner, on the other hand, the simplification is not so clear as the cost of learning many new definitions and theorems seems too great with regard to the use she or he can make of the new theory in

[1] The results of this work are gathered in a book [3] which also includes the presentation of other works (in France, Canada, USA and Morocco).

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contexts in which solving systems of linear equations is usually quite sufficient. In other words, the unifying and generalizing nature of linear algebra has a didactical consequence : it is difficult to motivate the learning of the new theory because its use will be profitable only after it may have been applied to a wide range of situations.

In order to make the introduction of the theory of vector spaces more meaningful for students, we have set up a teaching experiment, that will be shortly described below (see ([3] part II-3) and [18-21]).

On the other hand, we have used our epistemological analysis of the historical context as a means of better understanding the mistakes of the students, not only through Bachelard's epistemological obstacles but also in order to give more significance to the bare simplicity of formal modern definitions. The historical context provides us with possible ways of access to the knowledge but it is neither the best nor the shortest in terms of psychological cost. If history is a privileged source of inspiration for building teaching situations, it is not sufficient by itself, the historical context has to be adapted to the didactical context which is usually very different. However, the historical source has to be as complete as possible (we do not trust second hand summaries). In our work, we have explored in particular the role of the study of systems of linear equations and of geometry.

Moreover, besides specific results concerning the teaching and learning of linear algebra, our research led to the development of specific tools for the analysis of teaching experiments at university level. It pays particular attention to long term projects which induce new difficulties in particular with regard to the evaluation of experimental results. This is because the effect of a didactical device may only be visible a few weeks after it has been operative.

In this paper we will shortly present the teaching experiment, starting by the didactical hypotheses that we wanted to test. Then we will describe the type of evaluation we made and the difficulties we encountered.

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II. The Teaching Experiment

1. First hypothesis

Our first analyses led us to the hypothesis that the teaching of linear algebra needs to be built on several interwoven and long-term strategies, using the 'meta lever' as well as the change of settings (including within mathematics) and of points of view, in order to obtain substantial modifications for a sufficient number of students.

By long-term strategy (see [22]), we mean a type of teaching which cannot be divided up. The long-term is vital because the mathematical preparation and the changes in the didactical contract[2] have to operate over a long enough period to be efficient for the students, in particular regarding the evaluation. Finally the long-term is necessary in order to take into account of the non linearity of the teaching due to changes of points of view, which implies working on a subject matter more than once.

By 'meta lever', we mean that the teacher's use of information about students' sense of self-knowledge in mathematics or about their way of learning; or about mathematics (organization, different uses in other fields, differentiation between general and particular fields, etc.); or about mathematics as it is taught (for instance the use of methods, the different types of questioning in mathematics, how to organize a reflection on concepts, etc.). Moreover, we claim that this type of information used by the teacher must be followed or accompanied by activities involving its pertinent use by the students. We use what we called the 'meta lever'. 'Meta' means that a reflexive attitude from the student on his mathematical activity is expected, and 'lever' points out something which has to be used at the right moment in the right place to help the student get into this reflexive attitude while achieving a mathematical task which has been prepared carefully. The general idea is to put the students in a mathematical activity that can be solved by them and from there, to make them analyse,

[2] This term designates the rules established, most often implicitly, between the teacher and the students with regard to what is expected in the teaching.

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in a reflexive attitude, some possibilities of generalization or unification of the methods they have developed by themselves (see ([3], Part II-4) and [23-24]).

By change of settings or points of view (see [25]), we mean that the teaching is organized in such a way that the course and the exercises emphasize the translations of the same concept or question from one setting into another (from formal to numerical, from numerical to geometrical, etc.) or lead the students to a change of point of view on a notion (for instance seeing the linear equation $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0$ as the n-tuple $(a_{11}, a_{12}, \ldots, a_{1n})$, in order to go from the use of linear combinations of equations to the notion of rank of a set of n-tuples, etc.).

This hypothesis stated in II.1 is the main basis of our work: it led us to the building of a complete teaching project over one teaching semester. It has been tested during six years with several colleagues, for an audience of around 200 students.

2. Choices of strategy resulting from our didactical analysis

(a) One way of taking into account the specific epistemological nature of concepts is to introduce, at a favorable and precise time of the teaching, some activities which induce a reflection with use of the 'meta lever'. Such reflection should, on the one hand, clarify the specificity of the epistemological nature of these concepts (what is, in general, a linear equation ? What is the relation between the rank of vectors and the rank of equations ? etc.). On the other hand, it should create activities in which the students reflect on certain mathematical concepts (role of the axioms of vector-space, the two points of view used to represent a subspace of $\mathbb{R}^n$: equations and parameters, etc.). Finally, it should give students indicators or methods which can help to solve certain types of problems (which methods can I use to show the identity between two subspaces ? to determine the image and the kernel of an operator ? etc.).

(b) As a complement, it would be useful to set up a fairly long preliminary phase preceding the actual teaching of elementary concepts of linear algebra, which would prepare the

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students to understand, through 'meta' activities and presentations, the unifying role of these concepts. This means that various problems should be solved at a preliminary stage and then unified with use of vector-space theory, in a way that points out the simplification provided by the above solving.

(c) As far as possible, the changes of settings and points of view are put forward. This is especially the case in the beginning with geometrical and algebraical settings which allow the extension of the double point of view equation/parameters to $\mathbb{R}^n$. Moreover, in general, this is essential when linear algebra is a way to give a model in term of vector spaces of a problem which is issued from other parts of mathematics, this is what we call "intra-mathematical modeling.

(d) Finally, we chose to give the concept of rank a central position in our teaching. Indeed historical and epistemological works showed the importance of this concept as well as the difficulties encountered by students with this concept. This led us to give the theory of linear equations a privileged situation in the preliminary phase, as rank can be introduced quite naturally in this field.

It is important to see that our project does not induce a 'smooth progression' : $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^n$, general linear algebra. Even if the didactical difficulty increases, certainly, with the dimension and the abstraction, we think that the main difficulty is qualitative rather than quantitative. Therefore, although the organization we chose aims at putting forward similarities and changes of settings and points of view, it also aims at provoking a reflection on what is at stake in the new theories and at giving access to methodological processes which allow a better understanding of the concepts. Thus a paradox might arise : to obtain better understanding, we develop more demanding strategies, which is likely to induce, at least temporarily and apparently, worse scores during the evaluation.

3. The overall organization of the teaching

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(a) First step: The development of preliminary phases which include the organization of the changes of settings in order to facilitate the convergence of different points of view.

At the beginning of the course, we set up the 'circuit éléctrique' activity (see [26]), which aims to help students understand the rules of mathematical reasoning. We also give the basic language of set-theory, and some experience with space geometry.Then, we introduce during the first course Gauss' method of elimination for solving systems of *m* linear equations with *n* unknowns, by using $\mathbb{R}^n$ and its linear structure, as the set of reference in which the solutions must be found. This straight away gives a powerful tool for solving problems in algebra as well as in plane or space geometry. Moreover, this introduces what will be, for us, one of the central themes in linear algebra: the solving of systems of linear equations, from which the concepts of subspaces of $\mathbb{R}^n$, of rank, of double point of view equations/parameters will be drawn. The basic questions, which we try to induce our students to reflect upon and which will justify the introduction of the concepts of linear algebra are:

* do I have too many, just the right number, not enough equations ?

* How many parameters do I need to describe the set of solutions of a system of linear equations ?

We continue the teaching with Cartesian geometry in $\mathbb{R}^3$, with this double aspect equations/parameters as a central question.

(b) Second step: This starts by an explicit presentation to the students of all the common questions that can be asked about the preceding phase. This also implies the change of point of view consisting in seeing an equation as a n-tuple. Then we define linear independence and rank and we show the invariance of rank using the reversibility of certain linear combinations (as done in the exchange theorem). This is followed by the description, with parameters, of subspaces of $\mathbb{R}^n$, initially defined by their equations, and vice versa. Then, we explore the notions of dimension and bases of subspaces. In this phase, some results on the systems of linear equations, which were only conjectures in the first phase, are proved. The fact that the row and column rank of an array are the same is also proved.

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The proofs of fundamental results are made with use of abstract formulation, avoiding the use of coordinates. This is justified to the students by a systematic use of algebraic thinking (helped by some preliminary activities involving elements of group theory and illustrated byn some concrete finite groups). Another justification we use is the search for generality in order to ease the transition to further developments in abstract linear algebra.

Finally, we give some elements of methodology concerning the solving of problems about subspaces in $\mathbb{R}^n$: inclusion, equality, intersection, parameters and equations of subspaces.

During this phase we also apply linear methods in other fields : polynomials, linear recurrent series, linear differential equations.

(c) Third step: Here we teach abstract linear algebra (axiomatic theory of finite dimensional vector-spaces, linear operators) and its application as a modeling setting within mathematics. At the same time, we draw out several methods and point out some prototypical problems. These problems were solved during the previous phases, but they are now solved again in the setting of formal algebra. They have to be rich enough so that their generalization makes the use of the formal concepts almost compulsory (e.g. one can give 15 conditions for an interpolation, instead of the usual 3 or 4). The "contract" with the students is to model the problem in linear algebra terms, e.g. the problem has to be expressed in terms of a linear equation 'T(u) = v'. In this phase, problems are often raised in connection with other parts of the mathematical curriculum, outside linear algebra (e.g. differential equations).

(d) Fourth step: This phase is more technical and also shorter. It presents the matrices, the techniques associated with the change of bases and the inversion of square matrices. Of course, the tool 'matrix', introduced in relation to linear operator, is then used in several problems in relation to various other fields of mathematics. But matrix calculus is not an important goal for us.

III. The Evaluation and its Difficulties

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Essentially, we have to know whether the didactical engineering[3] is viable and does not, in fact, lead to an unacceptable number of failures.

More precisely we have to evaluate our specific hypotheses:

(a) The role of the 'meta lever'

We have to find some indicators that students use 'meta' themselves: What is the students' understanding of the nature of the concepts introduced ? What kind of questioning can they have about a given mathematical situation ? More precisely, do they use geometrical representations, do they check the coherence between the number of parameters and the number of equations, or between the dimension and the maximum number of independent vectors, etc. ? This led us to give the students situations in which the first goal is a form of questioning or checking.

(b) The changes of settings and points of view.

We must know if the notion of 'change of setting' itself has been understood, and in which way it is available or even just evoked. Is it used in problems involving modeling in terms of linear algebra, and how? Are the changes of points of view helpful to obtain better understanding? Do they not induce complications, at least for a certain time?

(c) The teaching of methods

We must evaluate its impact, especially by giving problems which have to be preceded by a reflection on the different types of methods that can be used to solve them. We also have to find ways of making students write or speak about this preliminary reflection.

It is clear from what we have explained of our experiment that the results of the evaluation cannot be as simple as 'our teaching design makes students understand linear algebra better'. As we tried to make fundamental changes to long term cognitive processes, classical evaluation designs would not be relevant. Before wondering about the results of the

[3] This term designate not only the description of the teaching experiment, but also the didactical reflection that led to it and what is esxpected in terms of learning.

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evaluation, we had to find out what aspects had to be evaluated and how these could be ascertained. Inevitably that led us to theoretical and practical difficulties. For a precise report of the difficulties, we refer to Dorier et al. (see [27]), here, we will give a short overview, listed in three points:

1) For an engineering involving long-term teaching, it is difficult to choose the appropriate time for its evaluation, as there are uncontrollable elements due to students' own organization of their time and work. Thus, phenomena of maturing, which are dependent on students' level of involvement (variable along the year) are difficult to take into account in the evaluation of the engineering.

2) It is quite difficult to trace indicators of students' reflection at a 'meta' level, especially in their writing: indeed in the usual teaching, the contract is to give the answer, never the means which led to it.

3) Several background "noises" often hide the phenomena we want to study. These include:

* whether or not the students have already worked on a problem of the type used in the evaluation;

* the variability in what is done during the tutoring;

* the recent democratization of university teaching and changes in secondary teaching; these two phenomena make the sociological impact sometimes more relevant than any didactical explanation.

Therefore, our next goal is to find a better means of access to the didactical variables we want to study, as it is difficult in the present state to come to entirely reliable conclusions.

Our conclusion will stress the most positive aspects of our experiment but we also want to point out its limitations. First of all, it is clear that the experiment is viable in the sense that our student had a good success rate at their final exam. Nevertheless this type of evaluation, even if it is essential, is not sufficient on a didactical point of view. A more precise evaluation shows that the double point of view equations/parameters in $\mathbb{R}^n$ is acquired

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by a significant proportion of students who are able to use it efficiently in order to solve problems. Furthermore, the notion of rank of a set of vector is understood and available for most students. It also seems that it is possible to make most students involve themselves in situations using the 'meta' lever, in which the main goal is epistemological as much as mathematical (see [17] and [28]). Nevertheless the impact on the learning is technically difficult to evaluate even if it seems to be positive. On the other hand, the chances of autonomous questioning by the students is very infrequent; and spontaneous modeling of a problem in the setting of linear algebra can only be expected from a minority of students.

It takes long to make students' practice change in such a radical way as expected by the introduction of meta lever actvities. However, our experiment leads us to think that it is possible, if the emphasis is stronger and cover different parts of the curriculum. Then, the impact of our didactical engineering, as it has been sketched here, could be much wider.

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