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On a Superfield Theoretical Treatment of the Higgs-Kibble Mechanism

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The superfield theoretical aspect of the Higgs-Kibble mechanism is discussed by realizing the supersymmetry of Becchi, Rouet and Stora (BRS) as translation and dilatation operations on the real elements of the Grassmann algebra. The Slavnov-Taylor identity becomes a statement on each connected Green's function separately in this formulation, and some of the pathological cases discussed in the literature can be avoided by imposing the manifest BRS covariance in terms of the superfields. This is applied to the discussion of non-linear (quadratic) gauge conditions. The manifestly BRS covariant treatment of non-linear gauge conditions requires a quartic self-coupling of Faddeev-Popov fields to ensure the multiplicative renormalization, although the physical S-matrix is independent of this ghost coupling. The proof of unitarity (i.e., the ghost cancellation) follows the same mechanism as the divergence cancellation in the ordinary superfield theory. The gauge independence is proved by a simple classification of operators according to their BRS transformation properties. We also briefly comment on the canonical treatment of superfields.

§ 1. Introduction

The renormalization of the Yang-Mills fields[1] and the Higgs-Kibble mechanism[2] has been extensively discussed in the literature.[3,4] The Faddeev-Popov Lagrangian[5] provides a starting point for these discussions of the renormalizability and unitarity. In connection with the Faddeev-Popov Lagrangian, Becchi, Rouet and Stora (BRS)[6] introduced a supersymmetry pseudo-algebra which gives rise to a very neat way to derive the Slavnov-Taylor identity[7] (i.e., the Ward-Takahashi identity associated with the BRS symmetry). It was also recognized that a detailed study of the BRS symmetry is sufficient to investigate the unitarity of the S-matrix elements.[8] Recently, Kugo and Ojima[9] further clarified the contents of the BRS symmetry and the structure of the physical Hilbert space. They made the following important observations: (i) The auxiliary Lagrangian multiplier field makes the BRS symmetry manifest, and (ii) the ghost number of the Faddeev-Popov fields should be properly associated with the scale transformation of the ghost fields. Although the latter point is not crucial in the perturbative treatment[10] of the Yang-Mills fields, the structure of the physical space is much simplified by this interpretation. It is also nicer to keep the Lagrangian manifestly Hermitian at all the stages of the calculation.

In this paper, we discuss the superfield theory[11)~14)] associated with the BRS symmetry. Some of the initial attempts[15] toward this direction have been made, but our approach differs from the previous attempts in that we treat the superfield as an element of the theory as much as possible.[13),14)] Although the superfield notation in the present case is not quite useful for practical calculations due to the non-linear realization of the supersymmetry, it is useful as a means to keep track of the very subtle BRS symmetry. Consequently, some of the "pathological" cases found in the literature[6),16)] can be avoided at the level of formalism.

§ 2. Superfields for BRS symmetry

We take the following pseudo-algebra as our starting point:

$$\begin{aligned}&\{Q, Q\}=0\,,\\&[D, Q]=-iQ\,,\\&[D, D]=0\,,\end{aligned}\tag{2·1}$$

where Q is the Hermitian generator of the *proper* BRS symmetry,[6)] and D is the generator of the dilatation for the ghost fields.[9)] Equation (2·1) can be converted into the ordinary algebra when multiplied by the real elements λ_1 and λ_2 of the Grassmann algebra (i.e., the anti-commuting c-numbers):

$$\begin{aligned}&[\lambda_1 Q, \lambda_2 Q]=0\,,\\&[D, \lambda_1 Q]=-i\lambda_1 Q\,,\\&[D, D]=0\,.\end{aligned}\tag{2·2}$$

Note that Q and the elements of the Grassmann algebra all anti-commute. We understand the algebra (2·2) as associated with the following operations on the *real elements* θ of the Grassmann algebra:

$$\begin{aligned}&Q: \theta\to\theta+\lambda\,,\\&D: \theta\to e^{\rho}\theta\,,\end{aligned}\tag{2·3}$$

where λ is again a real element of the Grassmann algebra and ρ a real number.

The basic idea of the superfield[12)] is to realize the algebra (2·2) as operations on the fields defined in the fictitious five-dimensional space (x_μ, θ), e.g., $\eta^a(x, \theta)$. Here the suffix a stands for a global symmetry (we take it to be a semi-simple group in the following). Following the discussion of the ordinary conformal symmetry,[17)] we define

$$U(\lambda)\,\eta^a(x,\theta)\,U(\lambda)^\dagger\equiv\eta^a(x,\theta+\lambda)\,,\tag{2·4}$$

$$U(\rho)\,\eta^a(x,\theta)\,U(\rho)^\dagger\equiv e^{d\rho}\eta^a(x,e^{\rho}\theta)\,,\tag{2·5}$$

where the unitary operators are defined by

$$\begin{aligned}&U(\lambda)\equiv e^{\lambda Q}\,,\\&U(\rho)\equiv e^{i\rho D}\,.\end{aligned}\tag{2·6}$$

Note that $U(\lambda)^\dagger = e^{Q\lambda} = e^{-\lambda Q} = U(\lambda)^{-1}$ as Q and λ anti-commute. We call d in (2·5) the BRS dimension characteristic of the superfield. The general form of the superfield can be written as

$$\eta^a(x,\theta) \equiv \eta^a(x) + \theta C^a(x), \tag{2·7}$$

where $\eta^a(x)$ and $C^a(x)$ are the ordinary fields, as a power series expansion of $\eta^a(x,\theta)$ terminates at the linear term in θ (note that $\theta^2=0$). Equation (2·4) then indicates

$$\begin{aligned} &U(\lambda)\eta^a(x)U(\lambda)^\dagger = \eta^a(x) + \lambda C^a(x),\\ &U(\lambda)C^a(x)U(\lambda)^\dagger = C^a(x), \end{aligned} \tag{2·8}$$

namely, the second component of a superfield is invariant under the *proper* BRS symmetry. Equation (2·5) gives

$$\begin{aligned} &U(\rho)\eta^a(x)U(\rho)^\dagger = e^{d\rho}\eta^a(x),\\ &U(\rho)C^a(x)U(\rho)^\dagger = e^{(d+1)\rho}C^a(x). \end{aligned} \tag{2·9}$$

In our approach we define the transformation laws by the right-hand sides of these equations, but these relations also become important when one attempts to express the generators Q and D in terms of the quantized component fields in the canonical approach.[9]

It turns out that the superfields with $d=$odd follow the Fermi statistics and with $d=$even the Bose statistics (the second component of a superfield obeys the opposite statistics due to the presence of θ). In our application, the superfields with $d=\pm1$ and 0 are important.

For the *Hermitian* field with $d=1$ belonging to an adjoint representation of some semi-simple group, we can construct a non-linear realization defined by[6]

$$\begin{aligned} \eta^a(x,\theta) &\equiv \eta^a(x) + i\theta(g/2)f^{abc}\eta^b(x)\eta^c(x)\\ &= \eta^a(x) + i\theta(g/2)f^{abc}\eta^b(x,\theta)\eta^c(x,\theta), \end{aligned} \tag{2·10}$$

where f^{abc} is the real, anti-symmetric structure constant for the semi-simple group, and g a fundamental *dimensionless* coupling constant. It is confirmed that (2·10) can be arranged to satisfy the basic requirements (2·8) and (2·9). We also need a Hermitian field*) with $d=-1$

$$\xi^a(x,\theta) \equiv \xi^a(x) + \theta B^a(x). \tag{2·11}$$

Note that the second component of a superfield with $d=-1$ is scalar under the full BRS symmetry (2·8) and (2·9). This provides a means to construct an invariant action later.

The composition law of two superfields is simply given by a *local* product of two superfields, and it gives rise to transformation laws for a new superfield. For example,

*) Note that $\eta^a(x)$ (2·10) and $\xi^a(x)$ (2·11) correspond to $c^a(x)$ and $\bar{c}^a(x)$ in the customary notation, respectively.

$$\xi(x,\theta)\partial_\mu\eta^a(x,\theta)$$
$$=\xi(x)\partial_\mu\eta^a(x)+\theta[B(x)\partial_\mu\eta^a(x)-igf^{abc}(\xi(x)\partial_\mu\eta^b(x))\eta^c(x)], \quad (2\cdot12)$$

where $\xi(x,\theta)\equiv\xi(x)+\theta B(x)$ is a superfield with $d=-1$ and without any group index, and $\eta^a(x,\theta)$ is defined in $(2\cdot10)$. If one sets $B(x)=\text{const}=1$ (a sort of spontaneous breakdown of BRS symmetry) and if one replaces $i\xi(x)\partial_\mu\eta^a(x)$ by $A_\mu{}^a(x)$, one obtains a superfield with $d=0$

$$A_\mu{}^a(x,\theta)=A_\mu{}^a(x)+i\theta[\partial_\mu\eta^a(x)-gf^{abc}A_\mu{}^b(x)\eta^c(x)]. \quad (2\cdot13)$$

We also construct two kinds of scalar superfields with $d=0$ from $\xi^a(x,\theta)$ in $(2\cdot11)$ and $\eta^a(x,\theta)$ in $(2\cdot10)$

$$\xi^a(x,\theta)\eta^a(x,\theta)=\xi^a(x)\eta^a(x)-i\theta(g/2)f^{abc}\xi^a\eta^b\eta^c+\theta B^a\eta^a,$$
$$f^{abc}\xi^b(x,\theta)\eta^c(x,\theta)=f^{abc}\xi^b(x)\eta^c(x)-i\theta(g/2)f^{abc}f^{cde}\xi^b\eta^d\eta^e+\theta f^{abc}B^b\eta^c. \quad (2\cdot14)$$

For the group $SU(2)$ where we have the relation

$$f^{abc}f^{cde}=f^{adc}f^{cbe}+\delta^{ad}\delta^{be}-\delta^{ab}\delta^{de}, \quad (2\cdot15)$$

$(2\cdot14)$ gives rise to interesting transformation laws of "composite" superfields. If one chooses a special kind of superfield $\xi^a(x,\theta)$ with $B^a(x)\equiv0$ (this is legitimate as the second component of $\xi^a(x,\theta)$ with $d=-1$ is BRS scalar), and if one identifies $i\xi^a(x)\eta^a(x)$ and $if^{abc}\xi^b(x)\eta^c(x)$ with $\phi(x)$ and $\phi^a(x)$ respectively, one obtains

$$\phi(x,\theta)=\phi(x)-i\theta(g/2)\phi^a(x)\eta^a(x), \quad (2\cdot16)$$
$$\phi^a(x,\theta)=\phi^a(x)+i\theta[-(g/2)f^{abc}\phi^b(x)\eta^c(x)+(g/2)\phi(x)\eta^a(x)]. \quad (2\cdot17)$$

These superfields $(2\cdot13)$, $(2\cdot16)$ and $(2\cdot17)$ give rise to the BRS transformation laws for the vector and scalar sector of the Weinberg-Salam model without photons and Fermions.[2] We adopt this simplest Higgs-Kibble model as our example in the following discussion. It is important to notice that the coupling constant g in $(2\cdot10)$ controls all the transformation laws of other superfields.

Before we proceed further we fix the notation. The derivative ∂_θ is defined as the left-derivative. For example,

$$\partial_\theta\xi^a(x,\theta)=B^a(x). \quad (2\cdot18)$$

The integration[18] is defined as identical to the left-derivative, and it satisfies the property expected for the integration, e.g.,

$$\int d\theta\xi^a(x,\theta)=\int d\theta\,\xi^a(x,\theta+\lambda)=B^a(x). \quad (2\cdot19)$$

The (proper) BRS invariant component of an arbitrary superfield is projected out by the operation

$$\int d\lambda\,U(\lambda)\eta^a(x,\theta)U(\lambda)^\dagger=\int d\lambda\,\eta^a(x,\theta+\lambda)=C^a(x) \quad (2\cdot20)$$

for the superfield of the form $(2\cdot7)$. This property is utilized later when we construct the BRS invariant states. The δ-function is defined by

$$\delta(\theta_1-\theta_2)\equiv-(\theta_1-\theta_2) \tag{2·21}$$

which satisfies the relation

$$\int d\theta_2\delta(\theta_1-\theta_2)f(x,\theta_2)=f(x,\theta_1). \tag{2·22}$$

§ 3. Slavnov-Taylor identities

3.1. *The model*

The invariant Lagrangian which consists of $A_\mu^a(x,\theta)$, $\phi^a(x,\theta)$ and $\psi(x,\theta)$ in (2·13)$\sim$(2·17) can be written as usual

$$\mathcal{L}_{\rm inv}(x,\theta)\equiv-\tfrac{1}{4}[\partial_\mu A_\nu^a(x,\theta)-\partial_\nu A_\mu^a(x,\theta)-gf^{abc}A_\mu^b(x,\theta)A_\nu^c(x,\theta)]^2$$
$$+|\nabla_\mu S(x,\theta)|^2-\lambda(|S(x,\theta)|^2-v^2/2)^2, \tag{3·1}$$

where

$$S(x,\theta)\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}\psi(x,\theta)+i\phi^3(x,\theta)\\ -\phi^2(x,\theta)+i\phi^1(x,\theta)\end{bmatrix} \tag{3·2}$$

and

$$\nabla_\mu S(x,\theta)\equiv\partial_\mu S(x,\theta)-i(g/2)\tau^a A_\mu^a(x,\theta)S(x,\theta). \tag{3·3}$$

As is well-known, $\mathcal{L}_{\rm inv}$ is in fact independent of θ variables due to the local gauge invariance. We can therefore regard $\mathcal{L}_{\rm inv}$ as an identically BRS invariant superfield with $d=0$. The scalar sector of (3·1) is actually invariant under the larger $SU(2)\times SU(2)$ symmetry, and after the spontaneous symmetry breaking

$$\langle\psi(x)\rangle=v\neq0 \tag{3·4}$$

the residual diagonal $SU(2)$ survives. This residual symmetry simplifies the discussion of the renormalizability and unitarity, but we note that it is not essential in some of the points we want to make in the following.

After the spontaneous breaking (3·4), it is convenient to redefine the fields in terms of the components which have no vacuum values:

$$\psi(x,\theta)=\psi(x)-i\theta(g/2)\phi^a(x)\eta^a(x), \tag{3·5}$$
$$\phi^a(x,\theta)=\phi^a(x)+i\theta[(g/2)(v+\psi(x))\eta^a(x)-(g/2)f^{abc}\phi^b(x)\eta^c(x)] \tag{3·6}$$

and the Lagrangian (3·1) becomes

$$\mathcal{L}_{\rm inv}=-\frac{1}{4}[\partial_\mu A_\nu^a-\partial_\nu A_\mu^a-gf^{abc}A_\mu^bA_\nu^c]^2$$
$$+\frac{1}{2}\{(\partial_\mu\phi^a)^2+(\partial_\mu\psi)^2+(gv/2)^2(A_\mu^a)^2+(gv)\phi^a\partial^\mu A_\mu^a$$
$$-gA_\mu^a(\psi\partial^\mu\phi^a-\phi^a\partial^\mu\psi)+gf^{abc}\partial^\mu\phi^aA_\mu^b\phi^c$$
$$+g(gv/2)(A_\mu^a)^2\psi+(1/4)g^2(A_\mu^a)^2[\psi^2+(\phi^a)^2]\}$$

$$-\frac{\lambda}{4}\{(2v)^2\psi^2+4v\psi[\psi^2+(\phi^a)^2]+[\psi^2+(\phi^a)^2]^2\}. \qquad (3\cdot7)$$

Here we note the crucial difference between the ordinary local or global gauge symmetry and the present (operator) BRS symmetry. The BRS transformation property among the *shifted* fields (3·5) and (3·6) are preserved as long as $\langle\eta^a(x)\rangle=0$. On the other hand, $\langle\phi^a(x)\rangle\neq0$ (i.e., the degeneracy of the vacua) results under the ordinary (global) gauge transformation once $\psi(x)$ is shifted.

3.2. *The gauge fixing Lagrangian*

We next add the most general gauge fixing terms constructed from $\xi^a(x,\theta)$ (2·11) and $\eta^a(x,\theta)$ (2·10) in addition to $A_\mu{}^a(x,\theta)$ and $S(x,\theta)$, which transform covariantly under the BRS transformation with $d=-1$ and have the *canonical* dimensions less than or equal to 4.

$$\mathcal{L}_g\equiv\alpha\xi^a(x,\theta)\partial^\mu A_\mu{}^a(x,\theta)+\beta\xi^a(x,\theta)\phi^a(x,\theta)+(\gamma/2)\xi^a(x,\theta)\partial_\theta\xi^a(x,\theta)$$
$$+\delta\xi^a(x,\theta)\phi^a(x,\theta)\psi(x,\theta)+i(\epsilon/2)f^{abc}\xi^a(x,\theta)\xi^b(x,\theta)\eta^c(x,\theta). \qquad (3\cdot8)$$

Here we assign the canonical dimensions 2 to $\xi^a(x,\theta)$, 1 to $A_\mu{}^a(x,\theta)$, $\phi^a(x,\theta)$ and $\psi(x,\theta)$, and 0 to $\eta^a(x,\theta)$. The canonical dimension of $\eta^a(x,\theta)$ is suggested by (2·10) and (2·13) to be 0. The transition between ξ and η then fixes the canonical dimension of ξ to be 2 (note that $\xi^a(x)$ and $\eta^a(x)$ correspond to the ghost fields $\bar{c}^a$ and c^a in the customary notation,[3),6),9)] respectively).

The BRS invariant action is obtained from $\mathcal{L}_g$ as

$$S_g\equiv\int dxd\theta\,\mathcal{L}_g(x,\theta) \qquad (3\cdot9)$$

which can be written in terms of *component* fields as

$$S_g=\int dx\{\alpha B^a\partial^\mu A_\mu{}^a+\beta B^a\phi^a+(\gamma/2)B^aB^a$$
$$+\delta B^a\phi^a\psi+i\epsilon f^{abc}B^a\xi^b\eta^c\}$$
$$+i\int dx\{\alpha\partial^\mu\xi^a(\partial_\mu\eta^a-gf^{abc}A_\mu{}^b\eta^c)-\beta\xi^a[(g/2)(v+\psi)\eta^a-(g/2)f^{abc}\phi^b\eta^c]$$
$$-\delta\xi^a\psi[(g/2)(v+\psi)\eta^a-(g/2)f^{abc}\phi^b\eta^c]+\delta\xi^a\phi^a[(g/2)\phi^b\eta^b]$$
$$+i(\epsilon g/4)f^{abc}f^{cde}\xi^a\xi^b\eta^d\eta^e\}. \qquad (3\cdot10)$$

This is Hermitian if one remembers that ξ and η anti-commute.

3.3. *The quantization*

We add the source terms to the Lagrangian

$$\mathcal{L}_S=-A_\mu{}^a(x,\theta)J^{a\mu}(x,\theta)+\phi^a(x,\theta)J^a(x,\theta)+\psi(x,\theta)J(x,\theta)$$
$$+\xi^a(x,\theta)J_\xi{}^a(x,\theta)+J_\eta{}^a(x,\theta)\eta^a(x,\theta), \qquad (3\cdot11)$$

where we may formally assign the BRS dimensions $d=-1$ to $J_a{}^\mu(x,\theta)$, $J^a(x,\theta)$

and $J(x,\theta)$, $d=0$ to $J_\xi^a(x,\theta)$ and $d=-2$ to $J_\eta^a(x,\theta)$, respectively. The entire action is then given by

$$S\equiv\int dxd\theta[\delta(-\theta)\mathcal{L}_{\mathrm{inv}}(x,\theta)+\mathcal{L}_g(x,\theta)+\mathcal{L}_S(x,\theta)], \qquad (3\cdot12)$$

where $\mathcal{L}_{\mathrm{inv}}$ is *formally* regarded to be the second component of some superfield with $d=-1$; note that $\delta(-\theta)\equiv\theta$.

The quantization is performed by the path integral method

$$W[J]\equiv\int[dA_\mu{}^a(x)\,d\phi^a(x)\,d\psi(x)\,d\xi^a(x)\,d\eta^a(x)\,dB^a(x)]\,e^{iS}. \qquad (3\cdot13)$$

It can be confirmed that the functional measure here is invariant under the full BRS transformation defined by the right-hand sides of (2·8) and (2·9). The connected Green's functions are generated by $Z[J]\equiv(-i)\ln W[J]$.

3.4. *Slavnov-Taylor identities*

The generating functional (3·13) is identical to the ordinary prescription[4] including the source terms for all the composite fields associated with the BRS transformation. It is, therefore, straightforward to derive the ordinary Slavnov-Taylor (ST) identities.[7] In the following discussion, however, the *integral* form of the Ward-Takahashi identity[13] is more convenient.

Consider, for instance,

$$\langle(\xi^a(x,\theta_1)\phi^b(y,\theta_2))_+\rangle\equiv\int[dA]\,\xi^a(x,\theta_1)\phi^b(y,\theta_2)\,e^{iS}\Big/\int[dA]\,e^{iS}|_{J=0}\,. \qquad (3\cdot14)$$

Since the action and the measure are invariant under the BRS transformation, this Green's function satisfies

$$\begin{aligned}\langle(\xi^a(x,\theta_1)\phi^b(y,\theta_2))_+\rangle&=\langle(\xi^a(x,\theta_1+\lambda)\phi^b(y,\theta_2+\lambda))_+\rangle\\&=e^{-\rho}\langle(\xi^a(x,e^\rho\theta_1)\phi^b(y,e^\rho\theta_2))_+\rangle,\end{aligned} \qquad (3\cdot15)$$

if there is no spontaneous breaking of the BRS symmetry. The θ-dependence of the Green's function is completely specified by these relations. The first relation of (3·15), which is a consequence of the proper BRS invariance, indicates that the Green's function is a function of $\theta_1-\theta_2$ (i.e., translation invariance). The second relation indicates that the Green's function is a first power in θ (dilatation invariance). Consequently we have

$$\langle(\xi^a(x,\theta_1)\phi^b(y,\theta_2)_+\rangle=(\theta_1-\theta_2)f(x-y)\,\delta^{ab}\,. \qquad (3\cdot16)$$

This gives four relations in terms of component fields

$$\begin{aligned}&\langle(\xi^a(x)\phi^b(y))_+\rangle=\langle(B^a(x)\,(g/2)\,[\,(v+\psi(y))\,\eta^b(y)-f^{bcd}\phi^c(y)\,\eta^d(y)\,])_+\rangle=0\,,\\&\langle(B^a(x)\phi^b(y))_+\rangle=\langle(\xi^a(x)\,(g/2)\,[\,(v+\psi(y))\,\eta^b(y)-f^{bcd}\phi^c(y)\,\eta^d(y)\,])_+\rangle\\&\qquad=f(x-y)\,\delta^{ab}\,.\end{aligned} \qquad (3\cdot17)$$

These are the ordinary *ST* identities for Green's functions.[6] In fact, the ordinary

ST identities are identical to the "differential" form

$$\partial_\lambda \langle (\xi^a(x,\theta_1+\lambda)\phi^b(y,\theta_2+\lambda))_+\rangle = 0\,. \tag{3·18}$$

The integral form of the ST identity is useful in two respects; firstly, it enforces the BRS symmetry unambiguously, and secondly it allows us to understand the unitarity and gauge independence in an intuitive manner. This second point will be discussed later. As for the first point, we note that the BRS invariance indicates, for example,

$$\begin{aligned}\langle (\xi^a(x,\theta_1)\xi^b(y,\theta_2))_+\rangle &= \langle (\xi^a(x,\theta_1+\lambda)\xi^b(y,\theta_2+\lambda))_+\rangle \\ &= e^{-2\rho}\langle (\xi^a(x,e^\rho\theta_1)\xi^b(y,e^\rho\theta_2))_+\rangle \\ &= 0\,,\end{aligned} \tag{3·19}$$

as the translation invariant second power in θ is given by $(\theta_1-\theta_2)(\theta_1-\theta_2)\equiv 0$. This means

$$\langle (\xi^a(x)\xi^b(y))_+\rangle = \langle (B^a(x)\xi^b(y))_+\rangle = \langle (B^a(x)B^b(y))_+\rangle = 0\,. \tag{3·20}$$

This last relation shows that

$$\langle B^a(x)B^a(x)\rangle \neq 0 \tag{3·21}$$

gives rise to an explicit or spontaneous breaking of the BRS symmetry, although $B^a(x)$ itself is a BRS scalar (we can similarly show $\langle B^a(x)\rangle = 0$ without using the global symmetry).

This rather subtle aspect of the BRS symmetry is closely related to the several "pathological" cases discussed in the literature.[6),16)] As an example, we discuss the extra mass term discussed in Ref. 16)

$$\mathcal{L}_m(x,\theta) \equiv m_0^2[A_\mu{}^a(x)A^{a\mu}(x) - 2i(\gamma/\alpha)\xi^a(x)\eta^a(x)]\delta(-\theta) \tag{3·22}$$

which corresponds to the spontaneous breaking of the BRS symmetry. This term is not included in the present formalism where only the identically BRS invariant term $\mathcal{L}_{\text{inv}}$ and the manifestly BRS covariant terms $\mathcal{L}_g$ (3·8) are allowed. Equation (3·22) becomes BRS invariant up to a divergence only if one imposes the equation of motion on the B-field

$$\alpha\partial^\mu A_\mu{}^a(x) + \gamma B^a(x) + i\epsilon f^{abc}\xi^b(x)\eta^c(x) = 0 \tag{3·23}$$

with $\beta=\delta=0$ and $\epsilon=-g\gamma/2$ in $\mathcal{L}_g$ (3·9). This equation is not manifestly BRS covariant, and it is not used in the present formalism. We instead implement the corresponding relation among the manifestly BRS covariant Green's functions by

$$\begin{aligned}&\alpha\partial^\mu\langle (A_\mu{}^a(x,\theta)\cdots)_+\rangle + \gamma\partial_\theta\langle (\xi^a(x,\theta)\cdots)_+\rangle \\ &\qquad + i\epsilon f^{abc}\langle (\xi^b(x,\theta)\eta^c(x,\theta)\cdots)_+\rangle = 0\,.\end{aligned} \tag{3·24}$$

Note that the second component of (3·24) implies the equation of motion for $\xi(x)$ without the mass term (3·22). The manifest BRS covariance thus automatically

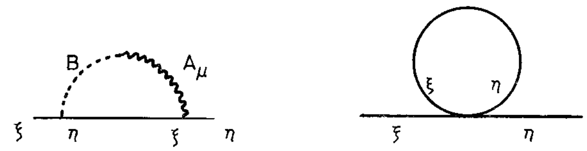

Fig. 1. Quadratically divergent mass terms which could give rise to the spontaneous breaking of the BRS symmetry.

excludes the unwanted term (3·22).

The next question is then whether a suitable regularization can ensure the absence of (3·22). We indicate that the dimensional regularization,[19] for example, can do this. In the one-loop level we have the formally quadratically divergent mass terms of the order of $\epsilon g/\alpha$ shown in Fig. 1. If one uses the relation $\epsilon = -g\gamma/2$, these quadratic divergences correspond to the second term in (3·22). In the well-regulated theory, these mass terms exactly cancel for $\beta = 0$ after a shift in the integration variable. For $\beta \neq 0$ a logarithmically divergent mass term survives, and it is absorbed by the renormalization of the β-term in (3·10).

This example shows that the manifest BRS covariance imposes an extra constraint on the theory other than the BRS invariance defined in Refs. 6) and 16), and that the manifest BRS covariance is preserved in perturbation theory.

§ 4. Unitarity and gauge independence

The renormalization of the Lagrangian (3·12) has been discussed by various authors[4),6),20)] and we do not repeat it here. We just note that if the gauge coupling is renormalized at the three-point vector vertex by $g_0 \to (Z_1/(Z_A)^{3/2})g$, the superfields including the vacume value are rescaled as

$$\begin{aligned}
A_\mu{}^a(x,\theta) &= \sqrt{Z_A}\{A_\mu{}^a(x) + i\theta Z_\eta[\partial_\mu\eta^a - (Z_1/Z_A)gf^{abc}A_\mu{}^b\eta^c]\},\\
\eta^a(x,\theta) &= Z_\eta[\eta^a(x) + i\theta Z_\eta(Z_1/Z_A)(g/2)f^{abc}\eta^b\eta^c],\\
\psi(x,\theta) &= \sqrt{Z_\phi}[\psi(x) + v_0 - i\theta Z_\eta(Z_1/Z_A)(g/2)\phi^a\eta^a],\\
\phi^a(x,\theta) &= \sqrt{Z_\phi}\{\phi^a(x) + i\theta Z_\eta(Z_1/Z_A)(g/2)[(\psi+v_0)\eta^a - f^{abc}\phi^b\eta^c]\},\\
\xi^a(x,\theta) &= \xi^a(x) + \theta B^a(x)
\end{aligned} \tag{4·1}$$

and the parameters in $\mathcal{L}_g$ are rescaled: $\alpha Z_\alpha/\sqrt{Z_A}$, $\beta Z_\beta/\sqrt{Z_\phi}$, γZ_γ, $\delta Z_\delta/Z_\phi$ and $\epsilon Z_\epsilon/Z_\eta$, respectively.*) The parameters in the last term in (3·1) are also rescaled as $\lambda \to \lambda Z_\lambda/(Z_\phi)^2$ and $v \to \sqrt{Z_\phi}v_0'$, and the difference $\delta v^2 \equiv v_0^2 - (v_0')^2$ with v_0 in (4·1) multiplies the so-called tadpole counter terms.[3),10)] As we include all the possible terms with canonical dimensions not greater than 4, the renormalizability of

*) It should be noted that (4·1) corresponds to the most general rescaling of component fields, which is consistent with the BRS symmetry. Although it is possible to transfer a part of the scaling factor Z_η to that of $\xi(x)$, we chose (4·1) so that all the superfields are made finite.

the model is more explicit than in the ordinary formulation, providing that one sufficiently regulates the theory so that all the ST identities are satisfied. The auxiliary field $B(x)$ also simplifies the discussion of non-linear gauge conditions. The necessity of the ϵ-term to ensure the multiplicative renormalization will be commented on later.

4.1. *Propagators*

As has been discussed in the previous section, the BRS covariance completely determines the θ-dependence of all the Green's functions. In particulatr

$$\begin{aligned}&\langle(\phi^a(x,\theta_1)\phi^b(y,\theta_2))_+\rangle=\delta^{ab}g(x-y),\\&\langle(\xi^a(x,\theta_1)A_\mu{}^b(y,\theta_2))_+\rangle=\delta^{ab}\partial_\mu h(x-y)\delta(\theta_1-\theta_2).\end{aligned}\tag{4·2}$$

It is, therefore sufficient to consider the sector formed by $B^a(x)$, $A_\mu{}^a(x)$ and $\phi^a(x)$. We can then recover the full propagators by means of the ST identities. The physical $\psi(x)$ sector disconnected from others should be added later.

As usual we have the renormalized two-point proper vertices

$$\begin{array}{cc} & \begin{array}{ccc} \quad A_\rho \quad & \quad\phi\quad & \quad B \quad \end{array} \\ \begin{array}{c} A_\nu \\ \phi \\ B \end{array} & \left(\begin{array}{ccc} (g^{\nu\rho}-k^\nu k^\rho/k^2)A(k^2)+k^\nu k^\rho/k^2\widetilde{B}(k^2) & ik^\nu D(k^2) & ik^\nu C(k^2) \\ -ik^\rho D(k^2) & k^2F(k^2) & E(k^2) \\ -ik^\rho C(k^2) & E(k^2) & G(k^2) \end{array}\right) \end{array}\tag{4·3}$$

and the propagators as an inverse of (4·3)

$$\begin{array}{cc} & \begin{array}{ccc} \quad A_\nu \quad & \quad\phi\quad & \quad B \quad \end{array} \\ \begin{array}{c} A_\mu \\ \phi \\ B \end{array} & \left(\begin{array}{ccc} (g_{\mu\nu}-k_\mu k_\nu/k^2)/A(k^2)+k_\mu k_\nu/k^2\,B(k^2) & \dfrac{-ik_\mu Y/C}{(k^2-X)^2} & \dfrac{-ik_\mu/C}{k^2-X} \\ \dfrac{ik_\nu Y/C}{(k^2-X)^2} & \dfrac{XY/E}{(k^2-X)^2}+\dfrac{1/F}{k^2-X} & \dfrac{-X/E}{k^2-X} \\ \dfrac{ik_\nu/C}{k^2-X^2} & \dfrac{-X/E}{k^2-X} & 0 \end{array}\right) \end{array},\tag{4·4}$$

where the factor δ^{ab} is suppressed, and

$$B(k^2)\equiv\frac{-X/\widetilde{B}}{k^2-X}+\frac{k^2YE/(XC^2)}{(k^2-X)^2},\tag{4·5}$$

$$X(k^2)\equiv E(k^2)D(k^2)/(C(k^2)F(k^2)),\tag{4·6}$$

$$Y(k^2)\equiv E(k^2)/F(k^2)-X(k^2)G(k^2)/E(k^2).\tag{4·7}$$

The ST identity $\langle(BB)_+\rangle=0$ (3·20) imposes the constraint

$$D(k^2)^2=\widetilde{B}(k^2)F(k^2).\tag{4·8}$$

Equation (4·3) indicates

$$B(0)=1/A(0)\tag{4·9}$$

as $F(0)=$finite. Consequently, the pole at $k^2=0$ completely decouples from the propagators (4·4) for the general gauge condition (3·8). We note that the β-term in (3·8) explicitly breaks the *global gauge* symmetry of the original Lagrangian, and the Goldstone theorem does not hold.

The salient feature of the non-linear gauge condition is that $C(k^2)$, $E(k^2)$ and $G(k^2)$ become k^2-dependent. We renormalize everything at $k^2=\mu^2$ for a suitable μ,

$$A(\mu^2)=-\mu^2+(gv/2)^2, \quad \widetilde{B}(\mu^2)=(gv/2)^2, \quad F(\mu^2)=1, \tag{4·10}$$

which normalize the vector meson and Goldstone boson wave functions and define the vacuum value $(gv/2)$. From (4·8), we have

$$D(\mu^2)=gv/2. \tag{4·11}$$

We further normalize as

$$\begin{aligned} C(\mu^2)&=\alpha, \\ E(\mu^2)&=\beta, \\ G(\mu^2)&=\gamma. \end{aligned} \tag{4·12}$$

Then

$$\begin{aligned} X(\mu^2)&=(\beta/\alpha)(gv/2), \\ Y(\mu^2)&=\beta-(\gamma/\alpha)(gv/2). \end{aligned} \tag{4·13}$$

To simplify the discussion of the gauge independence later, the mass of ψ is chosen at the *physical* value m_ψ, which in turn defines the coupling constant λ. Other coupling constants g, δ and ϵ are normalized at the symmetric point with $k_i^2=\mu^2$.

We next examine the pole structure of propagators: The physical vector meson mass is found from

$$\begin{aligned} A(M^2)&=0, \\ 1/Z_3&\equiv-\frac{\partial}{\partial s}A(s)\bigg|_{s=M^2}. \end{aligned} \tag{4·14}$$

Similarly we define the ghost pole position m^2 by

$$m^2\equiv X(m^2) \tag{4·15}$$

and expand $k^2-X(k^2)$ around $k^2=m^2$

$$k^2-X(k^2)=(1/Z)[(k^2-m^2)-Z_4(k^2-m^2)^2]+O(k^2-m^2)^3. \tag{4·16}$$

At the tree level $m^2=(\beta/\alpha)(gv/2)$. We also expand $C(k^2)$ and $E(k^2)$ as

$$C(k^2)\equiv\alpha_1+\alpha_2(k^2-m^2)+O(k^2-m^2)^2, \tag{4·17}$$

$$E(k^2)\equiv\beta_1+\beta_2(k^2-m^2)+O(k^2-m^2)^2. \tag{4·18}$$

We then obtain the *pole structure* of full propagators

$$\frac{1}{i}\langle(\xi^a(\theta_1)A_\mu{}^b(\theta_2))_+\rangle \cong \frac{-ik_\mu}{k^2-m^2}\left(\frac{Z}{\alpha_1}\right)\delta(\theta_2-\theta_1), \tag{4·19}$$

$$\frac{1}{i}\langle(\xi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong \frac{-m^2}{k^2-m^2}\left(\frac{Z}{\beta_1}\right)\delta(\theta_2-\theta_1), \tag{4·20}$$

$$\frac{1}{i}\langle(\phi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong \frac{ZN_1}{k^2-m^2}+\frac{L}{(k^2-m^2)^2}, \tag{4·21}$$

$$\frac{1}{i}\langle(\phi^a(\theta_1)A_\mu{}^b(\theta_2))_+\rangle \cong ik_\mu\left[\frac{ZN}{k^2-m^2}+\frac{(\beta_1/\alpha_1 m^2)L}{(k^2-m^2)^2}\right], \tag{4·22}$$

$$\frac{1}{i}\langle(A_\mu{}^a(\theta_1)A_\nu{}^b(\theta_2))_+\rangle \cong -Z_3\frac{g_{\mu\nu}-k_\mu k_\nu/M^2}{k^2-M^2}$$
$$+k_\mu k_\nu\left[\frac{ZN_2}{k^2-m^2}+\frac{(\beta_1/\alpha_1 m^2)^2 L}{(k^2-m^2)^2}\right], \tag{4·23}$$

where the factor δ^{ab} is suppressed, and we utilize a symbolic notation for the Fourier transform of the time-ordered product. The following parameters are also defined:

$$L\equiv(Z^2m^2/\beta_1)Y(m^2), \tag{4·24}$$

$$\begin{aligned}
K&\equiv(\beta_1/\alpha_1 m^2)^2\{1/F(m^2)+(L/Z)[\alpha_2/\alpha_1-\beta_2/\beta_1+(Z-1)/(m^2Z)]\},\\
N&\equiv Z\left[(Z_4/\alpha_1)Y(m^2)+\frac{\partial}{\partial s}(Y(s)/C(s))\Big|_{s=m^2}\right],\\
N_1&\equiv(\alpha_1 m^2/\beta_1)N+(\alpha_1 m^2/\beta_1)^2K,\\
N_2&\equiv(\beta_1/\alpha_1 m^2)N-K.
\end{aligned} \tag{4·25}$$

These parameters satisfy the crucial relation

$$(\alpha_1 m^2)^2N_2-2\alpha_1\beta_1 m^2N+\beta_1{}^2N_1=0. \tag{4·26}$$

If we define[9]

$$U_\mu{}^a(x,\theta)\equiv A_\mu{}^a(x,\theta)-(\beta_1/\alpha_1 m^2)\partial_\mu\phi^a(x,\theta)+\alpha_1 K\partial_\theta\partial_\mu\xi^a(x,\theta), \tag{4·27}$$

we find the following *pole structure* by using (3·19), (4·25) and (4·26):

$$\frac{1}{i}\langle(U_\mu{}^a(\theta_1)U_\nu{}^b(\theta_2))_+\rangle \cong -\delta^{ab}Z_3\frac{g_{\mu\nu}-k_\mu k_\nu/M^2}{k^2-M^2}, \tag{4·28}$$

$$\frac{1}{i}\langle(\xi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong -\delta^{ab}\frac{m^2}{k^2-m^2}\left(\frac{Z}{\beta_1}\right)\delta(\theta_1-\theta_2), \tag{4·29}$$

$$\frac{1}{i}\langle(\phi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong \delta^{ab}\left[\frac{ZN_1}{k^2-m^2}+\frac{L}{(k^2-m^2)^2}\right] \tag{4·30}$$

and all other propagators vanish except for

$$\frac{1}{i}\langle(\phi(\theta_1)\phi(\theta_2))_+\rangle\simeq\frac{\widetilde{Z}_\phi}{k^2-m_\phi{}^2}, \tag{4·31}$$

$$\frac{1}{i}\langle(\xi^a(\theta_1)\eta^b(\theta_2))_+\rangle\simeq\delta^{ab}\frac{i\widetilde{Z}_\eta}{k^2-m^2}. \tag{4·32}$$

Note that the propagator $\langle(\xi\eta)_+\rangle$ appears as an auxiliary device in the present formulation, and the precise form of $\widetilde{Z}_\phi$ and $\widetilde{Z}_\eta$ is not important in the following discussion.

We next define the "metric" by inverting (4·29) and (4·30)

$$iE^{(1)}(\theta_1,\theta_2)\equiv\begin{array}{c|cc} & \xi^b(\theta_2) & \phi^b(\theta_2) \\ \hline \xi^a(\theta_1) & (\beta_1/m^2Z)^2ZN_1 & -(\beta_1/m^2Z)\delta(\theta_1-\theta_2) \\ \phi^a(\theta_1) & +(\beta_1/m^2Z)\delta(\theta_1-\theta_2) & 0\end{array}(k^2-m^2)\delta^{ab} \tag{4·33}$$

and

$$iE^{(2)}(\theta_1,\theta_2)\equiv-\delta(-\theta_1)\delta(-\theta_2)\frac{(k^2-m^2)^2}{L}\delta^{ab}. \tag{4·34}$$

This metric satisfies, for example,

$$\langle(\phi(\theta_1)A_\mu(\theta_2))_+\rangle^{(1)}=\int d\theta_3d\theta_4\sum_{f,g}\langle(\phi(\theta_1)f(\theta_3))_+\rangle^{(1)}E^{(1)}(\theta_3,\theta_4)_{fg}$$
$$\times\langle(g(\theta_4)A_\mu(\theta_2))_+\rangle^{(1)}, \tag{4·35}$$

where the summation runs over f, $g=\xi$ and ϕ, and $\langle(\phi(\theta_1)A_\mu(\theta_2))_+\rangle^{(1)}$, for example, stands for the single-pole part of (4·22). This relation holds for all the propagators (4·19)$\sim$(4·23) including $\langle\xi\eta\rangle$ in (4·32) near the single-pole at $k^2=m^2$ if one remembers the relation (4·26). Similarly for the double-pole part, e.g.,

$$\langle(A_\mu(\theta_1)A_\nu(\theta_2))_+\rangle^{(2)}=\int d\theta_3d\theta_4\langle(A_\mu(\theta_1)\phi(\theta_3))_+\rangle^{(2)}E^{(2)}(\theta_3,\theta_4)$$
$$\times\langle(\phi(\theta_4)A_\nu(\theta_2))_+\rangle^{(2)}, \tag{4·36}$$

where $\langle(A_\mu(\theta_1)\phi(\theta_3))_+\rangle^{(2)}$, for example, stands for the double-pole part of (4·23).

These relations (4·35) and (4·36) show that the Green's functions of the form (with an arbitrary number of ψ and A_μ (or U_μ) at the physical pole position added)

$$\langle(\xi(\theta_1)\cdots\xi(\theta_l)\phi(\theta_{l+1})\cdots\phi(\theta_n))_+\rangle \tag{4·37}$$

and the "metric tensors" $E^{(1)}$ and $E^{(2)}$ are sufficient to generate a combined Green's function in the Landau-Cutkosky rule near the unphysical pole position $k_i{}^2=m^2$. (If one looks at the proper vertex, there is no such simplification.)

4.2. *Cutting rule* (*or Landau-Cutkosky rule*)[3]

In the proof of unitarity via the Landau-Cutkosky rule, one considers a diagram

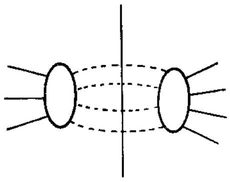

Fig. 2. Landau-Cutkosky diagram.

such as in Fig. 2, where all the external lines correspond to the physical particles (ϕ at $k^2=m_\phi{}^2$, and $A_\mu{}^a$ (or $U_\mu{}^a$) at $k^2=M^2$). The internal lines in general contain the physical as well as unphysical particles on the mass-shell.

We want to show that the intermediate states with unphysical particles give rise to a vanishing contribution. We dispense with the on-shell physical particles as they are BRS scalars. Consequently we consider to combine two Green's functions of the form (4·37) by the "metric tensors" $E^{(1)}$ and $E^{(2)}$.

Single-pole contribution

We first consider the single-pole contributions when they are jointed by the *off-diagonal* elements of $E^{(1)}$ in (4·33). Namely, we combine (4·37) with another Green's function of the form

$$\langle\,(\phi\,(\theta_1{}')\cdots\phi\,(\theta_l{}')\,\xi\,(\theta'_{l+1})\cdots\xi\,(\theta_n{}')\,)_+\rangle \tag{4·38}$$

by the off-diagonal elements of $E^{(1)}$, which have the θ-dependence of the form

$$\int d\theta_1 d\theta_1{}'\cdots d\theta_n d\theta_n{}'\,\delta\,(\theta_1-\theta_1{}')\cdots\delta\,(\theta_n-\theta_n{}')\,. \tag{4·39}$$

The translation and dilatation invariance in θ-variables determine (4·37) as a sum of l-th order monomials formed by the $(n-1)$ differences of θ-variables. Symbolically

$$\text{Eq. (4·37)} = \sum\{[\prod^{l}(\theta_i-\theta_j)]f(k_1,\cdots,k_n)\}\,. \tag{4·40}$$

Similarly

$$\text{Eq. (4·38)} = \sum\{[\prod^{n-l}(\theta_i{}'-\theta_j{}')]\,g\,(k_1,\cdots,k_n)\}\,. \tag{4·41}$$

The multiplication of these Green's functions with the metric (4·39) identically vanishes before the integration is performed, as one obtains a $2n$-th power in $(2n-1)$ independent anti-commuting variables of the form $(\theta_i-\theta_j)$. The simplest example is obtained by combining $\langle\,(\xi\,(\theta_1)\,\phi\,(\theta_2)\,)_+\rangle$ with $\langle\,(\phi\,(\theta_1{}')\,\xi\,(\theta_2{}')\,)_+\rangle$. In this case we have the θ-dependence

$$\int d\theta_1 d\theta_1{}' d\theta_2 d\theta_2{}'\,(\theta_1-\theta_2)\,(\theta_1{}'-\theta_2{}')\,(\theta_1-\theta_1{}')\,(\theta_2-\theta_2{}')\equiv 0\,. \tag{4·42}$$

This cancellation mechanism is analogous to the divergence cancellation in the ordinary supersymmetric field theory.[13),14)]

When the diagonal element of $E^{(1)}$ is involved, one combines two Green's functions of the form, e.g.,

$$\int d\theta \langle (\xi(\theta)\xi(\theta_1)\cdots\xi(\theta_l)\phi(\theta_{l+1})\cdots\phi(\theta_n))_+\rangle \tag{4·43}$$

instead of (4·37) and the one corresponding to (4·38) by the metric (4·39). As the second component of $\xi(x,\theta)$ is scalar under the full BRS symmetry, the θ-dependence of (4·43) is identical to (4·37) and, consequently, we again have a vanishing result. We note that

$$\langle (\xi(\theta_1)\xi(\theta_2)\cdots\xi(\theta_n))_+\rangle = 0 \tag{4·44}$$

due to the same reasoning as in (3·19) (this is valid for off-shell ξ's also). Therefore the combination of two Green's functions jointed by the diagonal element of $E^{(1)}$ alone also vanishes.

Double-pole contribution

Some of the $\phi(\theta)$'s in the Green's function can be bridged by the dipole ghosts in the intermediate summation via $E^{(2)}$. We here notice that the residues of $\langle(\phi(\theta_1)\phi(\theta_2))_+\rangle^{(2)}$ in (4·22) and $\langle(\phi(\theta_1)A_\mu(\theta_2))_+\rangle^{(2)}$ in (4·23) are proportional to those of $\int d\theta\langle(\xi(\theta)\phi(\theta_2))_+\rangle^{(1)}$ in (4·21) and $\int d\theta\langle(\xi(\theta)A_\mu(\theta_2))_+\rangle^{(1)}$ in (4·20), respectively. This means that the dipole contribution to the intermediate sum is proportional to the *diagonal* single-pole term such as (4·43) except for the difference in the powers appearing in the denominator. Consequently, these contributions again vanish.

This completes the proof of unitarity via the Landau-Cutkosky rule, and the essential ingredient is the full BRS covariance of all the Green's functions.

4.3. *Gauge independence*

We consider the *perturbative* construction of the connected physical T-matrix element

$$\langle (\psi(\theta_1)\cdots\psi(\theta_l)A_\mu(\theta_{l+1})\cdots A_\nu(\theta_n))_+\rangle, \tag{4·45}$$

where all the (amputated) ψ's are on the mass shell $k^2=m_\psi{}^2$, and the (amputated) vector particles are also restricted to the physical pole position by choosing the arbitrary renormalization point μ in (4·10) at $\mu^2=M^2$. Here we include the composite operators written in the interaction picture for *external* legs of the Green's function with appropriate renormalization factors added. It can be checked that the physical Green's function specified above is in fact independent of θ-variables for $m^2\neq M^2$.

We next examine the dependence of (4·45) on the *renormalized* parameter γ in $\mathcal{L}_g$ (3·8) by expanding it in powers in γ. The linear term in γ is in the symbolic notation

$$\gamma \int dx d\theta \langle (\xi(x,\theta)\partial_\theta \xi(x,\theta)\phi(\theta_1)\cdots A_\mu(\theta_n))_+\rangle$$
$$+\text{counter terms linear in } \gamma. \qquad (4\cdot 46)$$

The operator inserted into (4·46) transforms covariantly with BRS dimension $d=-1$, and the first term in (4·46) identically vanishes due to a reasoning similar to (3·19), if one remembers that (4·46) is independent of $\theta_1 \sim \theta_n$. Consequently, the corresponding counter terms linear in γ also vanish if renormalized in a BRS covariant manner. This argument can be continued to higher powers in γ, and the γ-independence of (4·45) is established.*) Similarly, any one of the terms in $\mathcal{L}_g$ (3·8) gives rise to a vanishing result when inserted into (4·45). The ϵ-independence, for example, can be shown by considering the insertion of the BRS covariant operator with $d=-1$

$$\epsilon \int dx d\theta f^{abc} \xi^a(x,\theta)\xi^b(x,\theta)[\eta^c(x)+i\theta(Z_\eta Z_1/Z_A)_0 (g/2) f^{cde}\eta^d\eta^e], \qquad (4\cdot 47)$$

where $(Z_\eta Z_1/Z_A)_0$ indicates the zeroth order in the renormalized parameter ϵ. See also (4·1).

We here note that there appear two basically different classes of BRS invariant operators in the present formulation; the first with $d=0$ such as $\mathcal{L}_{\rm inv}$ and the *on-shell* ϕ and A_μ (or U_μ), and the second with $d=-1$ such as $\mathcal{L}_g$ and $\xi(x,\theta)$ (or $B(x)$). It is the latter class of operators which give rise to a vanishing result when inserted into the on-shell T-matrix.

4.4. *Canonical treatment*

The proof of unitarity in the canonical treatment is based on the observation that all the (proper) BRS invariant states (i.e., invariant under $U(\lambda)$ (2·6)) formed by unphysical particles have the zero norm.[8),9)] One can see a close correspondence between our discussion of unitarity on the basis of the Landau-Cutkosky rule and the canonical treatment given by Kugo and Ojima.[9)] In the following we briefly sketch how the canonical treatment can be made more intuitively understandable in the present formulation.

All the steps up to (4·30) are identical in any approach. Equations (4·29) and (4·30) are simplified if one makes the finite renormalization,

$$\phi^a(\theta) \to \sqrt{ZN_1}\phi^a(\theta) \quad \text{and} \quad \xi^a(\theta) \to \sqrt{\frac{Z}{N_1}} m^2 (\beta_1(gv/2))^{-1} \xi^a(\theta),$$

we also set $L=0$ by suitably choosing the parameter γ; this avoids the unnecessary complications arising from the dipole ghost. We then have the pole structure

$$\frac{1}{i}\langle(\xi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong -\delta^{ab}\frac{(gv/2)}{k^2-m^2}\delta(\theta_1-\theta_2), \qquad (4\cdot 48)$$

*) This argument is essentially the same as the proof of the gauge independence in the case of the Abelian model given by B. W. Lee, Phys. Rev. **D5** (1972), 823.

$$\frac{1}{i}\langle(\phi^a(\theta_1)\phi^b(\theta_2))_+\rangle \cong \delta^{ab}\frac{1}{k^2-m^2}, \tag{4·49}$$

which can be converted into commutation relations among the asymptotic fields

$$[\xi^a_{as}(x,\theta_1), \phi^b_{as}(y,\theta_2)] = -i\delta^{ab}(gv/2)D(x-y;m^2)\delta(\theta_1-\theta_2), \tag{4·50}$$

$$[\phi^a_{as}(x,\theta_1), \phi^b_{as}(y,\theta_2)] = i\delta^{ab}D(x-y;m^2) \tag{4·51}$$

and

$$\{\xi^a_{as}(x,\theta_1),\ \xi^b_{as}(y,\theta_2)\}_+ = 0\,.$$

Expanding these into plane-wave solutions, one introduces the creation and annihilation operators, e.g.,

$$\begin{aligned}\phi^a(k,\theta)^\dagger &\equiv \phi^a(k)^\dagger + i\theta(gv/2)\eta^a(k)^\dagger\,,\\ \xi^a(k,\theta)^\dagger &\equiv \xi^a(k)^\dagger + \theta B^a(k)^\dagger\,.\end{aligned} \tag{4·52}$$

Here we suppress the suffix "asymptotic". The n-ghost state is constructed by

$$\phi(k_1,\theta_1)^\dagger\cdots\phi(k_l,\theta_l)^\dagger\xi(k_{l+1},\theta_{l+1})^\dagger\cdots\xi(k_n,\theta_n)^\dagger|\mathrm{phys}\rangle, \tag{4·53}$$

where $|\mathrm{phys}\rangle$ stands for the state formed by ψ and U_μ. The (proper) BRS invariant state is projected out by the operation (2·20)

$$\int d\lambda U(\lambda)\phi(k_1,\theta_1)^\dagger\cdots\xi(k_n,\theta_n)^\dagger U(\lambda)^\dagger|\mathrm{phys}\rangle$$

$$=\partial_\lambda\phi(k_1,\theta_1+\lambda)^\dagger\cdots\xi(k_n,\theta_n+\lambda)^\dagger|\mathrm{phys}\rangle, \tag{4·54a}$$

$$=Q\phi(k_1,\theta_1)^\dagger\cdots\xi(k_n,\theta_n)^\dagger|\mathrm{phys}\rangle, \tag{4·54b}$$

where (4·54b) follows from $U(\lambda)^\dagger|\mathrm{phys}\rangle = |\mathrm{phys}\rangle$. A set of n-ghost states can be generated from (4·54a) by performing the differentiation and expanding it into powers in θ-variables. By comparing the same powers in θ in (4·54b), one finds alternative expressions for these states. Equation (4·54b) indicates that all the BRS invariant states thus obtained have the zero norm and are orthogonal to the physical states.*)

The simplest example is given by considering the state $\phi(k_1,\theta_1)^\dagger\xi(k_2,\theta_2)^\dagger|\mathrm{phys}\rangle$. Namely,

$$[i(gv/2)\eta^+(k_1)\xi^+(k_2)+\phi^+(k_1)B^+(k_2)]|\mathrm{phys}\rangle = Q\phi^+(k^1)\xi^+(k^2)|\mathrm{phys}\rangle,$$

$$\begin{aligned}i(gv/2)\eta^+(k_1)B^+(k_2)|\mathrm{phys}\rangle &= Q(-igv/2)\eta^+(k_1)\xi^+(k_2)|\mathrm{phys}\rangle\\ &= Q\phi^+(k_1)B^+(k_2)|\mathrm{phys}\rangle,\end{aligned} \tag{4·55}$$

$$Q\eta^+(k_1)B^+(k_2)|\mathrm{phys}\rangle = 0\,.$$

*) What (4·54) means is that any n-ghost state $|n\rangle$ which satisfies the constraint $Q|n\rangle=0$ can be written as $|n\rangle=Q|n'\rangle$ by using a suitable n-ghost state $|n'\rangle$. Consequently, $Q|n\rangle=0$ automatically implies $\langle n|n\rangle=0$ and they amount to the basic algebra $\{Q,Q\}=0$. It is also possible to prove this by using the n-ghost projection operator $P^{(n)}$ introduced in Ref. 9). Written in our notation, $|n\rangle=P^{(n)}|n\rangle=Q\sum_k(-1/n)\{\xi(k)^\dagger P^{(n-1)}\phi(k)+\phi(k)^\dagger P^{(n-1)}\xi(k)+B(k)^\dagger P^{(n-1)}\xi(k)\}|n\rangle\equiv Q|n'\rangle$ if $Q|n\rangle=0$.

The first state in (4·55) corresponds to the example given in Ref. 9). As was shown by Kugo and Ojima,[9] one can consistently express the charge Q in terms of creation and annihilation operators, and the above argument can be refined.

4.5. *Multiplicative renormalization and the quartic ghost coupling*

We next comment that the quartic ghost coupling, the ϵ-term in (3·8), is required to multiplicatively renormalize the non-linear gauge condition with $\delta \neq 0$ in (3·8). See also Ref. 4). The divergent diagrams in the lowest order are shown in Fig. 3. All of these diagrams are of order δ^2 and logarithmically diver-

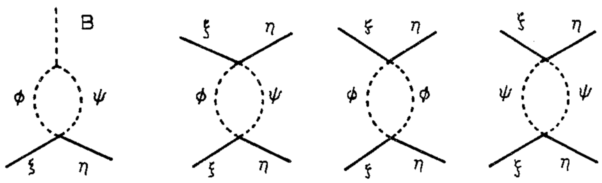

Fig. 3. Logarithmically divergent diagrams which are absorbed by the renormalization of the ϵ-term.

gent, and they can be simultaneously renormalized by an adjustment of ϵ in (3·8). It should be noted that all the terms in $\mathcal{L}_g$ (3·8) and $\mathcal{L}_{\text{inv}}$ (3·1) are separately BRS invariant and, consequently, the divergence in the ϵ-term cannot be regulated by other terms via *ST* identities. This can be confirmed by writing the *ST* identity for the proper vertices[7] in terms of the component fields. What happens then is that the *divergent part* of the $B\xi\eta$ three-point vertex, for example, completely decouples from the *ST* identity for three-point vertices.

In this connection, we note that some of the previous treatments[6),20)] of the non-linear gauge condition renormalized the ϵ-term *subtractively* by super-imposing the equation of motion for $\xi(x, \theta)$ with $\epsilon=0$ on the *ST* identity. The ϵ-term is somewhat akin to the graphic φ^4 coupling in the old Yukawa interaction, although the physical S-matrix is independent of it in the present case. The explicit presence of the ϵ-term allows us to discuss the renormalization solely on the basis of the BRS symmetry without any additional constraint. It is also gratifying that the fundamental coupling in (2·10) plays a non-trivial role in gauge theory.*)

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*) It would be interesting to investigate whether a non-linear gauge condition with the quartic ghost coupling gives any clue to the possible degeneracies in the Green's function defined by linear gauge conditions (the so-called Gribov ambiguity).

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