AMO - Advanced Modeling and Optimization, Volume 6, Number 2, 2004

ON A VOLUME FLEXIBLE INVENTORY MODEL

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Abstract.

An Economic Production Lot Scheduling model is generalized analytically in which time-dependent demand in the market is adjusted by finite rate of production. The rate of production depends upon modern technology, capital investment and number of labors. Also, the generalized EPLS model is simplified especially for constant demand, linear trend in demand, quadratic demand and exponentially demand pattern. The relevant cost function of this model is minimized analytically.

AMS Mathemarics Subject Classification : 90B05.

Key words : Inventory, rate of production, linear, quadratic, exponential, modern technology.

1. Introduction

The classical EOQ formula , which is also known as the Wilson's Square Root Formula[1] was derived long ago under the assupmtion of constant demand rate. In real market, the demand rate of any product is always in a dynamic state. Demand of product may vary with time or with price or even with the instanteneous level of inventory displayed in a retail shop. Several authors have enlightened the EOQ model with time-varying demand. It was started with the work of Silver and Meal[2] who developed a heuristic approach to determine EOQ in the general case of a deterministic time dependent demand pattern. Donaldson[3] came out with a full analytic solution of the inventory replenishment problem with a linear trend in demand over a finite time-horizon. Silver[4] used the Silver-Meal [2] heuristic to obtain a simple operating schedule for the same problem which incurs only negligible cost penalies. Other noteable works in this direction came from Ritche[5-7] , Kicks and Donalson[8] , Buchanan[9], Mitra et al.[10] , Ritchie and Tsado[11], Goyal[12], Goyal et al. [13], Deb and Chaudhuri[14], Murdeshwar[15], Dave[16], Goyal[17], Hariga[18], Goyal et al.[19], Dave and Patel[20], Bahari-Kashani[21], Hong et al.[22], Chung and Ting[23], Goswami and Chaudhuri[24], Hariga[25], Giri et al.[26], Teng[27], Jalan et al.[28], Chakrabarty et al.[29], Lin et al.[30], Goyal et al.[31], Chakrabarty and Chaudhuri[32], Hariga and Benkherouf[33], Wee[34], Khanra and Chaudhuri[35] among others.

In the Classical Economic Production Lot Scheduling model, the amount ordered becomes available at a constant supply rate[36]. But the machine production rate can easily be changed[37-39]. The models of Adler and Nanda[40], Sule[41, 42], Axsater and Elmaghraby[43], Muth and Spearmann[44] were concerned with learning effects on the optimal lot size. Proteus[45], Rosenblatt and Lee[46] extended the models to the imperfect production processes. Shweitzer and Seidmann[37] first enlightened the researchers about the concept of flexibility in the machine production rate and discussed optimization of processing rates for a FMS(flexible manufacturing system). Silver[47] discussed the effects of slowing down production in the context of a manufacturing equipment dedicated to the production of a family of items, assuming a common production cycle for all items. Gallego[48] extended the model of Silver[47] by applying different production cycles for different items. Moon, Gallego and Simchi-Levi[49] discussed controllable production rates in a family production context. Khouja and Mehrez[50] and Khouja[51] extended the EPLS model to an imperfect production process with a flexible production rate.

Under increased competition, inventory-based businesses are forced to better coordinate their procuremen and marketting decisions to avoid carrying excessive stock when sales are low or shortages when demands are high. An effective means of such coordination is to conduct the inventory control and manufacturing decision jointly. The main task in doing so is to determine the optimal rate of production and inventory policy in a given time varying demand.

In this paper, a manufacturing-inventory system is considered in which the time-varying demand(any continuous function of time "t") in market is met by its produced items. The rate of production is controlled by the applied technology, capital investment in variable factors of production and number of labours,

2. Fundamental Assumptions and Notations

Assumptions:

1. The model is developed for single item.
2. The rate of production is variable.
3. Shortages are not permitted.
4. Time-horizon is finite.
5. The lead time between production and supply to an enterprise is negligible small.

Notations:

P - The rate of production ;

T - Production technology which produces T times of production, keeping unchanged the another factors;

K - Capital investment in production system per unit time;

N - Number of labours;

$f(t)$ - Demand rate at time "t" ≥ 0 ;

C_L - Remuneration of each labour per unit time;

C_h - Inventory holding cost per unit per unit time;

C_s - Setup cost per cycle;

$Q(t)$ - On-hand inventory at time "t" ≥ 0 ;

n - Number of cycles;

H - Time horizon;

TAC - Total average profit.

3. Formulation of the Model

We consider a manufacturing system in which the demand in market is met by its produced items. Here the time horizon $[0, H]$ is divided into n cycles. In the i -th cycle ($i = 1, 2, ...n$) , production starts at T_{i-1} with zero inventory and continues upto time t_i and the level of inventory again reaches zero at time T_i. The inventory piles up during $[T_{i-1}, t_i]$, after adjusting the demand in market. Here n cycles are of equal length. Therefore,

$$T_i = i\frac{H}{n} \quad , \quad i = 1, 2, ...n \tag{1}$$

$$\begin{aligned} t_i &= T_{i-1} + r_i(T_i - T_{i-1}) \\ &= (i-1)\frac{H}{n} + r_i\frac{H}{n} \\ &= (r_i + i - 1)\frac{H}{n}, \end{aligned} \tag{2}$$

where r_i , $0 < r_i < 1$, is the service level of i - th cycle. Now the rate of production is considered as *Cobb - Doughlas* production function :

$$P = TK^{\alpha}N^{1-\alpha} \quad , \quad 0 < \alpha < 1 \tag{3}$$

Where T is the technology which accelerates the rate of production, K is the invested capital for production except investment in technology, N is the number of labours, $(1-\alpha)$ and α represent labour and capital elasticity of production respectively. Generally speaking, production is the result of combined efforts of the factors of production. These factors may be fixed or variable. A fixed factor is one, whose quantity cannot readily be changed in responce to desired changes in output or market conditions. Its quantity remains the same whether the level of output is more or less or zero. Buildings, land, machinery plants and top management are some common examples of fixed factors. A variable factor, on the otherhand, is one whose quantity may be changed in responce to a change in output. Raw materials, ordinary labour, power, fuel etc. are examples of variable factors. Such factors are required more, when output is more; less, when output is less and zero, when output is nil.

The governing equations of this model are :

$$\frac{dQ}{dt} = P - f(t) \quad , \quad T_{i-1} \leq t \leq t_i \quad with \quad Q(T_{i-1}) = 0 \tag{4}$$

and

$$\frac{dQ}{dt} = -f(t) \quad , \quad t_i \leq t \leq T_i \quad with \quad Q(T_i) = 0 \tag{5}$$

From equ.(4) and equ.(5) , we have

$$Q(t) = P(t - T_{i-1}) - \int_{T_{i-1}}^{t} f(t)\ dt \quad , \quad T_{i-1} \leq t \leq t_i \tag{6}$$

and

$$Q(t) = P(t_i - T_{i-1}) - \int_{T_{i-1}}^{t} f(t)\ dt \quad , \quad t_i \leq t \leq T_i \quad with \quad Q(T_i) = 0 \tag{7}$$

Using the condition $Q(T_i) = 0$ in equ.(7), we have

$P(t_i - T_{i-1}) - \int_{T_{i-1}}^{T_i} f(t)\ dt \ = 0$

or, $Pr_i \frac{H}{n} = \int_{T_{i-1}}^{T_i} f(t)\ dt = \int_{(i-1)\frac{H}{n}}^{i\frac{H}{n}} f(t)\ dt$

or,

$$r_i = \frac{1}{P}\frac{n}{H} A(i,n) \quad , \quad where \quad A(i,n) = \int_{(i-1)\frac{H}{n}}^{i\frac{H}{n}} f(t)\ dt \tag{8}$$

Therefore , the inventory of the i - th cycle is

$$Inv_i = \int_{T_{i-1}}^{t_i} Q(t)\ dt + \int_{t_i}^{T_i} Q(t)\ dt$$

$$
\begin{aligned}
&= \frac{P}{2}(t_i - T_{i-1})^2 - \int_{T_{i-1}}^{t_i} \{\int_{T_{i-1}}^{t} f(u)\ du\}\ dt \\
&+ P(t_i - T_{i-1})(T_i - t_i) - \int_{T_{i-1}}^{T_i} \{\int_{T_{i-1}}^{t} f(u)\ du\}\ dt \\
&= \frac{PH^2}{n^2} r_i (1 - \frac{r_i}{2}) - \int_{T_{i-1}}^{T_i} (T_i - t)\ f(t)\ dt \\
&= \frac{PH^2}{n^2} r_i (1 - \frac{r_i}{2}) - B(i,n), \qquad (9)
\end{aligned}
$$

where

$$
\begin{aligned}
B(i,n) &= \int_{T_{i-1}}^{T_i} (T_i - t) f(t)\ dt \\
&= \int_{(i-1)\frac{H}{n}}^{i\frac{H}{n}} (\frac{iH}{n} - t) f(t)\ dt
\end{aligned}
$$

Now the total average cost is

$$
\begin{aligned}
TAC &= \frac{nC_s}{H} + \frac{1}{H}\sum_{i=1}^{n}[(K + C_L N) r_i \frac{H}{n} + C_h\{P\frac{H^2}{n^2}(r_i - \frac{r_i^2}{2}) - B(i,n)\}] \\
&= \frac{nC_s}{H} + \frac{1}{H}\sum_{i=1}^{n}[(K + C_L N)\frac{A(i,n)}{P} \\
&+C_h\{\frac{H}{n}A(i,n) - \frac{1}{2P}A^2(i,n) - B(i,n)\}] \qquad (10)
\end{aligned}
$$

From equ.(3), we have

$$
K = (\frac{P}{TN^{1-\alpha}})^{\frac{1}{\alpha}}
$$

Using this in equ.(10), we have

$$
\begin{aligned}
TAC &= \frac{nC_s}{H} + \frac{1}{H}\sum_{i=1}^{n}[(C_L N P^{-1} + \frac{P^{\frac{1}{\alpha}-1}}{(TN^{1-\alpha})^{\frac{1}{\alpha}}})A(i,n) \\
&+C_h\{\frac{H}{n}A(i,n) - \frac{1}{2P}A^2(i,n) - B(i,n)\}] \qquad (11)
\end{aligned}
$$

Now we have to minimize TAC , *such that* $0 < r_i < 1$.

□ **<u>Theorem</u>** : There exist a global minimum value of TAC , for fixed "n"

and "N" iff $\frac{C_L}{C_h} > \frac{\sum_{i=1}^n A^2(i,n)}{2N\sum_{i=1}^n A(i,n)}$ is satisfied.

Proof:

Since

$$\frac{dTAC}{dP} = \frac{1}{H}\sum_{i=1}^{n}[\frac{(\frac{1}{\alpha}-1)P^{\frac{1}{\alpha}-2}}{(TN^{1-\alpha})^{\frac{1}{\alpha}}}A(i,n) - C_L N P^{-2}A(i,n) + \frac{C_h}{2}P^{-2}A^2(i,n)]$$

and

$$\frac{d^2TAC}{dP^2} = \frac{1}{H}\sum_{i=1}^{n}[\frac{(\frac{1}{\alpha}-1)(\frac{1}{\alpha}-2)P^{\frac{1}{\alpha}-3}}{(TN^{1-\alpha})^{\frac{1}{\alpha}}}A(i,n) + 2C_L N P^{-3}A(i,n) - C_h P^{-3}A^2(i,n)]$$

For extremum,

$$\frac{dTAC}{dP} = 0,$$

i.e., $\frac{(\frac{1}{\alpha}-1)P^{\frac{1}{\alpha}}}{(TN^{1-\alpha})^{\frac{1}{\alpha}}}\sum_{i=1}^n A(i,n) + \frac{C_h}{2}\sum_{i=1}^n A^2(i,n) = C_L N \sum_{i=1}^n A(i,n)$

or,

$$P = \frac{1}{\sum_{i=1}^n A(i,n)}(\frac{1}{\alpha}-1)^{\alpha}$$
$$.(TN^{1-\alpha}).[C_L N\sum_{i=1}^{n}A(i,n) - \frac{C_h}{2}\sum_{i=1}^{n}A^2(i,n)]^{\alpha} = P^* \quad (say).$$

For real value of P,

$$C_L N\sum_{i=1}^{n}A(i,n) > \frac{C_h}{2}\sum_{i=1}^{n}A^2(i,n)$$

must be satisfied.

Again,

$$\frac{d^2TAC}{dP^2}|_{P=P^*} = \frac{1}{HP^{\ 3}}[(\frac{1}{\alpha}-2)\sum_{i=1}^{n}A(i,n)\{C_L N\sum_{i=1}^{n}A(i,n)$$
$$- \frac{C_h}{2}\sum_{i=1}^{n}A^2(i,n)\} + 2C_L N\sum_{i=1}^{n}A(i,n) - C_h\sum_{i=1}^{n}A^2(i,n)]$$
$$= \frac{1}{HP^{\ 3}\alpha}\{C_L N\sum_{i=1}^{n}A(i,n)$$
$$-\frac{C_h}{2}\sum_{i=1}^{n}A(i,n)^2\}\sum_{i=1}^{n}A(i,n),$$

Therefore $\frac{d^2TAC}{dP^2}\ |_{P=P^*} > 0$ for $0 < \alpha < 1$ iff

$$C_L N \sum_{i=1}^{n} A(i,n) \quad > \quad \frac{C_h}{2} \sum_{i=1}^{n} A^2(i,n).$$

i.e.,

$$\frac{C_L}{C_h} \quad > \quad \frac{1}{2N} \frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)}.$$

Hence the Proof. □

The above problem can be solved by the following algorithm:

Algorithm :

Step 1 : Set $n = 1$

Step 2 : Set $N = 1$

Step 3 : Minimize $TAC(n, N, P)$; set $U_{old} = MinTAC(n, N, P)$

Step 4 : $N = N+1$; minimize $TAC(n, N, P)$ and set $U_{new} = MinTAC(n, N, P)$

Step 5 : Check the condition $U_{old} > U_{new}$; if it is true then go to Step 3; otherwise go to step 6

Step 6 : Set $n = n+1$; check the condition $MinTAC(n, N, P) > MinTAC(n+1, N, P)$; if it is true then go to step 2; otherwise go to step 7.

Step 7 : The required solution is $n = n^*$, $N = N^*$, $P = P^*$ and required $MinTAC = TAC(n^*, N^*, P^*)$

Step 8: Stop.

Corollary 1:
There exists a global minimum of TAC ; for fixed "n" and "N" , $f(t) = a + bt + ct^2$, a quadratic demand rate; iff

$$\begin{aligned} \frac{C_L}{C_h} > \frac{1}{2N}\{ & \frac{n(n+1)}{30}(6n^3+9n^2+n-1)\phi_3^2 + \frac{1}{2}n^2(n+1)^2\phi_2\phi_3 \\ & +\frac{1}{6}n(n+1)(2n+1)(2\phi_1\phi_3+\phi_2^2) + n(n+1)\phi_1\phi_2 \\ & +n\phi_1^2\} \frac{1}{n\phi_1 + \frac{1}{2}n(n+1)\phi_2^2 + \frac{1}{6}n(n+1)(2n+1)\phi_3} \text{ is satisfied .} \end{aligned}$$

Proof :

Since $f(t) = a + bt + ct^2$, a quadratic demand rate and

$$\begin{aligned} A(i,n) &= \int_{(i-1)\frac{H}{n}}^{i\frac{H}{n}} (a + bt + ct^2)\ dt \\ &= \phi_1(n) + \phi_2(n)i + \phi_3(n)i^2, \end{aligned}$$

where

$$\begin{aligned} \phi_1(n) &= \frac{aH}{n} - \frac{bH^2}{2n^2} + \frac{cH^3}{3n^3} \\ \phi_2(n) &= \frac{bH^2}{n^2} - \frac{cH^3}{n^3} \\ \phi_3(n) &= \frac{cH^3}{n^3}; \end{aligned}$$

$$\begin{aligned} A^2(i,n) &= \phi_3^2 i^4 + 2\phi_2\phi_3 i^3 + (2\phi_1\phi_3 + \phi_2^2)i^2 \\ &\quad +2\phi_1\phi_2 i + \phi_1^2. \end{aligned}$$

Therefore,

$$\begin{aligned} \sum_{i=1}^{n} A(i,n) &= n\phi_1 + \frac{1}{2}n(n+1)\phi_2 + \frac{1}{6}n(n+1)(2n+1)\phi_3; \\ \sum_{i=1}^{n} A^2(i,n) &= \frac{1}{30}n(n+1)(6n^3+9n^2+n-1)\phi_3^2 + \frac{1}{2}n^2(n+1)^2\phi_2\phi_3 \\ &+ \frac{1}{6}n(n+1)(2n+1)(2\phi_1\phi_3+\phi_2^2) + n(n+1)\phi_1\phi_2 + n\phi_1^2. \end{aligned}$$

Now,

$$\begin{aligned} \frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)} = \{&\frac{n(n+1)}{30}(6n^3+9n^2+n-1)\phi_3^2 + \frac{1}{2}n^2(n+1)^2\phi_2\phi_3 \\ &+\frac{1}{6}n(n+1)(2n+1)(2\phi_1\phi_3+\phi_2^2) + n(n+1)\phi_1\phi_2 \\ &+n\phi_1^2\}\frac{1}{n\phi_1 + \frac{1}{2}n(n+1)\phi_2^2 + \frac{1}{6}n(n+1)(2n+1)\phi_3} \end{aligned}$$

Therefore, $\frac{C_L}{C_h} > \frac{\sum_{i=1}^{n} A^2(i,n)}{2N\sum_{i=1}^{n} A(i,n)}$ implies

$$\begin{aligned}\frac{C_L}{C_h} > \tfrac{1}{2N}\{&\tfrac{n(n+1)}{30}(6n^3+9n^2+n-1)\phi_3^2 + \tfrac{1}{2}n^2(n+1)^2\phi_2\phi_3\\ &+\tfrac{1}{6}n(n+1)(2n+1)(2\phi_1\phi_3+\phi_2^2)+n(n+1)\phi_1\phi_2\\ &+n\phi_1^2\}\tfrac{1}{n\phi_1+\frac{1}{2}n(n+1)\phi_2^2+\frac{1}{6}n(n+1)(2n+1)\phi_3}.\end{aligned}$$

Hence the proof. □

Corollary 2:

There exists a global minimum of TAC ; for fixed "n" and "N" , $f(t) = a+bt$, a linear trend in demand , iff

$$\frac{C_L}{C_h} > \frac{1}{6N}[\frac{(n+1)\phi_2\{(2n+1)\phi_2+6\phi_1\}+6\phi_1^2}{2\phi_1+(n+1)\phi_2}]$$

is satisfied .

Proof :

Putting $c = 0$ in *Corollary 1* , we have,

$$\begin{aligned}\phi_1 &= \frac{aH}{n} - \frac{bH^2}{2n^2}\\ \phi_2 &= \frac{bH^2}{n^2}\\ \phi_3 &= 0;\end{aligned}$$

and

$$\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)} = \frac{(n+1)\phi_2\{(2n+1)\phi_2+6\phi_1\}+6\phi_1^2}{3\{2\phi_1+(n+1)\phi_2\}}$$

Therefore, $\frac{C_L}{C_h} > \frac{1}{2N}\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)}$ implies

$$\frac{C_L}{C_h} > \frac{1}{6N}[\frac{(n+1)\phi_2\{(2n+1)\phi_2+6\phi_1\}+6\phi_1^2}{2\phi_1+(n+1)\phi_2}]$$

Hence the proof.□

Corollary 3:

There exists a global minimum of TAC ; for fixed "n" and "N" , $f(t) = a$, a constant demand rate, iff

$$\frac{C_L}{C_h} > [\frac{aH}{2Nn}]$$

is satisfied .

Proof :

Putting $b = 0$, $c = 0$ in *Corollary 1*, we have

$$\begin{aligned}\phi_1 &= \frac{aH}{n},\\ \phi_2 &= 0,\\ \phi_3 &= 0,\end{aligned}$$

and

$$\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)} = a\frac{H}{n}$$

Therefore, $\frac{C_L}{C_h} > \frac{1}{2N}\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)}$ implies

$$\frac{C_L}{C_h} > \frac{aH}{2Nn}$$

Hence the proof. □

Corollary 4:

There exists a global minimum of TAC ; for fixed "n" and "N" , $f(t) = ae^{bt}$, $b > 0$, an exponential demand rate; iff

$$\frac{C_L}{C_h} > \frac{a}{2Nb}[\frac{(e^{\frac{bH}{n}} - 1)(e^{bH} - 1)}{(e^{\frac{bH}{n}} + 1)}]$$

is satisfied .

Proof :

Since

$$\begin{aligned}A(i,n) &= \int_{(i-1)\frac{H}{n}}^{i\frac{H}{n}} ae^{bt}\, dt\\ &= \frac{a}{b}[e^{i\frac{bH}{n}} - e^{(i-1)\frac{bH}{n}}]\\ \sum_{i=1}^{n} A(i,n) &= \frac{a}{b}(e^{bH} - 1)\\ \sum_{i=1}^{n} A^2(i,n) &= \frac{a^2}{b^2}(\frac{e^{\frac{bH}{n}} - 1}{e^{\frac{bH}{n}} + 1})(e^{2bH} - 1).\end{aligned}$$

Now,

$$\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)} = \frac{a}{b}\frac{(e^{\frac{bH}{n}} - 1)}{(e^{\frac{bH}{n}} + 1)}(e^{bH} + 1).$$

Therefore, $\frac{C_L}{C_h} > \frac{1}{2N}\frac{\sum_{i=1}^{n} A^2(i,n)}{\sum_{i=1}^{n} A(i,n)}$ implies

$$\frac{C_L}{C_h} > \frac{a}{2Nb}[\frac{(e^{\frac{bH}{n}}-1)(e^{bH}-1)}{(e^{\frac{bH}{n}}+1)}]$$

Hence the proof. □

4.Conclusion

Up to now, many inventory models have been considered in the literature. Some of them assumed the rate of production is inflexible and some of them assumed a decision variable. It is common belief that production is the result of combined efforts of the factors of production. These factors may be fixed or variable. The variable factors: raw materials, ordinary labour , power, fuel and modern technologies accelerate the rate of production. The capital investment for variable factors is required more when output is more; less when output is less. Generally, MRP(material requirement planning), OPT(optimized production technology), JIT(just-in-time) and FMS(flexible manufacturing system) offer the hope of eleminating many of the weaknesses of improving production effeiciency. In general, a production rate much higher than the demand rate leads to rapid accumulation of inventories resulting in higher holding costs and other related problems. If the machine production rate is less than the demand rate, the management has to face problems that are usually associated with stock-out situations. These inconveniences arise due to inability of the manufacturing setup to adjust its production rate in keeping with the variability in the market demand. But the machine production rate can easily be changed[37]. The treatment of machine production rate as a decision variable is especially appropriates for automated technologies that are volume flexible[38]. Indeed, in many aspects of industrial management one of the most difficult problems is to strike a proper balance between the scientific techniques on the one hand, and on the other hand, a complete dependence upon the decision and judgement of any or some people at the top echelon. In the former case, people with high salaries keep themselves busy and concerned over the inventory control problems and with so-called quantitive formulas. In the latter case, a small group of men generally much lesser degree of control and end up with high levels of inventory and low degree of coordination. For any manufacturing enterprise to be economically viable, it is of utmost importance to reach a working compromise between low control costs and maximum output costs, that is, low level of inventory with a high rate of inventory turnover, which is operationally feasible and economically sound. In this paper, we proposes an extension to the economic production lot size model in which the rate of production depends upon the technology of manufacturing system, capital investment for MRP, power fuel etc. and number of labours. In that sense, the proposed extension is reflective of the capability of modern production systems.

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