On Accelerated Flow for MHD Generalized Burgers' Fluid in a Porous Medium and Rotating Frame

Zainal Abdul Aziz, Faisal Salah, and Dennis Ling Chuan Ching

***Abstract*—The aim of this paper is to determine the exact steady-state solution of a magnetohydrodynamic (MHD) and rotating flow of generalized Burgers' fluid induced by a variable accelerated plate. This is attained by using the Fourier sine transform technique. This result is then presented in the equivalent forms in terms of exponential, sine and cosine functions. Similar solutions for Burgers', Oldroyd - B, Maxwell, Second grade and Navier - Stokes fluids are shown to emerge as the limiting cases of the present exact solution. The graphical results illustrate the velocity profiles which have been determined for the flow due to the variable acceleration of an infinite flat plate. Moreover, these figures are plotted to show the effects of different parameters on velocity field.**

***Index Terms*—Exact solution, Generalized Burgers' fluid, Steady - state solution, Fourier sine transform, Rotating frame, Porous space.**

I. Introduction

The studies of non-Newtonian fluids have received considerable attention because of numerous applications in industry, geophysics and engineering. Some investigations are notably important in industries related to paper, food stuff, personal care products, textile coating and suspension solutions [1]. A large class of real fluids do not exhibit the linear relationship between stress and rate of strain. Due to the non-linear dependence, the analysis of the behaviour of fluid motion of non Newtonian fluids tends to be much more complicated and subtle in comparison with that of Newtonian fluids. When the motion of a fluid is set up, the velocity field contains transients obtained by the initial conditions. These transients gradually disappear in time and the starting solution tends to the steady-state solution, which is independent of the initial conditions [2].

The generalized Burgers' fluids are in the category of non - Newtonian fluids and are the subclass of rate type fluids which is used to describe the motion of the earth's mantle [3]. It was employed to study various problems due to their relatively simple structure. Moreover, one can reasonably hope to obtain exact solutions from this type of generalized Burgers' fluid. This motivates us to choose the generalized Burgers' model in this study. The exact solutions are important as these provide standard for checking the accuracies of many approximate solutions which can be numerical or empirical. They can also be used as tests for verifying numerical schemes that are developed for studying more complex flow problems.

The analysis of the effects of rotation and magnetohydrodynamic (MHD) flows through a porous medium have gained an increasing interest due to the wide range of applications either in geophysics or in engineering such as the optimization of the solidification process of metals and metal alloys, the control underground spreading of chemical wastes, and pollutants. MHD is the study of the interaction of conducting fluids with electromagnetic phenomena. The flow of an electrically conducting fluid in the presence of magnetic field is of importance in various areas of technology and engineering such as MHD power generation and MHD pumps. Therefore, several researchers have discussed the flows of generalised Burgers fluid in different configurations and there are on hand few attempts which include the effects of rotation and MHD (for instances see studies in [4] - [16] and references therein).

The aim of the current study is to establish exact steady state solutions for the velocity field corresponding to flow induced by the variable accelerating plate in a generalized Burger fluid. The fluid is magnetohydrodynamic (MHD) in the presence of an applied magnetic field and occupying a half porous space, which is bounded by a rigid and non-conducting plate. Constitutive equations of a generalized Burgers fluid are used. Modified Darcy's law has been utilized. The steady-state solution to the resulting problem is achieved by Fourier sine transform, which contains as limiting cases the similar solutions for Burgers' fluid, Oldroyd-B, Maxwell, Second grade and Navier - Stokes fluids. The graphs are plotted in order to illustrate the effects of the variations of embedded flow parameters on the velocity field.

This paper is organised as follows, in section 2 we formulate our problem, and this problem is solved analytically in section 3. We write the results and discussions in section 4 and finally we conclude our deliberations in section 5.

II. Formulation of the Problem

We choose a Cartesian coordinate system by considering an infinite plate at $z = 0$. An incompressible fluid which occupies the porous space is conducting electrically by the exertion of an applied magnetic field $B_\circ$ which is parallel to the z axis. The electric field is not taken into consideration, the magnetic Reynolds number is small and such that the induced magnetic field is not accounted for. The Lorentz force $J \times B_\circ$ under these conditions is equal to $-\sigma B_\circ^2 V$. Here J is the current density, V is the velocity field, σ is the

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electrical conductivity of fluid. Both plate and fluid possess solid body rotation with a uniform angular Ω about the z axis.

The governing equations are

$$divV = 0 \tag{1}$$

$$\rho\left[\frac{\partial V}{\partial t} + (V.\nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r)\right] = -\nabla p + divS - \sigma B_\circ^2 V + D \tag{2}$$

where ρ is the fluid density, r is a radial vector with $r^2 = x^2 + y^2$, p is the pressure, S is the extra stress tensor and D is the Darcy's resistance.

The constitutive relationships for generalized Burgers' fluid are

$$T = -pI + S,$$

$$S\left(1 + \lambda_1 \frac{d}{dt} + \lambda_2 \frac{d^2}{dt^2}\right) = \mu\left[1 + \lambda_3 \frac{d}{dt} + \lambda_4 \frac{d^2}{dt^2}\right] X, \tag{3}$$

where T is the Cauchy stress tensor, I is the identity tensor, L is the velocity gradient, $X = L + L'$ is the first Rivlin - Eriksen tensor, λ_1 and λ_3 are correspondingly the relaxation and retardation times, μ is the dynamic viscosity of fluid, $\frac{d}{dt}$ represents the material derivative and λ_2, λ_4 are the material constants having the dimensions as the square of time. According to Tan and Masuoka [18], the Darcy's resistance in an Oldroyd-B fluid satisfies the following expression

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) D = -\frac{\mu\phi}{k}\left(1 + \lambda_3 \frac{\partial}{\partial t}\right) V, \tag{4}$$

where ϕ is the porosity and k is the permeability of the porous medium. For generalized Burgers' fluid, it has been proposed in [13] that

$$\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right) D = -\frac{\mu\phi}{k}\left[1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right] V \tag{5}$$

We seek a velocity field of the form

$$V = (u(z,t), v(z,t), w(z,t)), \tag{6}$$

which together with Eq. (1) yield $w = 0$. By using the equations (2), (3) and (6) we arrive at the following equations

$$\rho\left(\frac{\partial u}{\partial t} - 2\Omega v\right) = -\frac{\partial \hat{p}}{\partial x} + \frac{\partial S_{xz}}{\partial z} - \sigma B_\circ^2 u + D_x, \tag{7}$$

$$\rho\left(\frac{\partial v}{\partial t} + 2\Omega u\right) = -\frac{\partial \hat{p}}{\partial y} + \frac{\partial S_{yz}}{\partial z} - \sigma B_\circ^2 v + D_y, \tag{8}$$

where

$$\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right) S_{xz} = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial z}, \tag{9}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right) S_{yz} = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial v}{\partial z}. \tag{10}$$

The D_x and D_y are $x-$ and $y-$ components of the Darcy's resistance D , and $z-$ component of equation (2) shows that $\hat{p} \neq \hat{p}(z)$ and the modified pressure $\hat{p}$ is

$$\hat{p} = p - \frac{\rho}{2}\Omega^2 r^2$$

Invoking equations (5), (9) and (10) in equations (7) and (8), and then neglecting the pressure gradient, we now obtain the coupled governing equations as

$$\rho\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial t} - 2\Omega v\right) + \sigma B_\circ^2\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)u = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 u}{\partial z^2} - \frac{\mu\phi}{k}\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)u, \tag{11}$$

$$\rho\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial v}{\partial t} + 2\Omega u\right) + \sigma B_\circ^2\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)v = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\mu\phi}{k}\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)v, \tag{12}$$

where the appropriate initial and boundary conditions are

$$u = v = 0 \text{ at } t = 0, \; z > 0, \tag{13}$$

$$u(0,t) = At^2, \; v(0,t) = 0 \text{ for } t > 0, \tag{14}$$

$$u, \frac{\partial u}{\partial z}, \; v, \; \frac{\partial v}{\partial z} \to 0 \text{ as } z \to \infty; \; t > 0. \tag{15}$$

Letting $F = u + iv$ in equations (11) and (12), the problem is reduced by combining these two equations and it becomes

$$\rho\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial F}{\partial t} + 2i\Omega F\right) = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 F}{\partial z^2} - \sigma B_o^2\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)F - \frac{\mu\phi}{k}\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)F \tag{16}$$

The initial and boundary conditions for a variable accelerated plate are given as

$$F(0,t) = At^2, \text{ for } t > 0 \tag{17}$$

$$F(z,0) = \frac{\partial F(z,0)}{\partial t}, \; z > 0 \tag{18}$$

$$F(z,t), \frac{\partial F(z,t)}{\partial t} \to 0 \text{ as } z \to \infty; t > 0, \tag{19}$$

where A has dimension of $\frac{l}{t^3}$.

Introducing the following dimensionless quantities

$$\left.\begin{aligned} &\xi = z\left(\frac{A}{\nu^3}\right)^{\frac{1}{5}}, \; \tau = t\left(\frac{A^2}{\nu}\right)^{\frac{1}{5}}, \; G = \frac{F}{(\nu^2 A)^{\frac{1}{5}}}, \\ &\beta = \lambda_1\left(\frac{A^2}{\nu}\right)^{\frac{1}{5}}, \; \alpha = \lambda_2\left(\frac{A^2}{\nu}\right)^{\frac{2}{5}}, \\ &R = \lambda_4\left(\frac{A^2}{\nu}\right)^{\frac{2}{5}}, \; Q = \lambda_3\left(\frac{A^2}{\nu}\right)^{\frac{1}{5}}, \\ &M = \frac{\sigma B_o^2}{\rho}\left(\frac{\nu}{A^2}\right)^{\frac{1}{5}}, \; \omega = \left(\Omega\frac{\nu^{\frac{1}{5}}}{A^{\frac{2}{5}}}\right), \\ &\frac{1}{B} = \frac{\phi}{k}\left(\frac{\nu^3}{A}\right)^{\frac{2}{5}}, \; c = 2i\omega + M, \\ &\nu \text{ is the kinematic viscosity.} \end{aligned}\right\} \tag{20}$$

The problem statement according to the governing equation (16), is now reduced to

$$\left(\beta+\alpha\left(c+\frac{\partial}{\partial\tau}\right)\right)\frac{\partial^2 G(\xi,\tau)}{\partial\tau^2}+\left(1+\beta c+\frac{Q}{B}\right)\frac{\partial G(\xi,\tau)}{\partial\tau}+\left(c+\frac{1}{B}\right)G(\xi,\tau)=\left(1+Q\frac{\partial}{\partial\tau}+R\frac{\partial^2}{\partial\tau^2}\right)\frac{\partial^2 G(\xi,\tau)}{\partial\xi^2},\qquad(21)$$

$$G(0,\tau)=\tau^2,\ \ \tau>0,\qquad(22)$$

$$G(\xi,\tau),\ \ \frac{\partial G(\xi,\tau)}{\partial\xi}\to 0\ \text{as}\ \xi\to\infty;\ \ \tau>0.\qquad(23)$$

III. Solution of the Problem

Multiplying both sides of equation (21) by $\sqrt{\frac{2}{\pi}}\sin(\xi\eta)$, and integrating the result with respect to ξ from 0 to ∞ and taking into consideration the conditions (22) and (23), we find that

$$\alpha\frac{d^3G_s(\eta,\tau)}{d\tau^3}+(\beta+\alpha c+R\eta^2)\frac{d^2G_s(\eta,\tau)}{d\tau^2}+\left(1+\beta c+Q\left(\frac{1}{B}+\eta^2\right)\right)\frac{dG_s(\eta,\tau)}{d\tau}+\left(c+\frac{1}{B}+\eta^2\right)G_s(\eta,\tau)=\sqrt{\frac{2}{\pi}}\eta(\tau^2+2Q\tau+2R),\quad \text{for}\ \eta,\ \tau>0,\qquad(24)$$

where the Fourier sine transform $G_s(\eta,\tau)$ of $G(\xi,\tau)$ has to satisfy the conditions

$$G_s(\eta,0)=\frac{\partial G_s(\eta,0)}{\partial\tau}=\frac{\partial^2G_s(\eta,0)}{\partial\tau^2}=0,\eta>0.\qquad(25)$$

Solving the ordinary differential equation (24) subject to the initial condition (25) and inverting the result by means of the Fourier sine transform, we can write the velocity field $G(\xi,\tau)$ as a sum of the steady-state and transient solutions respectively, i.e.

$$G(\xi,\tau)=G_s(\xi,\tau)+G_\tau(\xi,\tau).$$

The steady-state solution of the MHD generalized Burgers' fluid, which is valid for large values of time, has the form

$$G(\xi,\tau)=\frac{2}{\pi}\int_0^\infty\Bigg(\eta((1-\alpha)c-\beta)(\tau^2+2Q\tau+2R)-2(\tau+Q)(1+\beta c-\alpha)+\frac{\tau^2+2R-2Q^2}{B}\Bigg)\Bigg(\left(1+\beta c+\frac{Q}{B}-\alpha+Q\eta^2\right)^2+\left((1-\alpha)c+\frac{1}{B}-\beta+(1-R)\eta^2\right)^2\Bigg)^{-1}\sin(\xi\eta)d\eta.\qquad(26)$$

The equation (26) can also be written as [see Appendix for the standard integrals]

$$G(\xi,\tau)=e^{-\xi Y}(\theta\sin(\xi E_\circ)+M_\circ\cos(\xi E_\circ)),\qquad(27)$$

where

$$\theta=(((1-\alpha)c-\beta)(\tau^2+2Q\tau+2R)-2\tau+Q)(1+\beta c-\alpha)+\frac{\tau^2+2R-2Q^2}{B}-b^2((\tau^2+2R-2Q^2)-R(\tau^2+2Q\tau+2R)))(U(Q^2+(1-R)^2))^{-1}.\qquad(28)$$

$$M_\circ=\frac{(\tau^2+2R-2Q^2)-R(\tau^2+2Q\tau+2R)}{Q^2+(1-R)^2},\qquad(29)$$

$$U^2=(Q((1-\alpha)c+\frac{1}{B}-\beta)-(1-R)(1+\beta c+\frac{Q}{B}-\alpha))^2(Q^2+(1-R)^2)^{-2},\qquad(30)$$

$$b^2=(Q(1+\beta c+\frac{Q}{B}-\alpha)+(1-R)((1-\alpha)c+\frac{1}{B}-\beta))(Q^2+(1-R)^2)^{-1},\qquad(31)$$

$$2Y^2=\sqrt{b^4+U^2}+b^2,2E_\circ^2=\sqrt{b^4+U^2}-b^2.\qquad(32)$$

Limiting Cases

1) The above expressions (26) to (32) can also be written for MHD Burgers' fluid, by letting $\lambda_4=0$ in a porous space, which takes the form

$$G(\xi,\tau)=e^{-\xi Y_1}(\theta_1\sin(\xi E_1)+M_1\cos(\xi E_1))\qquad(33)$$

where

$$\theta_1=(((1-\alpha)c-\beta)(\tau^2+2Q\tau)-2(\tau+Q)(1+\beta c-\alpha)+(\tau^2-2Q^2)(\frac{1}{B}-{b_1}^2))(U_1(Q^2+1))^{-1},\qquad(34)$$

$$M_1=\frac{\tau^2-2Q^2}{Q^2+1},\qquad(35)$$

$${U_1}^2=(Q((1-\alpha)c+\frac{1}{B}-\beta)-(1+\beta c+\frac{Q}{B}-\alpha))^2(Q^2+1)^{-2},\qquad(36)$$

$${b_1}^2=\frac{Q\left(1+\beta c+\frac{Q}{B}-\alpha\right)+\left((1-\alpha)c+\frac{1}{B}-\beta\right)}{Q^2+1},\qquad(37)$$

$$2{Y_1}^2=\sqrt{{b_1}^4+{U_1}^2}+{b_1}^2,\ \ 2{E_1}^2=\sqrt{{b_1}^4+{U_1}^2}-b_1^2.\qquad(38)$$

2) Similarly the result (26) to (32) for MHD Oldroyd-B fluid (letting $\lambda_2=\lambda_4=0$) in a porous space takes the form

$$G(\xi,\tau)=(\tau^2-2Q^2+Q\xi(\tau+Q)\sqrt{c+\frac{1}{B}}-\frac{\xi((1+\beta c)(\tau+Q)+\beta)}{\sqrt{c+\frac{1}{B}}}-\frac{Q\xi(\tau+Q)}{B(\sqrt{c+\frac{1}{B}})}+\frac{\xi(1+\beta c)^2(1+\xi\sqrt{c+\frac{1}{B}})}{4(\sqrt{c+\frac{1}{B}})^3})e^{-\xi\sqrt{c+\frac{1}{B}}}+\frac{8Q(1+\beta c)}{\pi}\int_0^\infty\frac{\eta(\eta^2+\frac{1}{B})\sin(\xi\eta)}{(\eta^2+\frac{1}{B}+c)^3}d\eta$$

$$+\frac{4Q^2}{\pi}\int_0^\infty \frac{\eta(\eta^2+\frac{1}{B})^2\sin(\xi\eta)}{(\eta^2+\frac{1}{B}+c)^3}d\eta. \quad (39)$$

3) The above expression (39) can also be written for MHD Maxwell fluid (letting $\lambda_2=\lambda_3=\lambda_4=0$) in a porous space, which is now of the form

$$\begin{aligned}G(\xi,\tau) =&(\tau^2-\xi((1+\beta c)\tau+\beta)(c+\frac{1}{B})^{-1/2}\\&+\frac{1}{4}\xi(1+\beta c)^2(1+\xi\sqrt{c+\frac{1}{B}})(c\\&+\frac{1}{B})^{-3/2})e^{-\xi\sqrt{c+\frac{1}{B}}}. \end{aligned}\quad (40)$$

4) The result (26) to (32) for MHD second grade fluid (letting $\lambda_1=\lambda_2=\lambda_4=0$) in a porous space takes the form

$$G(\xi,\tau)=e^{-\xi Y_2}\left(\theta_2\sin(\xi E_2)+M_2\cos(\xi E_2)\right), \quad (41)$$

where

$$\theta_2=\frac{b_2^2\left(2Q^2-\tau^2\right)-\left(2Q\varphi+\tau\left(2\varphi-2\phi Q-\tau\phi\right)\right)}{U_2\left(Q^2+1\right)}, \quad (42)$$

with

$$\phi=c+\frac{1}{B},\ \varphi=1+\frac{Q}{B}, \quad (43)$$

$$M_2=\frac{\tau^2-2Q^2}{Q^2+1}, \quad (44)$$

$${U_2}^2=\left(\frac{\varphi-Q}{Q^2+1}\right)^2,\ {b_2}^2=\left(\frac{Q\varphi+\phi}{Q^2+1}\right), \quad (45)$$

$$2{Y_2}^2=\sqrt{{b_2}^4+{U_2}^2}+{b_2}^2,\ 2{E_2}^2=\sqrt{{b_2}^4+{U_2}^2}-{b_2}^2. \quad (46)$$

5) Similar solution for viscous fluid can be obtained as a limiting case of the MHD generalized Burgers' fluid solution. By letting $\lambda_1,\lambda_2,\lambda_3$ and $\lambda_4\to 0$ in equations (26) to (32), we find that

$$G(\xi,\tau)=e^{-\xi f}\left(\tau^2\cos(\xi L_\circ)-2\tau\sin(\xi L_\circ)\right), \quad (47)$$

where

$$s=c+\frac{1}{B}, \quad (48)$$

$$2f^2=\sqrt{s^2+1}+s, \quad (49)$$

$$2L_\circ^2=\sqrt{s^2+1}-s. \quad (50)$$

The above solution (47) can be shown to be equivalent to the starting solution of the Newtonian fluid.

IV. Results and Discussion

In this section, we present the graphical illustrations of the velocity profiles which have been determined for the flow due to the variable acceleration of an infinite flat plate. The emerging parameters here are the rotating parameter $W=\omega$, magnetic field parameter M and parameter of the porous medium B, the material constants parameters are $E=\alpha$ and R. In order to illustrate the role and influence of these parameters on the real and imaginary parts of the velocity profile $G(\xi,\tau)$, the figs. 1 to 5 have been displayed. In these figs., panels (a) depict the variations of $-\mathrm{Re}[G(\xi,\tau)]$ for MHD generalized Burgers' fluid and similarly panels (b) indicate the variations of $\mathrm{Im}[G(\xi,\tau)]$.

Fig. 1(a) shows that the real part of the velocity profile decreases for various values of rotation W, with respect to the increase in ξ. As W increases, the velocity profile decreases. Fig. 1(b) indicates that the magnitude of imaginary part of the velocity profile increases initially and later decreases for various values of rotation W, with respect to the increase in ξ. As W increases, the velocity profile also increases. This result can be compared with that obtained by Hayat et al. [14].

Fig. 2(a) is prepared to see the effects of the applied magnetic field M on the real part of the velocity profile. Keeping R, E, B, Q, β, W, τ fixed and varying M, it is noted that the real part of the velocity profile decreases as the magnetic field parameter increases. Fig. 2(b) is also graphed to see the effects of the applied magnetic field on the imaginary part of the velocity profile. Keeping R, E, B, Q, β, W, τ fixed and varying M, it is noted that the imaginary part decreases initially and later increases.

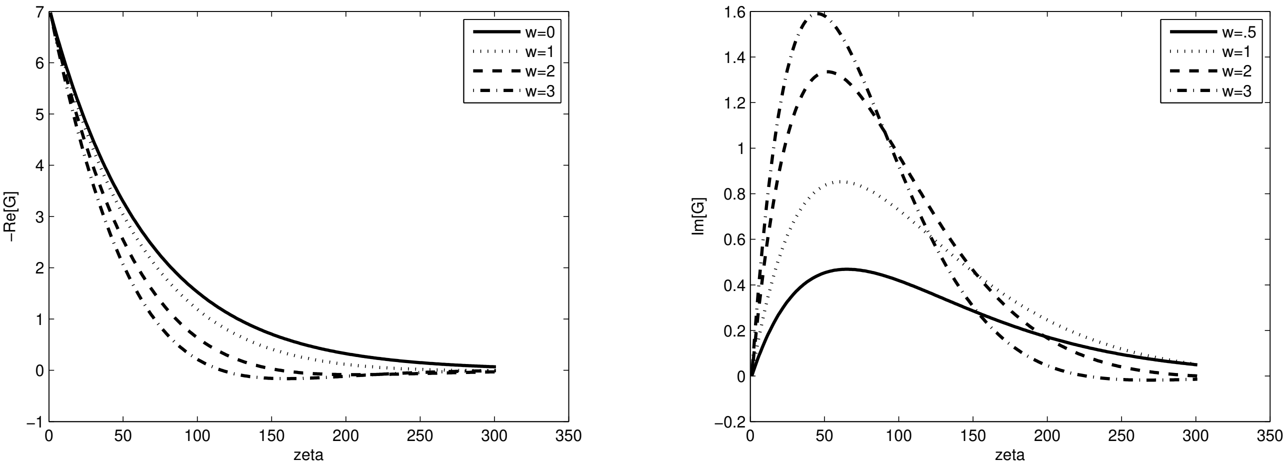
Fig. 3(a) indicates the variations of the porosity parameter B. Keeping R, E, M, Q, β, W, τ fixed, it is observed that by increasing in the porosity parameter B would then lead to increase in the real part of the velocity profile. Fig. 3(b) Keeping R, E, M, Q, β, W, τ fixed and varying B, it is noted that the imaginary part increases initially and later decreases.

Fig. 4(a) shows the effects of material parameter E of MHD generalized Burgers' fluid on the real part of velocity profile when R, B, M, Q, β, W, τ are fixed. It is interesting to note that by increasing in the material constant parameter E, this leads to increase in the real part of the velocity profile. Fig. 4(b) is obtained when R, B, M, Q, β, W, τ are fixed and the material constant parameter E is increased. This would lead to imaginary part of the velocity profile increases initially and later decreases.

Fig. 5(a) shows the effects of material parameter R of MHD generalized Burgers' fluid on the real part of velocity profile and keeping B, E, M, Q, β, W, τ fixed. It is found that by increasing in the parameter would lead to decrease in the real part of the velocity profile. Fig. 5(b) is prepared to see the influence of material parameter R of MHD generalized Burgers' fluid on the imaginary part of velocity profile and keeping B, E, M, Q, β, W, τ fixed. It is found that by increasing in the material constant parameter R would then lead to a decrease in the imaginary part of the velocity profile.

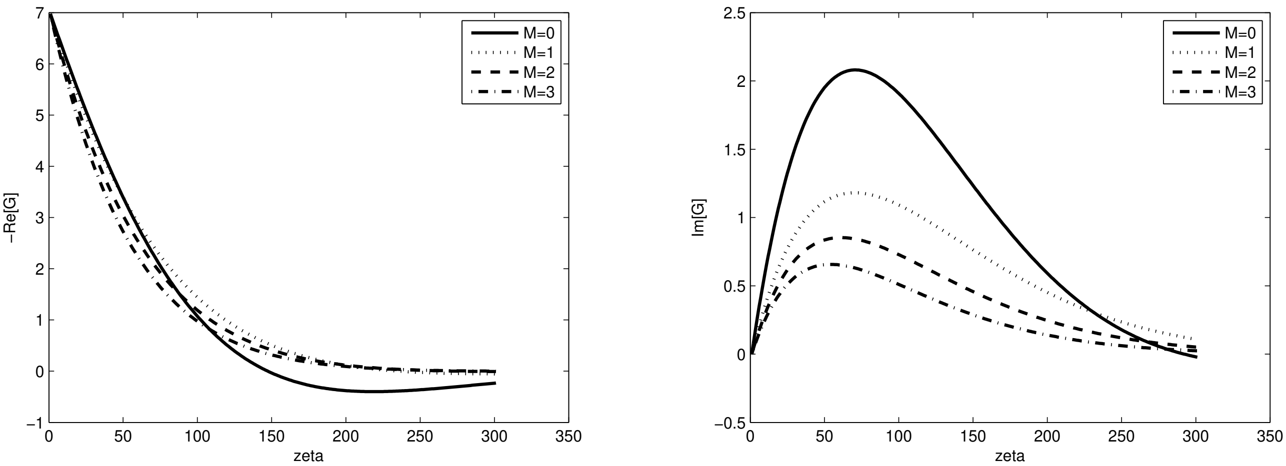
V. Conclusion

The exact steady - state solution, corresponding to the motion of the MHD generalized Burgers' fluid due to the variable acceleration of an infinite flat plate, is established by means of the Fourier sine transforms. The solution for MHD generalized Burgers' fluid and similar solutions (i.e. the limiting cases) for Burgers', Maxwell, Second grade and Navier - Stokes fluids are presented here in simple forms, i.e. in terms of the elementary exponential and trigonometric functions. These satisfy all the above governing equations and all the above imposed boundary conditions.



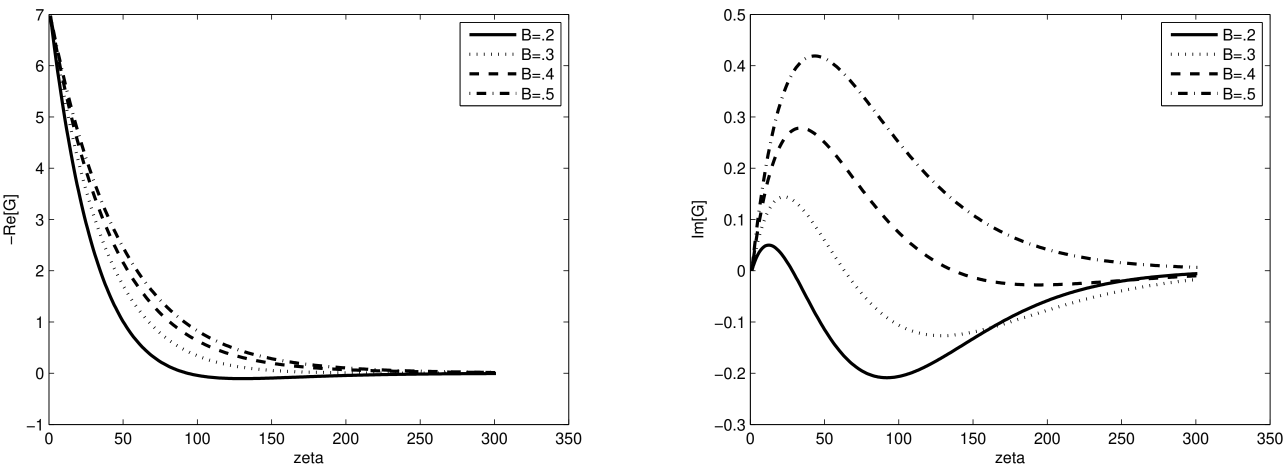
(a) (b)

Fig. 1. The variation of velocity profile $G(\xi, \tau)$ for various values of rotation W when $R = 1.3$, $E = 1.5$, $B = 1$, $Q = 1$, $\beta = 1$, $M = 2$, $\tau = pi/2$.



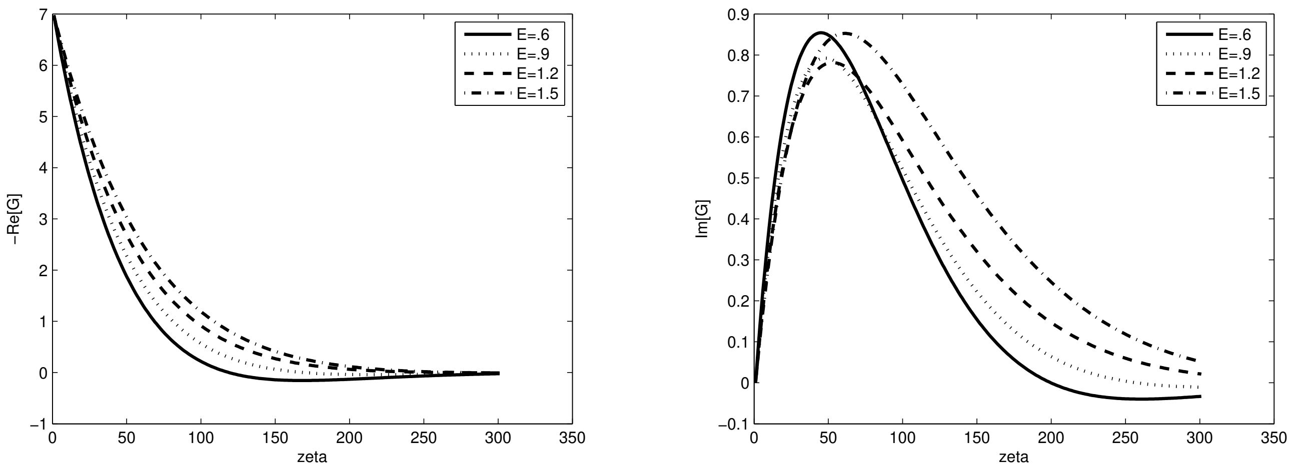
(a) (b)

Fig. 2. The variation of velocity profile $G(\xi, \tau)$ for various values of (MHD) M when $R = 1.3$, $E = 1.5$, $B = 1$, $Q = 1$, $\beta = 1$, $W = 1$, $\tau = pi/2$.



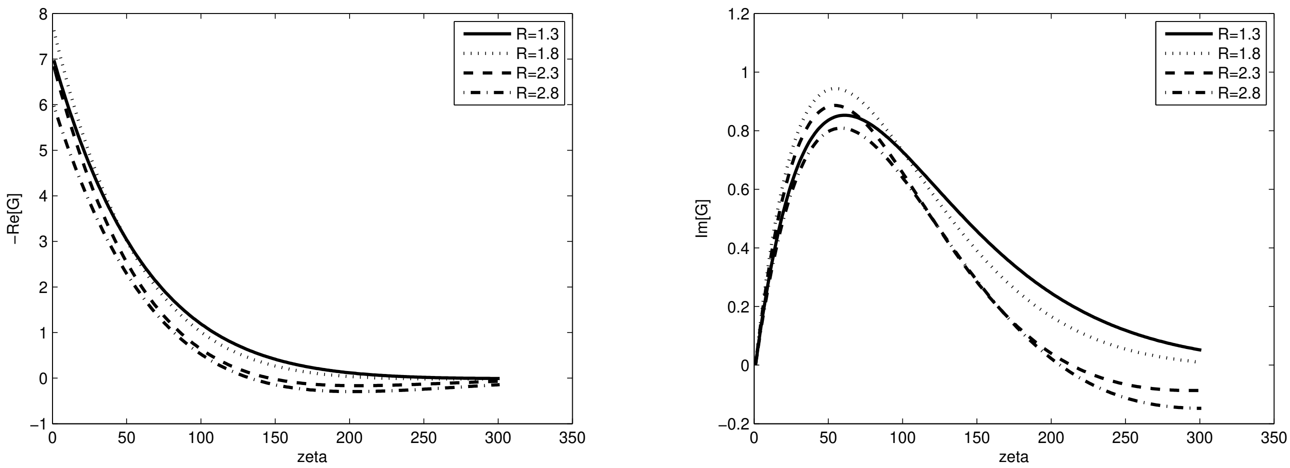
(a) (b)

Fig. 3. The variation of velocity profile $G(\xi, \tau)$ for various values of porosity parameter B when $R = 1.3$, $E = 1.5$, $M = 2$, $Q = 1$, $\beta = 1$, $W = 1$, $\tau = pi/2$.



(a) (b)

Fig. 4. The variation of velocity profile $G(\xi,\tau)$ for for various values parameter E when $R = 1.3$, $B = 1$, $M = 2$, $Q = 1$, $\beta = 1$, $W = 1$, $\tau = pi/2$.



(a) (b)

Fig. 5. The variation of velocity profile $G(\xi,\tau)$ for for various values parameter R when $E = 1.5$, $B = 1$, $M = 2$, $Q = 1$, $\beta = 1$, $W = 1$, $\tau = pi/2$.

VI. Appendix

In order to obtain equalities (27) from equation (26) and (39) we use results from [17]:

$$\int_0^\infty \frac{x\sin(ax)}{q^2+x^2}dx = \frac{\pi}{2}e^{-aq} \qquad a>0, \mathrm{Re}\ q>0 \tag{51}$$

$$\int_0^\infty \frac{x\sin(ax)}{\left(x^2+\in q^2\right)^2+n^2}dx = \frac{\pi}{2n}e^{-aK}\sin(aN), \tag{52}$$

$$\int_0^\infty \frac{x\left(x^2+\in q^2\right)\sin(ax)}{\left(x^2+\in q^2\right)^2+n^2}dx = \frac{\pi}{2}e^{-aK}\cos(aN), \tag{53}$$

where

$$2K^2 = \sqrt{q^4+n^2}+\in q^2, 2N^2 = \sqrt{q^4+n^2}-\in q^2 \tag{54}$$

and

$$\in = \pm 1. \tag{55}$$

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