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On Algebraic Identification of Critical States for Deadlock Control in Automated Manufacturing Systems Modeled With Petri Nets

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ABSTRACT Petri nets are an important and popular tool to model and analyze deadlocks in automated manufacturing systems. The state space of a Petri net model can be divided into two disjoint parts: a live-zone and a dead-zone. A first-met bad marking (FBM) is a marking in the dead-zone, representing the very first entry from the live-zone to the dead-zone, and the calculation of FBMs to a large extent contributes to the complexity of designing optimal liveness-enforcing supervisors. Most existing studies have to fully enumerate the reachable markings of a Petri net model to obtain the FBMs, which exacerbates the computational overheads. This paper first explores a variation mechanism of calculating FBMs with respect to the resource capacity in a class of S^3PR (Systems of Simple Sequential Processes with Resources) from the structural analysis perspective, which contains a ξ-resource. More generally, for the class of S^3PR with an η-resource as defined in this paper, the FBMs can be calculated in an algebraic way by a customized structural analysis technique without enumerating all the reachable markings. Finally, the variation mechanism of calculating FBMs is revealed for these considered classes of Petri net models. Examples are given to demonstrate the proposed method.

INDEX TERMS Petri net, first-met bad marking, deadlock control, automated manufacturing system.

I. INTRODUCTION

Over the passed two decades, the development of science and technology has reached an unprecedented height. Traditional manufacturing systems cannot meet the demands of human society in terms of production throughput or diversity of products. Nowadays, increasing importance has been given to automated manufacturing systems for their quick response to the market and production efficiency. For highly automated manufacturing systems with resource-sharing, a series of mechanisms are needed to deal with the deadlock phenomenon [1] that arises in a running system.

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System blocking caused by deadlocks may lead to catastrophic consequences and enormous economic losses, which should be considered when designing a supervisory controller for automated manufacturing systems (AMSs). In [2], the author proposes four necessary conditions for the occurrence of deadlocks: mutual exclusion, hold and wait, no preemption, and circuit wait. The first three conditions depend on a system's structure and its resources, while the last one is decided by the request, allocation and release of system resources. Once a deadlock occurs, the above four conditions must hold.

Petri nets [3], as a major mathematical model, have been applied to many problems in discrete event systems [4]–[10], such as modeling and analysis [11], [52], control [12]–[17], opacity verification and enforcement [18], [19], scheduling [20]–[28], [50], [51], performance evaluation [29], [30], [53], fault identification and diagnosis [32], [33], [61], [62], validation of various properties [18], [19], [54], and deadlock analysis and control [31], [32], [34], [35], [45]–[48].

To manage the deadlock problem in a system, three types of methods have been developed by researchers: deadlock detection and recovery, deadlock avoidance, and deadlock prevention [35]–[41], [49]. A deadlock avoidance approach dynamically examines each system state to guarantee that deadlocks never happen. The implementation of deadlock avoidance is usually done by an online control policy. During the operation of a system, a deadlock controller can disable the occurrence of some events such that a deadlock never occurs. Preventing a system from entering the dead-zone (DZ) of the state space of a system by deadlock prevention is an effective method to design a deadlock controller. Therefore, the first-met bad markings (FBMs) [43], defined as the states in the dead-zone (DZ), representing the very first entries from the live-zone (LZ) to the DZ, need to be calculated in advance for a system model [42]. The notion of FBMs has received much attention, based on which various deadlock resolution methods are reported [4], [42], [44], [52], [57].

Reachability analysis is a common and powerful means to explore the behavior of a Petri net model. The reachability graph of a Petri net can reflect its complete behavior, from which one identifies intuitively the states that can lead a system to a deadlock state. The calculation of FBMs to a large extent contributes to the computational complexity of designing a deadlock controller. Chen et al. [44] develop a method that can derive a maximally permissive liveness-enforcing supervisor for Petri nets based on the pre-computed FBMs if such a supervisor exists, where a maximally permissive control place is devised via a place invariant (PI) which forbids one or more FBMs and none of markings in the LZ is forbidden.

Traditionally, before the FBMs are computed, one has to enumerate all the reachable markings of a Petri net. With the increase of the initial marking or the size of a system model, the computational cost of FBMs grows dramatically and sometimes their computation becomes impossible because of the limited memory of computers. From the structural perspective, this paper primarily focuses on the analysis of subclasses of S^3PR with and without ξ-resources and proposes a number of ways to identify FBMs.

In [38], [41], a resource-transition circuit (RT-circuit) is defined as a kind of structure in a Petri net modeling a flexible manufacturing system, which consists of resource places and transitions only. A maximal perfect resource transition circuit (MPRT-circuit) whose definition is given in Section III in this paper is said to be *perfect* if the output transitions of the activity places on the circuit are exactly the transitions of this circuit. A resource is said to be a ξ-resource if its capacity is one and shared by two or more MPRT-circuits that do not contain each other. In this work, the existence of a ξ-resource [55], [56] plays an important role in determining the FBMs for a kind of S^3PRs (η-S^3PR) as described later, and divides the FBMs into several types for analysis.

The rest of the paper is organized as follows. Section II reviews some basic notions and definitions of Petri nets and S^3PR. Section III defines and analyzes an η-S^3PR to identify its FBMs. Section IV gives an example to demonstrate the proposed approach. Conclusions and future work are summarized in Section V. Appendix shows the outcome of an example given in Section IV.

II. PRELIMINARIES

A. BASICS OF PETRI NETS

A Petri net is a directed bipartite graph consisting of two components – a net structure and an initial marking. The net structure includes two types of nodes: places and transitions, linked by directed arcs. A directed arc cannot link the same type of nodes. Arcs are labeled by positive integers to represent their weights. In a Petri net representation, places are graphically shown as circles and transitions as boxes or bars. A place can hold tokens, graphically denoted by black dots, or a non-negative integer to represent their number. The distribution of tokens over the places of a net is called a marking that corresponds to a state of a modeled system. Token distribution at the initial state is called the initial marking. Let $\mathbb{N}$ denote the set of non-negative integers and $\mathbb{N}^+$ the set of positive integers.

Definition 1: A Petri net structure is a quadruple $N = (P, T, F, W)$, where

1) *$P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places, $n \in \mathbb{N}^+$;*
2) *$T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions with $P \cap T = \emptyset$, $m \in \mathbb{N}^+$;*
3) *$F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places.*
4) *$W: (P \times T) \cup (T \times P) \to \mathbb{N}$ is a mapping that assigns a weight to an arc: $W(f) > 0$ if $f \in F$ and $W(f) = 0$ otherwise. It is called the weighted function of a Petri net.*

Let $N = (P, T, F, W)$ be a Petri net (structure). N is called an ordinary net if for any $f \in F$, $W(f) = 1$; a generalized one if there exists at least an arc $f \in F$ with $W(f) > 1$. The Petri nets considered in this paper are ordinary ones. An ordinary Petri net can be denoted as $N = (P, T, F)$.

A marking of Petri net $N = (P, T, F, W)$ is a mapping $M: P \to \mathbb{N}$. (N, M_0) is referred to as a net system or marked net with M_0 being its initial marking. In a Petri net, a place $p \in P$ is said to be marked at M if $M(p) > 0$. A set of places $D \subseteq P$ is marked at M if at least one place is marked. $M(D) = \sum_{p \in D} M(p)$ is the total number of tokens in D at M.

Let $x \in P \cup T$ be a node in a Petri net $N = (P, T, F, W)$. The preset of x, denoted by $^\bullet x$, is defined as $^\bullet x = \{y \in P \cup T | (y, x) \in F\}$, and its postset, denoted by $x^\bullet$, is defined as $x^\bullet = \{y \in P \cup T | (x, y) \in F\}$. Given a place (transition) p (t), the nodes in its preset are called the pre-transitions (pre-places) of p (t) and the nodes in its postset are called the post-transitions (post-places) of p (t).

Given a Petri net N and a marking M, a transition $t \in T$ is enabled at M if for all $p \in {}^\bullet t$, $M(p) \geq W(p,t)$, denoted by $M[t\rangle$. When an enabled transition fires, the Petri net evolves to a new marking M' according to $M'(p) = M(p) - W(p,t) + W(t,p)$, $\forall p \in P$, which is denoted as $M[t\rangle M'$. Marking M'' is reachable from M if there exist a sequence of transitions $\sigma = t_0t_1 \ldots t_n$ and markings $M_1, M_2, \ldots, M_n$ such that $M[t_0\rangle M_1[t_1\rangle M_2[t_2 \ldots M_n[t_n\rangle M''$ holds, which is denoted by $M[\sigma\rangle M''$. The set of markings reachable from M in N is called the reachability set of Petri net (N, M), denoted as $R(N, M)$.

Definition 2: Let $N = (P, T, F, W)$ be a Petri net. A nonempty set $S \subseteq P$ is said to be a siphon (trap) if ${}^\bullet S \subseteq S^\bullet$ ($S^\bullet \subseteq {}^\bullet S$). A siphon (trap) is minimal if the removal of any place from S makes the fallacy of ${}^\bullet S \subseteq S^\bullet$ ($S^\bullet \subseteq^\bullet S$). A minimal siphon S is said to be strict if ${}^\bullet S \subset S^\bullet$. The set of strict minimal siphons in a Petri net is denoted by Π.

Definition 3: The output and input incidence matrices of a net N are defined as $[N^+](p,t) = W(t,p)$ and $[N^-](p,t) = W(p,t)$, respectively. The incidence matrix of a net N is defined as $[N] = [N^+] - [N^-]$, where $[N](p,t) = W(t,p) - W(p,t)$.

Definition 4: A P-vector is a column vector I: $P \to Z$ indexed by P and a T-vector is a column vector J: $T \to Z$ indexed by T, where Z is the set of integers. P-vector I is called a P-invariant (place invariant, PI for short) if $I \neq 0$ and $I^T[N] = 0^T$. T-vector J is called a T-invariant (transition invariant) if $J \neq 0$ and $[N]J = 0$. P-invariant I is a P-semiflow if every element of I is non-negative. $||I|| = \{p|I(p) \neq 0\}$ is called the support of I.

B. S^{3}PR MODELS

This section mainly reviews the basic concepts and notions of a system of simple sequential processes with resources, called an S^{3}PR [37]. An S^{3}PR is an ordinary net modeling a flexible manufacturing system producing multiple products with sequential processing stages by using different resource types, in which each processing stage needs one unit of a resource type only and one resource cannot participate in two or more consecutive processing stages.

Definition 5: A simple sequential process (S^{2}P) is a Petri net $N = (P_A \cup \{p^0\}, T, F)$, satisfying the following statements:

1) *$P_A \neq \emptyset$ is called the set of the activity (operation) places;*
2) *$p^0 \notin P_A$ is called the process idle place or idle place;*
3) *N is a strongly connected state machine;*
4) *Every circuit of N contains place p^0;*

Definition 6: An S^{2}P with resources (S^{2}PR) is a Petri net $N = (\{p^0\} \cup P_A \cup P_R, T, F)$, satisfying:

1) *The subnet generated from $X = P_A \cup \{p^0\} \cup T$ is an S^{2}P;*
2) *$P_R \neq \emptyset$; $(P_A \cup \{p^0\}) \cap P_R = \emptyset$;*
3) *$\forall p \in P_A, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, \exists r_p \in P_R, {}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$;*
4) *$\forall r \in P_R$, ${}^{\bullet\bullet} r \cap P_A = r^{\bullet\bullet} \cap P_A \neq \emptyset$; $\forall r \in P_R$, ${}^\bullet r \cap r^\bullet = \emptyset$;*
5) *${}^{\bullet\bullet}(p^0) \cap P_R = (p^0)^{\bullet\bullet} \cap P_R = \emptyset$.*

Definition 7: Given an S^{2}PR $N = (\{p^0\} \cup P_A \cup P_R, T, F)$, an initial marking M_0 is called an acceptable one for N if:

1) *$M_0(p^0) \geq 1$;*
2) *$M_0(p) = 0, \forall p \in P_A$;*
3) *$M_0(r) \geq 1, \forall r \in P_R$;*

Definition 8: An S^{3}PR, i.e., a system of S^{2}PR, can be defined recursively as follows:

1) *An S^{2}PR is an S^{3}PR;*
2) *Let $N_1 = (P_{A_1} \cup \{p_1^0\} \cup P_{R_1}, T_1, F_1)$ and $N_2 = (P_{A_2} \cup \{p_2^0\} \cup P_{R_2}, T_2, F_2)$ be two S^{3}PRs, satisfying $(P_{A_1} \cup \{p_1^0\}) \cap (P_{A_2} \cup \{p_2^0\}) = \emptyset$, $P_{R_1} \cap P_{R_2} = P_C \neq \emptyset$, and $T_1 \cap T_2 \neq \emptyset$. The Petri net $N = (P_A \cup P^0 \cup P_R, T, F)$ composed by N_1 and N_2 through P_C, denoted by $N = N_1 \circ N_2$, is still an S^{3}PR, defined as $P_A = P_{A_1} \cup P_{A_2}$, $P^0 = \{p_1^0\} \cup \{p_2^0\}$, $P_R = P_{R_1} \cup P_{R_2}$, $T = T_1 \cup T_2$, and $F = F_1 \cup F_2$.*

Definition 9: Let N be an S^{3}PR. (N, M_0) is called an acceptably marked S^{3}PR if one of the following conditions is satisfied:

1) *(N, M_0) is an acceptably marked S^{2}PR.*
2) *$N = N_1 \circ N_2$, where (N_i, M_{0_i}) $(i = 1, 2)$ is an acceptably marked S^{3}PR. Moreover, for all $i \in \{1, 2\}$ and $p \in P_{A_i} \cup \{p_i^0\}$, $M_0(p) = M_{0_i}(p)$; for all $i \in \{1, 2\}$ and $r \in P_{R_i} \backslash P_C$, $M_0(r) = M_{0_i}(r)$; for all $r \in P_C$, $M_0(r) = \max\{M_{0_1}(r), M_{0_2}(r)\}$.*

Transitions in $(P^0)^\bullet$ (${}^\bullet(P^0)$) are called source (sink) transitions, representing the entry (exit) of raw materials (completed products) when a manufacturing system is modeled with an S^{3}PR.

C. RT-CIRCUIT, η-S^{3}PR, AND FBMS

In [36], a resource-transition circuit (RT-circuit) Θ in an S^{3}PR is defined as a circuit that contains resource places and transitions only. We use $\Lambda(\bullet)$ to denote the set of all nodes on a circuit "$\bullet$". An RT-circuit is said to be a perfect one (PRT-circuit) if the output transitions of the activity places on the circuit are exactly the transitions on it. Let R_0 be a set of resource places and $\mathcal{G}(R_0)$ the set of all PRT-circuits formed by resource set R_0. If $\Theta_1, \Theta_2 \in \mathcal{G}(R_0)$, then $\Theta_1 \cup \Theta_2 \in \mathcal{G}(R_0)$. Therefore, $\mathcal{G}(R_0)$ contains a unique maximal PRT-circuit (MPRT-circuit). A resource in an S^{3}PR is said to be a ξ-resource if its capacity is one and shared by two or more MPRT-circuits that do not contain each other.

Definition 10: Given an S^{3}PR (N, M_0), if there exist two MPRT-circuits Θ_1 and Θ_2 that do not contain each other such that $(\Lambda(\Theta_1) \cap \Lambda(\Theta_2)) \cap P_R = \{r\}$, then r is called an

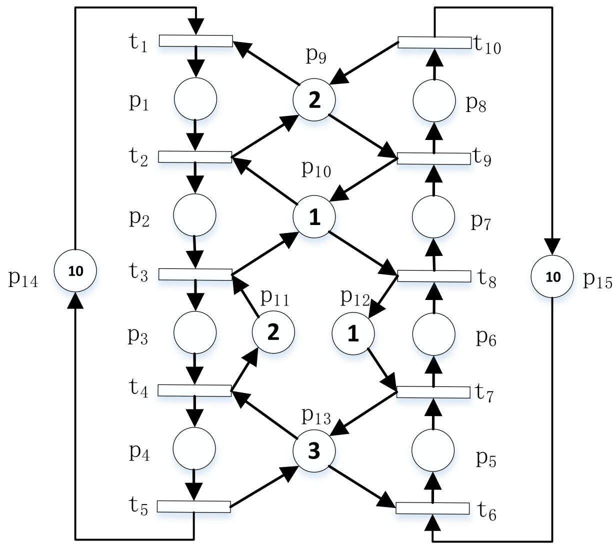

FIGURE 1. An η-S^{3}PR.

η-resource. An η-resource is said to be a ξ-resource if $M_0(r) = 1$.

Definition 11: Given a resource $r \in P_R$ in an S^{3}PR, the set of its holders is defined as $H(r) = (^{\bullet\bullet}r) \cap P_A$. For a siphon S in an S^{3}PR, $S = S_R \cup S_A$, where $S_R = S \cap P_R$ and $S_A = S \cap P_A$.

Definition 12: A holder-resource circuit (HR-circuit) with respect to resource r in an S^{3}PR is a simple circuit if it contains resource r, an activity place $p \in H(r)$, and transitions. If a resource is associated with one holder-resource circuit only, i.e., $|H(r)| = 1$, the holder-resource circuit is said to be monoploid. Let $[S] = (\bigcup_{r \in S_R} H(r)) \backslash S$ denote the complentary of siphon S.

Definition 13: Let $S \in \Pi$ be a strict minimal siphon in an S^{3}PR. Resource $r \in P_R$ is said to be independent if there does not exist a strict minimal siphon S such that $H(r) \cap S_A \neq \emptyset$; otherwise, it is said to be a dependent resource. Let S^R_{in} denote the set of independent resources in P_R.

In the rest of this paper, subclasses of S^{3}PR, depending on different initial token distributions, called η-S^{3}PR with or without a ξ-resource, are discussed from the Petri net structural perspective. Motivated by the work in [55], an η-S^{3}PR is redefined as follows.

Definition 14: An S^{3}PR (N, M_0) is said to be an η-S^{3}PR if the following three conditions are satisfied:

1) *There is only one η-resource;*
2) *$\forall r \in P_R$, there exists an RT-circuit Θ such that $r \in \Lambda(\Theta)$;*
3) *$\forall r \in S^R_{in}$, r is associated with a monoploid HR-circuit.*

In an η-S^{3}PR (N, M_0), if an η-resource r is of one-unit, i.e., $M_0(r) = 1$, then it is a ξ-resource. Accordingly, $M_0(r) > 1$ indicates that r is not a ξ-resource. For example, Fig. 1 shows an S^{3}PR, where p_1–p_8, p_9–p_{13}, and p_{14}, p_{15} are activity, resource, and idle places, respectively. Circuit $t_3p_3t_4p_{11}t_3$ is a monoploid HR-circuit. There are three strict minimal siphons: $S_1 = \{p_2, p_8, p_9, p_{10}\}$, $S_2 = \{p_4, p_7, p_{10}, p_{11}, p_{12}, p_{13}\}$, and $S_3 = \{p_4, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}\}$, i.e., $\Pi = \{S_1, S_2, S_3\}$. According to Definition 13, $S^R_{in} = \{p_{11}, p_{12}\}$. There are two MPTR-circuits, $p_9t_9p_{10}t_2p_9$ and $p_{10}t_8p_{12}t_7p_{13}t_4p_{11}t_3p_{10}$. Place p_{10} is an η-resource shared by these two MPRT-circuits. According to Definition 14, this S^{3}PR is an η-S^{3}PR. If η-resource p_{10} contains one token initially, then this net is an η-S^{3}PR with a ξ-resource.

Property 1: In an η-S^{3}PR, there are only three shared resources denoted by r_a, r_b, and r_η that are in strict minimal siphons with one of them being an η-resource.

Proof: It follows from Definition 14. □

A reachability graph of a Petri net model for an AMS can be classified into two parts: a live-zone (LZ) and a deadlock-zone (DZ). In the case of no confusion, we use *LZ* (*DZ*) to indicate the set of markings in the LZ (DZ). The *LZ* contains all the markings that form the maximal strongly connected component in the reachability graph including the initial marking. A marking in the DZ inevitably leads to deadlock states. An FBM (First-met bad marking) is a marking in the DZ, representing the very first entry from LZ to DZ. The set of FBMs is defined as: $\mathcal{M}_{\text{FBM}} = \{M | M \in DZ, \exists M' \in LZ, \exists t \in T, M'[t\rangle M\}$. A system cannot reach DZ if all FBMs are forbidden. That is to say, when designing a liveness-enforcing supervisor, we need to consider the FBMs only, whereas other markings in DZ can be ignored. This is the reason why the concept of FBMs is of interest.

According to Section IV.C in [36], in an η-S^{3}PR with a ξ-resource, there exists a marking M in DZ, for all $S \in \Pi$, $M(S) \neq 0$, where $S \in \Pi$ is a strict minimal siphon. These kinds of markings are called spurious-safe markings whose set is denoted by DZ^*. Let DZ^*_f represent the markings that are FBMs and belong to DZ^* and DZ_f the markings that are FBMs, at which there exists an empty siphon. In this case, we have $\mathcal{M}_{\text{FBM}} = DZ^*_f \cup DZ_f$ and $DZ^*_f \cap DZ_f = \emptyset$.

III. IDENTIFICATION OF FBMS IN η-S^{3}PR

This section explores the laws to identify FBMs in a class of S^{3}PR, called η-S^{3}PR. The traditional way of obtaining FBMs is to analyze the reachability by enumerating all the markings of a system. The degree of difficulty in enumerating all the reachable states depends on the structural complexity of a Petri net model and its initial marking. Generally, it is infeasible to practically generate all the states due to the limited computer memory, which is known as the state explosion problem. For the class of Petri nets, η-S^{3}PR, we show that the FBMs can be directly computed in an algebraic way, without enumerating all reachable markings.

The Petri net in Fig. 2 is an η-S^{3}PR with r_η being an η-resource, where idle places are omitted. We assume that the number of tokens in the idle places is initially big enough. In this sense, the removal of idle places does not impact deadlock analysis and supervisor design. In the case of no confusion, in the rest of this paper, we do not consider

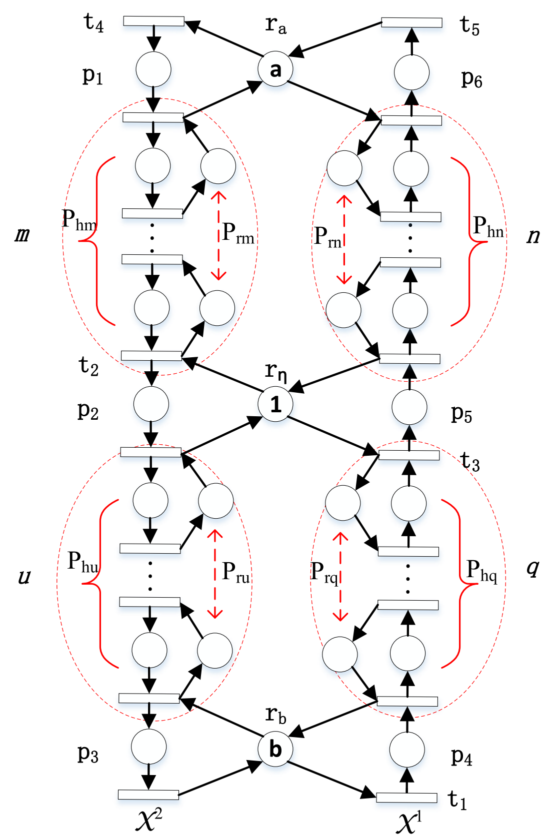

FIGURE 2. An η-S^{3}PR model.

idle places in a marked η-S^{3}PR that is then denoted by $(P_A \cup P_R, T, F, M_0)$. In Fig. 2, let $\alpha \in \{m, n, u, q\} \subseteq \mathbb{N}$ and the α-part represent the number and the location of the monoploid HR-circuits in the net. Let $P_{r\alpha}$ and $P_{h\alpha}$ represent the sets of resource places and holder places of the α monoploid HR-circuits, respectively. For example, m-part consists of P_{h_m}, P_{r_m}, and its associated transitions. In this net, there are two RT-circuits Θ_1 and Θ_2. RT-circuit Θ_1 consists of r_a, P_{r_n}, r_η, P_{r_m}, and transitions linking these places. RT-circuit Θ_2 consists of r_η, P_{r_u}, r_b, P_{r_q}, and transitions linking them. Assume that $M_0(r_\eta) = 1$. Then, r_η is a ξ-resource. The initial marking can be expressed as

$$M_0 = ar_a + \sum_{i=1}^{m} \Delta_i^m p_{r_i}^m + \sum_{i=1}^{n} \Delta_i^n p_{r_i}^n + r_\eta + \sum_{i=1}^{u} \Delta_i^u p_{r_i}^u + \sum_{i=1}^{q} \Delta_i^q p_{r_i}^q + br_b$$

In this expression, $M_0(r_a) = a$ and $M_0(r_b) = b$, where $a, b \in \mathbb{N}^+$. $p_{r_x}^\alpha$'s represent resource places in the monoploid HR circuits and $\alpha \in \{m, n, u, q\}$, the superscript of place $p_{r_x}^\alpha$, indicates the location of this place. Let Δ_x^α denote the number of resource units in $p_{r_x}^\alpha$, where $\Delta_x^\alpha \in \mathbb{N}$, $x \in \{1, 2, \dots, \alpha\}$, $\alpha \in \{m, n, u, q\}$. There are three strict minimal siphons, i.e., $S_1 = \{p_2, p_6, r_a, r_\eta\} \cup P_{r_m} \cup P_{r_n}$, $S_2 = \{p_3, p_5, r_b, r_\eta\} \cup P_{r_u} \cup P_{r_q}$, and $S_3 = \{p_3, p_6, r_a, r_\eta, r_b\} \cup P_{r_m} \cup P_{r_n} \cup P_{r_u} \cup P_{r_q}$.

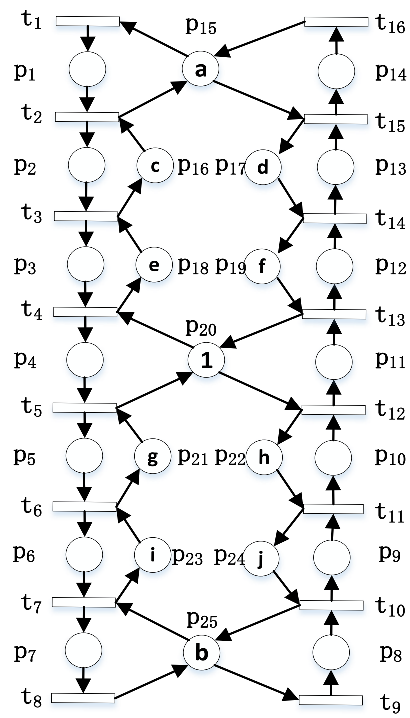

FIGURE 3. A parameterized η-S^{3}PR model (N, M_0).

The process that contains p_4, p_5, p_6, r_a, r_η, r_b, $q + n$ monoploid HR-circuits and corresponding transitions is named $\mathcal{X}^1$, while the process that contains p_1, r_a, p_2, r_η, p_3, r_b, and $m + u$ monoploid HR-circuits and corresponding transitions is called $\mathcal{X}^2$, denoted by $\mathcal{X}^1 = (P_{A_1} \cup P_{R_1}, T_1, F_1)$ and $\mathcal{X}^2 = (P_{A_2} \cup P_{R_2}, T_2, F_2)$, respectively.

Any reachable marking M in an η-S^{3}PR consists of two parts: the submarking of the activity places and that of the resource places. Let M^A indicate the submarking of activity places and M^R the one of other part. Then, a marking M in an η-S^{3}PR can be described as $M = M^A \oplus M^R$. Given an η-S^{3}PR with a ξ-resource as shown in Fig. 2, the set of places in each HR-circuit forms the support of a P-semiflow. Hence, given an initial marking, any reachable marking M can be decided if M^A is known. For the sake of clarity and visualization, we use submarking of activity places instead of the marking of all places when a marking is mentioned.

A. IDENTIFICATION OF FBMS IN AN η-S^{3}PR WITH A ξ-RESOURCE

As stated above, if $M_0(r_\eta) = 1$, then the η-resource in an η-S^{3}PR is a ξ-resource. We aim to find the relationship between the structure of an η-S^{3}PR and its FBMs. Consider the η-S^{3}PR shown in Fig. 3, where $m = 2$, $n = 2$, $u = 2$, and $q = 2$. In Fig. 3, we assume that $M_0(p_{20}) = 1$, i.e., p_{20} is the ξ-resource. There are 10 parameters $a, b, \dots,$ and j that are all positive integers, i.e., $M_0(p_{15}) = a$, $M_0(p_{16}) = c$, $M_0(p_{17}) = d$, $M_0(p_{18}) = e$, $M_0(p_{19}) = f$, $M_0(p_{21}) = g$, $M_0(p_{22}) = h$, $M_0(p_{23}) = i$, $M_0(p_{24}) = j$, and $M_0(p_{25}) = b$.

TABLE 1. DZ_f^* **of the net in Fig. 3.**

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}
M_1^A	2	4	5	0	3	4	0	3	4	2	0	3	3	0
M_2^A	2	4	5	0	3	4	0	3	4	2	1	2	3	0
M_3^A	2	4	5	0	3	4	0	3	4	2	1	3	2	0
M_4^A	2	4	5	1	2	4	0	3	4	2	0	3	3	0
M_5^A	2	4	5	1	3	3	0	3	4	2	0	3	3	0

We set $a = 2, b = 3, c = 4, d = 3, e = 5, f = 3, g = 3, h = 2, i = 4$, and $j = 4$ as the initial marking of this net. Up to now, there is no study that directly exposes the relationship between M_0 and FBMs in an S^{3}PR. With $\mathcal{M}_{\text{FBM}} = DZ_f^* \cup DZ_f$, according to [36], the existence of an emptied siphon can separate DZ_f from $\mathcal{M}_{\text{FBM}}$. However, there is no distinctive feature to specify DZ_f^*. First, we use a traditional way to enumerate the markings in $\mathcal{M}_{\text{FBM}}$ to obtain DZ_f^* as shown in TABLE 1.

There are only five markings in DZ_f^*. Note that, at these markings, activity places p_1–p_3 and p_8–p_{10} hold all tokens of their associated HR-circuits, respectively. We can find M^R via M^A. Thus, at these markings, for resources p_{15}–p_{25}, we have $M_1(p_{20}) = 1$, $M_2(p_{19}) = 1$, $M_3(p_{17}) = 1$, $M_4(p_{21}) = 1$, $M_5(p_{23}) = 1$, and the resources are emptied for all other cases. It can be noted that the number of states in DZ_f^* is $n + q + 1$ and at these markings there is only one token held by P_{r_n}, r_η, and P_{r_u}, i.e., for all $M \in DZ_f^*$, $M(P_{r_n} \cup r_\eta \cup P_{r_u}) = 1$. To show the relationship between a parametered initial marking in Fig. 3 and its DZ_f^*, we use different initial markings to compute its DZ_f^*, and the five parametered markings are shown in TABLE 2.

Next, another example is used to expound the rules among the structure of the η-S^{3}PR, initial marking, and DZ_f^*. In Fig. 4, we have $m = 1$, $n = 3$, $u = 3$, and $q = 1$, and parameters $M_0(p_{15}) - M_0(p_{19})$ and $M_0(p_{21}) - M_0(p_{25})$ are denoted by $a - e$ and $f - j$, respectively, which are all in $\mathbb{N}^+$. Compared with the net in Fig. 3, in Fig. 4, we decrease m and q from 2 to 1, respectively, to test if the places in these locations affect the number of states in DZ_f^*, and increase n and u from 2 to 3, respectively, to uncover the relationship between states in DZ_f^* and places in the $n-$ and u-parts. Its DZ_f^* is shown in TABLE 3. In this case, we have the results similar to those in Fig. 3.

By comparing TABLEs 2 with 3, two rules can be summarized: 1) the number of states in DZ_f^* is $n + u + 1$; 2) for all $M \in DZ_f^*$, $M(P_{r_n} \cup P_{r_u} \cup r_\eta) = 1$, $M(P_R \backslash (P_{r_n} \cup P_{r_u} \cup r_\eta)) = 0$. In order to further confirm these findings, in what follows, we analyze the structure of the η-S^{3}PR by first introducing a net called $\mathcal{X}$-activity linear system with resources.

Definition 15: A Petri net $\mathcal{X} = (P_A, T, F)$ is said to be an $\mathcal{X}_h$-activity linear system ($\mathcal{X}_h$-ALS, $h \in \mathbb{N}$, $h \geq 3$, indicating that it has h activity places) if it satisfies:

1) *$P_A = \{p_1, p_2, \ldots, p_h\}$ is a set of activity places;*
2) *$T = \{t_1, t_2, \ldots, t_{h+1}\}$ is a set of transitions;*

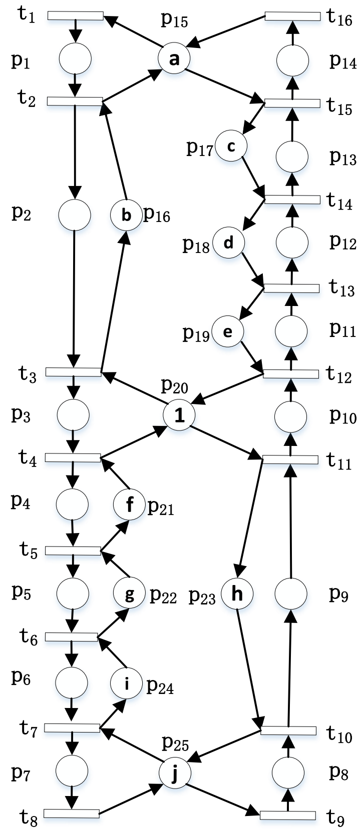

FIGURE 4. A parameterized η-S^{3}PR model (N, M_0).

3) $F \subseteq (P_A \times T) \cup (T \times P_A)$, $\forall k, l \in \{1, 2, \ldots, h\}$, $k \neq l$, $|p_k^\bullet| = |{}^\bullet p_k| = 1$, $p_k^\bullet \neq p_l^\bullet$, ${}^\bullet p_k \neq {}^\bullet p_l$, $p_k^\bullet = \{t_{k+1}\}$, ${}^\bullet p_k = \{t_k\}$.

Definition 16: An $\mathcal{X}_h$-ALS with resources ($\mathcal{X}_h$-ALSR) $\mathcal{X} = (P_A \cup P_R, T, F)$ is a Petri net satisfying:

1) *The subnet generated by $X = P_A \cup T$ is an $\mathcal{X}_h$-ALS.*
2) *$P_A \cap P_R = \emptyset$, $|P_A| = |P_R|$;*
3) *$\forall p \in P_A, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, \exists r_p \in P_R, {}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$;*
4) *$\forall r \in P_R$, ${}^{\bullet\bullet} r \cap P_A = r^{\bullet\bullet} \cap P_A \neq \emptyset$; $\forall r \in P_R$, ${}^\bullet r \cap r^\bullet = \emptyset$.*

In the case of no confusion, an $\mathcal{X}_h$-ALSR can also be called an $\mathcal{X}$-ALSR when the number of activity places is of no interest. The net in Fig. 5 is an $\mathcal{X}_h$-ALSR ($\mathcal{X}$-ALSR), where p_1–p_h and r_1–r_h are activity and resource places, respectively. t_1 and t_{h+1} are called the source and sink transitions, respectively.

TABLE 2. DZ_f^* **of the net in Fig. 3.**

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}
M_1^A	a	c	e	0	g	i	0	b	j	h	0	f	d	0
M_2^A	a	c	e	0	g	i	0	b	j	h	1	$f-1$	d	0
M_3^A	a	c	e	0	g	i	0	b	j	h	1	f	$d-1$	0
M_4^A	a	c	e	1	$g-1$	i	0	b	j	h	0	f	d	0
M_5^A	a	c	e	1	g	$i-1$	0	b	j	h	0	f	d	0

TABLE 3. DZ_f^* **of the net in Fig. 4.**

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}
M_1^A	a	b	0	f	g	i	0	j	h	0	e	d	c	0
M_2^A	a	b	0	f	g	i	0	j	h	1	$e-1$	d	c	0
M_3^A	a	b	0	f	g	i	0	j	h	1	e	$d-1$	c	0
M_4^A	a	b	0	f	g	i	0	j	h	1	e	d	$c-1$	0
M_5^A	a	b	1	$f-1$	g	i	0	j	h	0	e	d	c	0
M_6^A	a	b	1	f	$g-1$	i	0	j	h	0	e	d	c	0
M_7^A	a	b	1	f	g	$i-1$	0	j	h	0	e	d	c	0

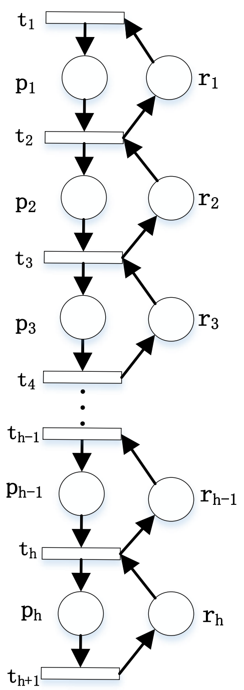

FIGURE 5. An $\mathcal{X}_h$**-ALSR system** $(\mathcal{X}, M_0)$**.**

According to Definition 14, combined with the general η-S^{3}PR as shown in Fig. 2, an η-S^{3}PR can be divided into two parts, i.e., two $\mathcal{X}$-ALSRs that share the same shared resource places. The two $\mathcal{X}$-ALSRs in an η-S^{3}PR can be called $\mathcal{X}^1$-component and $\mathcal{X}^2$-component ($\mathcal{X}^1$ and $\mathcal{X}^2$ for short), respectively. For the convenience of notation, let $T^{\mathcal{X}^i}$ and $P^{\mathcal{X}^i}$ be the sets of transitions and places in $\mathcal{X}^i$-component, respectively, and $M^{\mathcal{X}^i}$ represents a marking of $\mathcal{X}^i$-component, $i \in \{1, 2\}$. For example, in Fig 2, $\mathcal{X}^1$-component consists of p_4–p_6, r_a, r_η, r_b, all places in P_{hn}, P_{hq}, P_{rn}, and P_{rq}, and the associated transitions, and $\mathcal{X}^2$-component consists of p_1–p_3, r_a, r_η, r_b, all places in P_{hm}, P_{hu}, P_{rm}, and P_{ru}, and the associated transitions. These two parts share the same resources r_a, r_b, and r_η. The concept "composition" in an S^{3}PR context can be used to describe this kind of net construction that an η-S^{3}PR N is composed of two $\mathcal{X}$-ALSRs $\mathcal{X}^1$ and $\mathcal{X}^2$ through $P_r = \{r_a, r_b, r_\eta\}$, denoted by $N = \mathcal{X}^1 \circ \mathcal{X}^2$. A marking $M^{\mathcal{X}^i}$ of an $\mathcal{X}^i$-component is said to be safe if for all $r \in P_R^{\mathcal{X}^i}$, $M^{\mathcal{X}^i}(r) + M^{\mathcal{X}^i}(H(r)) > 0$, $i \in \{1, 2\}$; otherwise, it is unsafe.

Lemma 1: Let M be a reachable marking of an η-S^{3}PR (N, M_0) with shared resources r_j, $j \in \{a, \eta, b\}$. If $M^{\mathcal{X}^i}$ is safe, there exists a sequence of transitions $\sigma \in (T^{\mathcal{X}^i})^*$, such that $M^{\mathcal{X}^i}[\sigma\rangle M'$ with $M'(P_A^{\mathcal{X}^i}) = M_0(P_A^{\mathcal{X}^i})$ and $M'(r_j) = M^{\mathcal{X}^i}(r_j) + M^{\mathcal{X}^i}(H(r_j))$, $j \in \{a, \eta, b\}$, $i \in \{1, 2\}$.

Proof: If $M^{\mathcal{X}^i}$ is safe, then an $\mathcal{X}$-ALSR $(\mathcal{X}, M_0)$ can be used to model this subnet. This lemma indicates that there exists a sequence of transitions σ, $M \in R(\mathcal{X}, M_0)$, such that $M[\sigma\rangle M_0$. We use induction to prove this with an $\mathcal{X}$-ALSR, as shown in Fig. 5,

1) $M[\sigma_1\rangle M_1$ where $\sigma_1 = t_{h+1}t_{h+1}\dots t_{h+1}$, $|\sigma_1| = M(p_h)$, $M_1(r_h) = M_0(r_h)$.

2) $M_1[\sigma_2\rangle M_1'$, if $M_1(p_{h-1}) < M_1(r_h)$, $\sigma_2 = t_h t_h \dots t_h$, $|\sigma_2| = M_1(p_{h-1})$, $M_1'(r_{h-1}) = M_0(r_{h-1})$. If $M_1(p_{h-1}) > M_1(r_h)$, σ_2 consists of several t_h and t_{h+1}, such that $M_1'(r_{h-1}) = M_0(r_{h-1})$. Then, similar to 1), $M_1'[\sigma_3\rangle M_2$, $\sigma_3 = t_{h+1}t_{h+1}\dots t_{h+1}$, $|\sigma_3| = M_1'(p_h)$, $M_2(r_h) = M_0(r_h)$ and $M_2(r_{h-1}) = M_0(r_{h-1})$. Similarly, we could empty $p_{h-2}, p_{h-3}, \dots, p_1$ sequentially and get the initial state. □

A path $\mathcal{L}$ is a string of nodes that are all different, i.e., $\mathcal{L} = x_1x_2\dots x_k$, where $x_i \in P \cup T$ and $(x_i, x_{i+1}) \in F$, $i \in \{1, 2, \dots, k-1\}$. In a path $\mathcal{L} = x_1x_2\dots x_k$, x_1 is said to be the head and x_k the tail of $\mathcal{L}$. In an $\mathcal{X}$-ALSR $\mathcal{X}$, let $\overrightarrow{H(r)}(\mathcal{X})$ denote the set of activity places on a path whose head is in $H(r)$ and the tail is the sink transition of $\mathcal{X}$, and $\overrightarrow{H(r)}(\bullet)$ denote the union of all $\overrightarrow{H(r)}(\mathcal{X}^i)$, where $\mathcal{X}^i$ is an $\mathcal{X}$-ALSR system associated with r. In Fig. 5, $\overrightarrow{H(r_3)}(\mathcal{X}) = \{p_3, p_4, \ldots, p_h\}$. In Fig. 2, $\overrightarrow{H(r_\eta)}(\mathcal{X}^2) = \{p_2\} \cup \{p_3\} \cup P_{h_u}$, $\overrightarrow{H(r_\eta)}(\mathcal{X}^1) = \{p_5\} \cup \{p_6\} \cup P_{h_n}$, and $\overrightarrow{H(r_\eta)}(\bullet) = \overrightarrow{H(r_\eta)}(\mathcal{X}^1) \cup \overrightarrow{H(r_\eta)}(\mathcal{X}^2)$.

On the basis of what have been expounded above, after analyzing the markings in TABLEs 2 and 3, we find that there is only one of the resource places associated with the activity places in $\overrightarrow{H(r_\eta)}(\bullet)\backslash(H(r_a) \cup H(r_b))$ that is marked with one token at each marking in DZ_f^*, i.e., for all $M \in DZ_f^*$, $M((\bigcup_{H(r)\subseteq\overrightarrow{H(r_\eta)}(\bullet)}\{r\})\backslash\{r_a, r_b\}) = 1$. According to this finding, a rule of algebraically computing DZ_f^* is obtained. Before showing this conclusion, we present a lemma first.

Lemma 2: Let M be a reachable marking of an η-S^3PR (N, M_0) with shared resources r_i, $i \in \{a, \eta, b\}$. If $M(r_i) > 0$ and $M(r_j) > 0$, then there exists a sequence of transitions $\sigma \in T^*$ such that $M[\sigma\rangle M_0$, where $i, j \in \{a, \eta, b\}$, $i \neq j$.

Proof: Let $M \in R(N, M_0)$ with $M(r_i) > 0$ and $M(r_j) > 0$. For a shared resource r_k ($k \in \{a, \eta, b\}$, $k \neq i$, $k \neq j$), we have two cases:

1) *$M(r_k) > 0$: This indicates that both $M^{\mathcal{X}^1}$ and $M^{\mathcal{X}^2}$ are safe. According to Lemma 1, there exists a sequence of transitions $\sigma_1 \in (T^{\mathcal{X}^1})^*$ such that $M[\sigma_1\rangle M'$ and $M'(P_A^{\mathcal{X}^1}) = M_0(P_A^{\mathcal{X}^1})$. That is, the markings of all activity places in $\mathcal{X}^1$-component are at their initial state, i.e., $M'(P_A^{\mathcal{X}^1}) = 0$. In this case, $M'^{\mathcal{X}^2}$ is safe. Similarly, there exists a sequence of transitions $\sigma_2 \in (T^{\mathcal{X}^2})^*$ such that $M'[\sigma_2\rangle M''$ and $M''(P_A^{\mathcal{X}^2}) = M_0(P_A^{\mathcal{X}^2}) = 0$. All transitions in σ_1 and σ_2 are in $T^{\mathcal{X}^1}$ and $T^{\mathcal{X}^2}$, respectively. Thus, $M'(P_A^{\mathcal{X}^1}) = 0$ holds during the firing of σ_2. That is, we have $M'' = M_0$. Therefore, there exists a transition sequence $\sigma \in T^*$ such that $M[\sigma\rangle M_0$, where $\sigma = \sigma_1\sigma_2$.*
2) *$M(r_k) = 0$: Under this circumstance, there is at least one of $\mathcal{X}^1$ and $\mathcal{X}^2$ whose marking is safe. Assume the marking of $\mathcal{X}^i$ is safe. Similar to the proof of (1), the result holds.* □

Lemma 2 can be used to decide whether a marking M satisfying some particular condition is in LZ or not. For example, in Fig 3, if $M(p_{20}) = 1$ and $M(p_{25}) \neq 0$, then M is in LZ.

Proposition 1: Given an η-S^3PR (N, M_0) with r_η being a ξ-resource, r_a and r_b being dependent non-ξ-resources, if there exists $M \in R(N, M_0)$, $M(((\bigcup_{H(r)\in\overrightarrow{H(r_\eta)}(\bullet)}\{r\})\backslash\{r_a, r_b, r_\eta\}) \cap P_R^{\mathcal{X}_i}) = M(H(r_\eta)\cap P_A^{\mathcal{X}_i}) = 1$, $i \in \{1, 2\}$, and for all $p \in P_A\backslash\overrightarrow{H(r_\xi)}(\bullet)$, $M(p) = M_0(^{\bullet\bullet}p \cap P_R)$, then, $M \in DZ_f^*$.

Proof: For the sake of visualization, we use a general η-S^3PR as shown in Fig. 2 to prove this result. According to Lemma 2, suppose that M is in DZ. Then, there is at most one shared resource r such that $M(r) > 0$. Under this condition, there exist at most four kinds of markings in DZ: 1) $M(r_a) > 0$, $M(r_\eta) = 0$, and $M(r_b) = 0$; 2) $M(r_a) = 0$, $M(r_\eta) = 1$, and $M(r_b) = 0$; 3) $M(r_a) = 0$, $M(r_\eta) = 0$, and $M(r_b) > 0$; and 4) $M(r_a) = 0$, $M(r_\eta) = 0$, and $M(r_b) = 0$.

1) $M(r_a) > 0$, $M(r_\eta) = 0$, and $M(r_b) = 0$: If M is in DZ, then $M(p_4) = M_0(r_b)$; otherwise, $M(p_3) > 0$ indicates that $p_3^\bullet$ is firable, which meets Lemma 2 and M is in LZ. The existence of tokens in p_3 means that r_b can be marked by firing the output transition of p_3. Furthermore, $M(p_2) = M_0(r_\eta) = 1$ means that the marking of $\mathcal{X}^2$-component is safe. In order to hold tokens in places p_2 and p_4, $M(P_{h_u}) = M_0(P_{r_u})$ and $M(P_{h_q}) = M_0(P_{r_q})$ must hold, leading to an emptied siphon $S = \{p_3, p_5, r_\eta, r_b\} \cup P_{r_u} \cup P_{r_q}$, which contradicts the definition of DZ_f^*. Then, we conclude $M \notin DZ_f^*$.

2) $M(r_a) = 0$, $M(r_\eta) = 1$, and $M(r_b) = 0$: In the case that r_a or r_b is marked, we have $M(p_4) = M_0(r_b)$, $M(p_1) = M_0(r_a)$, $M(P_{h_m}) = M_0(P_{r_m})$, and $M(P_{h_q}) = M_0(P_{r_q})$. However this resource configuration would cause the fact that t_2 and t_3 are firable. Assume that t_2 fires. Then $M(p_2) = 1$, implying that there exists a transition sequence $\sigma_m \in (^\bullet P_{h_m})^*$ such that $M[\sigma_m\rangle M_1$ with $M_1(r_a) > 0$. In order not to make false Lemma 2, the transitions in $p_2^\bullet$ should be prohibited from firing. This indicates that $M(P_{h_u}) = M_0(P_{r_u})$. Similarly, $M(P_{h_n}) = M_0(P_{r_n})$ holds.

If $M'(S_{in}^R) = 0$, $M'(r_\eta) = 1$, $M'(r_a) > 0$, and $M'(r_b) > 0$, then M' is in LZ. There exists a sequence of transitions $\sigma = t_1t_1 \ldots t_1t_4t_4 \ldots t_4$ which contains $M'(r_a)$ t_4 and $(M'(r_b) - 1)$ t_1. $M'[\sigma\rangle M''$, $M'' \in LZ$, $M''[t_1\rangle M$, $M \in DZ$ with $M(S_{in}^R) = 0$. Thus, M is in $\mathcal{M}_{\text{FBM}}$.

3) $M(r_a) = 0$, $M(r_\eta) = 0$, and $M(r_b) > 0$: Similar to 1), we have $M \notin DZ_f^*$.

4) $M(r_a) = 0$, $M(r_\eta) = 0$, and $M(r_b) = 0$: This indicates that $M(p_1) = M_0(r_a)$, $M(p_4) = M_0(r_b)$, $M(P_{h_m}) = M_0(P_{r_m})$, and $M(P_{h_q}) = M_0(P_{r_q})$, which can prevent r_a and r_b from being marked. Since the markings in DZ_f^* are considered instead of those in DZ_f, if $M(p_2) = 1$, then $M(P_{r_u}) \neq 0$ ($M(P_{r_u}) = 0$ indicates that siphon $S = \{p_3, p_5, r_\eta, r_b\} \cup P_{r_u} \cup P_{r_q}$ is emptied). Thus, there exists a transition sequence $\sigma_u \in (^\bullet P_{h_u})^*$ such that $M[\sigma_u\rangle M_2$ with $M_2(r_\eta) = 1$, which is the same as the condition of Case 2) if M and M_2 belong to DZ. Therefore, M_2 should meet constraints in 2): $M_2(S_{in}^R) = 0$. For this reason, $M(P_{r_u}) = 1$. Similarly, if $M(p_5) = 1$, then $M(S_{in}^R) = 1$ with $M(P_{r_n}) = 1$. The approach to prove that these states are FBMs is similar to that in 2).

According to Conditions 2) and 4) above, we can get that there exists $M \in R(N, M_0)$, if $M(((\bigcup_{H(r)\in\overrightarrow{H(r_\eta)}(\bullet)}\{r\})\backslash\{r_a, r_b, r_\eta\}) \cap P_R^{\mathcal{X}_i}) = M(H(r_\eta) \cap P_A^{\mathcal{X}_i}) = 1$, $i \in \{1, 2\}$, and for all $p \in P_A\backslash\overrightarrow{H(r_\xi)}(\bullet)$, $M(p) = M_0(^{\bullet\bullet}p \cap P_R)$, then $M \in DZ_f^*$. □

Corollary 1: Given an η-S^3PR N with r_η being a ξ-resource, we have $|DZ_f^*| = n + u + 1$.

Proof: According to Conditions 2) and 4) in the proof of Proposition 1, if $M(r_a) = 0$, $M(r_\eta) = 1$, and $M(r_b) = 0$, then $M(S_{in}^R) = 0$ and $M \in DZ_f^*$. For Condition 4), $M(r_a) = 0$, $M(r_\eta) = 0$, and $M(r_b) = 0$, if $M(P_{r_n}) = 1$, then there are n markings which are in DZ_f^*. Similarly, if $M(P_{r_u}) = 1$, there

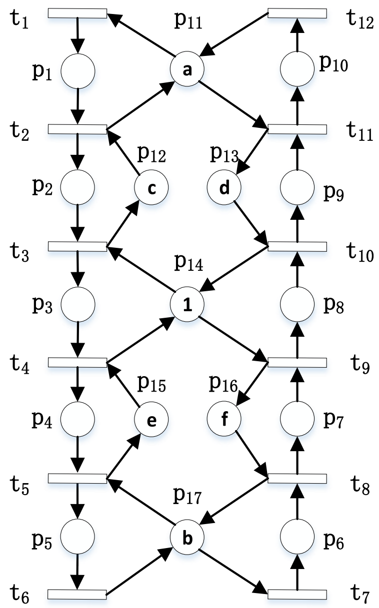

FIGURE 6. A parameterized η-S^3PR model (N, M_0).

are u markings which are in DZ_f^*. With the above analysis, we have $|DZ_f^*| = n + u + 1$. □

Proposition 1 shows the relationship between M_0 and DZ_f^*. However, denoted by DZ_f, the markings in $\mathcal{M}_{\mathrm{FBM}} \backslash DZ_f^*$ that have not been considered share the same characteristic of an emptied siphon at these markings. We take the net in Fig. 6 as an example. There are six parameters with p_{14} being a ξ-resource, i.e., $M_0(p_{11}) = a$, $M_0(p_{17}) = b$, $M_0(p_{12}) = c$, $M_0(p_{13}) = d$, $M_0(p_{15}) = e$, $M_0(p_{16}) = f$, and $M_0(p_{14}) = 1$. Assume that $a = 2$, $b = 2$, $c = 1$, $d = 1$, $e = 1$, and $f = 1$. We use traditional way to calculate DZ_f first, as shown in Table 4.

From TABLE 4, we can see that M_1^A–M_{23}^A and M_{24}^A–M_{46}^A are markings, at which siphons $S_1 = \{p_5, p_8, p_{14}, p_{15}, p_{16}, p_{17}\}$ and $S_2 = \{p_3, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$ are emptied, respectively. Let us consider M_1^A–M_{23}^A. For M_1^A–M_6^A, only markings of p_1, p_{10} and p_{11} change while those of the other two monoploid HR-circuits associated with p_{12} and p_{13} keep unchanged. The place set at the HR-circuit associated with p_{11} is the support of a P-semiflow I_1, i.e., $||I_1|| = \{p_1, p_{10}, p_{11}\}$. M_1^A–M_6^A consist of all the possible token distributions at $||I_1||$. Similarly, all these 23 markings are combinations of token distributions in the support of each P-semiflow, i.e., $||I_1|| = \{p_1, p_{10}, p_{11}\}$, $||I_2|| = \{p_2, p_{12}\}$, and $||I_3|| = \{p_9, p_{13}\}$. By $M_0(p_{11}) = 2$, there are six ways to randomly distribute tokens at $||I_1||$, denoted by $\overline{I_1} = 6$. Then, there should be $\overline{I_1} \times \overline{I_2} \times \overline{I_3} = 6 \times 2 \times 2 = 24$ markings. However, there is no state at which a trap is emptied, i.e., $M(S_1^*) \neq 0$ with $S_1^* = \{p_1, p_{11}, p_{12}, p_{13}, p_{14}, p_8\}$. This is the reason why there are 23 markings. The same findings can be elaborated from M_{24}^A–M_{46}^A where trap $S_2^* = \{p_3, p_{14}, p_{15}, p_{16}, p_{17}, p_6\}$ cannot be emptied. There also exists a third emptied siphon, i.e., $S_3 = \{p_5, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}\}$. Markings in DZ_f associated with emptied siphon S_3 are included when S_1 and S_2 are analyzed, i.e., M_{23}^A and M_{44}^A. If we use the net in Fig. 6 with a parameterized initial marking, then $|DZ_f| = \overline{I_1} \times \overline{I_2} \times \overline{I_3} + \overline{I_4} \times \overline{I_5} \times \overline{I_6} - 1 = (1 + 2 + \ldots (a + 1)) \times (c + 1) \times (d + 1) + (1 + 2 + \ldots (b + 1)) \times (e + 1) \times (f + 1) - 1$, where $(1 + 2 + \ldots (a + 1))$ is the number of different token distributions in the support of I_1, $c + 1$ is the number of different token distributions in the support of I_2, and the rest can be done by the same reasoning. According to these findings, we have the following results.

Proposition 2: Given an η-S^3PR (N, M_0) with r_η being a ξ-resource, and r_a and r_b being dependent non-ξ-resources, for all $M \in DZ_f$, there exists $S_0 \in \Pi$ such that $M(S_0) = 0$, then, for all $r \in P_R \backslash S_{R_0}$, $M(r) + M(H(r)) = M_0(r)$ with $M(r) \in \{0, 1, \ldots, M_0(r)\}$ and $M(H(r)) \in \{0, 1, \ldots, M_0(r)\}$, where there does not exist trap S^* such that $M(S^*) = 0$.

Proof: This proposition expresses the markings of all r's in $P_R \backslash S_{R_0}$ and the associated $H(r)$ in the marking M which can be any token distribution satisfying $M(r) + M(H(r)) = M_0(r)$, where no trap can be emptied since all traps are marked at M_0.

A general η-S^3PR as shown in Fig. 2 is used to prove this conclusion. There exist three strict minimal siphons, i.e., $\Pi = \{S_1, S_2, S_3\}$. There exists a transition sequence $\sigma_1 \in (T^{\mathcal{X}^2})^*$ such that $M_0[\sigma_1\rangle M_1$ with $M_1(r_a) = a$, $M_1(r_\eta) = 1$, $M_1(r_b) = 1$, and $M_1(P_{r_u}) = 0$, where for all $r \in P_{r_m}$, $M_1(r) + M_1(H(r)) = M_0(r)$, $M_1(r) \in \{0, 1, \ldots, M_0(r)\}$ and $M_1(H(r)) \in \{0, 1, \ldots, M_0(r)\}$. This kind of markings of $\mathcal{X}^2$-component is denoted by $\mathcal{X}^2(M_1)$.

Similarly, there exists a transition sequence $\sigma_2 \in (T^{\mathcal{X}^1})^*$ such that $M_1[\sigma_2\rangle M_2$ with $M_2(r_a) + M_2(p_6) = a$, $M_2(r_\eta) = 1$, $M_2(r_b) = 1$, and $M_2(P_{r_q}) = 0$, where for all $r \in P_{r_n}$, $M_2(r) + M_2(H(r)) = M_0(r)$, $M_2(r) \in \{0, 1, \ldots, M_0(r)\}$ and $M_2(H(r)) \in \{0, 1, \ldots, M_0(r)\}$. This kind of markings of $\mathcal{X}^1$-component is denoted by $\mathcal{X}^1(M_2)$.

The flow of tokens in $\mathcal{X}^1$-component does not interfere with the token configuration in $\mathcal{X}^2$-component. For the marking of the whole net, there exists a transition sequence $\sigma \in T^*$ such that $M_0[\sigma\rangle M_3$ where $M_3^{\mathcal{X}^1} \in \mathcal{X}^1(M_2)$ and $M_3^{\mathcal{X}^2} \in \mathcal{X}^2(M_1)$. The resource units in r_η and r_b can be transmitted into p_2 and p_4, respectively. The tokens in p_6 can flow into p_1 and p_a at any time. If $\{r_\eta, r_b\}$ is not emptied by firing t_1 and t_2, the system is in LZ. Thus, the last firing of t_1 and t_2 leads the system to DZ from LZ where these markings in DZ are in DZ_f and the conclusion is true. No matter which one fires finally, there does not exist trap $S^* = \{p_1, p_5, r_a, r_\eta\} \cup P_{r_m} \cup P_{r_n}$ that is emptied.

Similarly, the result holds when siphon $S_1 = \{p_2, p_6, r_a, r_\eta\} \cup P_{r_m} \cup P_{r_n}$ is emptied. If strict minimal siphon $S_3 = \{p_3, p_6, r_a, r_\eta, r_b\} \cup P_{r_m} \cup P_{r_n} \cup P_{r_u} \cup P_{r_q}$ is emptied, the markings in $\mathcal{M}_{FBM}$ have been analyzed in the case that S_1 or S_2 is emptied, i.e., $M(S_3) = 0$, $M(p_5) = 1$ (S_1 is emptied) or $M(p_2) = 1$ (S_2 is emptied). □

In summary, this subsection discusses $\mathcal{M}_{\mathrm{FBM}}$ of an η-S^3PR with a ξ-resource. According to Propositions 1 and 2,

TABLE 4. DZ_f **of the net in Fig. 6.**

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
M_1^A	0	0	1	1	0	2	1	0	0	0
M_2^A	0	0	1	1	0	2	1	0	0	1
M_3^A	0	0	1	1	0	2	1	0	0	2
M_4^A	1	0	1	1	0	2	1	0	0	0
M_5^A	1	0	1	1	0	2	1	0	0	1
M_6^A	2	0	1	1	0	2	1	0	0	0
M_7^A	0	0	1	1	0	2	1	0	1	0
M_8^A	0	0	1	1	0	2	1	0	1	1
M_9^A	0	0	1	1	0	2	1	0	1	2
M_{10}^A	1	0	1	1	0	2	1	0	1	0
M_{11}^A	1	0	1	1	0	2	1	0	1	1
M_{12}^A	2	0	1	1	0	2	1	0	1	0
M_{13}^A	0	1	1	1	0	2	1	0	0	0
M_{14}^A	0	1	1	1	0	2	1	0	0	1
M_{15}^A	0	1	1	1	0	2	1	0	0	2
M_{16}^A	1	1	1	1	0	2	1	0	0	0
M_{17}^A	1	1	1	1	0	2	1	0	0	1
M_{18}^A	2	1	1	1	0	2	1	0	0	0
M_{19}^A	0	1	1	1	0	2	1	0	1	0
M_{20}^A	0	1	1	1	0	2	1	0	1	1
M_{21}^A	1	1	1	1	0	2	1	0	1	0
M_{22}^A	1	1	1	1	0	2	1	0	1	1
M_{23}^A	2	1	1	1	0	2	1	0	1	0
M_{24}^A	2	1	0	0	0	0	0	1	1	0
M_{25}^A	2	1	0	0	0	1	0	1	1	0
M_{26}^A	2	1	0	0	0	2	0	1	1	0
M_{27}^A	2	1	0	0	1	0	0	1	1	0
M_{28}^A	2	1	0	0	1	1	0	1	1	0
M_{29}^A	2	1	0	0	2	0	0	1	1	0
M_{30}^A	2	1	0	0	0	0	1	1	1	0
M_{31}^A	2	1	0	0	0	1	1	1	1	0
M_{32}^A	2	1	0	0	0	2	1	1	1	0
M_{33}^A	2	1	0	0	1	0	1	1	1	0
M_{34}^A	2	1	0	0	1	1	1	1	1	0
M_{35}^A	2	1	0	0	2	0	1	1	1	0
M_{36}^A	2	1	0	1	0	0	0	1	1	0
M_{37}^A	2	1	0	1	0	1	0	1	1	0
M_{38}^A	2	1	0	1	0	2	0	1	1	0
M_{39}^A	2	1	0	1	1	0	0	1	1	0
M_{40}^A	2	1	0	1	1	1	0	1	1	0
M_{41}^A	2	1	0	1	2	0	0	1	1	0
M_{42}^A	2	1	0	1	0	0	1	1	1	0
M_{43}^A	2	1	0	1	0	1	1	1	1	0
M_{44}^A	2	1	0	1	0	2	1	1	1	0
M_{45}^A	2	1	0	1	1	0	1	1	1	0
M_{46}^A	2	1	0	1	1	1	1	1	1	0

we can use Algorithm 1 to compute $\mathcal{M}_{\text{FBM}}$ in Fig. 2. The computational complexity of Algorithm 1 depends on the number of HR-circuits and their initial markings due to the structure of an η-S^{3}PR. In line 8 of Algorithm 1, we assume that there exist n resources where the number of initial tokens in each resource r_i is m_i, $n, m_i \in \mathbb{N}$, $i \in \{1, 2, \ldots, n\}$. The complexity of computing marking vectors restricted to $\{r_i\} \cup H(r_i)$ is $O(m_i^{|H(r_i)|})$. This indicates that the complexity of computing all r in line 8 is $O(\Sigma_{i=1}^{n} m_i^{|H(r_i)|})$, which is polynomial since $|H(r_i)| \leq 2$ in an η-S^{3}PR. Line 10 of

Algorithm 1 Computation of $\mathcal{M}_{\text{FBM}}$ in an η-S^{3}PR With a ξ-Resource

Require: A marked η-S^{3}PR $(P_R \cup P_A, T, F, M_0)$ with a ξ-resource r_ξ.
Ensure: $\mathcal{M}_{\text{FBM}}$.

1: $\mathcal{M}_{\text{FBM}} := \emptyset$.
2: Compute strict minimal siphon set Π for $(P_R \cup P_A, T, F)$.
3: Find a siphon $S_\xi \in \Pi$, such that $S_\xi \cap H(r_\xi) = \emptyset$.
4: **for** each $S \in \Pi \backslash \{S_\xi\}$. **do**
5: $\quad M'_S = 0$. /* M'_S is a marking vector restricted to S.*/
6: $\quad$ Compute $M'_{[S]}$.
7: $\quad MM = \{M'_S \oplus M'_{[S]}\}$. /* For two vectors x and y, $x \oplus y = [x|y]$.*/
8: $\quad$ **for** each $r \in P_R \backslash S_R$ **do**
9: $\quad\quad$ Compute $\mathcal{M}_{r,H(r)}$ according to Proposition 2. /* $\mathcal{M}_{r,H(r)}$ is a set of marking vectors restricted to $\{r\} \cup H(r)$.*/
10: $\quad\quad MM = MM \otimes \mathcal{M}_{r,H(r)}$. /* For vector sets X and Y, $X \otimes Y = \{x \oplus y | x \in X \wedge y \in Y\}$, by convention, $\emptyset \otimes Y = \{y | y \in Y\}$.*/
11: $\quad$ **end for**
12: $\quad$ Delete the marking in MM at which there is a trap that is empty.
13: $\quad \mathcal{M}_{\text{FBM}} = \mathcal{M}_{\text{FBM}} \cup MM$.
14: **end for**
15: Compute DZ_f^* in accordance with Proposition 1.
16: $\mathcal{M}_{\text{FBM}} = \mathcal{M}_{\text{FBM}} \cup DZ_f^*$.
17: Output $\mathcal{M}_{\text{FBM}}$.

TABLE 5. $\mathcal{M}_{\text{FBM}}$ of the net in Fig. 6 with $M(p_{14}) = 5$.

$\mathcal{M}$ \ P	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
M_1^A	a	c	1	e	0	b	f	4	d	0
M_2^A	a	c	2	e	0	b	f	3	d	0
M_3^A	a	c	3	e	0	b	f	2	d	0
M_4^A	a	c	4	e	0	b	f	1	d	0

for-loop structure (Lines 8–11) means concatenating marking vectors of n HR-circuits which directly obtains FBMs. Note that the number of siphons in a Petri net, even in an S^{3}PR, is exponential with respect to its structural size. In summary, Algorithm 1 is of efficiency than traditional state enumeration methods.

Algorithm 2 Computation of $\mathcal{M}_{\text{FBM}}$ in an η-S^{3}PR Without a ξ-Resource

Require: A marked η-S^{3}PR $(P_R \cup P_A, T, F, M_0)$ without a ξ-resource r_η.
Ensure: $\mathcal{M}_{\text{FBM}}$.

1: $\mathcal{M}_{\text{FBM}} := \emptyset$.
2: Compute strict minimal siphon set Π for $(P_R \cup P_A, T, F)$.
3: Find a siphon $S_\eta \in \Pi$, such that $S_\eta \cap H(r_\eta) = \emptyset$.
4: **for** each $S \in \Pi \backslash \{S_\eta\}$. **do**
5: $\quad M'_S = 0$. /* M'_S is a marking vector restricted to S.*/
6: $\quad$ Compute $M'_{[S]}$.
7: $\quad MM = \{M'_S \oplus M'_{[S]}\}$. /* For two vectors x and y, $x \oplus y = [x|y]$.*/
8: $\quad$ **for** each $r \in P_R \backslash S_R$ **do**
9: $\quad\quad$ Compute $\mathcal{M}_{r,H(r)}$ according to Proposition 2. /* $\mathcal{M}_{r,H(r)}$ is a set of marking vectors restricted to $\{r\} \cup H(r)$.*/
10: $\quad\quad MM = MM \otimes \mathcal{M}_{r,H(r)}$. /* For vector sets X and Y, $X \otimes Y = \{x \oplus y | x \in X \wedge y \in Y\}$, by convention, $\emptyset \otimes Y = \{y | y \in Y\}$.*/
11: $\quad$ **end for**
12: $\quad$ Delete the marking in MM at which there is a trap that is empty.
13: $\quad \mathcal{M}_{\text{FBM}} = \mathcal{M}_{\text{FBM}} \cup MM$.
14: **end for**
15: Compute DZ_f for $S = S_\eta$ in accordance with case 2) in Proposition 3.
16: $\mathcal{M}_{\text{FBM}} = \mathcal{M}_{\text{FBM}} \cup DZ_f$.
17: Output $\mathcal{M}_{\text{FBM}}$.

B. IDENTIFICATION OF FBMS IN AN η-S^{3}PR WITHOUT ξ-RESOURCES

Let us assume that $M_0(p_{14}) = g$, $g > 1$, in the parameterized model as shown in Fig. 6. It is an η-S^{3}PR without a ξ-resource. There are three strict minimal siphons in Fig. 6, i.e., $S_1 = \{p_5, p_8, p_{14}, p_{15}, p_{16}, p_{17}\}$, $S_2 = \{p_3, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$, and $S_3 = \{p_5, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{14}, p_{15}, p_{16}, p_{17}\}$. According to [36], there are two kinds of reachable markings: safe ones and deadlocks only. Deadlocks occur if the entire system or part of it remains indefinitely blocked, i.e., at least one siphon is emptied.

Actually, there should be three types of markings in $\mathcal{M}_{FBM}$ when these three strict siphons are emptied. A traditional

TABLE 6. DZ_f^* of the net in Fig. 3.

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}
M_1^A	2	1	1	0	1	1	0	1	1	1	0	1	2	0
M_2^A	2	1	1	0	1	1	0	1	1	1	1	0	2	0
M_3^A	2	1	1	0	1	1	0	1	1	1	1	1	1	0
M_4^A	2	1	1	1	0	1	0	1	1	1	0	1	2	0
M_5^A	2	1	1	1	1	0	0	1	1	1	0	1	2	0

TABLE 7. $\mathcal{M}_{\mathrm{FBM}}$ **of the net in Fig. 3.**

Markings \ Places	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}
M_1^A	0	0	0	5	1	1	0	1	1	1	0	0	0	0
M_2^A	0	0	0	5	1	1	0	1	1	1	0	0	0	1
M_3^A	0	0	0	5	1	1	0	1	1	1	0	0	0	2
M_4^A	0	0	0	5	1	1	0	1	1	1	0	1	0	0
M_5^A	0	0	0	5	1	1	0	1	1	1	0	1	0	1
M_6^A	0	0	0	5	1	1	0	1	1	1	0	1	0	2
M_7^A	0	0	0	5	1	1	0	1	1	1	0	2	0	0
M_8^A	0	0	0	5	1	1	0	1	1	1	0	2	0	1
M_9^A	0	0	0	5	1	1	0	1	1	1	0	2	0	2
M_{10}^A	0	0	0	5	1	1	0	1	1	1	0	1	0	0
M_{11}^A	0	0	0	5	1	1	0	1	1	1	0	1	0	1
M_{12}^A	0	0	0	5	1	1	0	1	1	1	0	1	0	2
M_{13}^A	0	0	0	5	1	1	0	1	1	1	0	1	1	0
M_{14}^A	0	0	0	5	1	1	0	1	1	1	0	1	1	1
M_{15}^A	0	0	0	5	1	1	0	1	1	1	0	1	1	2
M_{16}^A	0	0	0	5	1	1	0	1	1	1	0	1	2	0
M_{17}^A	0	0	0	5	1	1	0	1	1	1	0	1	2	1
M_{18}^A	0	0	0	5	1	1	0	1	1	1	0	1	2	2
M_{19}^A	0	0	1	5	1	1	0	1	1	1	0	0	0	0
M_{20}^A	0	0	1	5	1	1	0	1	1	1	0	0	0	1
M_{21}^A	0	0	1	5	1	1	0	1	1	1	0	0	0	2
M_{22}^A	0	0	1	5	1	1	0	1	1	1	0	0	1	0
M_{23}^A	0	0	1	5	1	1	0	1	1	1	0	0	1	1
M_{24}^A	0	0	1	5	1	1	0	1	1	1	0	0	1	2
M_{25}^A	0	0	1	5	1	1	0	1	1	1	0	0	2	0
M_{26}^A	0	0	1	5	1	1	0	1	1	1	0	0	2	1
M_{27}^A	0	0	1	5	1	1	0	1	1	1	0	0	2	2
M_{28}^A	0	0	1	5	1	1	0	1	1	1	0	1	0	0
M_{29}^A	0	0	1	5	1	1	0	1	1	1	0	1	0	1
M_{30}^A	0	0	1	5	1	1	0	1	1	1	0	1	0	2
M_{31}^A	0	0	1	5	1	1	0	1	1	1	0	1	1	0
M_{32}^A	0	0	1	5	1	1	0	1	1	1	0	1	1	1
M_{33}^A	0	0	1	5	1	1	0	1	1	1	0	1	1	2
M_{34}^A	0	0	1	5	1	1	0	1	1	1	0	1	2	0
M_{35}^A	0	0	1	5	1	1	0	1	1	1	0	1	2	1
M_{36}^A	0	0	1	5	1	1	0	1	1	1	0	1	2	2
M_{37}^A	0	1	0	5	1	1	0	1	1	1	0	0	0	0
M_{38}^A	0	1	0	5	1	1	0	1	1	1	0	0	0	1
M_{39}^A	0	1	0	5	1	1	0	1	1	1	0	0	0	2
M_{40}^A	0	1	0	5	1	1	0	1	1	1	0	0	1	0
M_{41}^A	0	1	0	5	1	1	0	1	1	1	0	0	1	1
M_{42}^A	0	1	0	5	1	1	0	1	1	1	0	0	1	2
M_{43}^A	0	1	0	5	1	1	0	1	1	1	0	0	2	0
M_{44}^A	0	1	0	5	1	1	0	1	1	1	0	0	2	1
M_{45}^A	0	1	0	5	1	1	0	1	1	1	0	0	2	2
M_{46}^A	0	1	0	5	1	1	0	1	1	1	0	1	0	0
M_{47}^A	0	1	0	5	1	1	0	1	1	1	0	1	0	1
M_{48}^A	0	1	0	5	1	1	0	1	1	1	0	1	0	2

TABLE 7. *(Continued.)* $\mathcal{M}_{\mathrm{FBM}}$ of the net in Fig. 3.

M_{49}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	1	0
M_{50}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	1	1
M_{51}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	1	2
M_{52}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	2	0
M_{53}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	2	1
M_{54}^{A}	0	1	0	5	1	1	0	1	1	1	0	1	2	2
M_{55}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	0	0
M_{56}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	0	1
M_{57}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	0	2
M_{58}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	1	0
M_{59}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	1	1
M_{60}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	1	2
M_{61}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	2	0
M_{62}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	2	1
M_{63}^{A}	0	1	1	5	1	1	0	1	1	1	0	0	2	2
M_{64}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	0	0
M_{65}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	0	1
M_{66}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	0	2
M_{67}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	1	0
M_{68}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	1	1
M_{69}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	1	2
M_{70}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	2	0
M_{71}^{A}	0	1	1	5	1	1	0	1	1	1	0	1	2	1
M_{72}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	0	0
M_{73}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	0	1
M_{74}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	1	0
M_{75}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	1	1
M_{76}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	2	0
M_{77}^{A}	1	0	0	5	1	1	0	1	1	1	0	0	2	1
M_{78}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	0	0
M_{79}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	0	1
M_{80}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	1	0
M_{81}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	1	1
M_{82}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	2	0
M_{83}^{A}	1	0	0	5	1	1	0	1	1	1	0	1	2	1
M_{84}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	0	0
M_{85}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	0	1
M_{86}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	1	0
M_{87}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	1	1
M_{88}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	2	0
M_{89}^{A}	1	0	1	5	1	1	0	1	1	1	0	0	2	1
M_{90}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	0	0
M_{91}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	0	1
M_{92}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	1	0
M_{93}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	1	1
M_{94}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	2	0
M_{95}^{A}	1	0	1	5	1	1	0	1	1	1	0	1	2	1
M_{96}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	0	0
M_{97}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	0	1
M_{98}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	1	0
M_{99}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	1	1
M_{100}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	2	0

TABLE 7. ***(Continued.)*** $\mathcal{M}_{\mathrm{FBM}}$ **of the net in Fig. 3.**

M_{101}^{A}	1	1	0	5	1	1	0	1	1	1	0	0	2	1
M_{102}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	0	0
M_{103}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	0	1
M_{104}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	1	0
M_{105}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	1	1
M_{106}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	2	0
M_{107}^{A}	1	1	0	5	1	1	0	1	1	1	0	1	2	1
M_{108}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	0	0
M_{109}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	0	1
M_{110}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	1	0
M_{111}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	1	1
M_{112}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	2	0
M_{113}^{A}	1	1	1	5	1	1	0	1	1	1	0	0	2	1
M_{114}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	0	0
M_{115}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	0	1
M_{116}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	1	0
M_{117}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	1	1
M_{118}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	2	0
M_{119}^{A}	1	1	1	5	1	1	0	1	1	1	0	1	2	1
M_{120}^{A}	2	0	0	5	1	1	0	1	1	1	0	0	0	0
M_{121}^{A}	2	0	0	5	1	1	0	1	1	1	0	0	1	0
M_{122}^{A}	2	0	0	5	1	1	0	1	1	1	0	0	2	0
M_{123}^{A}	2	0	0	5	1	1	0	1	1	1	0	1	0	0
M_{124}^{A}	2	0	0	5	1	1	0	1	1	1	0	1	1	0
M_{125}^{A}	2	0	0	5	1	1	0	1	1	1	0	1	2	0
M_{126}^{A}	2	0	1	5	1	1	0	1	1	1	0	0	0	0
M_{127}^{A}	2	0	1	5	1	1	0	1	1	1	0	0	1	0
M_{128}^{A}	2	0	1	5	1	1	0	1	1	1	0	0	2	0
M_{129}^{A}	2	0	1	5	1	1	0	1	1	1	0	1	0	0
M_{130}^{A}	2	0	1	5	1	1	0	1	1	1	0	1	1	0
M_{131}^{A}	2	0	1	5	1	1	0	1	1	1	0	1	2	0
M_{132}^{A}	2	1	0	5	1	1	0	1	1	1	0	0	0	0
M_{133}^{A}	2	1	0	5	1	1	0	1	1	1	0	0	1	0
M_{134}^{A}	2	1	0	5	1	1	0	1	1	1	0	0	2	0
M_{135}^{A}	2	1	0	5	1	1	0	1	1	1	0	1	0	0
M_{136}^{A}	2	1	0	5	1	1	0	1	1	1	0	1	1	0
M_{137}^{A}	2	1	0	5	1	1	0	1	1	1	0	1	2	0
M_{138}^{A}	2	1	1	5	1	1	0	1	1	1	0	0	0	0
M_{139}^{A}	2	1	1	5	1	1	0	1	1	1	0	0	1	0
M_{140}^{A}	2	1	1	5	1	1	0	1	1	1	0	0	2	0
M_{141}^{A}	2	1	1	5	1	1	0	1	1	1	0	1	0	0
M_{142}^{A}	2	1	1	5	1	1	0	1	1	1	0	1	1	0
M_{143}^{A}	2	1	1	5	1	1	0	1	1	1	0	1	2	0
M_{144}^{A}	2	1	1	0	0	0	0	0	0	0	5	1	2	0
M_{145}^{A}	2	1	1	0	0	0	0	0	0	1	5	1	2	0
M_{146}^{A}	2	1	1	0	0	0	0	0	1	0	5	1	2	0
M_{147}^{A}	2	1	1	0	0	0	0	0	1	1	5	1	2	0
M_{148}^{A}	2	1	1	0	0	0	0	1	0	0	5	1	2	0
M_{149}^{A}	2	1	1	0	0	0	0	1	0	1	5	1	2	0
M_{150}^{A}	2	1	1	0	0	0	0	1	1	0	5	1	2	0
M_{151}^{A}	2	1	1	0	0	0	0	1	1	1	5	1	2	0
M_{152}^{A}	2	1	1	0	0	0	1	0	0	0	5	1	2	0

TABLE 7. ***(Continued.)*** **$\mathcal{M}_{\mathrm{FBM}}$ of the net in Fig. 3.**

M^A_{153}	2	1	1	0	0	0	1	0	0	1	5	1	2	0
M^A_{154}	2	1	1	0	0	0	1	0	1	0	5	1	2	0
M^A_{155}	2	1	1	0	0	0	1	0	1	1	5	1	2	0
M^A_{156}	2	1	1	0	0	1	0	0	0	0	5	1	2	0
M^A_{157}	2	1	1	0	0	1	0	0	0	1	5	1	2	0
M^A_{158}	2	1	1	0	0	1	0	0	1	0	5	1	2	0
M^A_{159}	2	1	1	0	0	1	0	0	1	1	5	1	2	0
M^A_{160}	2	1	1	0	0	1	0	1	0	0	5	1	2	0
M^A_{161}	2	1	1	0	0	1	0	1	0	1	5	1	2	0
M^A_{162}	2	1	1	0	0	1	0	1	1	0	5	1	2	0
M^A_{163}	2	1	1	0	0	1	0	1	1	1	5	1	2	0
M^A_{164}	2	1	1	0	0	1	1	0	0	0	5	1	2	0
M^A_{165}	2	1	1	0	0	1	1	0	0	1	5	1	2	0
M^A_{166}	2	1	1	0	0	1	1	0	1	0	5	1	2	0
M^A_{167}	2	1	1	0	0	1	1	0	1	1	5	1	2	0
M^A_{168}	2	1	1	0	1	0	0	0	0	0	5	1	2	0
M^A_{169}	2	1	1	0	1	0	0	0	0	1	5	1	2	0
M^A_{170}	2	1	1	0	1	0	0	0	1	0	5	1	2	0
M^A_{171}	2	1	1	0	1	0	0	0	1	1	5	1	2	0
M^A_{172}	2	1	1	0	1	0	0	1	0	0	5	1	2	0
M^A_{173}	2	1	1	0	1	0	0	1	0	1	5	1	2	0
M^A_{174}	2	1	1	0	1	0	0	1	1	0	5	1	2	0
M^A_{175}	2	1	1	0	1	0	0	1	1	1	5	1	2	0
M^A_{176}	2	1	1	0	1	0	1	0	0	0	5	1	2	0
M^A_{177}	2	1	1	0	1	0	1	0	0	1	5	1	2	0
M^A_{178}	2	1	1	0	1	0	1	0	1	0	5	1	2	0
M^A_{179}	2	1	1	0	1	0	1	0	1	1	5	1	2	0
M^A_{180}	2	1	1	0	1	1	0	0	0	0	5	1	2	0
M^A_{181}	2	1	1	0	1	1	0	0	0	1	5	1	2	0
M^A_{182}	2	1	1	0	1	1	0	0	1	0	5	1	2	0
M^A_{183}	2	1	1	0	1	1	0	0	1	1	5	1	2	0
M^A_{184}	2	1	1	0	1	1	0	1	0	0	5	1	2	0
M^A_{185}	2	1	1	0	1	1	0	1	0	1	5	1	2	0
M^A_{186}	2	1	1	0	1	1	0	1	1	0	5	1	2	0
M^A_{187}	2	1	1	0	1	1	0	1	1	1	5	1	2	0
M^A_{188}	2	1	1	0	1	1	1	0	0	0	5	1	2	0
M^A_{189}	2	1	1	0	1	1	1	0	0	1	5	1	2	0
M^A_{190}	2	1	1	0	1	1	1	0	1	0	5	1	2	0
M^A_{191}	2	1	1	1	1	1	0	1	1	1	4	1	2	0
M^A_{192}	2	1	1	2	1	1	0	1	1	1	3	1	2	0
M^A_{193}	2	1	1	3	1	1	0	1	1	1	2	1	2	0
M^A_{194}	2	1	1	4	1	1	0	1	1	1	1	1	2	0

way is to enumerate the markings in $\mathcal{M}_{FBM}$ of the net in Fig. 6. We find that the markings in $\mathcal{M}_{FBM}$ related to emptied siphons S_1 and S_2 are similar to those in DZ_f, as explored in the last section. At these markings, the number of tokens in the η-resource initially makes no difference to what we have presented in Proposition 2. By setting $g = 5$, the markings at which S_3 is emptied are shown in TABLE 5.

It is easy to find the property of the markings in $\mathcal{M}_{FBM}$ at which S_3 is emptied, i.e., $M(p_3)+M(p_8) = M_0(p_{20})$, $M(p_3) \in \{1, 2, \ldots, g-1\}$, and $M(p_8) \in \{1, 2, \ldots, g-1\}$. Actually, for the markings with S_3 being emptied in $\mathcal{M}_{FBM}$, $M(p_3) = g$ also indicates that S_1 is emptied. Similarly, $M(p_8) = g$ means that S_2 is emptied. Accordingly, we have the following result and Algorithm 2 based on it. The computational complexity analysis of Algorithm 2 is similar to Algorithm 1. They have the same computational complexity. Slight differences in these two algorithms come from the existence of a ξ-resource.

Proposition 3: Given an η-S^3PR (N, M_0) with η-resource r_η and $M_0(r_\eta) = c$, $(c > 1)$, r_a and r_b being dependent non-ξ-resources, for all $M \in \mathcal{M}_{\text{FBM}}$, there exists $S_0 \in \Pi$ such that $M(S_0) = 0$, we have two cases:

1) *if $H(r_\eta) \cap S_0 \neq \emptyset$, for all $r \in P_R \backslash S_{0R}$, $M(r) + M(H(r)) = M_0(r)$ with $M(r) \in \{0, 1, \ldots, M_0(r)\}$, $M(H(r)) \in \{0, 1, \ldots, M_0(r)\}$, where there does not exist a trap S^* such that $M(S^*) = 0$;*
2) *if $H(r_\eta) \cap S_0 = \emptyset$, then $M(H(r_\eta) \cap P^{\mathcal{X}^1}) + M(H(r_\eta) \cap P^{\mathcal{X}^2}) = M_0(r_\eta)$, where $M(H(r_\eta) \cap P^{\mathcal{X}^1}) \in \{1, 2, \ldots, M_0(r_\eta) - 1\}$, $M(H(r_\eta) \cap P^{\mathcal{X}^2}) \in \{1, 2, \ldots, M_0(r_\eta) - 1\}$.*

Proof: Similar to the proof of Proposition 2. □

IV. EXAMPLE

We use the net in Fig. 3 as an example. There are three strict minimal siphons: $S_1 = \{p_4, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$, $S_2 = \{p_7, p_{11}, p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{25}\}$, and $S_3 = \{p_7, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{25}\}$. We set $a = 2, b = 1, c = 1, d = 2, e = 1, f = 1, g = 1, h = 1, i = 1, j = 1$, and $M(p_{20}) = 5$ (indicating that it is not a ξ-resource). According to Algorithm 2, we have its all FBMs in Table 7 as shown in APPENDIX, where M_1^A–M_{143}^A, M_{144}^A–M_{190}^A, and M_{191}^A–M_{194}^A are markings at which siphons S_2, S_1, and S_3 are emptied, respectively. We reset $M_{20} = 1$ (indicating that it is a ξ-resource) where other resources' capacities remain the same. By Algorithm 1, when $S2$ ($S1$) is emptied, the markings in DZ_f are the same as the markings M_1^A–M_{143}^A (M_{144}^A–M_{190}^A) in Table 7, except that the marking of p_4 (p_{11}) is one in this situation, and DZ_f^* as shown in Table 6.

V. CONCLUSION

This paper shows that in an S^3PR with and without a ξ-resource, there are some laws to identify FBMs in an algebraic way as long as the initial marking is given. With these rules, some traditional methods of solving FBMs that need a complete enumeration of reachable states can be circumvented. The proposed methods can identify all FBMs of S^3PR models with described structures by determining whether a certain marking belongs to $\mathcal{M}_{\text{FBM}}$ or not. This can be used to design deadlock controllers to prevent a system from entering the deadlock-zone.

The limitation of this work is that only a class of S^3PR with an η-resource is analyzed. In the future work, we will extend the proposed methods to S^3PRs with more η-resources or more complex structures. We will explore state-tree strucutre to deal with deadlock avoidance problem in automated manufacturing systems [58], [59]. It is also interesting to use the subspace separation algorithm in [60] to find the deadlock zone.

APPENDIX

See Table 7.

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