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PROPERTIES OF ELASTICITIES DERIVED
FROM TRANSLOG AND GENERALIZED
LEONTIEF COST FUNCTIONS

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Working Paper No. 247

February, 1986

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Approximating the Statistical Properties of Elasticities
 Derived from Translog and Generalized Leontief Cost Functions[†]

Itzhak Krinsky and A. Leslie Robb

Empirical studies of consumer demand or of factor demand have now moved far beyond the Cobb-Douglas functional form and elasticities of interest are no longer estimated as parameters of the system. Instead, such elasticities are typically non-linear functions of the parameters that have been estimated and it is natural to want to be able to say something about the statistical properties of such elasticities. One way of dealing with this is to linearly approximate the elasticity formulas (in terms of the estimated parameters) and use classical statistical procedures to get approximations to the underlying variances. If $y=f(x)$ and x has a variance covariance matrix V , the linear approximation is given by:

$\text{Var}(y) = (\partial f/\partial x)V(\partial f/\partial x)$. The data needed for such an approximation are estimates of the parameters and of the associated variance-covariance matrix. Some of the earliest references that we have found to uses of this approximation technique in the elasticity context are to Griffin and Gregory (1976), Griffin (1977), and Fuss (1977), while the earliest references to the general method is to Klein (1953). Such an approximation procedure has also been recommended for the translog production function and cost function in recent papers by Alden L. Toevs (1980 and 1981).¹

The appropriateness of such a procedure depends, of course, on the nature of the non-linearity of the underlying elasticity formulae. Little or no attention has been devoted to this issue heretofore. Here, we propose a simulation exercise to establish the empirical distributions of a set of factor demand elasticities and compare our simulation results with the

linear approximation mentioned above. We start from the cost function and use either a translog (TL) or a generalized Leontief (GL) representation, the two most common functional forms in use these days.²

Given initial estimates of a parameter vector T , a corresponding variance-covariance matrix V , and a set of elasticity formulas

$$1) \quad E_i = f_i(T),$$

where E_i refers to the i th elasticity, and f_i indicates a non-linear function, we take random drawings for T from a multi-variate normal distribution with variance-covariance matrix V and mean \bar{T} .³ For each drawing, we calculate the elasticities and thus generate an empirical distribution for each elasticity. By varying the number of drawings, one can generate an empirical distribution to any desired degree of accuracy.⁴

We study the two examples mentioned above, which we believe to be representative, to illustrate the procedure. We compare the means and variances of the empirical distributions so generated with the linear approximations mentioned earlier. We find that serious errors can be made in statements about the precision of elasticities based on the approximations for both functional forms employed.

The particular example we use is a three factor production process characterized by a cost function (homogeneous of degree one in factor prices) of the form $K=g(Q,P_1,P_2,P_3)$, where K refers to total cost, P_i to the price of the i th factor, and Q to output. The two share equations (the third is implied by the adding up restrictions) that are estimated simultaneously with the cost function, depend on the explicit functional form for g . For the TL cost function, the two share equations can be written as;

$$2) \quad S_1 = A_1 + B_{11} * \ln (P_1/P_3) + B_{12} * \ln (P_2/P_3) + C_1 * \ln Q,$$

and,

$$3) \quad S_2 = A_2 + B_{21} * \ln(P_1/P_3) + B_{22} * \ln(P_2/P_3) + C_2 * \ln Q.$$

where S_i refers to the i th share, and the A 's, B 's and C 's are parameters.

The GL cost function is:

$$4) \quad K = \left(\sum_{ij} B_{ij} * (P_i P_j)^{1/2} \right) * Q * \text{Exp} \left(\delta + \frac{\theta}{2} \ln Q + \phi_1 \ln(P_1/P_3) + \phi_2 \ln(P_2/P_3) \right)$$

and the two share equations for the GL cost function are:

$$5) \quad S_1 = \left[B_{11} * P_1 + B_{12} * (P_2 P_1)^{1/2} + B_{13} * (P_3 P_1)^{1/2} \right] / \sum_{ij} B_{ij} * (P_i P_j)^{1/2} \\ + \phi_1 * \ln Q$$

$$6) \quad S_2 = \left[B_{22} * P_2 + B_{21} * (P_2 P_1)^{1/2} + B_{23} * (P_2 P_3)^{1/2} \right] / \sum_{ij} B_{ij} * (P_i P_j)^{1/2} \\ + \phi_2 * \ln Q$$

where, δ , θ , and the ϕ_i 's are parameters.

In addition, the cost functions must satisfy the property $B_{ij} = B_{ji}$.

The own and cross price elasticities of factor demand for these models are given by;

$$7) \quad E_{kk} = (P_k/S_k) * (\partial S_k / \partial P_k) + (S_k - 1.0) \text{ and}$$

$$8) \quad E_{kj} = (P_j/S_k) * (\partial S_k / \partial P_j) + S_j$$

where E_{kj} refers to the elasticity of the k th factor with respect to the j th price.⁵ These elasticity formulas are written in terms of the factor shares though these are endogenous and would need to be replaced by the appropriate share equations (e.g. 2 or 3 above for the TL case and 5 or 6 for the GL case) before calculation of the elasticities. The elasticities so calculated vary with the levels of the exogenous variables and one is required to pick a point of evaluation. Typically, the means of the data or some recent values (in the case of a time series) are chosen. For this paper, we work with the means.

For the TL cost function, the estimates of the 7 parameters of interest (equations 2 and 3 contain 8 parameters, but $B_{12} = B_{21}$) and the associated variance-covariance matrix for our particular example are given in

Table 1.⁶ We take these simply as a convenient example and make no particular claim for the estimates though we think them to be reasonably representative of such empirical models.

Table 2 reports a variety of statistics of interest related to 6 of the elasticities derived according to equations 7 and 8 above. We report on the three own-price elasticities and three of the cross-price elasticities (since E_{ij} is not equal to E_{ji} , there are, in fact, 6 cross-price elasticities, though 3 suffice for this example) calculated at the means of the data. The first column reports the elasticity calculated by inserting the parameter estimates into the elasticity formulas. The second column reports the average elasticity calculated over 1000 drawings of the parameter vector according to the variance-covariance matrix given in Table 1. Columns 3 through 7 report the 2.5, 5, 50, 95, and 97.5 percentile points according to the empirical distribution. Thus, for example, columns 3 and 7 can be interpreted as the 95 percent confidence limits for the elasticity, while column 5 can be interpreted as the median. Column 8 reports the standard deviation calculated from the empirical distribution while columns 9 and 10 report the mean and standard deviation calculated using the linear approximation mentioned earlier.⁷

For the GL cost function, the parameter information is recorded in Table 4 while the simulation results are recorded in Table 5.

The first thing one notices in Tables 2 and 5 is the dramatic differences between the standard deviations estimated by the linear approximation and those estimated from the simulations. Compared to the standard deviations derived from the simulations, the ones from the linear

approximations frequently appear to be dramatically understated.⁸ According to the simulation results, the standard deviations are frequently ten times too small and one is as much as a thousand times too small. The estimates of the means of the elasticities are much better, by comparison.⁹

Of course, the question arises whether the simulation results serve as a reasonable basis of comparison. If one accepts the estimates of the parameters of the model and the estimates of their variance-covariance matrix, then an infinite number of drawings should lead to the precise distribution of an elasticity measure. Additional experimentation reported elsewhere leads us to believe that our estimates from a thousand drawings should be sufficient to get a reasonably good estimate.¹⁰

In Tables 3 and 6 we report the results of an experiment designed to shed light on the issue of whether the standard deviations reported in Tables 2 and 5 are useful to the researcher interested in statistical testing. If the simulated distributions are approximately normal, then the standard deviations will be very useful in approximating confidence limits and the like. On the other hand, if the distributions are quite non-normal, then the standard deviations may not be very helpful in this regard. The columns in Table 3 and 6 report the difference between the upper and lower 2.5% tails (and 5% tails) and the mean, divided by the standard deviation (itself calculated from the simulation). If the distributions are normal these numbers should be ± 1.96 (± 1.65). As is easily seen, in the case of the translog the numbers are reasonably close to this standard, though in the case of the GL cost function, there is less accord. Whether such distributions will be approximately normal more generally will, no doubt, depend on the particular example and the non-linearities involved. Anyone using this simulation technique to find distributions will find it as easy

to generate the confidence limits as to calculate the standard deviation and we would recommend it.

In summary, however, the moral of the story is simple. Be wary of using linear approximations to get estimates of the dispersion of measures (e.g. elasticities) that are non-linear functions of random variables.

References

1. Appelbaum Elie, (1979), "On the Choice of Functional Forms", International Economic Review, 20, pp. 449-458.
2. Berndt, E.K. and M.S. Khaled, (1979), "Parametric Productivity Measurement and Choice Among Flexible Functional Forms", Journal of Political Economy, 87, pp. 1220-1245.
3. Freedman, D.A. and S.C. Peters, (1984), "Bootstrapping a Regression Equation: Some Empirical Results", Journal of the American Statistical Association, 79, pp. 97-106.
4. Fuss, Melvyn A., (1977), "The Demand for Energy in Canadian Manufacturing", Journal of Econometrics, 5, pp. 89-116.
5. Griffin, James M., (1977), "Long-Run Production Modelling with Pseudo Data: Electric Power Generation", Bell Journal of Economics, 8, pp. 112-127.
6. Griffin, James M. and Paul R. Gregory, (1976), "An Intercounty Translog Model of Energy Substitution Responses", American Economic Review, 66, pp. 845-857.
7. Klein, Lawrence R., (1953) A Textbook of Econometrics, Ross, Peterson and Co.
8. Krinsky Itzhak and A. Leslie Robb, (1985), "On Approximating the Statistical Properties of Elasticities", QSEP Research Report No. 146, McMaster University.
9. Toevs, Alden L., (1980), "Approximate Variance Formulas for the Elasticities of Substitution Obtained from Translog Production Functions", Economics Letters, 5, pp. 155-160.
10. _____, (1982), "Approximate Variance Formula for the Elasticities of Substitution Obtained from Translog Cost Functions", Economic Letters, 10, pp. 107-113.

FOOTNOTES

+We would like to thank Mr. F. Mahmud and Maria Berruti for research assistance in this project, and Mike Veall, Gordon Anderson and anonymous referees for comments on an earlier draft.

1. Toevs calculates the explicit derivatives necessary for the linear approximation. The TSP ANALYZ command will, however, do the calculations for you. It presumably approximates these derivatives by a numerical procedure.

2. See, for example, Applebaum (1979) or Berndt and Khaled (1979).

3. The approximation methods, of course, also assume the distribution of T to be multi-variate normal. In this exercise, we use the IMSL routine GGNSM to make the drawings.

4. The simulation nature of this exercise shares the philosophy of the Bootstrap (see, for example, Freedman and Peters (1984)) but is, in fact, somewhat different. To appreciate this, note that the Bootstrap could be used to generate the distribution of the parameters (T) initially. The parameter distributions so generated could then be used in place of our multivariate normal to calculate elasticities. The estimation of the parameters and their distributions is prior to the issues we raise here, though these two can obviously be combined.

5. One could also calculate elasticities of substitution if they were the object of interest.

6. The example is taken from estimates made by our research assistant in some of his thesis work. The factor shares are .002, .220 and .777, respectively.

7. In fact, we have used the ANALYZ command in TSP to effect this. Note that the mean from the linear approximation differs from Column 1. In Column 1, the parameter estimates are inserted into the elasticity formula while in Column 9, the linear approximation is applied first.

8. Note, though, in the TL case the linear approximation of the standard deviation of the last elasticity differs from the simulated one by only about 15% while in the GL case, the penultimate elasticity is within 2%. Although in all the cases reported here, the standard deviations from the 1000 draws are larger than those calculated by the linear approximation, there is nothing to require this. In fact, in some additional work we have found examples where the results go the other way.

9. The careful reader may notice and wonder at the differences in the elasticities estimated from the two different functional forms. The sensitivity of such elasticities to functional form is not unknown. See, for example, the tables in Berndt and Khaled (1979). The differences here may be greater than one commonly encounters, but, we have not made serious efforts to properly handle autocorrelation and to find the best specifications of each form. The results that we are interested in here do not depend on such considerations, in any event.

10. We have experimented with many drawings of 1000 and found the results to be very close to those reported here. In fact, many fewer than 1000 drawings give much the same estimate as the larger numbers. See Krinsky and Robb (1985) for details.

TABLE 1: PARAMETER INFORMATION - TRANSLOG COST FUNCTION

PARAMETER	MEAN	VARIANCE-COVARIANCE MATRIX						
		A1	A2	B11	B22	B12	C1	C2
A1	0.943562	9.359E-03						
A2	0.016581	2.594E-05	9.926E-06					
B11	0.156399	-1.87E-03	-5.93E-06	4.726E-04				
B22	0.003571	1.715E-06	1.278E-06	-6.44E-08	3.055E-07			
B12	-0.005287	-3.08E-06	-5.66E-08	1.386E-06	1.678E-07	2.875E-07		
C1	-0.112353	-5.96E-04	-1.23E-06	6.799E-05	-2.12E-07	-1.46E-07	6.513E-05	
C2	0.002008	-2.33E-06	-1.32E-06	2.561E-07	-2.28E-07	-9.91E-08	2.306E-07	2.177E-07

TABLE 2: ELASTICITIES FROM THE TRANSLOG COST FUNCTION -- BASED ON 1000 DRAWINGS

ELASTI- CITY	CALCULATED	MEAN	LOWER TAIL		MEDIAN	UPPER TAIL	
	FROM PARAMETER MEAN	ELASTICITY FROM 1000 DRAWS	2.5%	5%		5%	2.5%
E11	-0.02171	-.02624	-.081614	-.074099	-.025305	.018037	.029091
E22	-0.44410	-.45982	-.625690	-.593510	-.460320	-.316080	-.295810
E33	-0.09098	-.10765	-.303350	-.276940	-.104290	.050763	.090956
E12	-0.00019	-.00019	-.001598	-.001332	-.000170	.000972	.001180
E13	0.02111	.02643	-.030017	-.018541	.025487	.073771	.082000
E23	0.48105	.48213	.203200	.242660	.479570	.716230	.773360

STANDARD DEVIATION FROM 1000 DRAWS	RESULTS FROM LINEAR APPROXIMATION	
	MEAN	ST. DEV.
.02751	-0.021344	0.002183
.08458	-0.443957	0.000060
.09793	-0.092633	0.002186
.00070	-0.000307	0.000064
.02764	0.021686	0.008474
.14432	0.481716	0.122543

TABLE 3: COMPARISON OF THE RESULTS FOR THE TRANSLOG COST FUNCTION WITH THE NORMAL DISTRIBUTION

ELASTICITY	-2.5%/ST.DEV.	-5%/ST.DEV.	+5%/ST.DEV.	+2.5%/ST.DEV.
E11	-2.01	-1.74	1.61	2.01
E22	-1.96	-1.58	1.70	1.94
E33	-2.00	-1.73	1.62	2.03
E12	-2.01	-1.63	1.66	1.96
E13	-2.04	-1.63	1.71	2.01
E23	-1.93	-1.66	1.62	2.02

TABLE 4: PARAMETER INFORMATION - GENERALIZED LEONTIEF COST FUNCTION

VARIANCE-COVARIANCE MATRIX

PARAMETER	MEAN	B12	B13	B22	B23	B33
B12	0.167355	0.175E-02				
B13	0.513360	0.535E-02	0.717E-01			
B22	-0.094983	-0.15E-03	0.737E-02	0.134E-00		
B23	0.050003	-0.38E-02	-0.69E-02	-0.91E-01	0.725E-01	
B33	5.601208	0.112E-01	0.656E-01	0.408E-01	-0.26E-01	0.565E-00
ϕ_1	-0.000866	0.200E-04	0.150E-03	0.100E-04	0.380E-03	0.140E-03
ϕ_2	0.000694	-0.20E-04	-0.13E-03	0.543E-07	-0.40E-04	-0.12E-03
B	1.022877	-0.17E-02	-0.23E-01	-0.75E-03	-0.56E-03	-0.52E-01
θ	-0.104499	0.330E-03	0.519E-02	0.300E-04	0.380E-03	0.127E-01

ϕ_1	ϕ_2	B	θ
0.103E-07			
-0.94E-08	0.867E-08		
-0.40E-04	0.300E-04	0.124E-01	
0.616E-07	-0.60E-07	-0.31E-02	0.790E-03

TABLE 5: ELASTICITIES FROM THE GENERALIZED LEONTIEF COST FUNCTION -- BASED
ON 1000 DRAWINGS

ELASTICITIES

ELASTI- CITY	CALCULATED FROM PARAMETER MEAN	MEAN ELASTICITY FROM 1000 DRAWS	LOWER TAIL		MEDIAN	UPPER TAIL	
			2.5%	5%		5%	2.5%
E11	-1.3020	-1.3039	-1.3882	-1.3676	-1.3013	-1.2476	-1.2386
E22	-0.63192	-.57153	-.81418	-.79189	-.63759	-.23104	.05239
E33	0.05794	.06398	-.01559	.00162	.06091	.13565	.15603
E12	0.02320	.02329	.01421	.01552	.02336	.03075	.03254
E13	0.26143	.25907	.16725	.18173	.26147	.32858	.33712
E23	0.24621	.23543	-.07104	-.00762	.24658	.46294	.50444

STANDARD DEVIATION FROM 1000 DRAWS	RESULTS FROM LINEAR APPROXIMATION	
	MEAN	ST. DEV.
.03797	-1.301966	0.031785
.32092	-0.631930	0.074926
.04335	0.057889	0.039191
.00473	0.023231	0.002990
.04441	0.261357	0.046311
.17409	0.246194	0.126761

TABLE 6: COMPARISON OF THE RESULTS FOR THE GENERALIZED LEONTIEFF COST FUNCTION WITH NORMAL DISTRIBUTION

ELASTICITY	-2.5%/ST.DEV.	-5%/ST.DEV.	+5%/ST.DEV.	+2.5%/ST.DEV.
E11	-2.22	-1.68	1.48	1.72
E22	-0.76	-0.69	1.06	1.94
E33	-1.84	-1.45	1.65	2.12
E12	-1.92	-1.65	1.58	1.96
E13	-2.07	1.74	1.57	1.76
E23	-1.76	-1.40	1.31	1.55

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