Article

Nonlinear Theory of Elementary Particles Part XV: On Calculation of Elementary Particles' Masses

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Abstract

In a previous paper we showed that the existence of the size of the electron does not contradict the quantum field theory. This allows us to consider the electron and other particles as an electromagnetic volume resonator, capable of holding electromagnetic waves of a certain frequency. We assume that the reason of appearance of the mass spectra of elementary particles is the addition of nonlinear electromagnetic waves to this resonator. As it is known the solution of Schrödinger equation for electron in a potential well of final depth there has a limited number of own levels of energy. We assume that this limitation exists for the masses of some families of particles, e.g., for the family of leptons, which has only three generations.

Keywords: non-linear quantum theory, resonance mass-spectra, elementary particle mass spectra, particle generations.

1.0. Introduction

As we know neither classical nor quantum theory could yet explain the origin of the spectra of elementary particles masses and deduce numerical values for this masses.

The main experimental facts here are: 1) the masses of elementary particles arrange a discrete spectrum of mass values, 2) according to modern ideas, all elementary particles are excited states of a small set of stable particles, which have the lower value in the mass spectrum (electron neutrinos, quarks) (see also (Kyriakos , 2011a)).

The main difficulty in the classical electromagnetic (EM) theory, obviously, is that it does not take into account the quantum properties of elementary particles. However, there is an approach that allows us to create here the mass spectra of particles - the existence of resonant modes of EM waves in potential well. The essential difficulty of the quantum theory is that its fundamental particles are considered as pointlike (i.e., they do not have spatial dimensions). This deprives them of the degrees of freedom, needed to create a variety of masses of elementary particles.

In a previous article (Kyriakos, 2011b), we have shown that fundamental particles have a size and it does not contradict the quantum field theory. This makes it possible to return to the idea of the existence of the mass spectrum of elementary particles, based on the resonance. As is well known, de Broglie explained in such a way the existence of energy levels of electrons in the atom.

1.1. The reasons of quantization of mass-energy

According to Einstein's formula of equivalence of energy and mass $\varepsilon = mc^2$, to any stationary level of particle energy the certain rest mass corresponds. Therefore it is possible to assume that

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the discreteness of the mass spectrum of elementary particles corresponds to the discreteness of energy spectrum of electron in the atom.

The first calculation of energy-mass spectrum of electron in hydrogen atom has been based on the known Bohr atom theory. This approach allows the calculation of energy spectrum of electron, but it does not reveal the reasons of quantization.

The reasons of quantization have been specified by de Broglie. He showed that electron in a stationary state in orbits of hydrogen atom can be considered as standing de Broglie waves under conditions of the wavelength integrality. De Broglie has shown (Broglie, de, 1924; 1925;. Andrade e Silva and Loshak, 1972) that the length of any Bohr orbit L should contain an integer number of electron wavelengths $L = n\lambda$ (where $n = 1,2,3,...$ $\lambda = h/m\upsilon = 2\pi\hbar/p$, υ is the particle velocity) (see fig. 15.1):

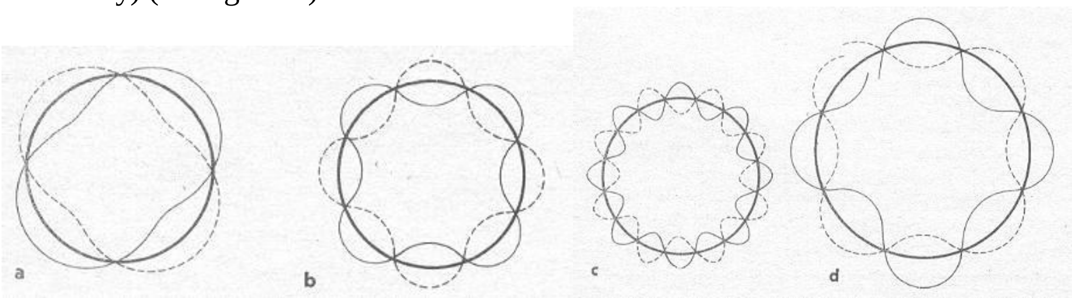

Fig. 15.1

For icons **a**, **b** and **c** of fig.1 this condition is carried out, when $n = 2,4,8$ respectively. In case of **d** this condition is not carried out and electron motion is unstable; this leads to self-destruction of a wave as a result of wave self-interference. Mathematically the integrality condition corresponds to the requirement of unambiguity of wave function.

A similar condition takes place also for elliptic orbits, but this case is more complex, since the length of de Broglie wave in different points of an elliptic orbit varies because the electron speed is not constant. In this case it is necessary to use the general quantization condition:

$$\int \frac{ds}{\lambda} = \int_0^T \frac{m\beta^2 c^2}{h\sqrt{1-\beta^2}} dt = n, \qquad (15.1.1)$$

where ds is the orbit length element, T is the period of motion, dt - time element, $\beta = c/\upsilon$.
From the above follows that the conditions of stationarity correspond to resonance conditions, which are adequate to conditions of integrality of the standing waves.

Such sight at the reason of appearance of quantum levels of electron energy also allows to calculate the lasts in other similar cases. For example, as approximate model of 3-dimensional short-range potential, can be considered the spherical potential well of some radius R (Naumov, 1984). According to de Broglie for the big circle of sphere of radius R we will have:

$$2\pi R = n\lambda = n\frac{2\pi\hbar}{p} = n\frac{2\pi\hbar}{\sqrt{2m\varepsilon}}, \qquad (15.1.2)$$

From here we obtain for energy levels: $\varepsilon_n = \frac{\hbar^2 n^2}{2mR^2}$. The Schrödinger wave equation (i.e., the Helmholz equation for de Broglie waves) gives in this case the exact result, which differs from the approximate one only by the factor π^2.

It follows that the reason for the quantization of energy-mass of an electron in the atoms are resonant conditions of the motion of de Broglie waves in a closed cavity (potential well). In this case, the sizes of the resonant cavity (potential well) and the medium, which fill the cavity, play a decisive role. Let us recall that according to the nonlinear theory of elementary particles (NTEP), de Broglie waves of massive particles are nonlinear (self-acting) electromagnetic waves.

In other words, the diversity of the quantization of energy takes place due to the difference of the dispersion relations for nonlinear electromagnetic waves in 3-dimensional space. For electromagnetic waves in a homogeneous medium the dispersion relation is independent from position and time; at the same time in field theory problems these relations are determined by the spatial dependence of potential energy, mathematical expressions of which can be quite diverse (conditionally speaking, in this case we have an EM wave in an inhomogeneous medium with the suitable dispersion relations).

Practically all the waves that we have considered have been "one-dimensional" (Crawford, 1968). That is, they have been waves propagating along a straight line, which we usually call the y axis. Now we must introduce three-dimensional waves.

We can see that there is something more to having extra dimensions than is implied by a mere change of variables. One has qualitatively new features because the extra dimensions give extra degrees of freedom. For example, in three dimensions and in vacuum one can have an electromagnetic wave that is a pure traveling wave in one direction, a pure standing wave in another, and an exponential wave in still another direction! In one dimension it is not possible to have exponential electromagnetic waves in vacuum, because the dispersion relation $\omega^2 = c^2 k^2$ cannot become $\omega^2 = -c^2 k^2$ for some frequency ranges. In order to have exponential waves in one dimension, one needs a cutoff frequency, i.e., one needs a dispersion relation like that of the ionosphere, $\omega^2 = \omega_0^2 + c^2 k^2$, which can become $\omega^2 = \omega_0^2 - c^2 k^2$ for sufficiently low frequency.

In three dimensions we will find that *k* is the magnitude of a vector, called the propagation vector. Thus the dispersion relation fur electromagnetic waves in vacuum becomes $\omega^2 = c^2\left(k_x^2 + k_y^2 + k_z^2\right)$. Under certain circumstances one can have one or two of the components k_z^2, elc., replaced by $-k_z^2$, etc., and still have the return force per unit displacement per unit inertia, ω^2, be positive, as it must be". As examples, we will be referring to the electromagnetic waves in three-dimensional waveguides and resonators.

We should note one remarkable feature, which has the solution of Schrödinger equation for electron in a potential well of final depth. In this case (Shiff, 1955; Matveev, 1989) there is a *limit* number of own levels of energy. A similar limitation exists for the masses of some families of particles. For example, the family of leptons has only three generations: electron, muon and taon and the corresponding neutrinos. If we will consider the electron self field as potential well of final depth, then heavy leptons can be examined as the electron excitations, whose number can be limited.

2.0. On present calculations of elementary particle masses

The attempts of calculation of mass-energy spectra on the basis of resonance behavior of particles exist for a long time. We will briefly mention the most consecutive of them. Unfortunately the existing calculations are based on assumptions and guesses, which cannot be proved enough within the framework of the quantum field theory.

2.1. Quasi-classical approaches to mass calculation

According to this approaches the basic particle is assimilated to a potential well (or, that is the same, to the resonator). The spectrum of masses of particles arises, when some additional resonance wave-particle (e.g. photon) is placed in this potential well. Characteristics of addition particle change the characteristics (mass, spin, charge, etc) of the basic particle and we can consider the last one as a new particle.

One of the first attempts of quasi-classical calculation (for masses of muon and pion) belongs to Putilov (Putilov, 1964). Note that this calculation does not take into account the experimental facts, which have been found out later (e.g., the existence of a tau-lepton, the law of lepton number conservation, etc.) and it should be considered only as an example of a corresponding computational procedure.

A second, much more detailed calculation (for the big number of particles, known at that time) is stated in paper (Kenny, 1974). Here is already the theoretical substantiation of a method of calculation and are obtained impressing results. But calculation is made by analogy to the theory of Bohr; therefore here was used Coulomb potential well. The obtained numerical values of masses, without serious substantiation, are corresponded with masses of known particles; this conduct to infringement of laws of conservation of quantum numbers, etc.

The calculations, based on idea of composite particles, has near connection to the resonance theory. One of the first possible approaches (Rivero and Gsponer, 2005) to an estimation of masses of elementary particles was based on the known composite model of Nambu-Barut (Nambu, 1952; Barut, 1979). In particular, for leptons Barut has obtained the following formula: $m(N) = m_e\left(1+\frac{3}{2\alpha}\sum_{n=0}^{n=N} n^4\right)$, which gives satisfactory values for both heavy leptons (here m_e is the electron mass, α is electromagnetic constant).

In this approach it is postulated that for calculation of masses of heavy leptons to the rest mass of electron the quantized magnetic energy $(3/2)\alpha^{-1}\sum_{n=0}^{n=N} n^4$ must be added, where n is a new quantum number, which for for n = 1 gives muon mass and for n = 2 – taon mass $m_\tau = 1786{,}08$ MeV.
A similar expression had been obtained from other reasons. In paper (Rodriguez and Vases, 1998) for muon mass as excited state of electron (which is allocated with properties of a quark) following formula is obtained: $m_l = \left(1+\frac{q_m^n}{e}\right)m_e$, where $q_m^n = \frac{3e}{2\alpha}n$. E.g., for muon at n = 1 turns out: $m_\mu = \left(1+\frac{3}{2\alpha}\right)m_e = 206{,}55 m_e$. Assuming that taon is the excited state of muon, authors

obtain also formula: $m_l = \left(1 + \frac{3}{2\alpha}\right)m_e + \frac{q_m^n}{e}m_e = \left(1 + \frac{3}{2\alpha}\right)m_e + \frac{3}{2\alpha}n$, which at n = 16 gives for taon mass the value, closed to experimental, namely $m_\tau = 3494m_e = 1781{,}9$ MeV.

In (Greulich, 2010) "*the masses of all fundamental elementary particles (those with a lifetime* > 10-24 sec) *can be calculated with an inaccuracy of approximation* 1% *using the equation* ***m/m***$_{\text{electron}}$ **= N/2*α*,** *where α is the coupling constant of quantum electrodynamics* (*also known as fine structure constant*) (= 1/137.036), *and* N *is an integer variable.*

Thus, the masses of the muon, charged pions, kaons can be expressed as m/m$_{\text{electron}}$ = N/2α , *with* N = 3 *for the muon,* N = 4 *for the pion and* N = 14 *for the kaon with a* 1% *inaccuracy except for eta and omega mesons* (2.22% *and* 1.6%, *respectively*), *and the* **xi** *baryon* (1.23%). *The masses of other particles may also be expressed in such simple terms. The results are listed in Table* 2 (see the paper (Greulich, 2010)). *Can this result have been obtained simply by chance*?"

Another approach is based on the quantization rules of Wilson-Sommerfeld. The group of scientists Yu. L. Ratic, F.A. Gareev et al. (Ratis and Garejev, 1992; Garejev, Kazacha, Ratis, 1996; Garejev, Kazacha, Barabanov, 1998; etc) has achieved impressive results, using the quantization condition for asymptotic momenta of decay products of the hadronic resonances.

2.2. Quantum approaches to mass calculation

2.2.1. *Kowalczynski model*

A simple theory of the elementary particle mass spectrum is proposed (Kowalczynski, 1988). "*It originates from the Dirac idea of the free electron motion and from the transformed Klein-Gordon equation. The theory is based on an equation that includes the squared mass operator having an infinite sequence of orthogonal eigenfunctions and a discrete spectrum of eigenvalues. A discrete mass formula is derived. It yields values of mass that are in agreement with present-day empiric data for elementary particles…*

All our considerations concern free particles. One of the known peculiarities of the Dirac electron is that its velocity operator has only two eigenvalues $\pm c$*. Together with other specific properties of the spinorial wave, this makes a classical description of the Dirac particle extremely difficult. If one insists, however, on giving such a description, then one may imagine a rapidly oscillating particle having, e.g., a periodic spiral-like trajectory determined by a classical motion equation including terms periodic in time. This kind of trajectory could only represent the classical rectilinear free motion as an average. Such a picture, though somewhat vague, is automatically contained in the Dirac equation and, consequently, in the Klein-Gordon equation, which, as yet, is obligatory for every free particle. As a free extension of the Dirac idea, it is therefore tempting to introduce an equation including terms periodic in time and being a modification of the Klein-Gordon equation.*

The slower-than-light free particle has a straight time-like world line in terms of relativity, and is represented by a plane harmonic de Broglie wave in terms of quantum mechanics. Thus, after applying the appropriate Lorentz transformation, the particle can be described by expressions with only one variable. The particle proper time s (affine parameter of the world line) is the most natural choice. Then the Klein-Gordon equation reduces to

$$-\frac{h^2}{c^4}\frac{d^2}{ds^2}\psi(s)=m^2\psi(s), \qquad (15.2.1)$$

If we observe the particle at rest, then s is, of course, the laboratory time. Thus, by Equation (2.1), operator $-h^2c^{-4}\ d^2/ds^2$ could be considered as a squared rest mass operator. However, its spectrum of eigenvalues is continuous, while the elementary particle masses form a discrete set of values. Basing upon our previous considerations, we can assume that this operator is only a zero approximation, being too coarse to perceive the frequent oscillations of the particle.
Let us consider the equation

$$\frac{h^2}{c^4}\left[-\frac{d^2}{ds^2}+\omega\tan(\omega s)\frac{d}{ds}+\frac{\omega^2a^2}{2(1-\sin\omega s)}+\frac{\omega^2b^2}{2(1+\sin\omega s)}\right]\psi(s)=m^2\psi(s), \qquad (15.2.2)$$

where $-\infty<s<+\infty$, a and b are dimensionless real constants, and ω is a constant frequency such that $\omega=2\pi/T$.

Thus, on the left-hand side of Equation (15.2.2) *we have a new squared mass operator with eigenfunctions ψ and eigenvalues m^2. Equation* (15.2.2) *is a generalization of Equation* (15.2.1), *since the terms added to the operator $-h^2c^{-4}$*d^2/ds^2 are periodically singular. Of course, its concrete form is arbitrarily postulated".

2.2.2. *Chiatti model*

"A model is conjectured in article (Chiatti, 2009) *for the calculation of the masses of elementary particles, based on the following assumptions.*

The localisation of the particle takes place in a region having minimum space size L. To bring about this localization, an energy ħc/L is required, by virtue of the uncertainty principle. Dividing this energy by c^2, the mass $m_s = \hbar/cL$ is obtained, which shall be called, in this context, the "skeleton" mass of the particle.

The lepton is delocalised over a spatial region whose size is $L = \hbar/m_s c$; however, the radius $r_0 = e^2/ m_s c^2 = \alpha L$

If we only consider electromagnetic interaction, the interaction energy can be estimated as:

$$\varepsilon_{\text{int}} \approx e^2/L = e^2/(\hbar/ m_s c) = (e^2/\hbar c)\, m_s c^2 = \alpha\, m_s c^2, \qquad (15.2.3)$$

*with the additional energy $k\alpha m_s c^2$ (where $|k| \approx 1$), equal to the self-*interaction energy of the virtual lepton. This is equivalent to saying that the effective rest mass of the lepton is not m_s, but $m_s + k\alpha m_s = m_s(1 + k\alpha)$.

One can suppose that the self-interaction term that must be added to the skeleton mass to obtain the effective mass is expressed by the integral:

$$m_{self} = k\alpha\, m_s = \frac{e}{c^2}\int\overline{\psi}\gamma^{\mu}\psi d\tau\,, \qquad (15.2.4)$$

where

$$\gamma^{\mu}\left(\partial_{\mu}-\frac{ie}{\hbar c}A_{\mu}\right)\psi=-\frac{m_{s}c}{\hbar}\psi\,, \tag{15.2.5}$$

$$\partial^{\nu}\partial_{\nu}A_{\mu}=-4\pi\, ie\,\bar{\psi}\gamma_{\mu}\psi\,, \tag{15.2.6}$$

In calculating the integrals that express m_{self} *and* $A\mu$, *it must be borne in mind that self-interaction is limited (in the particle rest frame of reference) to distances greater than* r_0; *at lower distances, only one lepton copy is present, which certainly cannot interact with itself".*

Below, we will show that all these approaches of calculation of the mass spectra of elementary particles are based on the resonance condition for nonlinear electromagnetic waves in the cavity, which is formed by means a stable particle.

3.0. Statement of problems of calculation of masses of particles in framework of NTEP. Elementary particles as resonators

In framework of NTEP we can suppose that:

1) all particles are divided into two groups: a) absolutely stable particles: photon, electron, neutrino, proton and their antiparticles; and b) metastable particles: all other particles;

2) the stable elementary particles are the simplest modes of non-linear waves;

3) the stable elementary particle are the box (potential well), which are the resonator for non-linear waves;

4) the metastable elementary particles are the composite non-linear waves, appearing as superposition of an absolutely stable non-linear waves and some additive non-linear waves;

5) the metastability of composite particles is ensured by resonance conditions and known conservation laws.

As a simplest reaction of formation of composite particle it is possible to consider the transition of electron e^- in hydrogen atom from a low level to higher level of energy:

$$e^-(\varepsilon_n)+N=\gamma+e^-(\varepsilon_b)+N\,, \tag{15.3.1}$$

where ε_n is any level of electron energy, ε_b is the base electron energy and $\varepsilon_n>\varepsilon_b$; γ is the photon (gamma-quantum); N means the field of the nucleus, which in this case works as a resonator. The solution of Schrödinger's equation gives spectra, which the resonator - potential well of proton Coulomb field - allows.

Note that the reaction of electron-positron pair production from a photon can be also described in the same way as (15.3.1):

$$\gamma+N=e^-+e^++N\,, \tag{15.3.2}$$

The record (15.3.1) is possible to be considered as an instruction that the electron in a mass-energy state ε_n "is composed" from the electron in the state ε_b and some photon γ inside the potential well (this problem we can conditionally name "direct")

As we noted above, the concrete result of calculation depends on distribution of the potential energy (i.e. on dispersion relationship and characteristics of well). Since we do not have information about the field distribution inside the elementary particles, we can rest only upon some (more or less) believable hypotheses.

In the case of electron as basic particle, it can be assumed that we deal with the toroidal resonator. What we can say about the properties of this resonator, i.e. about the dispersion relationship, which describes space inside this resonator?

If we consider an electron as circular high-frequency current, then its charge due to the skin effect is displaced to the surface; this means that the electric field is absent inside it. If we accept the circular current of electron as constant, then distribution can be other. In the case of such composite particles as mesons and baryons, the distribution can be even more complex.
Another statement of the problem (conditionally, an "inverse" problem) arises in case when we will write down the reaction (15.3.1) in the opposite direction:

$$\gamma + e^-(\varepsilon_b) + N = e^-(\varepsilon_n) + N\,, \tag{15.3.3}$$

Here initial particles γ-quantum and electron $e^-(\varepsilon_b)$ can be considered as the reason of generation of a composite particle $e^-(\varepsilon_n)$. In this case for calculation of energy-mass of a composite particle it is necessary to know the experimental characteristics of waves (particles), which have formed this level of energy.

For simplicity we will further consider the reactions with the participation of electron (15.3.1) and (15.3.2). In these reactions electron appears simultaneously as particle and the wave. The following paragraphs will be dedicated to the brief analysis of the kinematics and wave properties of electron and other particles.

4.0. Kinematics characteristics of a composite particle

The above-stated reactions in a general view can be presented as follows:

$$X_0 \Leftrightarrow X_1 + X_2\,, \tag{15.4.1}$$

where through X_0 we designated composite particle; indices 1 and 2 relate to the particles, whose superposition creates composite particle.

The law of conservation of the energy-momentum is valid for each of the particles:

$$\varepsilon_0^2 = c^2 p_0^2 + m_0^2 c^4\,, \tag{15.4.2}$$

$$\varepsilon_1^2 = c^2 p_1^2 + m_1^2 c^4\,, \tag{15.4.3}$$

$$\varepsilon_2^2 = c^2 p_2^2 + m_2^2 c^4\,, \tag{15.4.4}$$

where c is the speed of light, m means in this chapter the particle rest mass; the energies and momentums are defined by relativistic expressions:

$$\varepsilon = mc^2\delta\,,\ \ \vec{p} = m\vec{\upsilon}\delta\,, \tag{15.4.5}$$

where, $\delta = 1\big/\sqrt{1-\beta^2}$, $\beta = \vec{\upsilon}/c$, $\vec{\upsilon}$ is speed of a particle. Besides, in the relativistic mechanics the kinetic energy is entered by the following expression:

$$\varepsilon_k = mc^2\left(\delta - 1\right) = mc^2\delta - mc^2\,, \tag{15.4.6}$$

Since $\upsilon < c$, the expressions, containing δ, can be expanded to Maclaurin series (we take here into account only 4 terms):

$$\delta = 1 + \left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ...\right\}, \tag{15.4.7}$$

$$\beta\delta = 0 + \beta + 0 + \frac{1}{2}\beta^3 + ... \tag{15.4.8}$$

Thus we can obtain for energy and momentum the following expressions:

$$\varepsilon = mc^2 + mc^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ...\right\}, \tag{15.4.9}$$

$$\begin{aligned}\varepsilon_k &= \varepsilon - mc^2 = mc^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ...\right\} = \\ &= \frac{1}{2}m\upsilon^2 + \frac{3}{8}m\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16}m\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128}m\left(\frac{\upsilon^8}{c^6}\right) + ...\end{aligned}, \tag{15.4.10}$$

$$p = m\upsilon + \frac{1}{2}m\frac{\upsilon^3}{c^2} + ..., \tag{15.4.11}$$

At $\beta << 1$ we obtain from (15.4.9)-(15.4.11) as first approximation the non-relativistic expressions:

$$\varepsilon \approx mc^2 + \frac{1}{2}m\upsilon^2;\ \ p \approx m\upsilon;\ \varepsilon_k \approx \frac{1}{2}m\upsilon^2; \tag{15.4.12}$$

In this case according to conservation laws of energy and momentum we have:

$$\varepsilon_0 = \varepsilon_1 + \varepsilon_2\,, \tag{15.4.13}$$

$$\vec{p}_0 = \vec{p}_1 + \vec{p}_2\,, \tag{15.4.14}$$

Not narrowing the framework of the problem, we can consider a case, when the particle X_0 is motionless; that means $\vec{p}_0 = 0$. Then from (15.4.2) we obtain $\varepsilon_0 = m_0c^2$, and from (15.4.13):

$\vec{p}_1 = -\vec{p}_2$. Entering a designation $|\vec{p}_1| = |\vec{p}_2| = p_r$, from (15.4.15) we will obtain the known kinematic expression:

$$m_i c^2 = \sqrt{m_1^2 c^4 + c^2 p_r^2} + \sqrt{m_2^2 c^4 + c^2 p_r^2}\ , \qquad (15.4.15)$$

Considering the particles 1 and 2 as composite parts of particle 0, we can according to N. Bohr postulate here the quantization of momentum p_r and compare the solution with experimental data (see below the results of Yu. Ratis, F. Gareev et al.).

5.0. The wave characteristics of elementary particles

As it is known de Broglie's waves appearance is clearly relativistic effect, connected with motion of electron relative to other particles.

According to de Broglie the particle-wave with energy ε and momentum p has the following frequency and wavelength (taking into account (15.4.9)-(15.4.11)):

$$\nu = \frac{\varepsilon}{h} = \frac{mc^2}{h} + \frac{\varepsilon_k}{h} = \nu_0 + \nu(\upsilon)\,, \qquad (15.5.1)$$

$$\lambda = \frac{h}{p} = h \Big/ \left(m\upsilon + \frac{1}{2} m \frac{\upsilon^3}{c^2} + ... \right), \qquad (15.5.2)$$

where $\nu_0 = mc^2 / h$ and $\nu(\upsilon) = \frac{1}{h}\left\{ \frac{1}{2} m\upsilon^2 + \frac{3}{8} m\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16} m\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128} m\left(\frac{\upsilon^8}{c^6}\right) + ... \right\}$.

In the non-relativistic case $\upsilon << c$ we will obtain:

$$\nu = \frac{\varepsilon}{h} \approx \frac{mc^2}{h} + \frac{1}{2} m\upsilon^2 \Big/ h\,, \qquad (15.5.3)$$

$$\lambda = \frac{h}{p} \approx \frac{h}{m\upsilon}\,, \qquad (15.5.4)$$

Let's analyse these expressions.

Firstly we can note the interesting special feature of a de Broglie wave: in the general case it seemingly consists of the infinite sum of waves, whose frequencies are added arithmetically.
This series has always a certain "basic" wave with maximum frequency of very high value ν_0, which does not depend on particle motion.

The series remainder, which corresponds to expansion $\nu(\upsilon)$, contains the terms, whose frequencies depend on the speed of particle motion. The values of the frequencies of these waves are considerably less than the frequency of "basic" wave, so it seems these waves modulate the "basic" wave.

Secondly, the frequency ν_0 of the "basic" wave of particle is connected with the particle in rest and is determined by its self-energy (rest mass).

According to de Broglie's hypothesis (Broglie, 1923, 1924, 1925), to this frequency corresponds certain internal particle motion. From the other side, this "basic" frequency determines the Compton wavelength of "bare" rest particle; in accordance with our theory, it means that this frequency determines electromagnetic radius of particle. Actually in NTEP a radius of "bare" electron is equal:

$$r_e = \frac{\hbar}{m_e c} \equiv \bar{\lambda}_e = \frac{c}{\nu_0}, \qquad (15.5.5)$$

where $\bar{\lambda}_e$ is a Compton (bar) wavelength of electron.

Thus, hypothetical motion of rest particle is the rotation of the non-linear EM wave.
Usually in the expansion of frequency is considered only one wave, which has frequency and wavelength, which correspond to non-relativistic formulas (15.5.3)-(15.5.4). Specifically, these characteristics served for the experimental testing of hypothesis de Broglie about the particle as about the wave. In reality as it follows from formulas (15.5.1)-(15.5.2) besides this wave there is an infinite number of waves of smaller energy and longer wavelength.

What role these waves play in nature separately, we do not know, but they could be manifested with the high energy, if other more probable effects did not screen them. Thus, into some natural phenomena they could apparently play essential role.

Thirdly: De Broglie's wavelength of particle with zero speed is equal to infinity. For the classical oscillator such speed corresponds to the zero frequency. In contrast to this fact in this case de Broglie's wave has very high frequency ν_0 (about 10^{15} Hz).

It is easy to see that in the case of hydrogen atom the formation of the new levels of electron occurs not because electron itself, as resonator, absorbs a photon. The size of electron corresponds to the wavelengths of the photons, whose energies are considerably more than the energies of the electron between the levels of atom. The wavelengths of the photons, whose energies correspond to electron transitions in hydrogen atom, are considerably less. But as we know, these lengths are commensurate with the de Broglie wavelengths of the electron in hydrogen atom.

It is easy to see that in the case of hydrogen atom the formation of the new levels of electron occurs not because electron itself as resonator absorbs photon. Actually, according to NTEP the size of electron corresponds to the wavelength of the photon, whose energy is considerably bigger than the energies of the electron transition between the levels of atom. But as we know, the lasts are commensurate with the De Broglie wavelengths for the electron motion in hydrogen atom.
Thus, in the atom an electron itself does not carry out role of resonator, but resonator is formed by potential well of nuclear field.

It is possible to say that in this case the nature devised some "crafty" mechanism. The rest electron has high constant internal frequency. But the frequencies of attendant additional waves of electron in motion change from zero to infinity in dependence on the speed. Thus in dependence on the electron speed the wavelengths of these additional waves can be commensurate not only with the lengths of electron orbits in atoms, but also with the size of the slot of diffraction grating (which value is macroscopic).

Let us attempt to show now that the mass spectrum of elementary particles can be explained analogously with the mass spectrum of electron in hydrogen atom, taking into account the fact that in this case particle itself serves as resonator for the additional particles.

6.0. To the calculation of mass spectra of elementary particles

6.1. A direct problem

We will consider here a particle X_0 (see (15.4.1)) as the given resonator, and particles X_1, X_2 as the unknown waves, which satisfy to resonance conditions of this resonator.

The examples of the simple reactions (Review of Particle Properties, 1994) are:

1) reaction of electron-positron pair production $\gamma + N = e^- + e^+ + N$;

2) muon decay $\mu^\pm = e^\pm + \nu + \bar{\nu}(99\%)$, $\mu^\pm = e^\pm + \nu + \bar{\nu} + \gamma(1\%)$ (15.3.6 %) and taon decay $\tau^\pm = \mu^\pm + \nu_\tau + \bar{\nu}_\mu$ (17.37 %), $\tau^\pm = \mu^\pm + \nu_\tau + \bar{\nu}_\mu + \gamma$. Here: $m_\mu = 105.6 MeV$, $m_\tau = 1777 MeV$, $m_e = 0.51 MeV$, $m_{\nu_e} < 3 ev$, $m_{\nu_\mu} < 0.19 MeV$, $m_\nu \approx 1 eV$

We can consider these reactions as superposition of the non-linear photons and semi-photons. For instance, muon or taon are possibly to represent as superposition of the electronic linear polarized half-wave and two neutrino as circularly polarized half-waves with the opposite direction of rotation. (Here in Putilov's approach we will consider neutrino and antineutrino as one photon). Similarly it can be considered other reactions without the infringement of corresponding conservation laws, for example, the pions decay: $\pi^\pm = \mu^\pm + \nu_\mu (99.98\%)$, $\pi^0 = 2\gamma (98.79\%)$, $\pi^0 = e^+ + e^- + \gamma$ (15.1.19 %) ($m_{\pi^0} = 134.97 MeV$, $m_{\pi^\pm} = 139.57 MeV$).

Let us calculate accordingly this method the mass of heavy leptons as the excited states of free electron. The free Dirac electron equation

$$\left(\hat{\alpha}_0 \hat{\varepsilon} + c\hat{\vec{\alpha}}\hat{\vec{p}} + \hat{\beta} mc^2\right)\psi = 0, \qquad (15.6.1)$$

is satisfied by any mass, not only the electron mass. Therefore, the Dirac electron equation with an external field

$$\left[\hat{\alpha}_0 \left(\hat{\varepsilon} - \varepsilon_{ph}\right) + c\hat{\vec{\alpha}}\left(\hat{\vec{p}} - \vec{p}_{ph}\right) + \hat{\beta} mc^2\right]\psi = 0, \qquad (15.6.2)$$

after grouping the mass-energy part in brackets, we can rewrite as following:

$$\left\{\left(\hat{\alpha}_o\hat{\varepsilon}+c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)-\left[\left(\hat{\alpha}_o\varepsilon_{ph}+c\hat{\vec{\alpha}}\ \vec{p}_{ph}\right)+\hat{\beta}\ mc^2\right]\right\}\psi=0, \quad (15.6.3)$$

Here the expression in the brackets corresponds to the excited electron energy-masses:

$$M(n)=\left[\left(\hat{\alpha}_o\varepsilon_{ph}+c\hat{\vec{\alpha}}\ \vec{p}_{ph}\right)-\hat{\beta}\ mc^2\right]=\hat{\beta}(m_e+m_{ad})\ c^2, \quad (15.6.4)$$

where $m_{ad}=m_{ad}(n)$ is some additional masses, which are in the general case the discrete number of values depending on $n=1,2,3,...$. In other words we have here the transition of electron from the energy level of free electron $m_e c^2$ to the level of excited electron $(m_e+m_{ad})\cdot c^2$.

As we noted in the introduction, a resonance cavity can support standing waves in three different directions, and, in addition, these waves can be of different types, independent from each other. This means that the number of additional masses may be at least equal to three different values, but in general is far more:

$$M(n)=\beta(m_e+\sum_{i=1}^{3}(m_{ad})_i)c^2, \quad (15.6.4')$$

Substituting (15.6.4) in (15.6.2) and taking into account $\hat{\varepsilon}=i\hbar\partial/\partial\ t$, $\hat{\vec{p}}=-i\hbar\vec{\nabla}$, we will obtain:

$$\left[\left(\hat{\alpha}_o\frac{\partial}{\partial\ t}-c\hat{\vec{\alpha}}\ \vec{\nabla}\right)+i\hat{\beta}\ c\frac{(m_e+\sum_{i=1}^{3}(m_{ad})_i)c}{\hbar}\right]\psi=0, \quad (15.6.5)$$

6.1.1. *The examples of approximate calculation of particle masses*

Since we do not know the exact size and shape of particles of the lower level of mass, such as the electron, we can not construct a rigorous theory for calculating the mass spectrum of more massive particles. Therefore, we restrict ourselves to calculations using a spherical approximation of the electron.

In order to consider the resonance conditions we will try now to transform a mass term of Dirac's equation so that it included the wavelengths of particles.
It is possible to present the mass term in (15.6.5) (without coefficient $i\hat{\beta}\ c$) as follows:

$$\frac{(m_e+m_{ad})c}{\hbar}=\frac{m_e c}{\hbar}+\frac{m_{ad}c}{\hbar}=\frac{1}{\lambdabar_e}+\frac{1}{\lambdabar_{ad}}, \quad (15.6.6)$$

where $\lambda\!\!\!^{-}_e$, $\lambda\!\!\!^{-}_{ad}$ are the Compton waves' lengths (bar) of the electron and of the additional mass, respectively (by definition $\lambda\!\!\!^{-}_C = \hbar/mc = \lambda_C/2\pi mc$). Since the basic wave and additional waves should be commensurable, so the basic wave contains an integer number of the additional waves.

Thus they should satisfy the following condition of wave quantization:

$$\lambda_{ad} = \kappa \frac{\lambda_e}{n} \text{ or } \lambda\!\!\!^{-}_{ad} = \kappa \frac{\lambda\!\!\!^{-}_e}{n}, \qquad (15.6.7)$$

where κ is the number, describing a resonance condition at different motion of waves (longitudinal, cross-sectional resonance, etc.); $n = 1,2,3,...$ is an integer (quantum number).
In case of propagation of a wave along the circle (as in the above problem (15.1.2)) we have $\kappa = 2\pi$. In case of wave propagation along radius of the sphere: $\kappa = 4$; along the ring tube radius $\kappa = 2$, etc.

Thus, for mass term in Dirac equation (i.e. for mass of a composite elementary particle) we obtain:

$$\frac{c}{\hbar} m_{ep} = \frac{(m_e + m_{ad})c}{\hbar} = \frac{1}{\lambda\!\!\!^{-}_e} + \frac{1}{\lambda\!\!\!^{-}_{ad}} = \frac{1}{\lambda\!\!\!^{-}_e}\left(1 + \frac{\lambda\!\!\!^{-}_e}{\lambda\!\!\!^{-}_{ad}}\right), \qquad (15.6.8)$$

Since the value $\alpha = e^2/\hbar c = r_0/\lambda\!\!\!^{-}_e \approx 1/137$ represents an electromagnetic constant, we have $\lambda\!\!\!^{-}_e = r_0/\alpha$ (where $r_0 = e^2/m_e c^2$ is the classical electron radius). Then from (15.6.8) we will obtain:

$$\frac{c}{\hbar} m_{ep} = \frac{1}{\lambda\!\!\!^{-}_e}\left(1 + \frac{\lambda\!\!\!^{-}_e}{\lambda\!\!\!^{-}_{ad}}\right) = \frac{\alpha}{r_0}\left(1 + \frac{r_0}{\alpha \lambda\!\!\!^{-}_{ad}}\right), \qquad (15.6.9)$$

As we have shown, the "bare" size of electron corresponds to Compton wave length, which at polarization in physical vacuum decreases in $1/\alpha \approx 137$ times. Thus, taking into account the polarization of vacuum, instead of (15.6.7), we should write down:

$$\lambda_{ad} = \kappa \frac{r_0}{n} \text{ или or } \lambda\!\!\!^{-}_{ad} = \kappa \frac{r_0}{2\pi n}, \qquad (15.6.10)$$

From here $\dfrac{r_0}{\lambda\!\!\!^{-}_{ad}} = \dfrac{2\pi}{\kappa} n$, which by substitution in the formula (15.6.9), gives the final formula for mass of a composite particle, which are close to above results of A.O. Barut, K.A Putilov., W.A Rodrigues.- J. Vaz:

$$m_{ep} = \left(1 + \frac{2\pi}{\kappa\alpha} n\right) m_e \approx \left(1 + \frac{2\pi}{\kappa} 137 \cdot n\right) m_e, \qquad (15.6.11)$$

1) for $n = 0$ we obtain a trivial case of electron mass: $m_1 = m_e$;
2) for $\kappa = 4,\ n = 1$ we obtain $m_2 = 110.2$ MeV (that corresponds to $m_\mu = 105.6$ MeV);
3) for $\kappa = 4,\ n = 16$ we obtain $m_3 = 1755$ MeV (that corresponds to $m_\tau = 1777$ MeV);
4) for $\kappa = \pi,\ n = 1$ we obtain $m_4 = 140.25$ MeV (that corresponds to $m_{\pi^\pm} = 139.57$ MeV).

Despite the satisfactory coincidences we must not exclude that they are accidental. Nevertheless, these results confirm that the resonance theory of mass spectra has the good reasons.
It is also confirmed with calculations of particle masses according to the inverse problem.

6.2. Inverse problem.

We will consider here a particle X_0 of the reaction (15.4.1) as the unknown resonator, and particles $X_1,\ X_2$ as the initial waves, which create this resonator.

In each resonator there are at least three sizes L_j ($j = 1,2,3$), which define lengths of resonance waves. Thus we can write down at least three resonance conditions:

$$L_j / \lambda_i = \kappa n_{ij}, \qquad (15.6.12),$$

where i is number of a particle, participating in synthesis (in our example $i = 1,\ 2$), $n_{ij} = 1,2,3,...$ is an integer, κ_j is the dimensionless coefficient, defining resonance conditions. Since according to De Broglie $\lambda_i = h/p_i$, the formula (15.6.12) can be rewritten in the form:

$$p_{ij} \cdot L_j = \kappa_j h n_{ij}, \qquad (15.6.13)$$

From here at $n_{ij} = 1$ we obtain the condition for lowest states of a particle X_0:

$$p_{ij0} = \kappa_j \cdot h/L_j = const, \qquad (15.6.14)$$

Then for any other "excited" state of a particle X_0 we have:

$$p_{ij} = \kappa_j \frac{h}{L_j} n_{ij} = p_{ij0} \cdot n_{ij}, \qquad (15.6.15)$$

In case of two fussion particles ($i = 2$) we have for lowest momentums and quantum numbers the values, which depend only on the index j; namely: $p_{1j} = p_{2j} = p_{j0}$ and n_j. Thus, we can calculate the masses for different j.

In works of group of Ratis, Yu.L., Garejev, F.A. et al. (Ratis and Garejev, 1992;; Garejev, Kazacha et al, 1998;, etc.) values p_{j0} for the big group of hadron resonances were selected, which give encouraging confirmations to our approach.

In case if number of particles X_i is more than 2 (i.e. $i > 2$) for the calculation of spectra it is necessary to have additional correlations between p_{ij} with different i. Probably, the correlations, obtained in section 4, can be useful in this case.

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