On Capturing Rent from a Non-Renewable Resource International Monopoly: Prices versus Quantities*

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Abstract

In this paper we model the case of an international non-renewable resource monopolist as a dynamic game between a monopolist and n importing countries governments, and investigate whether a tariff on resource imports can be advantageous for consumers in importing countries. We analyze both the case of a price-setting monopolist and the case of a quantity-setting monopolist. We find that a tariff is advantageous for consumers, even when there is no commitment to the trade policy and importing countries do not coordinate their policies. Using a numerical example, we find that a tariff is more advantageous for the importing countries if the monopolist chooses the quantity instead of the price and that the optimal temporal path when the monopolist chooses the price is consistently below the optimal temporal path when the monopolist chooses the quantity for the entire period of exploitation of the resource. Nevertheless, the variation in total welfare between the two regimens is small.

Keywords: tariffs, non-renewable resources, depletion effects, price-setting monopolist, quantity-setting monopolist, differential games, linear strategies, Markov perfect Nash equilibrium.

JEL Classification System: C73, D42, F13, Q38

1 Introduction

An issue that is very important to Western countries is how to react optimally to the existence of a non-renewable natural resource cartel or monopoly. The OPEC vs. the West is still a good example. One option that the Western countries have is to strategically use a tariff on resource imports to affect the extraction behaviour of the monopolist and the international price of the resource, allowing them to capture part of the seller's rent.[1] Our paper analyses this issue, solving a differential game between a resource monopolist and the governments of large importing countries.

We assume that consumption takes place only in the importing countries which are not endowed with the resource and that the utility function is linear-quadratic.[2] The monopolist extracts the resource at an average cost that is constant with respect to the extraction rate but increasing with respect to the accumulated extractions (depletion effects) and sells it in an integrated international market.

In this framework, we address two cases. In the first case, the monopolist sets the price and the market sets the extraction rate, while the governments of the importing countries fix a tariff on resource imports. In the second case, we assume that the monopolist sets the quantity. For the first case, we find that for the Markov perfect Nash equilibrium in linear strategies the optimal tariffs are not zero, i.e. in other words, to fix a tariff on resource imports is advantageous for consumers in the importing countries, something that does not occur in a static setting.[3] The optimality of the tariff is ex-

[1] The idea that rent can be extracted from a foreign monopolist was presented for the first time by Katrak (1977) and Svedberg (1979).

[2] Brander and Djajic (1983) and, more recently, Njopmouo (2010) have explored other assumptions. Brander and Djajic (1983) analyse a bilateral monopoly, where the resource is an essential input in the production of a homogeneous consumption good in the two countries although only one is endowed with the resource. They find that rent extraction is limited but not eliminated by the exporter's ability to use the resource domestically. Njopmouo (2010) shows that, for the case of a monopsonist facing a competitive industry, the presence of the resource in the importing country essentially reinforces the ability of the importer to capture the foreign rent.

[3] As it is well known in a static setting if the monopolist chooses the price, the Nash equilibrium tariff of the game is zero and there is no place to use strategically a tariff. The strategic use of the

plained by the existence of an indirect strategic relationship that is not present in the static setting. For the case of a non-renewable resource, we can distinguish two types of strategic relationship between an importing country government and the monopolist, a direct or intratemporal relationship and an indirect or intertemporal relationship. The direct strategic interdependence, characterising the static model, appears because the consumer's welfare depends on the monopolist price, and the monopolist's profits depend on the tariff. The indirect strategic relationship appears because through the tariff, the governments of importing countries can influence the dynamics of the accumulated extractions and hence the costs of extractions and the evolution of the monopolist price. For this reason, in a dynamic context, like that analysed in the paper, the governments of importing countries can strategically use a tariff to capture part of the monopolist's rent. Our results show that, with depletion effects, the tariff decreases throughout the exploitation period of the resource and converges to zero in the long run. On the other hand, the international and consumer prices increase and converge to the *choke price*.

For the case of a quantity-setting monopolist we also find that a tariff is profitable for the consumers in the importing countries, which extends the result obtained in a static setting to a dynamic setting. When the monopolist chooses the quantity, the indirect or intertemporal strategic relationship between the importers and the monopolist disappears, but the direct or intratemporal relationship is still strong enough to make the use of a tariff profitable. To complete the analysis, the two cases are compared using a numerical example. According to the results of our numerical example, a tariff is more advantageous for the importing countries if the monopolist chooses the extraction rate instead of the price. This is consistent with what we know from the static model, although in our dynamic model, the use of the price does not completely eliminate the effectiveness of the tariff to capture a part of the monopolist's rent as occurs in the static model, instead only reducing it. The example also allows us to compare the tariff for the

tariff is not optimal because the only effect of a tariff would be to rise the price and this would further distort consumption in addition to monopolistic pricing. See Alepuz and Rubio (2009) for a study of the strategical use of a tariff against a monopolist under an integrated international market in a static framework.

two cases. Their comparison shows that the tariff is higher throughout the exploitation period of the resource when the monopolist chooses the quantity. Thus, we find that prices and quantities behave differently. The rationale behind the result is that when choices are made simultaneously, the accumulated extractions evolves differently. In the price setting game, the tariff rate appears in the equation of motion, while in the quantity setting model, only the monopolist's choice, the quantity, appears in the equation of motion. This difference in the dynamics of the stock changes the nature of the game and yields different tariffs, depending on whether the monopolist chooses the quantity or the price.

Finally, if we look at the other side of the market, it seems pretty clear that the optimal policy for the monopolist to face large importers that strategically use a tariff is to set the price. However, if we look at the total welfare, we find that the control variable used by the monopolist has no relevant effect on the total welfare obtained from the exploitation of the resource. For all the numerical simulations analysed in the paper, the reduction in total welfare is less than 1% when the monopolist chooses the quantity instead of the price.

The first comment we would like to make regarding the previous literature is that our results for the case of a quantity-setting monopolist are concordant with those obtained by Katrak (1977) and Svedberg (1979). Katrak and Svedberg show in a static setting and for the case of a linear demand that the optimal trade policy from a national point of view when there is only one seller in an international market is to set up a tariff. The argument is that a tariff will cause the consumer price to rise but by less than the full amount of the tariff so that on net, the loss in consumer surplus is more than compensated for the gain in tariff revenue.[4] We also use a linear demand and obtain that the optimal trade policy is to set a tariff. However, as our model is dynamic, the tariff must evolve over time according to the dynamics of the rent. The result is that the tariff decreases, since the rent tends to zero in the long run because of depletion effects. This explains why, in our model, the declining tariff rate does not become a subsidy in the long run. In

[4] Brander and Spencer (1984) prove that Katrak and Svedberg's result is true in general except when the demand is highly convex. In this case an import subsidy might be optimal.

other words, for each period we obtain Katrak and Svedberg's result but in each period the rent is lower and consequently the optimal tariff must be lower as well. The same argument can be applied for the case of a price-setting monopolist.

The issue of using an import tariff to reap part of the rent of a non-renewable resource has been also addressed by Kemp and Long (1980), Bergstrom (1982), Brander and Djajic (1983), Karp (1984, 1991), Maskin and Newbery (1990) and Karp and Newbery (1991, 1992). Among them only Bergstrom (1982), Brander and Djajic (1983) and Karp (1984) have focussed on the case of a non-renewable resource monopolist.[5] Bergstrom (1982) briefly discusses, in the framework of a partial equilibrium model where consumption takes place only in the importing countries and extraction costs are zero, the case in which all importing countries choose the same constant ad valorem tariff against a price-setting monopolist. He argues that almost all of the monopolist rent can be taxed away by the importing countries if they choose a sufficiently high tariff rate. His argument is based on the fact that with an *ad valorem* tariff if the extraction costs are zero, the profit-maximizing level of the consumer price is independent of the tariff. On the other hand, Brander and Djajic (1983) develop their analysis in the context of a simple two-country general equilibrium model of trade in exhaustible resources, where it is assumed that the resource is extracted costlessly and used by the two countries as an essential input in the production of a homogeneous consumption good. They find that the country without the resource has an incentive to impose a tariff so as to extract at least some of the available rent. The magnitude of the optimal tariff is found to be an increasing function of the relative size of the importing country, and approaches the confiscatory level as the resource importing country becomes very large. In Karp (1984), the interaction between a monopolist and a single buyer is modelled as a Stackelberg game where the extraction cost is inversely related to the stock and the buyer is the leader of the game.[6] The buyer

[5] In the rest of papers the extractive industry is competitive.

[6] Other papers, as those written by Robson (1983) and Lewis et al. (1986) have also analysed the interaction between two players that exploit a non-renewable resource. Robson (1983), in a two-periods model, supposes that both countries possess a stock of the resource but that consumption takes place only in one country, and focuses on the characterisation of the extraction pattern in each country. Lewis et al. (1986) analyse the interaction between a resource monopolist and a coalition of consumers that

chooses a tariff and the monopolist the rate of extraction. He shows that the open-loop tariff is temporally inconsistent because of the stock-dependence of extraction costs. Besides this, he also proposes a method of obtaining temporally consistent strategies and concludes that the consistent tariff against the monopolist is, in general, not identically zero. This implies that a consistent tariff allows the buyer to improve his position. Thus our paper can be seen as an extension of Bergstrom (1982) and Karp's (1984) papers. As regards Bergstrom's paper, we consider that extraction costs rise with cumulative extractions and that importing countries apply a *per unit* tariff. We also investigate more deeply the game theoretic aspects of the problem. In this framework, we find that the tariff must be decreasing since the rent converges to zero in the long run because of the economic exhaustion of the resource. Nevertheless, our analysis supports his conclusion that a tariff can be used to capture a part of the monopolist's rent - but not almost all the rent as he suggested - at least when the extraction costs are positive. Our paper also extends the results obtained by Karp (1984) showing that it is not necessary to have any kind of cooperation or coordination among the governments of importing countries to strategically use a tariff when the monopolist sets the quantity. Our findings establish that a tariff is advantageous for the importing countries even when the importing countries do not enjoy any strategic advantage. Thus, the main contribution of our paper is to show that when a set of large importers face a monopolist of a non-renewable resource, they can use a tariff to capture part of the monopolist's rent independently of the monopolist choosing the price or the quantity. We also show, that to obtain this result it is not necessary for the importing countries to have either a first mover advantage or the cooperation between each other, and that the interactions of the game are different in each case, although the difference in total welfare between the two cases is small.

The next section addresses the case of a price-setting monopolist, whereas the case of a quantity-setting monopolist is studied in Section 3. In Section 4, the two equilibria are compared. Section 5 offers concluding remarks.

act collectively to introduce a durable long-lived substitute. Two interesting extensions of this paper are Olsen (1993) and Harris and Vickers (1995).

2 The Case of a Price-Setting Monopolist

As in Bergstrom's (1982) analysis, we shall confine ourselves to a partial equilibrium model. Assuming that the representative consumer of an importing country acts as a price-taker agent, we can write the consumer's welfare function as $(1/b)(aq_i(t) - (1/2)q_i(t)^2) - (p(t)+\theta_i(t))q_i(t) + R_i(t)$, where $(1/b)(aq_i(t) - (1/2)q_i(t)^2)$ is the consumer's gross surplus, $q_i(t)$ the amount of the resource bought by the representative consumer of the importing country i, $p(t)$ the international price of the resource, $\theta_i(t)$ the *per unit tariff* on the resource imports fixed by the government of the importing country i, and $R_i(t)$ a lump-sum transfer that the consumer receives from the government.[7] Thus, the resource demand depends only on the consumer price: $q_i(t) = a - b(p(t) + \theta_i(t))$ so that the aggregate demand function can be written as $Q = \sum_{i=1}^{n}(a - b(p(t) + \theta_i(t)))$ where n is the number of importing countries. We also assume, for the purpose of simplifying the analysis, that the importing countries are not endowed with the resource.

The governments set the tariff with the aim of maximising the discounted present value of the welfare of the representative consumer. They reimburse tariff revenues as *lump-sum transfers*, so that finally the consumer's welfare does not depend on tariff revenue. The optimal time path for the tariff is thus given by the solution of the following optimal control problem:

$$\max_{\{\theta_i(t)\}} \int_0^{\infty} e^{-rt}\left(\left(\frac{a}{b} - p(t)\right)(a - b(p(t) + \theta_i(t))) - \frac{1}{2b}(a - b(p(t) + \theta_i(t)))^2\right) dt,$$

where r is the discount rate and $i = 1, ..., n$.

[7] We assume as in Bergstrom's (1982) analysis that the international market is integrated so all importing countries pay the same price for the resource.

Developing the expression between parenthesis, it is obtained[8]

$$\max_{\{\theta_i(t)\}} \int_0^{\infty} e^{-rt}\frac{b}{2}\left(\left(\frac{a}{b}-p(t)\right)^2-\theta_i(t)^2\right)dt, \quad i=1,...,n. \tag{1}$$

On the other side of the market, we have a monopolist extracting the resource at a cost equal to $c(x)Q$, where $c(x)=cx$ is the marginal extraction cost, x stands for the accumulated extractions and Q for the current extraction rate of the resource. Observe that, given the price and the tariffs, the extraction rate is determined by the aggregate demand function and consequently is equal to the resource bought by the representative consumers in the importing countries. The objective of the monopolist is to define a price strategy that maximises the present value of profits:

$$\max_{\{p(t)\}} \int_0^{\infty} e^{-rt}\left((p(t)-cx(t))\sum_{i=1}^{n}(a-b(p(t)+\theta_i(t)))\right)dt. \tag{2}$$

Finally, the dynamics of the accumulated extractions is given by the following differential equation:

$$\dot{x}=Q=\sum_{i=1}^{n}\left(a-b(p(t)+\theta_i(t))\right), \quad x(0)=x_0\geq 0. \tag{3}$$

Both types of players face the same dynamic constraint. The optimal time paths for the tariffs and the price are given by the solution of the *differential game* between the monopolist and the governments of the importing countries defined by (1)-(3). In the paper, it is assumed that the physical exhaustion of the resource is not profitable, i.e. that the marginal cost of extracting the last unit of the resource is greater than the choke price, which implies that the total accumulated extractions are lower than the initial stock of the resource.[9]

[8] Notice that in a static simultaneous game with a payoff function equal to $(b/2)((a/b-p)^2-\theta_i^2)$, the Nash equilibrium is zero for θ when the monopolist chooses the price. Next, we show that the optimal trade policy for the importing countries in the dynamic framework we are presenting is to set up a positive tariff.

[9] This assumption allows us to calculate the Markov perfect Nash equilibrium of the game in *stationary* strategies. If the physical exhaustion of the resource was profitable, the Markov perfect Nash equilibrium

Next we solve the differential game through the computation of a Markov perfect Nash equilibrium.[10] We use Markov strategies because they provide a subgame perfect equilibrium that is dynamically consistent.

Markov strategies must satisfy the following system of Hamilton-Jacobi-Bellman (HJB) equations

$$rW_i = \max_{\{\theta_i\}} \left\{ \frac{b}{2}\left(\left(\frac{a}{b} - p(t)\right)^2 - \theta_i(t)^2\right) + W_i' \sum_{j=1}^{n}(a - b(p+\theta_j)) \right\}, \ i = 1, ..., n, \quad (4)$$

$$rW_M = \max_{\{p\}} \left\{ (p - cx)\sum_{j=1}^{n}(a - b(p+\theta_j)) + W_M' \sum_{j=1}^{n}(a - b(p+\theta_j)) \right\}, \quad (5)$$

where $W_M(x)$ stands for the optimal current value function associated with the dynamic optimisation problem for the monopolist (2) and $W_i(x)$ for the optimal current value function associated with the dynamic optimisation problem for the importing country i (1); i.e., they denote the maxima of the objectives (1) and (2) subject to (3) for the current value of the state variable.[11]

If we focus on a symmetric equilibrium, we get the *instantaneous* reaction functions of the importing countries and the monopolist from the first-order conditions for the maximisation of the right-hand sides of the HJB equations:

$$\theta = -W_I', \quad (6)$$

$$p = \frac{1}{2}\left(\frac{a}{b} + cx - W_M' - \theta\right), \quad (7)$$

should be calculated using *non-stationary* strategies since in this case, the exhaustion of the resource will take a finite time. Nevertheless, the only qualitative change we expect, if the physical exhaustion is assumed, is that instead of having a tariff that converges to zero in the long-run, the tariff would converge to a positive value since when the physical exhaustion occurs, the rent of the resource does not vanish in the long-run. Given the extension of the present paper, we leave this issue for a future research.

[10] We have also computed the open-loop Nash equilibrium, see details in Rubio (2005), and the conclusion is that the tariff cannot be used strategically. The result is explained by the fact that in an open-loop Nash equilibrium, the players choose, in the initial moment, their temporal paths for all the game. Then, given a temporal path for the price, the optimal tariff is zero for the complete period of exploitation of the resource. In other words, the same argument that applies to explain why the optimal tariff is zero in a static game applies to explain why it is also zero when the equilibrium concept used to solve the differential game is the open-loop Nash equilibrium.

[11] Time arguments and the argument of the value function will be eliminated when no confusion arises.

where I stands for the representative importer.

These expressions establish that the optimal tariff is independent of the monopolist price and equal to the user cost of the resource for the importing countries, and that for a given value of the state variable the price is a *strategic substitute* of the tariffs.

By substitution of (6) in (7), we get the solution of the price as a function of the first derivatives of the value functions

$$p = \frac{1}{2}\left(\frac{a}{b} + cx - W_M' + W_I'\right). \tag{8}$$

Next we obtain the following system of two nonlinear differential equations after incorporating optimal strategies (6) and (8) into HJB Eqs. (4) and (5)

$$rW_I = \frac{b}{4}\left(\frac{1}{2}\left(\frac{a}{b} - cx + W_M'\right)^2 + (2n-1)\left(\frac{a}{b} - cx + W_M'\right)W_I' + \frac{4n-3}{2}\left(W_I'\right)^2\right), \tag{9}$$

$$rW_M = \frac{bn}{4}\left(\frac{a}{b} - cx + W_M' + W_I'\right)^2. \tag{10}$$

In order to derive the solution, we guess quadratic representations for the value functions W_I and W_M,

$$W_I(x) = \frac{1}{2}\alpha_I x^2 + \beta_I x + \mu_I,\ W_M(x) = \frac{1}{2}\alpha_M x^2 + \beta_M x + \mu_M, \tag{11}$$

which implies that $W_I'(x) = \alpha_I x + \beta_I$ and $W_M'(x) = \alpha_M x + \beta_M$.

The substitution of $W_I(x), W_M(x), W_I'(x)$ and $W_M'(x)$ into (9) and (10) yields a system of conditions over the parameters of the value functions that must hold for every x. Then if the system of equations for the coefficients of the value functions has a solution, the linear Markov-perfect Nash equilibrium strategies for the tariff and the price would be

$$\theta = -\alpha_I x - \beta_I, \tag{12}$$

$$p = \frac{1}{2}\left(\frac{a}{b} + \beta_I - \beta_M + (c + \alpha_I - \alpha_M)x\right), \tag{13}$$

which are obtained from (6) and (8). The consumer price could be calculated by adding the international price of the resource and the tariff

$$\pi = p + \theta = \frac{1}{2}\left(\frac{a}{b} - \beta_I - \beta_M + (c - \alpha_I - \alpha_M)x\right). \tag{14}$$

Finally, we would obtain the dynamics of the state variable in terms of the coefficients of the value functions using the consumer price in (3)

$$\dot{x} = \frac{bn}{2}\left(\frac{a}{b} + \beta_I + \beta_M - (c - \alpha_I - \alpha_M)x\right). \tag{15}$$

Thus, if we look for a stable solution, the following condition should be satisfied by the coefficients of the value functions:

$$\frac{d\dot{x}}{dx} < 0 \;\rightarrow\; \frac{d\dot{x}}{dx} = -\frac{bn}{2}(c - \alpha_I - \alpha_M) < 0 \rightarrow c - \alpha_I - \alpha_M > 0. \tag{16}$$

Applying this stability condition, we find that the system of equations defined by the HJB equations on the parameters of the value functions has only one stable solution, and we can conclude that

Proposition 1 *A tariff is advantageous for the consumers of the importing countries, i.e.* $\theta = -\alpha_I x - \beta_I$ *is positive in interval* $[0, a/bc)$ *where* a/bc *is the steady-state value of the accumulated extractions.*

Proof. See Appendix A. ■

The analysis establishes that, for the case of a non-renewable resource, two types of strategic relationship exist, one direct and the other indirect. The direct strategic relationship appears because the consumer's welfare depends on the monopolist price, see Eq. (1), and the monopolist's profits depend on the tariff, see Eq. (2). The indirect strategic relationship appears because through the tariff, the governments of the importing countries can influence the dynamics of the accumulated extractions and hence the extraction costs and optimal policy of the monopolist. In the model, this indirect strategic relationship operates through differential equation (3). Thus given the price, an increase in the tariff reduces the extraction rate, which determines the accumulation rate of the stock, finally yielding a reduction of the marginal extraction cost, which has a positive influence in the evolution of the price. For this reason, it is advantageous for the importing countries to set up a tariff on the resource imports, since the governments can influence the dynamics of the price through their influence on the dynamics of the stock using the tariff. Thus the profitability of the tariff for the importing countries is explained by the existence of an indirect strategic relationship that operates through the dynamic constraint.

In Appendix A, it is shown that the only stable solution for the system defined by the HJB equations satisfies that $\alpha_I, \alpha_M \in (0, c)$ and that $\beta_I = -a\alpha_I/bc$ and $\beta_M = -a\alpha_M/bc$ so that the equilibrium strategies for the tariff and the price given by (12) and (13) can be rewritten as

$$\theta = -\alpha_I x + a\alpha_I/bc = \alpha_I \left(\frac{a}{bc} - x\right), \tag{17}$$

$$p = \frac{1}{2}\left(\frac{a}{bc}(c - \alpha_I + \alpha_M) + (c + \alpha_I - \alpha_M)x\right) \tag{18}$$

which establish that the optimal tariff is decreasing with respect to the accumulated extractions and is zero for $x^\infty = a/bc$ whereas the monopolist price is increasing with respect to the accumulated extractions and equal to a/b for $x^\infty = a/bc$.[12]

Moreover, the consumer price equilibrium strategy can be written as

$$\pi = \frac{1}{2}\left(\frac{a}{bc}(c + \alpha_I + \alpha_M) + (c - \alpha_I - \alpha_M)x\right), \tag{19}$$

and the equilibrium strategy for the rate of extraction as

$$\dot{x} = Q = \frac{bn}{2}(c - \alpha_I - \alpha_M)\left(\frac{a}{bc} - x\right), \tag{20}$$

where $c - \alpha_I - \alpha_M$ is positive by the stability condition. Thus the consumer price is increasing with respect to the accumulated extractions and equal to a/b for $x^\infty = a/bc$ whereas the rate of extraction is decreasing with respect to the accumulated extractions and is zero for $x^\infty = a/bc$. In other words, we are assuming that the physical exhaustion of the resource is not profitable, i.e. that the marginal cost of extracting the last unit of the resource is greater than the choke price, which implies that the total accumulated extractions, a/bc, are lower than the initial stock of the resource. Moreover, as the marginal cost is linear with respect to the extraction rate, transversality conditions are satisfied for a rate of extraction equal to zero and the exploitation of the resource takes infinite time.

[12] Observe that as $\alpha_I, \alpha_M \in (0, c)$, expressions $c - \alpha_I + \alpha_M$ and $c + \alpha_I - \alpha_M$ cannot be negative or zero.

Next, we calculate the optimal path of the model variables. To obtain them, we need to solve differential equation (20). The solution to this equation for $x_0 = 0$ is[13]

$$x = \frac{a}{bc}\left(1 - \exp\left\{-\frac{bn}{2}\left(c - \alpha_I - \alpha_M\right)t\right\}\right). \tag{21}$$

Then substituting x in (17) and (18) we get

$$\theta = \alpha_I \frac{a}{bc}\exp\left\{-\frac{bn}{2}\left(c - \alpha_I - \alpha_M\right)t\right\}, \tag{22}$$

$$p = \frac{a}{b}\left(1 - \frac{c + \alpha_I - \alpha_M}{2c}\exp\left\{-\frac{bn}{2}\left(c - \alpha_I - \alpha_M\right)t\right\}\right). \tag{23}$$

Finally, the consumer price can be simply calculated by the addition of the monopolist price and the tariff

$$\pi = p + \theta = \frac{a}{b}\left(1 - \frac{c - \alpha_I - \alpha_M}{2c}\exp\left\{-\frac{bn}{2}\left(c - \alpha_I - \alpha_M\right)t\right\}\right) \tag{24}$$

so we can summarise these results as

Remark 1 *The Markov perfect Nash equilibrium tariff decreases throughout the exploitation period of the resource and converges to zero in the long run. Moreover, the monopolist and consumer prices are increasing and converge to the choke price.*

Now using $\beta_I = -a\alpha_I/bc$ and $\beta_M = -a\alpha_M/bc$, the independent terms of the value functions can be obtained so that finally the value functions can be written as

$$W_I(x) = \alpha_I\left(\frac{1}{2}x^2 - \frac{a}{bc}x + \frac{a^2}{2b^2c^2}\right), \quad W_M(x) = \alpha_M\left(\frac{1}{2}x^2 - \frac{a}{bc}x + \frac{a^2}{2b^2c^2}\right). \tag{25}$$

Thus, the value functions are decreasing with respect to the stock reaching a minimum equal to zero for $x^{\infty} = a/bc$.

The previous analysis allows us to conclude that the use of a tariff on non-renewable resource imports is profitable for consumers in the importing countries. However, it is not possible to obtain a closed form solution for the model, except if the importing countries

[13] In order to simplify the presentation we assume that $x_0 = 0$ which does not change the sign of the dynamics of the model variables. In any case, in this kind of non-renewable resources models it must be assumed that $x_0 < a/bc$ since $\dot{x} = Q \geq 0$.

coordinate their policies through a tariff agreement. If the importing countries sign an agreement to impose the same tariff on the resource imports with the aim of maximising the discounted present value of the sum of the representative consumer's welfare of each importing country, the interaction between $n+1$ players reduces to a differential game between two players: the coalition of importing countries and the monopolist, which has an explicit solution in terms of the parameters of the model. The details of the solution for this model can be found in Rubio (2005).

Thus, the analysis developed in this section shows that although an agreement will surely increase the part of the rent that can be captured through a tariff, the use of a tariff can be profitable for the consumers of the importing countries even if no agreement is signed.

Finally, it could be pointed out that the *bilateral monopoly* trade policies game is a particular case of the differential game we have just analysed. The solution for the bilateral monopoly is given by the expressions that appear above for $n = 1$.

3 The Case of a Quantity-Setting Monopolist

Until now, we have assumed that the monopolist chooses the price and the resource extraction rate is fixed by the market. In this section, we analyse the other possibility the monopoly has: to choose the extraction rate and leave the market to set the price. When it is the market that sets the price, this has to be substituted in the profits function by the inverse demand function, $p = a/b - (Q + b\sum_{i=1}^{n}\theta_i)/bn$, so that the optimal control problem for the monopolist must be rewritten as follows

$$\max_{\{Q\}} \int_0^\infty e^{-rt}\left(\frac{a}{b} - \frac{1}{bn}\left(Q + b\sum_{i=1}^{n}\theta_i\right) - cx\right)Qdt \tag{26}$$

$$s.t.\ \dot{x} = Q, \quad x(0) = 0. \tag{27}$$

Looking at the problem, we see that the governments of the importing countries do not have a direct influence on the dynamics of the stock, since the extraction rate is determined by the monopolist. This means that we have a differential game where the indirect strategic relationship disappears, and only an intratemporal or direct strategic relationship exists.

Now, the reaction function of the importing countries can be obtained from a static game played by the importing countries in each period taking the extraction rate as given. For the static game, the importing countries maximise the instantaneous consumer's welfare that, using the inverse demand function, can be written as follows

$$\max_{\{\theta_i\}} U_i = \frac{1}{2bn^2}\left(Q + b\sum_{j=1}^{n}\theta_j\right)^2 - \frac{b}{2}\theta_i^2, \quad i = 1, ...n. \tag{28}$$

Thus, when the monopolist chooses the quantity, the optimal tariff is obtained from the solution of a static game combined with the solution of a dynamic game, which simplifies the calculation of the Markov perfect Nash equilibrium, and shows that the nature of the game changes when the monopolist sets up the quantity instead of the price. Next, we explain how the equilibrium can be calculated.[14]

The procedure to calculate the equilibrium is the following: first for a given extraction rate we obtain the instantaneous reaction function of the importing countries which are implicitly defined by

$$\frac{1}{n^2}\left(Q + b\sum_{j=1}^{n}\theta_j\right) = b\theta_i, \quad i = 1, ..., n. \tag{29}$$

It is straightforward from this expression that if there exists an equilibrium, it must be symmetric[15] so that we can write the explicit reaction function as

$$\theta = \frac{Q}{bn(n-1)} \tag{30}$$

[14] The same kind of procedure we use in the paper has been applied by Karp and Newbery (1991). In their paper the Markov perfect Nash equilibrium of two games in which large importers of an exhaustible resource confront competitive suppliers is characterised. In the first game, it is assumed that the exporters move first which yields that the importing countries do not have a direct influence on the dynamics of the stock. In our paper, we solve a simultaneous game in which large importers confront a monopolist and the direct influence of the importing countries on the dynamics of the stock disappears, not because the monopolist moves first, but because it chooses the extraction rate. Notice that this feature of the game does not apply when the monopolist chooses the price.

[15] Notice that the left-hand side of (29) is the same for all i. Thus, the tariff must be the same for all the importers.

which establishes that in the static game the tariffs are *strategic complements* of the extraction rate for the importing countries.

Next, for a given set of tariffs, the optimal extraction rate must satisfy the HJB equation

$$rW_M = \max_{\{Q\}} \left\{ \left(\frac{a}{b} - \frac{1}{bn} (Q + bn\theta) - cx \right) Q + W_M' Q \right\}, \tag{31}$$

where again $W_M(x)$ stands for the optimal current value function associated with the dynamic optimisation problem for the monopolist defined by (26)-(27).

From the first-order condition for the maximisation of the right-hand sides of the HJB equation, we get the *instantaneous* reaction function of the monopolist that we write as follows:

$$Q = \frac{bn}{2} \left(\frac{a}{b} - \theta - cx + W_M' \right). \tag{32}$$

As it occurs for a price-setting monopolist, for a given value of the state variable the extraction rate is a *strategic substitute* of the tariffs for the monopolist.[16]

Computing the point of intersection of the reaction functions the optimal strategies are obtained

$$\theta = \frac{1}{2n-1} \left(\frac{a}{b} - cx + W_M' \right), \quad Q = \frac{bn(n-1)}{2n-1} \left(\frac{a}{b} - cx + W_M' \right). \tag{33}$$

Using the optimal strategies into Eq. (31), we eliminate the maximisation which yields the following nonlinear differential equation:

$$rW_M = \frac{bn(n-1)^2}{(2n-1)^2} \left(\frac{a}{b} - cx + W_M' \right)^2. \tag{34}$$

As in the previous section, we propose a quadratic function as a solution to the differential equation. Then if the system of equations for the coefficients of the value functions has a solution, the linear Markov-perfect Nash equilibrium strategies for the tariff and the quantity would be

$$\theta = \frac{1}{2n-1} \left(\frac{a}{b} + \beta_M - (c - \alpha_M)x \right), \tag{35}$$

$$Q = \frac{bn(n-1)}{2n-1} \left(\frac{a}{b} + \beta_M - (c - \alpha_M)x \right), \tag{36}$$

[16] Observe that the strategic relationship between the control variables of the model is not symmetric. For the importing countries, the tariff is a strategic complement of the extraction rate, whereas for the monopolist the extraction rate is a strategic substitute of the tariffs.

which are obtained from (33).

Thus, as $\dot{x} = Q$ if we look for a stable solution, the following condition should be satisfied by the coefficients of the value function:

$$\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = -\frac{bn(n-1)}{2n-1}(c-\alpha_M) < 0 \rightarrow c - \alpha_M > 0. \tag{37}$$

Using this condition, the coefficients of the value function have been calculated in Appendix B. Then substituting these coefficients in (35) and (36), the linear Markov perfect Nash equilibrium strategies for the extraction rate and the tariff can be written as

$$\theta = \frac{c-\alpha_M}{2n-1}\left(\frac{a}{bc} - x\right), \tag{38}$$

$$Q = \frac{bn(n-1)(c-\alpha_M)}{2n-1}\left(\frac{a}{bc} - x\right), \tag{39}$$

where

$$c - \alpha_M = -\frac{(2n-1)^2 r}{4bn(n-1)^2} + \frac{1}{2}\left(\frac{(2n-1)^2 r(8bcn(n-1)^2 + (2n-1)^2 r)}{4b^2n^2(n-1)^4}\right)^{0.5} > 0. \tag{40}$$

By visual inspection, it can be seen that both the tariff and the extraction rate are inversely related to the accumulated extractions, and that they converge to zero when the accumulated extractions tend to a/bc. Now using the equilibrium strategies, differential equation (27) can be solved, yielding

$$x = \frac{a}{bc}\left(1 - \exp\left\{-\frac{bn(n-1)(c-\alpha_M)}{2n-1}t\right\}\right), \tag{41}$$

which allows us to calculate the tariff and extraction rate dynamics by substitution in (38) and (39)

$$\theta = \frac{a(c-\alpha_M)}{bc(2n-1)}\exp\left\{-\frac{bn(n-1)(c-\alpha_M)}{2n-1}t\right\}, \tag{42}$$

$$Q = \frac{an(n-1)(c-\alpha_M)}{c(2n-1)}\exp\left\{-\frac{bn(n-1)(c-\alpha_M)}{2n-1}t\right\}, \tag{43}$$

so we can summarise the results as follows:

Proposition 2 *There exists a unique Markov perfect Nash equilibrium in linear strategies for which the tariff decreases throughout the exploitation period of the resource and converges to zero in the long run. Moreover, the extraction rate is also decreasing and converges to zero in the long run.*

Finally, we calculate the value function of the monopolist. See again Appendix B.

$$W_M(x) = \alpha_M \left(\frac{1}{2}x^2 - \frac{a}{bc}x + \frac{a^2}{2b^2c^2} \right). \tag{44}$$

However, for the importing countries we need to calculate the following integral

$$W_I(0) = \int_0^\infty e^{-rt} U_I(t) dt = \int_0^\infty e^{-rt} \left(\frac{1}{2bn^2} \left(Q(t) + bn\theta(t) \right)^2 - \frac{b}{2}\theta(t)^2 \right) dt$$

to obtain the discounted present value of the representative consumer's welfare in the initial moment.[17] Using (42) and (43), the integral can be rewritten as follows

$$W_I(0) = \frac{a^2(n^2-1)(c-\alpha_M)^2}{2bc^2(2n-1)^2} \int_0^\infty \left(\exp\left\{ -\left(r + \frac{2bn(n-1)(c-\alpha_M)}{2n-1} \right) t \right\} \right) dt,$$

whose result is

$$W_I(0) = \frac{a^2(n^2-1)(c-\alpha_M)^2}{2bc^2(2n-1)(r(2n-1)+2bn(n-1)(c-\alpha_M))}. \tag{45}$$

Next, we address the study of a tariff agreement. When the monopolist chooses the quantity and the importing countries coordinate their policies through a tariff agreement, the aggregate welfare of the agreement at a moment t is given by

$$W_A = \sum_{i=1}^{n} U_i = \frac{Q^2}{2bn} + \theta Q,$$

where θ is the common tariff selected by the importing countries.[18] The linearity of this expression with respect to the tariff tells us that the optimal policy consists of choosing, for a given quantity, a tariff such that the price be zero. The maximum tariff compatible with a non-negative price.[19] The explanation of why a Nash equilibrium with positive prices is obtained when the importing countries do not coordinate their tariffs is given by the fact that without coordination, the effects of an increase in the tariff by one country are different from the effects of an increase coordinated by all the importing countries. With cooperation, the effective demand is reduced by all countries in the same amount which

[17] Remember that we have assumed that the initial stock is zero.

[18] This expression is obtained from (28) doing $\theta_1 = ... = \theta_N = \theta$ and multiplying by n.

[19] The result is different if the agreement is the leader of the game as it is showed in Karp (1984). The interested reader in the issue can find in Rubio (2005) how to calculate the Markov perfect Stackelberg equilibrium in linear strategies when the importing countries coordinate their policy.

implies a reduction in the market demand that causes a reduction in the international price of the resource in the same amount than the increase in the tariff. However, as all the countries have diminished the demand by the same amount the distribution of the imports among the countries remains unchanged. This gives to the agreement the possibility of reaping all the rent from the monopolist. Without cooperation, one country reduces its demand assuming that the rest of countries do not change their tariffs which implies that the country must expect a reduction in the market demand that causes a reduction in the international price of the resource in an amount lower than the increase in the tariff. In fact, the reduction in price is going to be equal to the increase in the tariff divided by the number of countries. As consequence of this reduction in price imports increase in the rest of countries whereas imports decrease in the country that has augmented its tariff. Notice that total imports are constant and given by the extraction rate that is selected by the monopolist. Then an increase in the tariff has a positive effect on welfare because the international price of the resource is lower but has as well a negative effect because the consumption of the resource by the country is lower. The result is that there exists an optimal tariff for which the equilibrium price is positive.

However, if we take the result obtained in the previous section into account, it is clear that the monopolist will not play this kind of game against an agreement, since it has the alternative of setting the price and getting a positive rent. In other words, if the importing countries make an agreement, the best response of the monopolist is to select the price instead of setting the quantity. For this reason, we should not expect that a monopolist plays quantities against a tariff agreement.

4 Prices versus Quantities

Finally, we compare the two cases studied in the paper. Unfortunately, we cannot compare them analytically because when the monopolist sets up the price, it is not possible to obtain a closed form solution. For this reason, we have computed a numerical example to get an idea of the differences between the two equilibria.[20] Nevertheless, we would like to point out that the results obtained from the analysis of the tariff agreement suggest that the tariff is more effective when the monopolist chooses the quantity. The numerical example confirms this idea that, on the other hand, is consistent with what we know from the *static model*. As we have already commented, in a static game the optimal tariff is zero when the monopolist chooses the price. However, the optimal trade policy when the monopolist chooses the quantity is to set a positive tariff as can be easily checked from the static version of the model analysed in the previous section.[21] The difference is that in a dynamic setting as in the one studied in the paper, the asymmetry in the results is not as strong.

In the example, the market consists of two importing countries and a monopolist. In Table 1, the welfare levels of importing countries are shown for different values of a and c when the rate of discount is equal to 0.025 and b is equal to one. The different values for a appear in the first column and the different values for c in the first row. In the rest of boxes of the table, the first figure stands for the welfare when the monopolist chooses the price, and the second figure stands for the welfare when the monopolist chooses the quantity. In all the cases, the first figure is lower than the second one, indicating that the tariff allows the importing countries to capture a greater part of the monopolist's rent when it sets the quantity. The figures in the table also show that the welfare of the importing countries increases with the market size and decreases with the extraction costs. Remember that the steady state for the accumulated extractions in both cases is equal to a/bc so that an increment in a leads to an increase in the total welfare that can be obtained from the exploitation of the resource, whereas an increment in c has the contrary effect. The variations in the welfare of the importing countries go in the same direction as the variation in total welfare. Notice as well that an increase in a implies a

[20] Chou and Long (2009) also present a numerical solution for the case of a price-setting monopolist but not for the case of a quantity-setting monopolist. In their paper they focus on the effects that the market size and asymmetry between the importing countries have on welfare under three regimes: free trade, non-cooperative use of tariffs and tariff agreement.

[21] For the static version of the model, the reaction function of the monopolist would be $Q = bn((a/b) - \theta - c))/2$ and the optimal tariff $\theta = ((a/b) - c)/(2n - 1)$.

lower elasticity of demand for each price so that it can be stated that the effectiveness of the tariff is inversely related to the elasticity of demand both when the monopolist chooses the price and when it selects the quantity.

⇒ TABLE 1: Welfare levels of importing countries ($r = 0.025,\ b = 1$). ⇐

The difference in welfare for the importing countries is explained by the fact that the tariff is higher when the monopolist chooses the quantity as it is illustrated in the next figure[22]

⇒ FIGURE 1: Comparing the two tariff rates. ⇐

Notice that the tariff is higher in a situation where the importing countries cannot directly influence the dynamics of the accumulated extractions. This leads us to think that when the monopolist chooses the quantity, the direct strategic relationship between the monopolist and the importing countries is strong enough as to more than compensate for the disappearance of the indirect effect that the tariff has on the price through its influence on the dynamics of the stock when the monopolist chooses the price.

In Table 2, we again present the welfare levels of importing countries but for a discount rate equal to 0.05. As it is easy to check, the properties of this numerical example are the same as those we have just commented on for Table 1 except that for all the boxes the welfare is greater. In other words, an increase in the discount rate gives the importing countries greater power to strategically use a tariff.

⇒ TABLE 2: Welfare levels of importing countries ($r = 0.05,\ b = 1$). ⇐

The positive effect of an increase of the discount rate in the welfare of the importing countries comes from the fact that the increment in the discount rate has a different effect on the values of the parameters α_I and α_M. On one hand, α_I increases with the rate of discount. On the other hand, α_M decreases, although the net effect is a reduction

[22]The dot-line stands for the case of a quantity-setting monopolist. We have calculated the optimal temporal paths of the tariffs for $a = 100$ and $c = b = 1$. The initial value of the tariff for the case of a quantity-setting monopolist is 7.02 and for the case of a price-setting monopolist 3.06.

of $\alpha_I + \alpha_M$ which implies an increment in $c - \alpha_I - \alpha_M$, that according to Eq. (22), supposes a greater discount of the initial tariff for the complete period of exploitation of the resource. However, the initial tariff increases with α_I. The result is a tariff that is higher during an initial period but lower for the rest of the time as it is shown in the following figure[23]

$\Rightarrow$ FIGURE 2: The effect of an increase in the discount rate. $\Leftarrow$

In other words, for a greater discount rate, the optimal trade policy of the importing countries consists of augmenting the tariff in the initial periods to take advantage of the fact that with a greater discount rate, the welfare of the initial periods have a bigger weight in the discount present value of the representative consumer's welfare. The increment in the tariff in the initial periods is enough to more than compensate for the negative effects that the reduction of the tariff has for the more distant periods in the discount present value of the representative consumer's welfare. For this reason, an increase in tariff in the initial periods, when the rate of discount increases, allows the importing countries to capture a greater part of the monopolist's rent, yielding a greater welfare for the consumers.

The following tables, 3 and 4, show the monopolist's rent. The first figure in each box of the tables stands for the rent when the monopolist chooses the price, and the second figure stands for the rent when the monopolist chooses the quantity. In all the boxes, the first figure is greater than the second one, which is consistent with the figures obtained for the welfare of the importing countries. The tables also show that the rent of the monopolist increases with the market size and decreases with the extraction costs, as it occurs with the welfare of the importing countries and for the same reason. Finally, in accordance with the argument presented above, the monopolist's rent is lower when the rate of discount is 0.05. Nevertheless, the total welfare of the market decreases

[23] The dot-line represents the temporal path of the tariff for $r = 0.05$ whereas the solid-line stands for the temporal path of the tariff for $r = 0.025$. Both temporal paths have been calculated for $a = 100$ and $b = c = 1$ and for the case of a price-setting monopolist. The initial value of the tariff when the rate of discount is 0.05 is 3.89 and when the rate of discount is 0.025 is 3.06.

when the rate of discount increases. The same occurs when the monopolist chooses the quantity instead of the price but for all the cases computed in this section, the variation in total welfare is less than 1%. In other words, although the control variable used by the monopolist significantly affects the distribution of the total welfare between the importing countries and the monopolist, it has no relevant effect on the total welfare obtained from the exploitation of the resource.

$\Rightarrow$ TABLE 3: Rent of the monopolist ($r = 0.025,\ b = 1$). $\Leftarrow$

$\Rightarrow$ TABLE 4: Rent of the monopolist ($r = 0.05,\ b = 1$). $\Leftarrow$

The effects of changes in b have also been analysed , but as the effects of an increase (decrease) in b are equivalent to the effects of a decrease (increase) in a, we have decided to omit them to avoid reiterations. Nevertheless, it should be pointed out that the effects of changes in a and b are not completely equivalent. As it can be easy checked from the expressions of the value functions (25), (44) and (45) the distribution in percentage terms of the market welfare between the importers and the exporter is independent of parameter a as Table 5 shows.[24]

$\Rightarrow$ TABLE 5: Percentage of importers welfare over total welfare ($r = 0.025,\ b = 1$). $\Leftarrow$

Thus, a change in a increases the welfare level of importing countries and the rent of the monopolist proportionally so that the relative share of the importers welfare on the total welfare of the market remains unchanged. However, this is not the case when a change in b occurs, as it is shown in Table 6.

$\Rightarrow$ TABLE 6: Percentage of importers welfare over total welfare ($r = 0.025, a = 100$). $\Leftarrow$

Thus, a decrease in b, which implies a greater demand for the same price, has a stronger effect on the welfare level of importing countries than on the rent of the monopolist, yielding an increase of the relative share of the importers welfare on the total welfare of

[24]This result is independent of the rate of interest.

the market. For instance, in Table 6, for $b = 2$, $c = 0.5$, $a = 100$ and $r = 0.025$, the welfare of the importing countries stands for 7% of the total welfare when the monopolist chooses the price and for 11% of the total welfare when the monopolist selects the quantity. However, when $b = 0.5$, the first percentage is 12% and the second is 20%. With regard to the variations of c and r, it can be added that they have the same kind of effect on the relative welfare of the importing countries than on the absolute welfare. An increase in c decreases both the absolute and relative welfare of the importing countries and an increase in r increases both the absolute and relative welfare of the importing countries.

Summarizing, the numerical example suggests that the effectiveness of the tariff to capture a part of the monopolist's rent is greater when the monopolist selects the price instead of the quantity, that is independently of the level of parameter a and that increases with a reduction of b and c and with an increase of the discount rate.

Finally, it seems pretty clear from the comparison we have just developed that the optimal policy for the monopolist to face large importers that use a tariff strategically is to set the price; although it should be highlighted that the use of the price does not eliminate the effectiveness of the tariff to reap a part of the monopolist's rent as it occurs in the static model, instead only reducing it.

5 Conclusions

In the paper we have revisited the issue, first tackled by Bergstrom (1982), of using a tariff on non-renewable resource imports in order to appropriate part of the monopolist's rent. We extend his analysis to take into account that the exploitation of non-renewable resources is characterised by the presence of depletion effects, and to consider that the monopolist can choose the price, as in Bergstrom's paper, or the extraction rate, as in Karp's (1984) paper. Moreover, we investigate the game theoretic aspects of the problem more deeply using the differential game theory.

Our results establish that it is not necessary to either commit to the trade policy or to cooperate to use a tariff strategically. The optimality of the tariff is explained, for the case of a price-setting monopolist, by the fact that, given the price, the governments of the importing countries can influence the dynamics of the accumulated extractions through the tariff and hence the extraction costs and subsequent evolution of the monopolist price. However, the case of a quantity-setting monopolist is explained by the fact that the direct (intratemporal) strategic relationship is strong enough to make a tariff profitable for consumers in the importing countries even though in this case, given the quantity, the importing countries cannot directly influence the dynamics of the accumulated extractions. Finally, using a numerical analysis, we also find that the importing countries can capture a greater part of the monopolist's rent when the monopolist chooses the extraction rate instead of the price, although the variation in total welfare between the two regimens is small. The difference in the welfare of the importing countries is explained by the fact that the tariff is higher throughout the exploitation period of the resource when the monopolist chooses the quantity. Obviously, from the perspective of the monopolist, the best policy to face large importers that strategically use a tariff is to set the price of the resource instead of choosing the quantity.

Several issues are left for future research. For instance, we have assumed that the budgets of governments must be balanced at any date so that the tariff revenues are equal in each period to the lump-sum transfer that the consumer receives from the government. Clearly this eliminates the possibility of using an intertemporal budget constraint, which in fact would give the governments more flexibility. In this framework, it would be interesting to investigate whether the use of a *decreasing* tariff and a *constant* lump-sum transfer to the representative consumer could dominate the type of policy analysed in the paper. The use of a constant lump-sum transfer lower than the initial tariff would yield, during a first period of the resource exploitation, a surplus that the governments could lend in the capital markets to finance the deficit that would appear later on when the tariff will be bellow the lump-sum transfer. The conjecture to check is whether the use of the capital markets would allow to set up a greater enough lump-sum transfer that yields a larger welfare for the consumer than that obtained applying the policy studied in the paper. Another interesting issue to address would be to study the trade-policy component of a carbon tax for a polluting resource market with large importers and a monopolist. Wirl (1994), Tahvonen (1996), Rubio and Escriche (2001) and Liski and

Tahvonen (2004) have studied the issue in the framework of a bilateral monopoly, where the seller sets the price and the buyer sets a carbon tax to control pollution damage from resource consumption.[25] As Liski and Tahvonen (2004) have clarified, coordinating carbon taxation creates a strategic incentive to use the tax as a partial import tariff, so that the optimal carbon tax generally deviates from the neutral Pigovian tax which would only internalise the damage cost of pollution. The issue to analyse would be to investigate how the lack of coordination between the importing countries could affect the trade-policy component of the optimal tax for a stock pollutant. Finally, a natural extension of the model would be to consider other market structures different from the monopoly that is an extreme case of concentration of the extractive activity.

A Proof of Proposition 1

For a symmetric equilibrium, the system of two nonlinear differential equations (9) and (10) yields the following system of equations for the parameters of the quadratic value

[25] More recently, Wirl (2011) has presented a very interesting paper where each player can choose either the quantity or the price strategy. He calculates both the Markov perfect Nash equilibrium and the Markov perfect Stackelberg equilibrium for the different cases analysed in the paper. The overall conclusion is that both players should prefer the price instrument which agrees with the result obtained in this paper. Wirl's (2011) conclusion is in line with the result derived by Strand (2011) in the framework of a static model. Strand (2011) shows that the equivalence between a carbon tax and a cap-and-trade scheme vanishes when the exporters respond strategically to the policy adopted by the importers. Moreover, he obtains that the application of a cap-and-trade scheme is the worst strategy the importer group can follow.

functions

$$\begin{aligned}
4r\alpha_I &= b\left((c-\alpha_M)^2 - 2(2n-1)(c-\alpha_M)\alpha_I + (4n-3)\alpha_I^2\right), && (46)\\
2r\alpha_M &= bn(c-\alpha_I-\alpha_M)^2, && (47)\\
4r\beta_I &= b\left((2n-1)\left(\left(\frac{a}{b}+\beta_M\right)\alpha_I - (c-\alpha_M)\beta_I\right) - \left(\frac{a}{b}+\beta_M\right)(c-\alpha_M)\right. && (48)\\
&\quad \left. +(4n-3)\alpha_I\beta_I\right), \\
2r\beta_M &= -bn\left(\frac{a}{b}+\beta_I+\beta_M\right)(c-\alpha_I-\alpha_M), && (49)\\
8r\mu_I &= b\left(\left(\frac{a}{b}+\beta_M\right)^2 + 2(2n-1)\left(\frac{a}{b}+\beta_M\right)\beta_I + (4n-3)\beta_I^2\right), && (50)\\
4r\mu_M &= bn\left(\frac{a}{b}+\beta_I+\beta_M\right)^2. && (51)
\end{aligned}$$

Next, we want to show that this system has a unique solution for which β_I is negative and that α_I and α_M satisfy the stability condition (16). As the stability condition (16) requires a positive value for the difference $c-\alpha_I-\alpha_M$ we can obtain α_I from (47)

$$\alpha_I = c - \alpha_M - \left(\frac{2r\alpha_M}{bn}\right)^{1/2}, \tag{52}$$

and use this expression to eliminate α_I from (46) to get an equation only for α_M :

$$b(n-1)(8bnr)^{1/2}\alpha_M^{3/2} + 3(2n-1)br\alpha_M + (r-bc(n-1))(8bnr)^{1/2}\alpha_M^{1/2} - 2bcnr = 0. \tag{53}$$

It is easy to show that this equation has a unique positive solution. The first derivative of the l.h.s. with respect to α_M is

$$\frac{3b}{2}(n-1)(8bnr)^{1/2}\alpha_M^{1/2} + 3(2n-1)br + \frac{1}{2}(r-bc(n-1))(8bnr)^{1/2}\alpha_M^{-1/2},$$

so that we have two possible behaviours for the l.h.s. of the equation. First, if $r-bc(n-1) \geq 0$, the l.h.s. of the equation is increasing for all α_M. Then, as the l.h.s. of the equation is negative for $\alpha_M = 0$, there will be a unique positive value of α_M for which the equation is zero. In the other case, if $r-bc(n-1) < 0$ the l.h.s. of the equation presents a unique extreme that is a minimum since the l.h.s is a strictly convex function with respect to α_M. Thus, the l.h.s. is first decreasing to increase after reaching the minimum. Then as the l.h.s. of the equation is negative for $\alpha_M = 0$ we obtain that the l.h.s. of the equation must be equal to zero for a unique positive value of α_M. Moreover, it is also easy to check that α_M is lower than c. Substituting c in the l.h.s. of (53) and developing the expression, we obtain that the l.h.s. of (53) for $\alpha_M = c$ is

$$(4n-3)bcr + r(8bnr)^{1/2}c^{1/2} > 0.$$

Therefore the value of α_M that satisfies equation (53) must be lower than c since the l.h.s. of (53) is increasing on the right of the equation root.

For the solution to (53), expression (52) would allow us to calculate the corresponding value for α_I. To find out the sign of this coefficient we look at equation (46) that can be written as

$$b(4n-3)\alpha_I^2 - 2(2r + b(2n-1)(c-\alpha_M))\alpha_I + b(c-\alpha_M)^2 = 0. \tag{54}$$

As α_M is lower than c and the independent term is positive, according to the Descartes' rule of signs, the sign of α_I that satisfies (46) and (47) must be positive. Finally, as we have selected the positive value for $c - \alpha_I - \alpha_M$ in (47) in order to satisfy the stability condition, we can conclude that α_I must not only be positive but also lower than c. Notice that equation (54) could have two positive solutions but in any case the value selected by (52) would be lower than c for the reason we have just presented.

Summarising, we find that the system (46) and (47) has a unique solution that satisfies the stability condition for which α_I and α_M belong to interval $(0, c)$ and $c - \alpha_I - \alpha_M > 0$.

Now we need to show that β_I is negative and that (12), the tariff, is positive in interval $[0, x^\infty)$ where x^∞ is the steady-state value of the accumulated extractions, to conclude that a tariff is advantageous for the consumer. Thus we continue the proof calculating x^∞. According to differential equation (15)

$$x^\infty = \frac{\frac{a}{b} + \beta_I + \beta_M}{c - \alpha_I - \alpha_M},$$

that using (47) and (49) yields

$$x^\infty = -\beta_M/\alpha_M \tag{55}$$

which establishes that

$$W_M'(x^\infty) = \beta_M + \alpha_M x^\infty = 0. \tag{56}$$

Now we calculate β_M from (48) and (49)

$$\beta_M = -\frac{1}{N} an(c - \alpha_I - \alpha_M)\left(4r + 2b(n-1)(c - \alpha_I - \alpha_M)\right),$$

where

$$N = 2r(4r + b(2n-1)(c - \alpha_M) - b(4n-3)\alpha_I) + 4bnr(c - \alpha_I - \alpha_M) + 2b^2 n(n-1)(c - \alpha_I - \alpha_M)^2,$$

that allows us to write (55) as

$$x^{\infty} = \frac{1}{N\alpha_M} an(c - \alpha_I - \alpha_M)\left(4r + 2b(n-1)(c - \alpha_I - \alpha_M)\right).$$

According to (47) we can substitute $2r\alpha_M$ in the denominator by $bn(c - \alpha_I - \alpha_M)^2$ obtaining after some simplifications

$$x^{\infty} = \frac{a\left(4r + 2b(n-1)(c - \alpha_I - \alpha_M)\right)}{b((c - \alpha_I - \alpha_M)(4r + b(2n-1)(c - \alpha_M) - b(4n-3)\alpha_I + 2b(n-1)\alpha_M) + 4r\alpha_M)}. \tag{57}$$

Now developing the denominator, the following expression is obtained

$$\begin{aligned} &-4r\alpha_I + 4r(c - \alpha_M) - b(2n-1)(c - \alpha_M)\alpha_I + 2bn(c - \alpha_M)^2 - b(c - \alpha_M)^2 \\ &+ b(4n-3)\alpha_I^2 - b(4n-3)(c - \alpha_M)\alpha_I + 2b(n-1)(c - \alpha_I - \alpha_M)\alpha_M + 4r\alpha_M, \end{aligned}$$

where $4r\alpha_I$ can be substituted by $b((c - \alpha_M)^2 - 2(2n-1)(c - \alpha_M)\alpha_I + (4n-3)\alpha_I^2)$ according to (46) yielding the following expression for the denominator after simplifying terms

$$2b(n-1)(c - \alpha_M)^2 - 2b(n-1)(c - \alpha_M)\alpha_I + 4rc + 2b(n-1)(c - \alpha_I - \alpha_M)\alpha_M.$$

Developing this expression we obtain finally that the denominator of (57) is equal to

$$c(4r + 2b(n-1)(c - \alpha_I - \alpha_M))$$

and, consequently, that $x^{\infty} = a/bc$ which establishes according to (55) that $\beta_M = -a\alpha_M/bc < 0$ since we have showed above that α_M is positive.

Now we use differential equation (15) to end the proof. The steady-state value of the accumulated extractions must satisfy

$$\dot{x} = 0 = \frac{bn}{2}\left(\frac{a}{b} + \beta_I + \beta_M - (c - \alpha_I - \alpha_M)x^{\infty}\right)$$

where $\beta_M + \alpha_M x^\infty = 0$ according to (56) so that $a/b + \beta_I - (c - \alpha_I)x^\infty$ must be equal to zero. As $x^\infty = a/bc$ we obtain that $W_I'(x^\infty) = \beta_I + \alpha_I(a/bc) = 0$ which implies that $\beta_I = -a\alpha_I/bc < 0$ since we have established above that α_I is positive. This concludes the proof.

B Derivation of the Stationary Linear Markov Strategies for the Case of a Quantity-Setting Monopolist

If we guess a quadratic representation for the value function W_M as that proposed in Section 2.1, the HJB equation for the monopolist yields the following system equations for the parameters of the value functions:

$$\frac{r}{2}\alpha_M = \frac{bn(n-1)^2}{(2n-1)^2}(c-\alpha_M)^2, \tag{58}$$

$$r\beta_M = -\frac{2bn(n-1)^2}{(2n-1)^2}\left(\frac{a}{b}+\beta_M\right)(c-\alpha_M), \tag{59}$$

$$r\mu_M = \frac{bn(n-1)^2}{(2n-1)^2}\left(\frac{a}{b}+\beta_M\right)^2. \tag{60}$$

The solution for the first equation is

$$\alpha_M = c + \frac{(2n-1)^2 r}{4bn(n-1)^2} \pm \frac{1}{2}\left(\frac{(2n-1)^2 r\left(8bcn(n-1)^2 + (2n-1)^2 r\right)}{4b^2n^2(n-1)^4}\right)^{0.5}. \tag{61}$$

To choose between the two roots, the stability condition (37) is used. It is easy to check that only the lowest root of (58) satisfies this condition yielding

$$c - \alpha_M = -\frac{(2n-1)^2 r}{4n(n-1)^2} + \frac{1}{2}\left(\frac{(2n-1)^2 r\left(8cn(n-1)^2 + (2n-1)^2 r\right)}{4n^2(n-1)^4}\right)^{0.5} > 0, \tag{62}$$

that is expression (40) in the main text. Now adding ra/b to the two sides of (59) and taking common factor $a/b + \beta_M$ we obtain

$$\frac{a}{b} + \beta_M = \frac{ar(2n-1)^2}{b(r(2n-1)^2 + 2bn(n-1)^2(c-\alpha_M))},$$

that multiplying the numerator and denominator by $c - \alpha_M$ yields

$$\frac{a}{b} + \beta_M = \frac{ar(2n-1)^2(c-\alpha_M)}{br(2n-1)^2(c-\alpha_M) + 2b^2n(n-1)^2(c-\alpha_M)^2}.$$

The expression can be simplified taking into account that $r(2n-1)^2\alpha_M = 2bn(n-1)^2(c-\alpha_M)^2$ according to (58). Thus, we get

$$\frac{a}{b} + \beta_M = \frac{a}{bc}(c - \alpha_M),$$

so that $\beta_M = -a\alpha_M/bc$. Finally, by substitution in (60), the independent term of the value function can be calculated. The result is

$$r\mu_M = \frac{bn(n-1)^2a^2}{(2n-1)^2b^2c^2}(c - \alpha_M)^2,$$

that taking into account that

$$\frac{r\alpha_M}{2} = \frac{bn(n-1)^2}{(2n-1)^2}(c - \alpha_M)^2,$$

according to (58), reduces to $\mu_M = \alpha_M a^2/2b^2c^2$ so that finally the value function can be written as

$$W_M(x) = \alpha_M\left(\frac{1}{2}x^2 - \frac{a}{bc}x + \frac{a^2}{2b^2c^2}\right) \tag{63}$$

according to (62).

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$a\backslash c$	0.5	1	1.5	2
50	97, 157	38, 60	22, 34	15, 23
100	389, 627	153, 242	87, 137	58, 91
150	876, 1410	344, 544	196, 308	131, 204
200	1558, 2507	612, 967	349, 547	233, 363

TABLE 1: Welfare levels of importing countries (r=0.025, b=1).

$a\backslash c$	0.5	1	1.5	2
50	119, 195	49, 78	28,45	19, 30
100	475, 781	195, 313	113, 180	76, 121
150	1070, 1758	438, 705	255, 405	172, 272
200	1902, 3125	779, 1254	453, 721	306, 484

TABLE 2: Welfare levels of importing countries (r=0.05, b=1).

$a\backslash c$	0.5	1	1.5	2
50	1914, 1790	1032, 987	712, 687	545, 529
100	7654, 7162	4129, 3946	2848, 2747	2181, 2114
150	17222, 16114	9289, 8879	6408, 6181	4906, 4757
200	30617, 28646	16515, 15786	11393, 10989	8722, 8458

TABLE 3: Rent of the monopolist (r=0.025, b=1).

$a\backslash c$	0.5	1	1.5	2
50	1724, 1562	957, 895	669, 634	516, 493
100	6898, 6250	3827, 3581	2675, 2537	2064, 1973
150	15521, 14063	8611, 8057	6018, 5708	4645, 4440
200	27592, 25000	15308, 14323	10700, 10148	8257, 7893

TABLE 4: Rent of the monopolist (r=0.05, b=1).

$a\backslash c$	0.5	1	1.5	2
50	0.09, 0.15	0.07, 0.10	0.06, 0.09	0.05, 0.08
100	0.09, 0.15	0.07, 0.11	0.06, 0.09	0.05, 0.08
150	0.09, 0.15	0.07, 0.11	0.06, 0.09	0.05, 0.08
200	0.09, 0.15	0.07, 0.11	0.06, 0.09	0.05, 0.08

TABLE 5: Percentage of importers welfare over total welfare (r=0.025, b=1)

$b\backslash c$	0.5	1.0	1.5	2.0
0.5	0.12, 0.20	0.09, 0.15	0.08, 0.12	0.07, 0.11
1.0	0.09, 0.15	0.07, 0.11	0.06, 0.09	0.05, 0.08
1.5	0.08, 0.12	0.06, 0.09	0.05, 0.07	0.04, 0.06
2.0	0.07, 0.11	0.05, 0.08	0.04, 0.06	0.04, 0.06

TABLE 6: Percentage of importers welfare over total welfare (r=0.025, a=100)

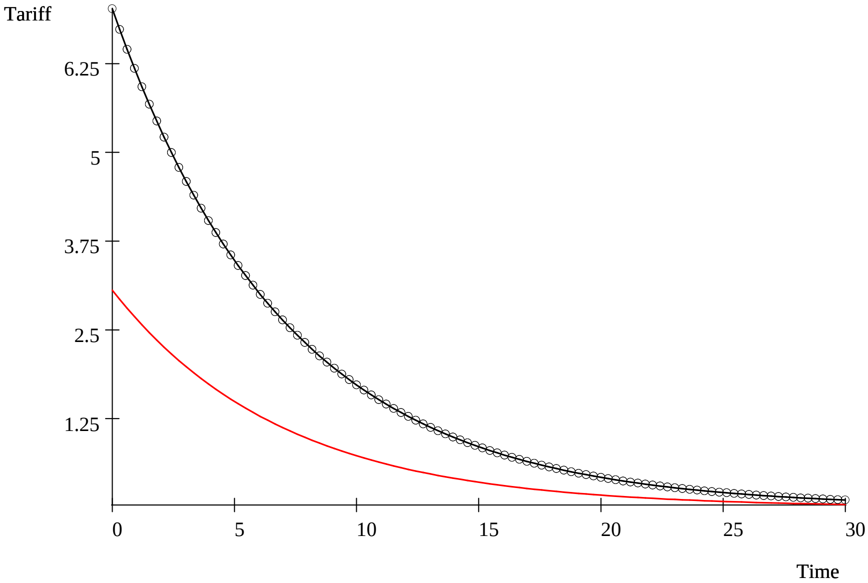

FIGURE 1: Comparing the two tariffs.

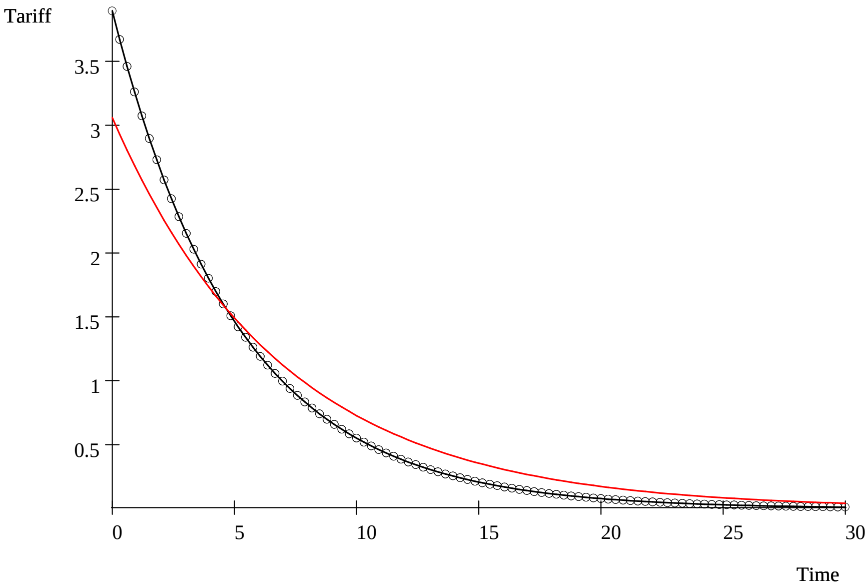

FIGURE 2: The effect of an increase in the discount rate.