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ON COLLINEAR WAKE FIELD ACCELERATION*

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Introduction

In the Voss-Weiland scheme of wake field acceleration[1] a high current, ring-shaped driving bunch is used to accelerate a low current beam following along on axis. In such a structure, the transformer ratio, *i.e.* the ratio of the maximum voltage that can be gained by the on-axis beam and the voltage lost by the driving beam, can be large. In contrast, it has been observed that for an arrangement in which driving and driven bunches follow the same path, and where the current distribution of both bunches is gaussian, the transformer ratio is not normally greater than two.[2] This paper explores some of the possibilities and limitations of a collinear acceleration scheme. In addition to its application to wake field acceleration in structures, this study is also of interest for the understanding of the plasma wake field accelerator.[3,4]

Consider a driving bunch with current distribution $I(t)$ which extends from time $t = 0$ to time $t = T$ and with total charge Q. After traversing some length L of a structure, there will be a retarding potential $V^-(t)$ within the bunch, with a minimum value V_m^-. The energy U extracted from the bunch by the retarding potential goes into one or more modes of the structure, producing a potential $V^+(t)$ behind the bunch which reaches some maximum accelerating value V_m^+. The gradient that can be used to accelerate a trailing bunch is therefore $E_a = V_m^+/L$. The transformer ratio is defined as $R = -V_m^+/V_m^-$.

Let us assume that all the electrons in the driving bunch have the same initial energy eV_i, and that the electrons in the distribution which see the maximum retarding field are brought to rest in distance L. Thus $V_i = -V_m^-$. The efficiency for extracting energy from the driving beam is given by

$$\eta = \frac{U}{QV_i} = \frac{\int_0^T I(t)V^-(t)\,dt}{QV_m^-} \,. \tag{1}$$

It is useful to define a total loss factor $k_{tot} = U/Q^2$, which depends both on the structure and on the bunch distribution. Thus the charge required in the driving bunch to reach an accelerating gradient E_a is $Q = \eta E_a L/(Rk_{tot})$. If there is only a single mode with loss factor $k = E_a^2L^2/4U$, then $k_{tot} = 4\eta^2 k/R^2$ and

$$Q = \frac{E_a LR}{4\eta k} \,. \tag{2}$$

Note that in this case R can be written as $R = -\sqrt{2kU}/V_m^-$.

Four important parameters in a collinear wake field acceleration scheme are the transformer ratio, the efficiency, the total charge and peak current in the driving bunch. In the following we will focus mostly on optimizing R. In the final section we will briefly consider some of the implications of a high R scheme for other machine parameters.

Symmetric Bunches

It can be proved[5] that for a current distribution symmetric about its midpoint in a single mode cavity the tranformer ratio can be no larger than two. The upper limit can be achieved if and only if $V^-(0) = V_m^-$ for a bunch symmetric about $t = 0$. Even for symmetric bunches in actual accelerating structures, which have many modes, this limitation still tends to apply.[2] However, it is possible in principle to design a structure in which the accelerating potentials for several modes superimpose maximally behind the bunch, resulting in $R > 2$ even for a symmetric bunch. Consider a two mode structure with frequencies ω_0 and ω_1 related by $\omega_1 = 3\omega_0 + \delta$, where δ is a small quantity, and with loss factors k_0 and k_1. The potential due to a bunch with current density $I(t)$ is

$$V(t) = -\int_{-\infty}^{t} I(t')\,W_z(t-t')\,dt' \tag{3}$$

where $W_z(t)$ is the longitudinal wake potential. The contribution of mode n to $W_z(t)$ is[6] $2k_n \cos\omega_n t$. Therefore, inside a rectangular bunch extending from $-T$ to T, with constant $I(t) = I$ we have the retarding potential

$$V^-(t) = -2I\left[\frac{k_0}{\omega_0}\sin\omega_0(t+T) + \frac{k_1}{\omega_1}\sin\omega_1(t+T)\right] . \tag{4}$$

Behind the bunch ($t > T$) we have

$$V^+(t) = -4I\left[\frac{k_0}{\omega_0}\cos\omega_0 t\sin\omega_0 T + \frac{k_1}{\omega_1}\cos\omega_1 t\sin\omega_1 T\right] . \tag{5}$$

If we choose $\omega_0 T = \pi/2$ and $k_1 = k_0$ we find the mimimum potential inside and the maximum potential behind the bunch are, respectively,

$$V_m^- \approx -\frac{8}{3\sqrt{2}}\frac{k_0 I}{\omega_0} \quad \text{and} \quad V_m^+ = \frac{16}{3}\frac{k_0 I}{\omega_0} \,. \tag{6}$$

Therefore we get $R \approx 2\sqrt{2} > 2$.

This calculation can be generalized to structures with many modes where the frequencies are related by $\omega_n = (2n+1)\omega_0 + \delta_n$ and all the loss factors are equal. It is straight forward to show that V^- turns out to be the Fourier expansion of a rectangular pulse, and $V_m^- \approx \pi k_0 I/(2\omega_0)$. The maximum accelerating potential behind the bunch corresponds to the sum of all amplitudes, so $V_m^+ = (4k_0I/\omega_0)(1+1/3+1/5+\ldots)$. Thus the transformer ratio becomes $R \approx (8/\pi)(1+1/3+1/5+\ldots)$. It can be rightly argued that such a structure is unphysical. On the other hand there is no reason to believe that the two-mode structure described above is not realizable. However, the $\sqrt{2}$ gain in transformer ratio over the single mode case is still quite modest.

Asymmetric Bunches

We next consider the case of an asymmetric driving bunch. Take as an example a growing triangular bunch in a single mode cavity. In practice such a driving bunch has been used in autoacceleration experiments[7] to accelerate trailing particles. Let $I(t) = I\omega t$ for $0 < t < T$ and $I(t) = 0$ otherwise. For simplicity let the bunch length be $T = 2\pi N/\omega$, where N is a positive integer. Then within the bunch

$$V^-(t) = -2kI\omega\int_0^t t'\cos\omega(t-t')\,dt' = -\frac{2kI}{\omega}(1-\cos\omega t)\,, \tag{7}$$

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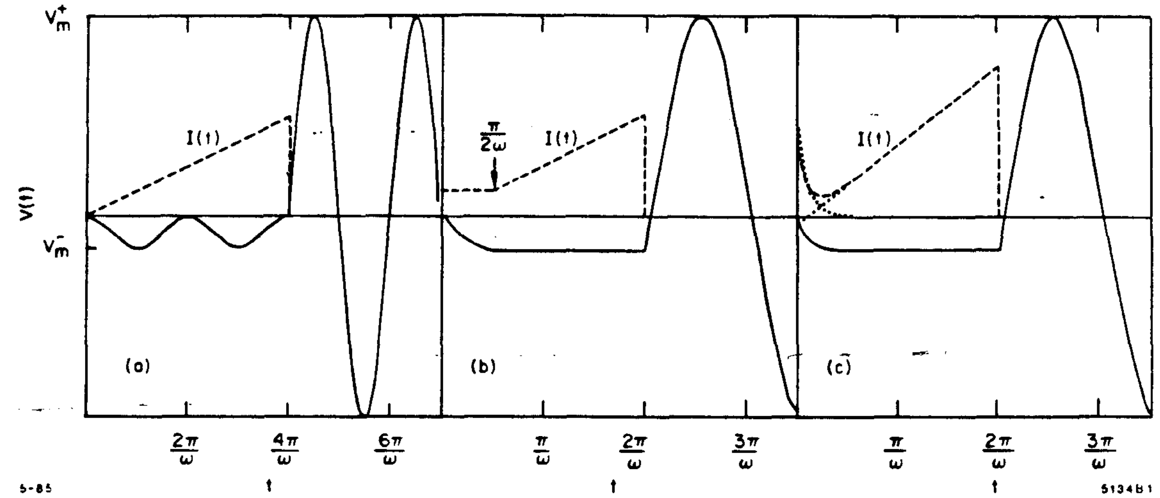

Fig. 1. The voltage induced by three different asymmetric current distributions.

whereas behind the bunch

$$V^+(t) = -2kI\omega \int_0^T t' \cos\omega(t-t')\,dt' = 2kIT \sin\omega t \quad . \tag{8}$$

Thus $R = -V_m^+/V_m^- = \pi N$. Notice that R is proportional to the number of ripples N in $V^-(t)$ (See Fig. 1a). The longer the current pulse we choose, the larger the transformer ratio. To understand this, notice that in this case all particles in the bunch give energy to the cavity mode: $V^-(t) \le 0$ for all t within the bunch. Therefore since $V_m^+ = 2\sqrt{kU}$ where U is the energy stored in the mode, we see that the more energy stored in the mode, the higher V_m^+ we have behind the bunch. And since for $\omega t > \pi$, V_m^- no longer changes, the transformer ratio will continue to increase. In contrast, for a long rectangular pulse from $-T$ to T we find that $V^-(t) \propto -\sin\omega(t+T)$ (See previous section). Thus, the first half wavelength of charge gives energy to the mode, the next half wavelength of charge takes the same amount of energy from the mode, and so forth. Therefore making the bunch longer than half the wavelength will not improve the transformer ratio.

Consider next the following current distribution: $I(t) = I$, a constant, for $0 \le t \le \pi/2\omega$ and $I(t) = (2/\pi)I\omega t$ for $\pi/2\omega \le t \le T$. This represents a growing triangular bunch preceeded by a quarter wavelength rectangular pulse. (We will call this the doorstep distribution. See Fig. 1b.) In this case the transformer ratio becomes $R = \sqrt{1 + (1 - \pi/2 + \omega T)^2}$. For $T = 2\pi N/\omega$, $R \approx 2\pi N$. We see that the transformer ratio here is approximately twice that of the triangular bunch. Except for particles in the first quarter wavelength of the bunch all particles experience the same retarding potential.

The Optimal Transformer Ratio In A Single Mode Structure

What is the optimal transformer ratio for a single mode structure? From the foregoing examples we would guess that it would be the current distribution that causes all particles in the bunch to see the same retarding potential, an assertion that can be proven.[8]

The general features of the proof are as follows: Assuming a case with $V^-(t) = V_0$, a constant, exists, we denote its stored energy by U_0 and its transformer ratio by $R_0 = -2\sqrt{kU_0}/V_m^-$. Suppose there is a perturbation of the current $\delta I(t)$ localized to the interval $[t_0 - \epsilon, t_0 + \epsilon]$ that keeps the charge Q unchanged. When ϵ is small this will result in a bump in voltage $\delta V(t)$ whose magnitude is of order ϵ and that is also localized to the same interval. It also results in an energy perturbation δU of order ϵ^2, with $\delta U \gtrless 0$ for $\delta V \lessgtr 0$. For $\delta V < 0$ we get

$$R = -2\frac{\sqrt{k(U_0 + \delta U)}}{V_0 + \delta V_0} < R_0 \ , \tag{9}$$

where δV_0 is the minimum of δV. On the other hand if $\delta V > 0$ then, since $\delta U < 0$,

$$R = -2\frac{\sqrt{k(U_0 + \delta U)}}{V_0} < R_0 \ . \tag{10}$$

A slight extension of this argument shows that in fact any deviation from a constant $V^-(t)$ reduces R.

Let us now find the current that gives a constant retarding potential across the bunch. From Eq. (3) we see that it is not possible to have a constant retarding potential starting exactly at the head of the bunch for regular current distributions. We therefore parameterize the optimal $V^-(t)$ as

$$V^-(t) = \begin{cases} (1 - e^{-\alpha t})\,V_0, & \text{if } 0 \le t \le T; \\ 0 & \text{otherwise} \ . \end{cases} \tag{11}$$

(See Fig. 1c.) As $\alpha \to \infty$, $V^-(t)$ approaches the constant V_0.

By the use of the Laplace transform Eq. (3) can be inverted to give the current that produces a given $V(t)$ when $W_z(t)$ is known,[9] *i.e.*

$$I(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\mathcal{L}\{V(t)\}}{\mathcal{L}\{W_z(t)\}} e^{st}\,ds \ , \tag{12}$$

where γ is some positive quantity which avoids integration along the imaginary axis of the complex plane. Applying this method to Eq. (11) gives

$$I(t) = -\frac{V_0}{2k\alpha}\left[(\alpha^2 + \omega^2)e^{-\alpha t} + \omega^2(\alpha t - 1)\right] , \quad \text{for } 0 \le t \le T \ . \tag{13}$$

This current is a superposition of two currents: one decays exponentially from $t = 0$, the other is a triangular bunch. In the asymptotic limit ($\alpha \to \infty$) the decaying exponential becomes a δ-function and we get $R \to \sqrt{1 + (\omega T)^2}$. Given a bunch length T this is the ultimate transformer ratio for a single mode cavity. In addition, since $V^-(t)$ is constant we see from Eq. (1) that the efficiency η is 100% in this case. Note also that for ωT sufficiently large the doorstep bunch shape gives very similar results to the optimal case.

Numerical Examples

We now compare our theoretical analysis with two numerical examples: the SLAC accelerating structure (many modes) and a plasma wake field accelerator (single mode).

The SLAC Structure

The SLAC structure is far from being an ideal single-mode structure. Nevertheless, it is still interesting to see how it can be used as a collinear wakefield accelerator. The SLAC structure is a constant gradient disk loaded structure, with fundamental frequency $\omega_0/2\pi = 2856$ MHz and with a cell length of 3.5 cm. $V(t)$ was calculated from Eq. (3) using a table of the longitudinal wake field $W_z(t)$ for SLAC.[10]

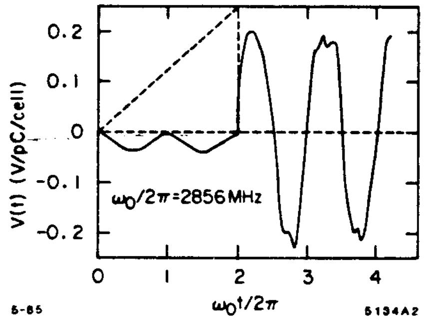

Fig. 2. A triangular bunch in the SLAC structure.

Figure 2 shows the retarding and accelerating potentials due to a triangular bunch in units of volts per pico-coulomb per cell. The bunch length was chosen to be twice the fundamental wavelength. Within the bunch the retarding potential behaves very closely to the single mode calculation, i.e. $V^-(t) \propto 1 - \cos\omega t$. However, some energy goes into the higher modes, as is evident by ripples on the cosine wave behind the bunch. This causes a degradation of the transformer ratio from the single mode prediction $R = 2\pi$ to $R = 4.86$. The degradation worsens as the bunch length gets longer, as can be seen in Fig. 3.

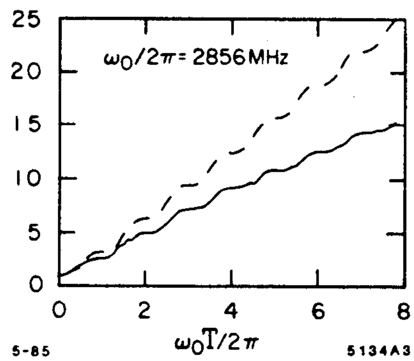

Fig. 3. R for a triangular bunch in SLAC *vs.* bunch length. The dashed line gives single mode results.

The Plasma Wake Field Accelerator(PWFA)

The plasma wake field accelerator is another type of collinear wake field acceleration scheme which replaces the RF cavities by a plasma medium. Since the plasma in this scheme is assumed to be cold, one expects that only a single mode, *i.e.* the oscillation at the plasma frequency, will be excited. A one and two-halves dimensional (x, v_x, v_y, v_z) relativistic, electromagnetic particle code is used to simulate the PWFA.[9] Physically this code corresponds to a one dimensional system where both the plasma and the beam extend infinitely in the transverse directions.

Fig. 4 shows the wake electric field excited in the plasma due to a triangular bunch which is one wave length long. The system has 512 grid points and the number of charges for the beam and plasma are 80 and 16384, respectively. Because of the single mode nature of the system, the transformer ratio as measured from the figure is $R \simeq \pi$, in good agreement with theoretical prediction. Other simulations with different bunch shapes also show good agreement with the theory.[11]

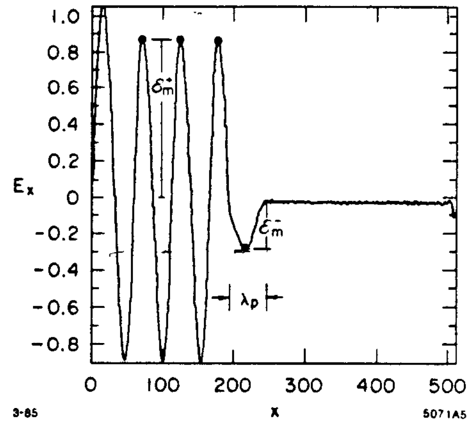

Fig. 4. Wake field of a triangular bunch in a plasma.

Discussion

As a practical example, consider an accelerator operating at $\lambda = 5$ cm with a desired gradient of 100 MV/m. We choose a transformer ratio of 100, so that an energy loss by the driving bunch of 1MV/m must be made up, possibly by induction units or low-frequency RF accelerator cells spaced periodically along the accelerator. In order to achieve $R = 100$, a doorstep bunch with length $cT \approx \lambda R/(2\pi) = 80$ cm is needed. From Eq. (2) we see that the charge in the driving bunch is about 35μC for a typical value of $k/L = 75$ V/pC/m at $\lambda = 5$cm, assuming also that $\eta \approx 100\%$. The peak current at the tail of the driving bunch is about 25 kA.

In this paper we have looked at some aspects of a collinear acceleration scheme, focusing mostly on the concept of transformer ratio. A more complete study of the usefulness of such a scheme needs to address further questions such as transverse effects and the feasability of creating the very high peak current bunches required for high transformer ratios.

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