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On Completely ρ-Irresolute and Weakly ρ-Irresolute Functions

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Abstract. The purpose of this paper is to introduce two new types of irresolute functions called, completely ρ-irresolute functions and weakly ρ-irresolute functions. We obtain their characterizations and their basic properties.

1. Introduction and preliminaries

Functions and of course irresolute functions give new path towards research. In 1972, Crossley and Hildebrand [2] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years, for example see [7,8,10]. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences.

Recently, as generalization of closed sets, the notion of ρ -closed sets were introduced and this notion was further studied by the authors [3,4,5]. In this paper, We will continue the study of related irresolute functions with ρ -open sets. We introduce and characterize the concepts of completely ρ-irresolute functions and weakly ρ-irresolute functions.

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and f : $(X,\tau) \to (Y,\sigma)$ (or simply f : $X \to Y$) denotes a function f of a space (X,τ) into space(Y,σ). Let A be a subset of a space X. The closure and the interior of A are denoted by Cl(A) and Int(A), respectively.

Definition 1.1. Let (X,τ) be a topological space. A subset A of the space X is said to be

1. preopen [17] if $A\subseteq$ int(cl(A)) and preclosed [17] if cl(int(A))$\subseteq$ A.
2. semi-open [14] if $A\subseteq$ cl(int(A)) and semi-closed [1] if int(cl(A)) $\subseteq$A.
3. regular open [21] if A=int(cl(A)) and regular closed [21] if A=cl(int(A)).

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Definition 1.2([7]). Let (X,τ) be a topological space and A$\subseteq$ X.

1. The pre-interior of A, denoted by pint(A), is the union of all preopen subsets of A.

2. The pre-closure of A, denoted by pcl(A), is the intersection of all preclosed sets containing A.

Definition 1.3. Let (X,τ) be a topological space. A subset A$\subseteq$X is said to be

1. $\hat{g}$-closed [23] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is semi open in X.

2. *g-closed [25] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\hat{g}$-open in X.

3. #g- semi closed (briefly #gs-closed)[24] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is *g-open in X.

4. $\tilde{g}$-closed set [12] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is #gs-open in X.

5. ρ-closed set [3] if pcl(A)$\subseteq$Int(U) whenever A$\subseteq$U and U is $\tilde{g}$-open in X.

The complements of the above mentioned sets are called their respective open sets.

The family of all ρ -open subsets of (X,τ) is denoted by $\rho O(X)$. We set $\rho O(X, x)$ = {V$\in\rho$O(X)|x $\in$V} for x $\in$X.

The intersection of all ρ-closed sets containing a set A in a space X is called the ρ-closure of A and is denoted by ρ-cl(A)[3].

Theorem 1.4([3]). *Every preclosed subset of X is ρ-closed.*

Lemma 1.5([3]). *If A is a ρ-closed set then ρ-cl(A) = A. Converse need not be true.*

Theorem 1.6([3]). *A subset A of (X,τ) is regular open if A is both open and ρ-closed.*

Definition 1.7. A space (X,τ) is called locally indiscrete space [18] if every open subset of X is closed.

Definition 1.8. A function f : (X,τ) $\rightarrow$ (Y,σ) is called:

1. strongly continuous [13]if f $^{-1}$(V) is both open and closed in X for each subset V of Y.

2. perfectly ρ-continuous [4] if f $^{-1}$(V) is clopen in X for every ρ-closed (resp. ρ-open) subset V of Y.

3. ρ -irresolute [3] if f^{-1}(V) is ρ -closed (resp. ρ -open) in X for every ρ-closed (resp.ρ-open) subset V of Y.

4. contra-ρ-irresolute if f $^{-1}$(V) is ρ-open (resp. ρ-closed) in X for every ρ-closed (resp.ρ-open) subset V of Y.

2. Completely ρ-irresolute functions

Definition 2.1. A function f : X$\rightarrow$Y is called completely ρ-irresolute if the inverse image of each ρ-open subset of Y is regular open in X.

Example 2.2. Let X = { a, b, c } = Y with topologies τ = { ϕ,{a},{b,c},X } and

σ = {ϕ,{a},{c},{c,a},Y}. Define f : (X,τ) → (Y,σ) by f(a) = a; f(b) = f(c) = c. Then the function f is completely ρ-irresolute.

Theorem 2.3. *Every strongly continuous function is completely ρ-irresolute.*

Proof. Let V be ρ-open subset of Y. By hypothesis, f $^{-1}$(V) is both open and closed in X. Since f $^{-1}$(V) is preclosed and by Theorem 1.4, f $^{-1}$(V) is ρ-closed in X and also by Theorem 1.6, f $^{-1}$(V) is regular open in X. Hence f is completely ρ-irresolute. □

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.4. Let X = {a,b,c,d}, Y = {a,b,c} with topologies τ = {ϕ,{b},{c},{b,c}, X} and σ = {ϕ,{c},{a},{c,a},Y}. Define f : (X,τ) → (Y,σ) by f(a) = f(b) = f(d) = b; f(c) = c is completely ρ-irresloute but not Strongly continuous.

Theorem 2.5. *Every completely ρ-irresolute function on a locally indiscrete space is contra-ρ-irresolute.*

Proof. Let V be ρ-open subset of Y. By hypothesis, f $^{-1}$(V) is regular open in X. Since X is a locally indiscrete space, f^{-1}(V) is closed and hence it is preclosed in X . Then by Theorem 1.4, f $^{-1}$(V) is ρ-closed in X and so f is contra-ρ-irresolute. □

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.6. Let X = { a, b, c } = Y with topologies τ = { ϕ, {a}, {b,c}, X } and σ = {ϕ, {a,b}, Y}. Then the identity function f : (X,τ) → (Y,σ) is contra-ρ-irresloute but not completely ρ-irresloute.

Theorem 2.7. *The following statements are equivalent for a function f : X → Y*

(*i*) *f is completely ρ-irresolute;*

(*ii*) *f $^{-1}$(F) is regular closed in X for every ρ-closed set F of Y.*

Proof. (i)⇒(ii): Let F be any ρ-closed set of Y. Then Y\F is ρ-open set in Y. By (i), f $^{-1}$(Y\F) = X - f $^{-1}$(F) is regular open in X. We have f $^{-1}$(F) is regular closed in X.

Converse is similar. □

Lemma 2.8([5]). *Let S be an open subset of a space (X, τ). Then the following hold:*

(*i*) *If U is regular open in X, then so is U∩S in the subspace (S, τ_s).*

(*ii*) *If B⊂S is regular open in (S, τ_s). then there exists a regular open set U in (X, τ) such that B = U∩S.*

Theorem 2.9. *If f : (X, τ) →(Y, σ) is a completely ρ-irresolute function and A is any open subset of X, then the restriction $f|_A$: A →Y is completely ρ-irresolute.*

Proof. Let F be a ρ-open subset of Y. By hypothesis, f $^{-1}$(F) is regular open in X.

Since A is open in X, it follows from the previous lemma that $(f|_A)^{-1}(F) = A \cap f^{-1}(F)$, which is regular open in A. Therefore, $f|_A$ is completely ρ-irresolute. □

Theorem 2.10. *The following hold for function $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$:*

(*i*) *If f is completely ρ-irresolute and g is perfectly ρ-continuous, then $g \circ f$ is completely ρ-irresolute.*

(*ii*) *If f is completely ρ-irresolute and g is ρ-irresolute, then $g \circ f$ is completely ρ-irresolute.*

(*iii*) *If f is completely ρ-irresolute and g is strongly continuous functions, then $g \circ f$ is completely ρ-irresolute.*

Proof. The proof of the theorem is obvious and hence omitted. □

Definition 2.11. A space X is said to be ρ-connected [4] (resp. almost connected [9]) if there does not exist disjoint ρ-open (resp. regular open) sets A and B such that $A \cup B = X$.

Theorem 2.12. *If $f : (X,\tau) \to (Y,\sigma)$ is completely ρ-irresolute surjective function and X is almost connected, then Y is ρ-connected.*

Proof. Suppose that Y is not ρ-connected, Then there exist disjoint ρ-open sets A and B of Y such that $A \cup B = Y$. Since f is completely ρ-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X. Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is ρ-connected. □

Definition 2.13. A space *X* is said to be

(i) nearly compact [19, 20] if every regular open cover of X has a finite subcover;

(ii) nearly Lindelof [9] if every cover of X by regular open sets has a countable subcover;

(iii) nearly countably compact [11] if every countable cover by regular open sets has a finite subcover;

(iv) ρ-compact [4] if every ρ-open cover of X has a finite subcover;

(v) countably ρ-compact if every ρ-open countable cover of X has a finite subcover;

(vi) ρ-Lindelof if every cover of X by ρ-open sets has a countable subcover.

Theorem 2.14. *Let $f : (X, \tau) \to (Y, \sigma)$ be a completely ρ-irresolute surjective function. Then the following statement hold:*

(*i*) *If X is nearly compact, then Y is ρ-compact.*

(*ii*) *If X is nearly Lindelof, then Y is ρ-Lindelof.*

(*iii*) *If X is nearly countably compact, then Y is countably ρ-compact.*

Proof. (i) Let $f : X \to Y$ be a completely ρ-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha : \alpha \in \Delta\}$ be any ρ-open cover of Y. Then, $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X. Since X is nearly compact, there exists a finite subfamily, { $f^{-1}(U_{\alpha_i})$|i=1,2,.....,n} of {$f^{-1}(U_\alpha)$: $\alpha\in\Delta$} which cover X. It follows that {U_{α_i} : i = 1,2,.....,n} is a finite subfamily of { U_α : $\alpha\in\Delta$} which cover Y. Hence, space Y is a ρ-compact space.

The proof of other cases are similar. □

Definition 2.15. A space (X, τ) is said to be:

(i) ρ-closed compact (resp. S-closed [22]) if every ρ-closed (resp. regular closed) cover of X has a finite subcover;

(ii) countably ρ-closed compact (resp. countably S-closed compact [6]) if every countable cover of X by ρ-closed (resp. regular closed) sets has a finite subcover;

(iii) ρ-closed Lindelof (resp.S-Lindelof [16]) if every cover of X by ρ-closed(resp. regular closed) sets has a countable subcover.

Theorem 2.16. *Let f : (X, τ) → (Y, σ) be a completely ρ-irresolute surjective function. Then the following statements hold:*

(*i*) *If X is S-closed, then Y is ρ-closed compact.*

(*ii*) *If X is S-Lindelof, then Y is ρ-closed Lindelof.*

(*iii*) *If X is countably S-closed-compact, then y is countably ρ-closed compact.*

Proof. (i) Let f : X → Y be a completely ρ-irresolute surjective function. Let {U_α: $\alpha\in\Delta$} be any ρ-closed cover of Y. Then, by Theorem 2.7, {f $^{-1}(U_\alpha)$: $\alpha\in\Delta$} is a regular closed cover of X. Since X is S-closed, there exists a finite subfamily, {f $^{-1}(U_{\alpha_i})$|i=1,2,.....,n} of {f $^{-1}(U_\alpha)$: $\alpha\in\Delta$} which cover X. It follows that {U_{α_i} : i = 1,2,.....,n} is a finite subfamily of { U_α : $\alpha\in\Delta$} which cover Y. Hence, space Y is a ρ-closed compact space.

The proof of other cases are similar. □

Definition 2.17. A space (X,τ) is said to be ρ-T_1(resp. r-T_1 [9]) if for each pair of distinct points x and y of X, there exist ρ-open (resp. regular open) sets $U_1$and U_2 such that $x \in U_1$and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 2.18. *If f : (X,τ) → (Y,σ) is completely ρ-irresolute injective function and Y is ρ-T_1, then X is r-T_1.*

Proof. Let x,y be any distinct points of X. Since f is injective,we have $f(x) \neq f(y)$. Since Y is ρ-T_1, there exist ρ-open sets V and W in Y such that f(x)∈V, f(y)∈W, f(x)∉W and f(y)∉V. Since f is completely ρ-irresolute, f^{-1}(V) and f^{-1}(W) are regular open sets in X such that $x\in f^{-1}(V)$, y∈f^{-1}(W), $x\notin f^{-1}(W)$ and $y\notin f^{-1}(V)$. This shows that X is r-T_1. □

Definition 2.19. A space (X,τ) is said to be ρ-T_2 if for each pair of distinct points x and y in X, there exist disjoint ρ-open sets $U_1$and U_2 in X such that $x\in U_1$and $y\in U_2$.

Theorem 2.20. *If f : (X,τ) → (Y,σ) is completely ρ-irresolute injective function and Y is ρ-T_2, then X is T_2.*

Proof. Let x,y be any distinct points of X. Since f is injective, we have f(x)≠f(y).

Since Y is ρ-T_2, there exist ρ-open sets V and W in Y such that f(x)∈V, f(y)∈W and V∩W = ϕ. Since f is completely ρ-irresolute, f^{-1}(V) and f^{-1}(W) are regular open sets in X such that x∈f^{-1}(V), y∈f^{-1}(W) and f^{-1}(V)∩f^{-1}(W) = ϕ. This shows that X is T_2. □

Definition 2.21. A function f : X → Y is called ρ*-closed [5] if the image of each ρ-closed set of X is a ρ-closed set in Y.

Theorem 2.22. *If a mapping f : X → Y is ρ*-closed, then for each subset B of Y and each ρ-open set U of X containing f^{-1}(B), there exists a ρ-open set V in Y containing B such that f^{-1}(V)⊂ U.*

Proof. Suppose that f is ρ*-closed map. Let B⊂Y and U be ρ-open set of X such that f^{-1}(B)⊂U. Since f is ρ*-closed, $(f(U^c))^c$ = V is a ρ-open set in Y containing B such that f^{-1}(V)⊂U. □

Definition 2.23. A space X is said to be strongly ρ-normal (resp. mildly ρ-normal) if for each pair of distinct ρ-closed (resp. regular closed) sets A and B of X, there exist disjoint ρ-open sets U and V such that A⊂U and B⊂V.

Theorem 2.24. *If f : X → Y is completely ρ-irresolute, ρ*-closed function from a mildly ρ-normal space X onto a space Y, then Y is strongly ρ-normal.*

Proof. Let A and B be two disjoint ρ-closed subsets of Y. Since f is completely ρ-irresolute, f^{-1}(A) and f^{-1}(B) are disjoint regular closed subsets of X. Since X is mildly ρ-normal space, there exist disjoint ρ -open sets U and V in X such that f^{-1}(A)⊂ U and f^{-1}(B)⊂V. Since f is ρ*-closed, f(X\U) and f(X\V) are ρ-closed sets in Y. Then by Theorem 2.22, there exist ρ-open sets G = Y \f(X\U) and H = Y\f(X\V) containing A and B such that f^{-1}(G)⊂U and f^{-1}(H)⊂V. Clearly, G and H are disjoint ρ-open subsets of Y. Hence, Y is strongly ρ-normal. □

Definition 2.25. (i) A space X is said to be strongly ρ-regular if for each ρ-closed set F and each point x∉F, there exists disjoint ρ-open sets U and V in X such that x∈U and F⊂V.

(ii) A space X is said to be almost ρ-regular if for each regular closed subset F and each point x∉F, there exists disjoint ρ-open sets U and V in X such that x∈U and F⊂V.

Theorem 2.26. *If f is a completely ρ-irresolute, ρ*-closed injection of an almost ρ-regular space X onto a space Y, then Y is strongly ρ-regular space.*

Proof. Let F be a ρ-closed subset of Y and let y∉F. Then, f^{-1}(F) is regular closed subset of X such that $f^{-1}(y) = x$∉f^{-1}(F). Since X is almost ρ -regular space, there exists disjoint ρ -open sets U and V in X such that $f^{-1}(y)$ ∈ U and f^{-1}(F) ⊂ V. Since f is ρ∗-closed and by Theorem 2.22, there exist ρ -open sets G = Y\f(X\U) such that f^{-1}(G)⊂U, y∈G and H = Y\f(X\V) such that f^{-1}(H)⊂V, F⊂H. Clearly, G and H are disjoint ρ-open subsets of Y. Hence, Y is strongly ρ-regular space.

3. Weakly ρ-irresolute functions

Definition 3.1. A function f :X $\rightarrow$ Y is said to be weakly ρ-irresolute if for each point $x$$\in$X and each V$\in\rho$O(Y, f($x$)), there exists a U$\in\rho$O(X,$x$) such that f(U) $\subset$ ρ-cl(V).

Example 3.2. Let X = {a,b,c} = Y with topologies τ = {ϕ,{a,b},X} and σ = {ϕ,{a},{b},{a,b},Y}. Then the identity function f : (X,τ) $\rightarrow$ (Y,σ) is weakly ρ-irresloute.

It is evident that every ρ-irresolute function is weakly ρ-irresolute but the converse is not true.

Example 3.3. Let X = Y = {a,b,c} with topologies τ= {ϕ,{c},{b,c},X} and σ= {ϕ,{a},{b},{a,b},Y}. Then the identity function f : (X,τ) $\rightarrow$(Y,σ) is clearly weakly ρ-irresolute but not ρ-irresolute. Observe that for the ρ-closed set V = {c} in (Y,σ), f^{-1}(V) = {c} is not ρ-closed in (X,τ).

Definition 3.4. A space (X,τ) is called a ρ-$T_s$space [4] if every ρ-closed (resp. ρ-open) set is closed (resp. open).

Theorem 3.5. *A function f : X $\rightarrow$ Y is weakly ρ-irresolute if the graph function, defined by g(x) = (x,f(x)) for each x$\in$X, is weakly ρ-irresolute.*

Proof. Let $x \in$ X and V $\in \rho$O(Y,f(x)). Then X $\times$ V is a ρ-open set of X $\times$ Y containing g(x). Since g is weakly ρ-irresolute, there exists U$\in\rho$O(X,x) such that g(U)$\subset\rho$-cl(X $\times$V)$\subset$X $\times\rho$-cl(V). Therefore, we have f(U)$\subset\rho$-cl(V). □

Lemma 3.6. *A locally indiscrete ρ-$T_s$topological space is ρ-T_2 if and only if for each pair of distinct points x,y$\in$X, there exist U$\in\rho$O(X,x) and V$\in\rho$O(X,y) such that ρ-cl(U)$\cap\rho$-cl(V) = ϕ.*

Proof. This follows immediately from the definitions of locally indiscrete ρ-T_s space and Lemma 1.5. □

Theorem 3.7. *If a locally indiscrete ρ-$T_s$space Y is ρ-T_2space and f : X $\rightarrow$Y is a weakly ρ-irresolute injection, then X is ρ-T_2.*

Proof. Let x,y be any two distinct points of X. Since f is injective, we have f(x)$\neq$f(y). Since Y is locally indiscrete ρ-T_s and ρ-T_2 space, by Lemma 3.6 there exists V$\in\rho$O(Y,f(x)) and W$\in\rho$O(Y,f(y)) such that ρ-cl(V)$\cap\rho$-cl(W) = ϕ. Since f is weakly ρ-irresolute, there exists G$\in\rho$O(X,x) and H$\in\rho$O(X,y) such that f(G) $\subset\rho$-cl(V) and f(H) $\subset\rho$-cl(W). Hence we obtain G$\cap$H = ϕ. This shows that X is ρ-T_2.□

Definition 3.8. A function f : X $\rightarrow$Y is said to have strongly ρ-closed graph if for each (x,y)$\in$(X$\times$Y)-G(f), there exist U$\in\rho$O(X,x) and V$\in\rho$O(Y,y) such that [ρ-cl(U)$\times\rho$-cl(V)]$\cap$G(f) = ϕ.

Theorem 3.9. *If a function f : X $\rightarrow$ Y is weakly ρ-irresolute, injective and G(f) is strongly ρ-closed, then X is ρ-T_2.*

Proof. Let x,y be a pair of distinct points of X. Since f is injective, f(x)$\neq$f(y)

and $(x,\mathrm{f}(y))\notin\mathrm{G(f)}$. Since G(f) is strongly ρ-closed, there exist $\mathrm{G}\in\rho\mathrm{O}(\mathrm{X},x)$ and $\mathrm{V}\in\rho\mathrm{O}(\mathrm{Y},\mathrm{f}(y))$ such that $\mathrm{f}(\rho\text{-cl}(\mathrm{G}))\cap\rho\text{-cl}(\mathrm{V}) = \phi$. Since f is weakly ρ-irresolute, there exists $\mathrm{H}\in\rho\mathrm{O}(\mathrm{X},y)$ such that $\mathrm{f(H)}\subset\rho\text{-cl}(\mathrm{V})$. Hence we have $\mathrm{f}(\rho\text{-cl}(\mathrm{G}))\cap\mathrm{f(H)} = \phi$; hence $\mathrm{G}\cap\mathrm{H} = \phi$. This shows that X is ρ-T_2. □

4. Conclusion

The authors introduce the classes of completely ρ-irresolute functions and weakly ρ-irresolute functions. Properties of completely ρ-irresolute functions and weakly ρ-irresolute functions are investigated. In the literature, many topologists investigated several irresoluteness for some questions in topology.

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