

ON DECOMPOSITION OF FUZZY A -CONTINUITY

S.JAFARI¹, K.VISWANATHAN^{2*}, M.RAJAMANI² AND S.KRISHNAPRAKASH²

ABSTRACT. In this paper, we introduce and study the notion of fuzzy C -sets and fuzzy C -continuity. We also prove a mapping $f : X \rightarrow Y$ is fuzzy A -continuous if and only if it is both fuzzy semi-continuous and fuzzy C -continuous.

1. INTRODUCTION

In the classical paper [10] of 1965, Zadeh generalized the usual notion of a set by introducing the important and useful notion of fuzzy sets. Since then, this notion has had tremendous effect on both pure and applied mathematics in different respects. Recently El-Naschie has shown in [4] and [5] that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and ε^∞ theory. In 1986, Tong [9] introduced the notion of A -sets and A -continuous mappings in topological spaces and proved that a mapping is continuous if and only if it is both α -continuous and A -continuous. In 1990, Ganster [7] established a decomposition of A -continuity: A mapping $f : X \rightarrow Y$ is A -continuous if and only if it is both semi-continuous and LC-continuous. Erguang and Pengfei [6] introduced the notion of C -sets and C -continuity and obtained another decomposition of A -continuity: A mapping $f : X \rightarrow Y$ is A -continuous if and only if it is both semi-continuous and C -continuous. Recently, Rajamani and Ambika [8] introduced the notion of fuzzy A -sets and fuzzy A -continuity and obtained a decomposition of fuzzy continuity. In this paper, we transform the notions of C -set and C -continuity to fuzzy topological settings and obtain a decomposition of fuzzy A -continuity.

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* Corresponding author.

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2. PRELIMINARIES

Throughout this paper, X and Y denote fuzzy topological spaces (X, τ) and (Y, σ) respectively on which no separation axioms are assumed. Let λ be a fuzzy set in a fuzzy topological spaces X . The fuzzy interior of λ , fuzzy closure of λ and fuzzy preclosure of λ are denoted by $int(\lambda)$, $cl(\lambda)$ and $pcl(\lambda)$ respectively.

Now, we recall some definitions and results which are used in this paper.

DEFINITION 2.1: A fuzzy set λ in a fuzzy topological space X is called

- (1) fuzzy semi-open [1] if $\lambda \leq cl(int(\lambda))$;
- (2) fuzzy pre-open [2] if $\lambda \leq int(cl(\lambda))$;
- (3) fuzzy regular-open [1] if $\lambda = int(cl(\lambda))$.

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

DEFINITION 2.2: A fuzzy set λ in a fuzzy topological space X is called a fuzzy A -set [6] if $\lambda = \alpha \wedge \beta$, where α is a fuzzy open set and β is a fuzzy regular closed set.

DEFINITION 2.3: A map $f : X \rightarrow Y$ is said to be

- (1) fuzzy continuous [3] if $f^{-1}(\mu)$ is fuzzy open in X , for every fuzzy open set μ in Y ;
- (2) fuzzy semi-continuous [1] if $f^{-1}(\mu)$ is fuzzy semi-open in X , for every fuzzy open set μ in Y ;
- (3) fuzzy pre-continuous [2] if $f^{-1}(\mu)$ is fuzzy pre-open in X , for every fuzzy open set μ in Y ;

The collection of all fuzzy C -sets and fuzzy semi-open sets in X will be denoted by $FC(X, \tau)$ and $FSO(X, \tau)$ respectively.

3. FUZZY C -SETS

DEFINITION 3.1: A fuzzy set λ in a fuzzy topological space X is called a fuzzy C -set if $\lambda = \alpha \wedge \beta$, where α is fuzzy open and β is fuzzy pre-closed in X .

PROPOSITION 3.2: Every fuzzy A -set is a fuzzy C -set.

REMARK 3.3: The converse of the Proposition 3.2. need not be true as seen from the following example.

EXAMPLE 3.4: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \rightarrow [0, 1]$ by

$$\alpha_1(a) = 0.3 \quad \alpha_2(a) = 0.4 \quad \alpha_3(a) = 0.7$$

$$\alpha_1(b) = 0.4 \quad \alpha_2(b) = 0.5 \quad \alpha_3(b) = 0.6$$

$$\alpha_1(c) = 0.4 \quad \alpha_2(c) = 0.5 \quad \alpha_3(c) = 0.6$$

Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X, τ) is a fuzzy topological space. Now, α_3 is a

fuzzy C -set but not a fuzzy A -set.

REMARK 3.5: The concepts of fuzzy C -sets and fuzzy semi-open sets are independent as shown by the following examples.

EXAMPLE 3.6: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \rightarrow [0, 1]$ by

$$\alpha_1(a) = 0.2 \quad \alpha_2(a) = 0.3 \quad \alpha_3(a) = 0.3$$

$$\alpha_1(b) = 0.3 \quad \alpha_2(b) = 0.3 \quad \alpha_3(b) = 0.3$$

$$\alpha_1(c) = 0.3 \quad \alpha_2(c) = 0.4 \quad \alpha_3(c) = 0.3$$

Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X, τ) is a fuzzy topological space. Now, α_3 is a fuzzy semi-open set but not a fuzzy C -set.

EXAMPLE 3.7: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \rightarrow [0, 1]$ by

$$\alpha_1(a) = 0.4 \quad \alpha_2(a) = 0.6 \quad \alpha_3(a) = 0.5$$

$$\alpha_1(b) = 0.5 \quad \alpha_2(b) = 0.7 \quad \alpha_3(b) = 0.6$$

$$\alpha_1(c) = 0.6 \quad \alpha_2(c) = 0.8 \quad \alpha_3(c) = 0.7$$

Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X, τ) is a fuzzy topological space. Now, α_3 is a fuzzy C -set but not a fuzzy semi-open set.

LEMMA 3.8: Let α be a fuzzy set in a fuzzy topological space X . Then $\alpha \in FC(X, \tau)$ if and only if $\alpha = \lambda \wedge pcl(\alpha)$ for some fuzzy open set λ .

Proof: Let $\alpha \in FC(X, \tau)$. Then $\alpha = \lambda \wedge \mu$ where λ is fuzzy open and μ is fuzzy pre-closed. Now, $\alpha \leq \lambda$ and $\alpha \leq \mu$, we have $pcl(\alpha) \leq pcl(\mu) = \mu$, since μ is fuzzy pre-closed in X . Thus $pcl(\alpha) \leq \mu$. Therefore $\lambda \wedge pcl(\alpha) \leq (\lambda \wedge \mu) = \alpha \leq \lambda \wedge pcl(\alpha)$. (i.e.,) $\lambda \wedge pcl(\alpha) = \alpha$.

Converse part is obvious.

THEOREM 3.9: Let α be a fuzzy set in a fuzzy topological space X . Then $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ if and only if $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$.

Proof: Let $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ in X . Then $\alpha \leq cl(int(\alpha))$. So α is fuzzy semi open in X . Let $\beta = cl(int(\alpha))$, then β is fuzzy regular closed. Since every fuzzy regular closed set is fuzzy pre-closed, β is fuzzy pre-closed which implies α is fuzzy C -set. Thus $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$.

Conversely, let $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$. Then $\alpha \in FC(X, \tau)$ and $\alpha \in FSO(X, \tau)$. Since $\alpha \in FC(X, \tau)$, $\alpha = \lambda \wedge pcl(\alpha)$, using Lemma 3.8. Thus $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ .

4. DECOMPOSITION OF FUZZY A -CONTINUITY

DEFINITION 4.1: A mapping $f : X \rightarrow Y$ is called fuzzy A -continuous if $f^{-1}(\mu)$ is a fuzzy A -set in X , for every fuzzy open set μ in Y .

DEFINITION 4.2: A mapping $f : X \rightarrow Y$ is called fuzzy C -continuous if $f^{-1}(\mu)$ is a fuzzy C -set in X , for every fuzzy open set μ in Y .

PROPOSITION 4.3: Every fuzzy A -continuous function is fuzzy C -continuous.

REMARK 4.4: The converse of Proposition 4.3 need not be true as shown by the following example.

EXAMPLE 4.5: Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :

$$\alpha_1(a) = 0.3 \quad \alpha_2(a) = 0.4 \quad \alpha_3(a) = 0.7$$

$$\alpha_1(b) = 0.4 \quad \alpha_2(b) = 0.5 \quad \alpha_3(b) = 0.6$$

$$\alpha_1(c) = 0.4 \quad \alpha_2(c) = 0.5 \quad \alpha_3(c) = 0.6$$

Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}$, $\tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is fuzzy C -continuous but not fuzzy A -continuous.

REMARK 4.6: The concepts of fuzzy C -continuity and fuzzy semi-continuity are independent as shown by the following examples.

THEOREM 4.7: Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :

$$\alpha_1(a) = 0.2 \quad \alpha_2(a) = 0.3 \quad \alpha_3(a) = 0.3$$

$$\alpha_1(b) = 0.3 \quad \alpha_2(b) = 0.3 \quad \alpha_3(b) = 0.3$$

$$\alpha_1(c) = 0.3 \quad \alpha_2(c) = 0.4 \quad \alpha_3(c) = 0.3$$

Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}$, $\tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is fuzzy semi-continuous but not fuzzy C -continuous.

EXAMPLE 4.8: Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :

$$\alpha_1(a) = 0.4 \quad \alpha_2(a) = 0.6 \quad \alpha_3(a) = 0.5$$

$$\alpha_1(b) = 0.5 \quad \alpha_2(b) = 0.7 \quad \alpha_3(b) = 0.6$$

$$\alpha_1(c) = 0.6 \quad \alpha_2(c) = 0.8 \quad \alpha_3(c) = 0.7$$

Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}$, $\tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is fuzzy C -continuous but not fuzzy semi-continuous.

THEOREM 4.9: A mapping $f : X \rightarrow Y$ is fuzzy A -continuous if and only if it is both fuzzy semi-continuous and fuzzy C -continuous.

Proof: Follows from Theorem 3.9.

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¹ COLLEGE OF VESTSJAELLAND SOUTH
HERRESTRAD 11
4200 SLAGELSE
DENMARK.

E-mail address: jafari@stofanet.dk

² POST GRADUATE AND RESEARCH DEPARTMENT OF MATHEMATICS
N G M COLLEGE
POLLACHI-642 001
TAMILNADU, INDIA.

E-mail address: visu_ngm@yahoo.com