On-Demand Charging in Wireless Sensor Networks: Theories and Applications

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***Abstract*—Recently, adopting mobile energy chargers to replenish the energy supply of sensor nodes in wireless sensor networks has gained increasing attention from the research community. The utilization of the mobile energy chargers provides a more reliable energy supply than the systems that harvested dynamic energy from the surrounding environment. While pioneering works on the mobile recharging problem mainly focus on the optimal *offline* path planning for the mobile chargers, in this work, we aim to lay the theoretical foundation for the *on-demand* mobile charging problem, where individual sensor nodes request charging from the mobile charger when their energy runs low. Specifically, in this work we analyze the on-demand mobile charging problem using a simple but efficient *Nearest-Job-Next with Preemption* (NJNP) discipline for the mobile charger, and provide analytical results on the system throughput and charging latency from the perspectives of the mobile charger and individual sensor nodes, respectively. To demonstrate how the actual system design can benefit from our analytical results, we present an example on determining the optimal remaining energy level for individual sensor nodes to send out their recharging requests. Through extensive simulation with real-world system settings, we verify our analysis matches the simulation results well and the system designs based on our analysis are effective.**

I. Introduction

For real-world sensor network applications, sensor nodes are usually powered by on-board batteries or super-capacitors [1]–[6]. This limited energy supply makes energy the most precious resource in the system, and thus its efficient usage is highly crucial.

Recently, research efforts begin to explore the concept of mobile energy chargers to replenish the energy supply of individual sensor nodes to improve the system sustainability [7]–[9]. Prototypes of such mobile charging are implemented in [10], [11]. For most existing research, researchers have mainly focused on the *offline* scenarios [7], [8], in which the charging of individual nodes is carried out in a *periodic* and *deterministic* manner. However, due to the close interaction with the surrounding environment, the energy consumption profiles of the nodes in the network demonstrate high diversity. Furthermore, for sensor nodes integrated with energy-harvesting modules, the amount of the harvested energy also fluctuates greatly [1], [2]. The uncertainty in both the energy demand and supply indicates that existing periodic charging solutions may suffer from non-negligible performance degradation.

Observing the limitation of existing solutions, in this paper we investigate the on-demand energy replenishment in wireless sensor networks, and aim to lay the theoretical foundation for such an on-demand charging process. Specifically, we are interested in answering questions such as *how the mobile charger should schedule and carry out the charging of individual nodes without a priori knowledge of the demands raised in the near future, and what the achievable performance is*. We analytically investigate a simple but efficient *Nearest-Job-Next with Preemption* (NJNP) discipline for the on-demand mobile charging problem, which schedules the charging of individual nodes according to their spatial and temporal properties. We prove the performance of NJNP within constant factors of the optimal solutions. We also present an example on how these analyses guide the system implementations.

Specifically, our contributions in this paper include:

- We mathematically formulate the on-demand mobile charging problem in wireless sensor networks, and theoretically establish the charging performance with the *Nearest-Job-Next with Preemption* discipline. These analytical results not only shed light on the performance of NJNP, but also provide useful guidances for the design of more sophisticated charging schemes. To the best of our knowledge, this is the first in-depth work to analytically evaluate the performance of the on-demand mobile charging in sensor networks.
- We present four theorems that demonstrate the asymptotic performance of NJNP is within constant factors of the optimal results with respect to the system throughput and charging latency of individual nodes, under both light and heavy charging demands in the system.
- With these analytical results on NJNP, we use an example to demonstrate how our results can guide the system design choices for real-world sensor networks. Specifically, we discuss how to determine the optimal remaining energy level for individual nodes to send out their recharging requests.

We emphasize that while we demonstrate a promising performance of NJNP, we are not suggesting that NJNP is the *best* discipline and should be directly adopted for system implementations. Instead, the purpose of this work is to establish a solid foundation for understanding the performance of NJNP-based on-demand charging processes and provide guidances on designing more advanced charging schemes and

Fig. 1. Lab settings for WISP-based charging efficiency measurements.

determining optimal system parameters. Furthermore, with the theoretically proven performance, NJNP also serves as a perfect benchmark to evaluate more advanced schemes.

The paper is organized as follows. The on-demand mobile charging problem is formally presented in Section II. In Section III, we analytically evaluate the performance of NJNP and prove its optimality. An example on guiding the system design is presented in Section IV. The evaluations of our work are presented in Section V. Section VI briefly reviews the literature, and we conclude this paper in Section VII.

II. Preliminaries

A. Problem Statement

With the advance of energy transferring technologies, the time to replenish the energy supply of sensor nodes has been dramatically reduced [12].

In our previous work [2], we implemented an energy sharing system (using capacitor-array powered MicaZ motes) transfers energy from energy-rich to energy-hungry nodes with an efficiency of around 90%. Furthermore, from our empirical results in [1], the time to charge a 10 F super-capacitor to a voltage of 2.5 V is normally in the order of tens of seconds. This continuously shortened charging time makes adopting mobile chargers to replenish the energy supply of individual nodes a promising approach to achieving a stable and sustainable network operation.

Most existing research efforts adopt the wireless charging technology for the mobile charger to replenish node energy [8], [10], [11]. Although the wireless charging technology has advanced, it still suffers from a low charging efficiency. Experiment measurement results show that the efficiency of the PowerCast wireless charging solution [13] is only around 1.5% even with a short distance of 10 cm between the transceiver and receiver [10]. Our own measurements on the Wireless Identification and Sensing Platform (WISP) [14] also demonstrate this extremely low efficiency (Fig. 1). On the other hand, the *direct-contact* based energy transferring technologies can achieve a much higher efficiency. For example, it is reported in [15] that the 2-D Waveguide Power Transmission technology can achieve an energy efficiency of 87.7%. The contact-based charging technology is also adopted in *off-the-shelf* products such as iRobot Roomba [16]. Observing the efficiency issue, in this work we focus on the scenario where the mobile charger replenishes the energy supply of nodes with direct-contact charging technologies.

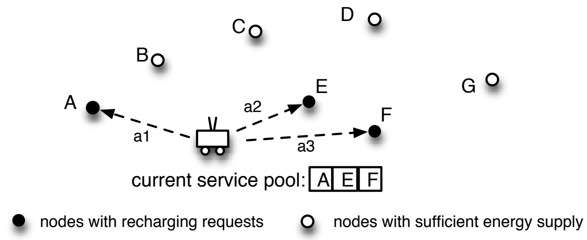

Fig. 2. On-demand charging of nodes: a_1, a_2, and a_3 represent the corresponding distances to requesting nodes A, E, and F.

For the on-demand mobile charging problem, we consider wireless sensor networks where nodes are randomly deployed to monitor the environment. Nodes actively monitor their residual energy [10], which can be realized either by the nodes themselves, or by adopting specialized companion modules to improve the accuracy [2]. Nodes send out charging requests to mobile chargers when the nodes energy levels fall below a certain threshold. Such lightweight communications can be accomplished by adopting the *state-of-the-art* protocols for tracking and communicating with the mobile charger [17]. The request delivery time is assumed to be negligible when compared with the travel and charging times [3]. The mobile charger maintains a service pool to store the received requests, and serves them according to its charging discipline. By serving a request, we mean the mobile charger moves to the corresponding node and fully charges it. For the ease of presentation, the terms *serve a charging request* and *replenish the energy supply of the corresponding node* are used interchangeably. An example of the on-demand mobile charging process is presented in Fig. 2, where three charging requests from nodes A, E, and F have been received, and the charger needs to select one of them as the next node to charge.

Different from the offline charging scenario, the charging tasks in the on-demand scenario exhibit highly dynamic properties in both the *temporal* dimension, i.e., when a new charging request arrives, and the *spatial* dimension, i.e., where the new request comes from (or which request is sent by which node). Such dynamic properties suggest that our design for the on-demand mobile charging should shift from the optimal *path planning* as that in the offline scenarios [7], [8], [18], to the design of efficient scheduling disciplines to select the next to-be-charged node.

B. Nearest Job Next with Preemption

The simplest and most intuitive scheduling discipline is *First-Come-First-Serve* (FCFS), whose performance has been extensively studied by the queuing theory community [19]. However, FCFS schedules the incoming charging requests based only on their *temporal* property and could lead to the back-and-forth charger movement in the *spatial* dimension [10].

Observing the limitation of FCFS, in this work, we explore another discipline, *Nearest-Job-Next with Preemption* (NJNP), which considers both the *spatial* and *temporal* properties of incoming requests. NJNP allows the mobile charger to switch to a spatially closer target node if the new requesting node is closer to the mobile charger. More specifically, under NJNP, *each charging completion of nodes and the arrival of new charging requests trigger the re-selection of the next to-be-charged node, and the mobile charger selects the spatially closest requesting node at that time as the next node to charge*. Clearly, with the contact-based charging technologies, the preemption of charging tasks can occur only before the mobile charger reaches the target node, after which the distance between them can be mathematically treated as zero.

For clarity, in this paper we consider the case where only one mobile charger is available in the network. This single mobile charger scenario also serves as a foundation to address the mobile charging problem when multiple mobile chargers are available, which is left as a future work.

III. On-Demand Charging with NJNP

In this section, we investigate the on-demand mobile charging with NJNP, which can be evaluated from two aspects.

- **Throughput**: From the view of the mobile charger, the *throughput* of the charging process defined as the number of requests the mobile charger can serve during a given time period is the essential metric to evaluate the capability of the system in providing the charging service to individual nodes. In general, a higher system throughput indicates a shorter charger travel time, which in turn means a lower charger energy consumption.
- **Charging Latency**: On the other hand, the *charging latency* of the request defined as the time since the request is sent by a node to the time it is fully charged is what the nodes care about the most.

In what follows, we analytically investigate the charging process from these two aspects, respectively.

For the mobile charger, the selection of requests from its service pool demonstrates a clear queuing behavior, which inspires us to adopt a queuing model to investigate and analyze the charging process: the mobile charger serves as the *server* and the charging requests sent by nodes are treated as *clients*. More specifically, an $M/G/1/NJNP$ queuing model is adopted, as shown in Fig. 3. The soundness of this model in capturing the on-demand mobile charging problem is further verified in Section V. Similar queue-based approaches have been adopted in [20], [21] on the problem of mobile data collection in sensor networks. However, besides the different service disciplines adopted, the underlying service processes are different, e.g., the communication time to collect data could be negligible while the charging time is definitely not.

A. System Throughput with NJNP

To serve a request, the mobile charger needs to first move to the corresponding node and then charge it. Thus, the time to serve each request consists of two parts: the travel time and the charging time.

For the charging time, we simplify the analysis by considering the worst-case charging time, T_c, which is needed for the mobile charger to fully charge an energy-depleted node. Our analysis can be easily extended for more dynamic charging times, as will be explained in Section III-A1.

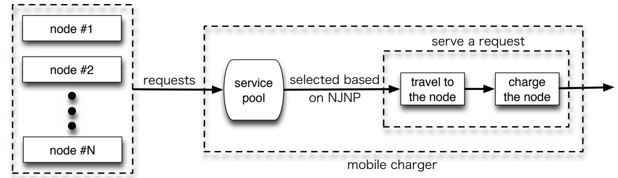

Fig. 3. Demonstration of the queuing model.

Due to the dynamics in both the charger's movement and the requesting nodes set at random time instances, when selecting the next node to charge, the charger's current location and that of the selected node can be viewed as two random locations in the deployment area. The distance between random locations in a specific shape is a well-studied topic in geometric probability. Without loss of generality, we assume a unit square deployment area in this work, and the distance d between two random locations follows the distribution [21]

$$f_D(d) = \begin{cases} 2d(\pi - 4d + d^2) & d \in [0,1] \\ 2d[2\sin^{-1}(\frac{1}{d}) - 2\sin^{-1}\sqrt{1-\frac{1}{d^2}} \\ +4\sqrt{d^2-1} - d^2 - 2] & d \in (1, \sqrt{2}] \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where the second case differs from the first one because for d to be larger than 1, at least one of the two points must fall outside the inscribed circle of the square. The cumulative distribution function (CDF) of the above distribution can be easily calculated as $F_D(d) = \int_0^d f_D(x)dx$. In the case where the deployment field cannot be approximated by a square, we can substitute (1) with corresponding distance distributions for that field without affecting the following analysis.

With NJNP, the mobile charger always selects the nearest requesting node as the next to charge. Intuitively, the more nodes waiting to be charged, the more likely for the mobile charger to find a spatially closer node to charge. Thus we investigate the system throughput under NJNP in two steps: first focus on the case where the number of waiting requests is given, and then extend the analysis to a more general case.

1) Charging with a Given Number of Requests: Consider the case that l requests are waiting to be served when the mobile charger just accomplishes the service of the current request (or a new request is just received by the charger). With the current location of the mobile charger, we can approximately treat the distances from the l requesting nodes to the mobile charger as l independent and identically distributed (i.i.d.) random variables conforming to $f_D(d)$[1]. We

[1] In systems which require the coordinated operation among spatially closely located nodes, the requests of energy replenishment from these nodes may show certain correlation. Although this correlation may make our analysis deviate from the reality, it actually improves the performance of NJNP because requesting nodes will be more closely located, and thus our analytical results can be treated as performance lower bounds of NJNP.

will further verify this i.i.d. condition in Section V. Among these l distances (e.g., a_1, a_2, and a_3 in Fig. 2), the mobile charger selects the node (node E in Fig. 2) with the shortest distance (a_2 in Fig. 2) as the next node to charge. Clearly, this is the *first-order statistic* [22], i.e., finding the smallest one of l i.i.d. variables. The distribution of the shortest distance to these l nodes can be calculated by

$$\begin{aligned} F_D(d,l) &= \sum_{i=0}^{l-1} \binom{l}{i} (1-F_D(d))^i F_D(d)^{l-i} \\ &= 1-(1-F_D(d))^l . \end{aligned} \tag{2}$$

Thus the time for the mobile charger to travel through this distance follows a distribution of

$$F_T(t,l) = F_D(vt,l) \quad (0 \le t \le \sqrt{2}/v), \tag{3}$$

where v is the charger travel speed. The maximal possible travel time $\frac{\sqrt{2}}{v}$ happens when the mobile charger locates at one corner of the square area while the requesting node is located at the diagonal corner, and no preemption happens before the mobile charger arrives at that node. Denote the corresponding probability density function (PDF) as $f_T(t,l) = \partial F_T(t,l)/\partial t$.

Still let us assume l requests are in the service pool and the distance between the mobile charger and the closest requesting nodes is d. Therefore, the time for the mobile charger to reach this closest requesting node is $t = \frac{d}{v}$. We discretize t into a sequence of short time slots with a duration of δ_t and define $z = \frac{t}{\delta_t}$, and thus at most one new arrival can occur during each time slot. For the first preemption to occur at the ith time slot ($1 \le i \le z-1$), two conditions must hold: first, a new request has to be received during the ith slot, which happens with probability $\lambda\delta_t$, and λ is the aggregated arrival rate of charging requests at the mobile charger; second, the node sending the new request has to be within distance $d - iv\delta_t$ to the mobile charger, which happens with probability $F_D(d - iv\delta_t)$. Define $q_d(i) = \lambda\delta_t F_D(d-iv\delta_t)$, then the probability for a charging task with an initial travel distance d to be preempted is

$$q_d = 1 - \sum_{i=1}^{z-1} (1-q_d(i)) . \tag{4}$$

If the mobile charger arrives at a requesting node at the ith time slot ($1 \le i \le z-1$), it indicates there exist a requesting node with a distance no longer than $v\delta_t$ from the mobile charger during the $(i-1)$th time slot. Let $p_d^l(i)$ and l_i be the probability that the charger arrives at the destination node at time slot i and the service pool size at time slot i respectively, we observe the following recursive relationship

$$p_d^l(i) = \prod_{j=1}^{i-1} \left(1-p_d^l(j)\right) F_D(v\delta_t, l_{i-1}). \tag{5}$$

For the service pool sizes $\{l_0, l_1, ..., l_{i-1}\}$, because no departure of charging requests occurs between time 0 to time $i-1$, the sequence of service pool sizes is non-decreasing

$$0 \le l_1 - l_0 \le l_2 - l_0 \le \cdots \le l_{i-1} - l_0 \le X_j,$$

where X_j is the number of new charging requests arrived during this time. In a stable system, the number of new charging requests between two consecutive service completions is limited, and thus X_j is normally a small number. Furthermore, because we discretize the time into a sequence of short time slot δ_t, $F_D(v\delta_t, l)$ is a small positive value. Based on the above observations, we use $\beta = F_D(v\delta_t, l)$ to approximate $F_D(v\delta_t, l_i)$ in (5), and thus $p_d^l(i)$ can be calculated by considering another fact that $\sum_{i=1}^{z} p_d^l(i) = 1$. Specifically, we have

$$p_d^l(i) \approx \begin{cases} \prod_{j=1}^{i-1}(1-p_d^l(j))\beta & 1 \le i \le z-1 \\ \\ 1-\sum_{i=1}^{z-1} p_d^l(i) & i = z, \end{cases}$$

and the first and second case occur with probability q_d and $1-q_d$ respectively.

Thus the distribution of the travel time with service pool size l can be calculated as

$$f_{\mathcal{S}'_l}(t) = \int_t^{\sqrt{2}/v} p_x^l(t/\delta_t) \cdot f_T(x,l) dx,$$

and with charging time T_c, the time to replenish the energy supply of a node follows the distribution of

$$f_{\mathcal{S}_l}(t) = f_{\mathcal{S}'_l}(t - T_c) \quad (T_c \le t \le T_c + \sqrt{2}/v). \tag{6}$$

Note that when the charging time of a node is dynamic, we can empirically estimate the charging time profile $f_{\mathcal{C}}(t)$ and adopt the convolution theorem to substitute (6) as $f_{\mathcal{S}_l}(t) \sim f_{\mathcal{S}'_l}(t) * f_{\mathcal{C}}(t)$, where $*$ represents the convolution operation.

2) Charging in the General Case: So far all our analysis is based on the conditional service pool size l, and thus we need to investigate l to generalize our results. An *embedded discrete-time Markov chain* can be observed if we view the service pool size only at requests departure times, and thus the departure-time system size probabilities can be obtained with a Markov chain-based approach by defining [19]

$$\Pr\{X_j = i\} = a_i^j = \int_{T_c}^{\sqrt{2}/v+T_c} \frac{e^{-\lambda t}(\lambda t)^i}{i!} f_{\mathcal{S}_j}(t) dt. \tag{7}$$

Denote the obtained departure-time system size probabilities as $\boldsymbol{\pi}$. While $\boldsymbol{\pi}$ in general is different from the probabilities of steady-state system size, for the $M/G/1$ queue, these two quantities have been proved to be asymptotically identical [19].

Applying $\boldsymbol{\pi}$ to (6), we can derive the general service time distribution of charging requests as

$$F_{\mathcal{S}}(t) = \pi_0 \int_0^t f_{\mathcal{S}_1}(x)dx + \sum_{l=1}^{N} \pi_l \int_0^t f_{\mathcal{S}_l}(x)dx, \tag{8}$$

where N is the number of nodes in the system and $f_{\mathcal{S}}(t) = \partial F_{\mathcal{S}}(t)/\partial t$. The first term on the right side of (8) accounts for the requests arrive at an empty service pool, and the second term corresponds to the requests that experience a busy system upon arrival.

The system throughput with NJNP, $\mathcal{H}_{njnp}$, follows

$$\Pr\{\mathcal{H}_{njnp} < h\} = 1 - F_{\mathcal{S}}(1/h), \tag{9}$$

and the expected system throughput with NJNP is

$$\mathrm{E}[\mathcal{H}_{njnp}] = \frac{1}{\int_{T_c}^{T_c+\sqrt{2}/v} t \cdot f_{\mathcal{S}}(t)dt}. \tag{10}$$

For any stable queuing system, a well-known result is its system utility ρ, defined as the ratio between the clients arrival rate and system service rate, should be smaller than 1. In our on-demand data collection problem, this condition takes the form of $\rho = \lambda \mathrm{E}[\mathcal{H}_{njnp}] < 1$.

3) Optimality w.r.t. System Throughput: In the following we will show that although simple, NJNP can achieve a system throughput that is close to the optimal. Let us denote $\mathrm{E}[\mathcal{H}^*]$ as the optimal system throughput achievable with any online schedule schemes.

First, we have the following theorem on $\mathcal{H}_{njnp}$ with light requests intensity, i.e., when the aggregated request arrival rate at the mobile charger $\lambda \to 0$.

Theorem 1: In terms of the asymptotic system throughput, the performance of NJNP is within a constant factor of the optimal results when the requests intensity in the network is light. Specifically

$$\frac{\mathrm{E}[\mathcal{H}_{njnp}]}{\mathrm{E}[\mathcal{H}^*]} \quad \geq \quad \frac{d_2 + vT_c}{d_1 + vT_c} \quad \text{as} \quad \lambda \to 0,$$

where $d_1 \approx 0.52$ and $d_2 \approx 0.38$.

Proof: Clearly, comparing with the non-preemptive NJN discipline, NJNP further reduces the travel time to reach the target node. Thus the system throughput is also increased. Furthermore, the asymptotical service time with NJN is shown to be shorter than that with FCFS in [20], and thus we have the following relationship

$$\mathrm{E}[\mathcal{H}_{njnp}] \geq \mathrm{E}[\mathcal{H}_{njn}] \geq \mathrm{E}[\mathcal{H}_{fcfs}] = \frac{1}{d_1/v + T_c}, \tag{11}$$

where $d_1 \approx 0.52$ is the expected distance between two random locations in the system area, which can be obtained by (1).

When the charging request intensity in the system is light ($\lambda \to 0$), the asymptotically shortest travel distance for the mobile charger to reach the charging node is lower bounded by the expected distance between the field center and a random location in the network ((17) in [23]). Denote this distance as d', its distribution can be obtained by

$$f'_D(d') = \begin{cases} \pi d'^2 & 0 \leq d' \leq 1/2 \\ d'^2 \left(\pi - 4\cos^{-1}\left(\frac{1}{2d'}\right)\right) & \\ +2d' \sin\left(\cos^{-1}\left(\frac{1}{2d'}\right)\right) & 1/2 < d' \leq \sqrt{2}/2 \\ 0 & \text{otherwise} \end{cases},$$

where the first two cases are based on whether the random location falls in the inscribed circle of the square area. Similar results can be found in [24]. Denote $d_2 = E[d'] \approx 0.383$, then

$$\mathrm{E}[\mathcal{H}^*] \leq \frac{1}{d_2/v + T_c}. \tag{12}$$

Theorem 1 follows by combining (11) and (12). ■

Next we consider the scenario of heavy requests intensity in the network, namely, when $\lambda \to \frac{1}{T_c}$.

Theorem 2: In terms of the asymptotic system throughput, the performance of NJNP is within a constant factor of the optimal results when the requests intensity in the network is heavy. Specifically

$$\frac{\mathrm{E}[\mathcal{H}_{njnp}]}{\mathrm{E}[\mathcal{H}^*]} \quad \geq \quad \frac{d_4 + vT_c}{d_3 + vT_c} \quad \text{as} \quad \lambda \to \frac{1}{T_c}.$$

where $d_3 \approx 0.64$ and $d_4 \approx 0.27$.

Theorem 2 can be proved based on (21) and (45) in [23]. We do not include the details here due to the space limit.

Theorem 1 and 2 reveal that when either the travel speed of the mobile charger is fast ($v \to \infty$) or the time to charge nodes is long ($T_c \to \infty$), the performance of NJNP approaches the optimal results. This is because the scheduling of the charging tasks only affects the travel distance of the mobile charger, so when the time to charge a node is much longer than the travel time of the charger, all scheduling disciplines achieve comparable results.

B. Charging Latency with NJNP

1) Charging Latency Distribution: The most essential metric for nodes to evaluate the mobile charging process is the charging latency $\mathcal{R}$. For a given charging request arrives at the mobile charger at time t_0, let us denote the completion time of requests after its arrival as $\{t_1, t_2, ..., t_k\}$, where t_k is the completion time of this particular request. Then clearly, $t_k - t_0$ is the charging latency of this request, and asymptotically $(t_i - t_{i-1}) \sim f_{\mathcal{S}}(t) \quad (1 \leq i \leq k)$. Thus by the convolution theorem, we have

$$\mathcal{R} \quad = \quad \sum_{i=1}^{k} (t_i - t_{i-1}) \sim f_{\mathcal{S}}^{(k)}(t), \tag{13}$$

where $f^{(k)}(\cdot)$ is the k-fold convolution of $f(\cdot)$.

To further investigate the number of departures k, we again consider the scenario that l requests are waiting in the service pool. As discussed in (3), these l distances from the mobile charger to each of the requesting nodes can be viewed as l i.i.d. random variables, and thus asymptotically, the probability for any one of them to be the smallest is $\frac{1}{l}$. As a result, if the mobile charger starts to charge a node during the next time slot, the probability for each of the l requesting node to be the charged node is also $\frac{1}{l}$. Thus k follows the distribution of

$$\Pr\{K = k\} = \left(1 - \sum_{l=1}^{N} \frac{\pi_l}{l(1-\pi_0)}\right)^{k-1} \sum_{l=1}^{N} \frac{\pi_l}{l(1-\pi_0)}, \tag{14}$$

where $\sum_{l=1}^{N} \frac{\pi_l}{l(1-\pi_0)}$ is the probability that a given request leaves the system each time when a departure occurs.

Combining (13) and (14), the charging latency conforms to

$$F_{\mathcal{R}}(t) \quad = \quad \sum_{k=1}^{N} \Pr\{K = k\} \int_0^t f_{\mathcal{S}}^{(k)}(x)dx. \tag{15}$$

2) Optimality w.r.t. Charging Latency: Similar to the system throughput, we have the following two theorems on NJNP with respect to the charging latency under both light and heavy requests intensities. Let $\mathcal{R}^*$ denote the minimal charging latency achievable for any online schedule schemes.

Theorem 3: In terms of the asymptotic charging latency of requests, the performance of NJNP is within a constant factor of the optimal results when the requests intensity in the *network is light. Specifically*

$$\frac{\mathrm{E}[\mathcal{R}_{njnp}]}{\mathrm{E}[\mathcal{R}^*]} \quad \leq \quad \frac{d_1 + vT_c}{d_2 + vT_c} \quad \text{as} \quad \lambda \to 0.$$

Theorem 4: In terms of the asymptotic charging latency of requests, the performance of NJNP is within a constant factor of the optimal results when the requests intensity in the network is heavy. Specifically

$$\frac{\mathrm{E}[\mathcal{R}_{njnp}]}{\mathrm{E}[\mathcal{R}^*]} \quad \leq \quad \left(\frac{d_3}{d_4}\right)^2 \quad \text{as} \quad \lambda \to \frac{1}{T_c}.$$

The detailed proofs of the two theorems are not included due to the space limit. Essentially, when the intensity of requests is heavy, d_3 and d_4 correspond to the longest distance the mobile charger has to travel to charge a node with NJNP and the shortest distance the mobile charger has to travel in the optimal case, respectively. These travel distances have a twofold effect on the charging latency: on the time to serve one request and on the number of requests in the pool. This explains the quadric form of the constant in the theorem.

IV. Determining the Remaining Energy Level

The analytical results on NJNP not only reveal insights on the on-demand charging process, but also guide system design choices in practical implementations. We use an example to show how the analytical results can assist the system implementation in this section.

Consider a sensor network with N nodes deployed in an area of $L \times L\ m^2$. The energy capacity of each node is C_s, and nodes can actively estimate their energy consumption rate r_e. When the residual energy is below $\theta_e C_s$ $(0 < \theta_e < 1)$, node will send a charging request to the mobile charger. The mobile charger serves the received requests with NJNP. The request *missing ratio*, defined as the probability that nodes fail to be charged before energy depletion, is required to be no larger than θ_p.

The remaining energy level θ_e plays a critical role in the charging process. If the charging request is sent too early (with a too large θ_e), it is very likely that when the mobile charger arrives at the target node, the node still has a sufficient energy supply. This not only degrades the efficiency of the mobile charger but also increases the charging latency for other requesting nodes. However, if the node sends the charging request too late (with a too small θ_e), the time left for the charger to travel to and charge the node is limited, which increases the requests missing ratio.

In the following, we demonstrate how our analytical results on the charging latency distribution can guide the setting of a proper θ_e. In stable systems with a large number of nodes, the fact that a specific node is waiting to be charged has a negligible effect on the aggregated request arrival rate at the mobile charger. Thus, the following relationship between the aggregated request arrival rate λ and the energy level θ_e exists

$$\lambda \approx \frac{r_e N}{(1-\theta_e)C_s}, \tag{16}$$

Algorithm 1 Find the optimal θ_e

```
θe = 0, θ̂p = 1, Δ = 0.01;
while θ̂p > θp and θe < Cs do
    θe = θe + Δ; λ = re N / ((1 − θe) Cs);
    calculate fR(t) with λ;
    θ̂p = 1 − ∫_0^{θe Cs / re} fR(t) dt;
end while
```

where $(1-\theta_e)C_s$ is the energy that a fully charged node can use before sending out its charging request. Based on the charging latency distribution with NJNP, we can set the optimal θ_e under the maximal tolerable charging missing ratio θ_p by following a *fixed-point iteration* approach. Start with $\theta_e = 0$ and calculate the corresponding estimated $\hat{\theta}_p$ according to the latency distribution shown in (15). If $\hat{\theta}_p > \theta_p$, then increase θ_e with a small step length Δ. Repeat the process until we find the smallest θ_e satisfying the requirement on $\hat{\theta}_p$. The detailed description is shown in Algorithm 1. More efficient methods, such as binary search, could be adopted to further accelerate the calculation.

The algorithm returns a proper value for θ_e if a feasible solution exists, otherwise (if returns $\theta_e = 1$) we need to either increase the nodes energy capacity, or lower the required θ_p.

Homogeneous energy consumption rates of all nodes are assumed in the above example. In a more general case with heterogeneous energy consumption rates, it is also possible to find θ_e^i for each node i by modifying (16) accordingly

$$\lambda = \frac{\sum_{i=1}^{N} r_e^i}{(1-\theta_e^i)C_s}.$$

V. Evaluations

The evaluations of our work consist of two parts. We first verify the accuracy of our analytical results on the system throughput and charging latency with NJNP. Then we show the effectiveness of our proposed methods on guiding the system design. We implement the simulation with Matlab. Unless specified otherwise, we consider a wireless sensor network with 100 randomly deployed nodes in a $100 \times 100\ m^2$ square area, and the charger travel speed is $1\ m/s$ based on the parameters from real-world robots [3].

A. Verification of Analytical Results

1) Model Verification: Our analysis on NJNP is based on the $M/G/1/NJNP$ queuing model. Thus, before evaluating the analysis accuracy, we first verify the soundness of the model. Specifically, we verify the assumption on the Poisson arrival of charging requests to the mobile charger, which means their inter-arrival time is exponentially distributed.

We adopt an event-driven simulator to verify the Poisson arrival. When an event occurs at a random location in the area, nodes within a distance of $10\ m$ can detect it, and the corresponding information is generated and forwarded to the control center through multi-hop communications. Nodes have a full energy capacity of $C_s = 100$. Two cases of the transmission/reception energy costs of $e_{tx} = e_{rx} = 0.02$

TABLE I
CORRELATION BETWEEN THE l DISTANCES.

l	2	3	4	5	6
Max	0.0219	0.0299	0.0199	0.0186	0.0177
Mean	0.0087	0.0097	0.0057	0.0071	0.0080

and $e_{tx} = e_{rx} = 0.05$ per packet are explored, respectively. Nodes send out charging requests when their remaining energy is below a pre-defined threshold $0.044C_s$ (this value is set according to the guidance on determining θ_e introduced in Section IV, and will be explained in details in Section V-B). The charger serves these requests and fully charges nodes according to NJNP. We record the request arrival time at the charger, calculate their inter-arrival times, and compare them with two exponential distributions with the same mean values.

The verification results are shown in Fig. 4, where the average of the inter-arrival time with $e_{tx} = e_{rx} = 0.02$ is around 250 s, indicating an arrival rate λ at the mobile charger of roughly 0.004, and those for $e_{tx} = e_{rx} = 0.05$ are around 100 s and 0.01, respectively. The consistency between exponential distributions and simulation results indicates our model is sound.

The Poisson arrival is further verified by the Kolmogorov-Smirnov (K-S) test [25] with a significance level of 0.05. With a total number of 50×2 tests on the request inter-arrival times obtained with 50 different topologies, only 1 of them with $e_{tx} = e_{rx} = 0.02$ rejects the hypotheses that the inter-arrival time is exponentially distributed, and all those with $e_{tx} = e_{rx} = 0.05$ are accepted.

Note that the independence between consecutive service times also needs to be verified to validate the queuing model. Because it can be proven with a similar approach as in [26], we do not repeat it here due to the space limit.

2) System Throughput: Two cases of the node-level request arrival rates, i.e., the frequency for individual nodes to send out its charging request, are explored with $\lambda' = 0.00005$ and $\lambda' = 0.00025$, indicating an average node lifetime of about $\frac{1}{0.00005}$ $s \approx 5.5$ h and $\frac{1}{0.00025}$ $s \approx 1.1$ h, respectively. These node-level arrival rates are chosen according to the empirical results from [1]: an ultra-capacitor with a capacitance of 22 F indicates a typical lifetime of around 2.15 h for MicaZ motes at 10% duty cycle. With a total number of 100 nodes, these roughly correspond to request arrival rates at the mobile charger of 0.005 and 0.025, respectively.

The analysis on the shortest distance from requesting nodes to the mobile charger is based on the first-order statistic, which requires all these variables to be i.i.d. We run the simulation with a given location of the mobile charger and l requesting nodes, and calculate these l distances. We repeat this calculation for 10,000 times with different value of $l = 2, 3, \cdots, 6$, and calculate the pairwise correlation coefficients of these l distance sequences. The maximal and mean correlation of the $\frac{l(l-1)}{2}$ sequence pairs are shown in Table I. The small correlation supports our methodology.

The verification results on the probability that a service is preempted are shown in Fig. 5, where the x-axis is the distance between the mobile charger and the selected target node, and y-axis is the probability that this charging is preempted before the mobile charger reaches the node. Our analytical results can capture the preemption probability reasonably well. The increase of the preemption probability with the initial distance is intuitive, because the longer travel time offers more chances for the preemption to happen. Furthermore, the verification results indicate that our results are actually a lower bound of the preemption probability. This is because our analysis is based on a continuous distance distribution $F_D(d)$, which suggests every location in the area should be occupied by a node. As a result, a term of $1 - q_d(j)$ for every possible j is included in (4). However, not all of these $1 - q_d(j)$ need to be considered in practice, because there may not exist node of distance $d - jv\delta_t$ to the current location of the charger.

Figure 6 shows the evaluation results on the system throughput, where the time to fully charge an energy-depleted node, T_c, is 10 s. The match between the analysis and simulation results indicates a good accuracy of our analysis. Furthermore, the throughput increases as the requests intensity becomes heavier, which can be explained by two reasons. First, a higher demands intensity leads to more requests in the service pool, and thus makes it more likely for the mobile charger to select a closer target node. This in turn increases the system throughput. Second, from (4), it is clear that a larger request arrival rate increases the preemption probability, which reduces the service time and thus increases the system throughput as well. The system throughput under FCFS with identical settings is shown for comparison. A clear advantage of NJNP over FCFS can be observed, especially when the requests arrival rates are high. This is because more pending requests make it more likely for NJNP to find closer nodes to charge.

A well-known condition for stable queuing systems is that the system utilization ratio ρ should be smaller than 1. This implies that for our analysis to hold, the condition that $\lambda' N \mathrm{E}[\mathcal{H}_{njnp}] < 1$ must be guaranteed. To verify this, we increase the number of nodes to 200 and repeat the simulation. The results are shown in Fig. 7. We can see that our analysis is still accurate when λ' is relatively small, e.g., below 1.5×10^{-4} in Fig. 7. However, the deviation between the analysis and simulation results increases quickly when λ' increases from 1.5×10^{-4}, because the stable condition of the system does not hold anymore.

To investigate the effect of T_c on the charging process, we record the system throughput when T_c varies from 10 s to 80 s. The results with the node-level request arrival rates of 0.00005 and 0.00025 are shown in Fig. 8 and Fig. 9, respectively. The match between the analysis and simulation results when $\lambda' = 0.00005$ verifies the accuracy of our analysis. However, when λ' is 0.00025, our analysis tends to underestimate the system throughput when T_c is large. This is again due to the invalidation of the stable system condition. Note that the advantage of NJNP over FCFS is not so obvious in Fig. 8, because with such a light request intensity, the number of requests in the service pool is limited, which offers little chances for the greedy feature of NJNP to take effect.

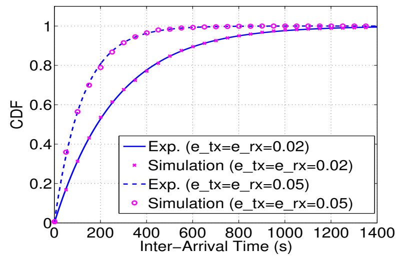

Fig. 4. Verify Poisson arrival.

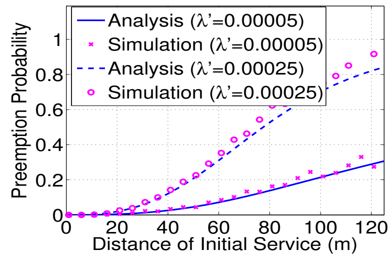

Fig. 5. Preemption probability.

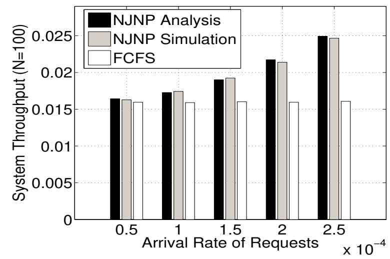

Fig. 6. System throughput with 100 nodes.

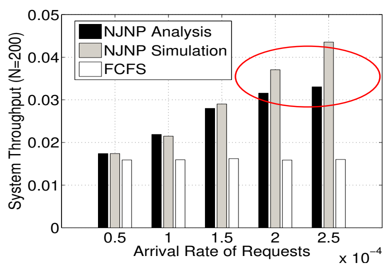

Fig. 7. System throughput with 200 nodes.

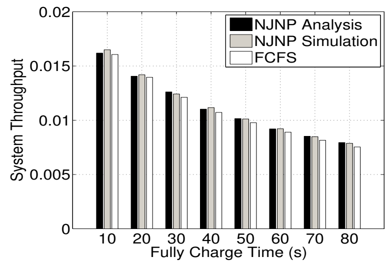

Fig. 8. System throughput with T_c ($\lambda' = 0.00005$).

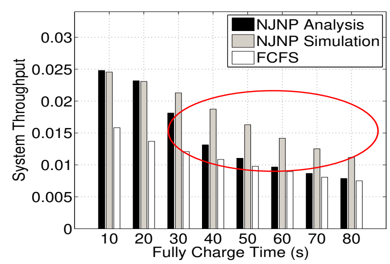

Fig. 9. System throughput with T_c ($\lambda' = 0.00025$).

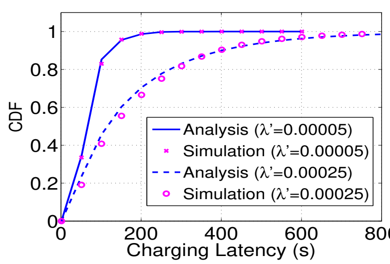

Fig. 10. Charging latency with 100 nodes.

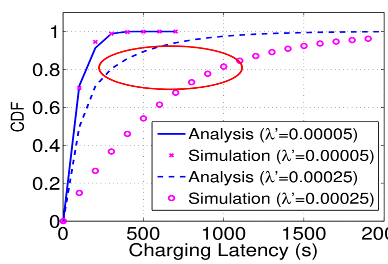

Fig. 11. Charging latency with 200 nodes.

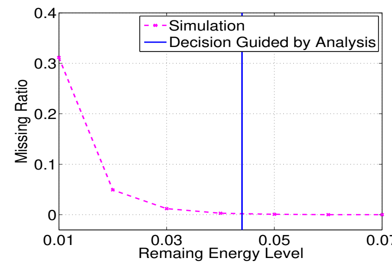

Fig. 12. Determine θ_c.

3) Charging Latency: Figure 10 shows the evaluation results on the charging latency distribution. The match of the analysis and simulation results not only verifies our results, but also indicates that the proposed guidance on determining the optimal node remaining energy level should perform well, as we will see in Section V-B. Furthermore, we can see the tail of the charging latency distribution is not excessively long, especially when compared with the typical lifetime of nodes, i.e., $20,000$ s and $4,000$ s with λ' of 0.00005 and 0.00025, respectively. This observation indicates the unfairness issue with NJNP is not severe.

The charging latency distribution with $N = 200$ is shown in Fig. 11. The results with $\lambda' = 0.00005$ are still accurate. However, due to the invalidation of the stable condition, our analysis deviates from the simulation when $\lambda' = 0.00025$.

B. Effectiveness of System Design Guidances

We consider an energy capacity $C_s = 100$, and the long-term energy consumption rate is $r_e = 0.01$, which indicates a node lifetime of around $\frac{100}{0.01}$ $s \approx 1.5$ h. We require that $\hat{\theta}_p < \theta_p = 0.99$. We run the simulation with different remaining energy levels, and record the resultant $\hat{\theta}_p$. Then we use the guidance presented in Section IV to estimate the optimal value of θ_e. The simulation results and the optimal setting returned by the proposed guidance are shown in Fig. 12. We can see the θ_e returned by the proposed guidance satisfies our requirement pretty well: it has to be as small as possible while guaranteeing the requirement of $\hat{\theta}_p < 0.99$. The returned θ_e (4.4% of the full energy capacity of nodes) is adopted in the simulation when verifying the arrivals, as mentioned in Section V-A1.

VI. Related Work

A number of research works have been devoted to adopting nodes with controlled mobility to accomplish the data collection in wireless sensor networks [3], [4], [20]. Although introducing mobility is shown to be able to improve the system performance, only a few work is done on utilizing the controlled mobility to replenish the energy supply of individual nodes [7], [10], [11].

Yet, replenishing node energy by harvesting energy from surrounding environment has been extensively studied [27], [28]. Many of these studies observed that the harvested energy is unevenly distributed among nodes [29], and how to use the concept of energy sharing to improve the node lifetime has attracted more and more attention [2]. An obvious difference between these works with ours is the lack of controlled mobility in consideration.

For those limited works that exploited mobility to accomplish the energy replenishment of the nodes in sensor networks, most of them focused on the offline scenario [7]–[9], which is quite different from the on-demand mobile charging problem in our consideration, in terms of the application scenarios, problem settings, and research objectives.

The most closely related works are [10], [11], where the mobile charger shares its limited energy supply with individual nodes through wireless recharging. The objective there is to maximize the system operation time with a given energy capacity of the mobile charger, while our focus is on improving the efficiency of the mobile charging process to realize the long-term operation of the system.

NJNP is similar to the preemptive version of the *Shortest-Seek-Time-First* (SSTF) discipline in disk scheduling [30]. However, the diversity with SSTF resides in a one dimensional space, i.e., the tracks of the disk, while that of NJNP is in a two dimensional space, i.e., the field. Clearly, this difference makes the analysis on NJNP more challenging.

The on-demand mobile charging problem investigated in this paper is similar to the dynamic scheduling problems such as *dynamic vehicle routing* [31] and *dynamic traveling salesman problem* [32], [33]. Greedy schemes similar to the non-preemptive version of NJNP have been explored in [34], [35], and our analysis advances the investigation by incorporating the preemption and obtaining the probability distributions of critical performance metrics.

VII. Conclusions

In this paper, we have analytically evaluated the on-demand mobile charging problem under the discipline of NJNP. Analytical results on the system throughput and charging latency have been presented and their closeness to the optimal solutions have been proved. Furthermore, we have demonstrated an example on how to use the analysis to guide the system design in practice. The accuracy of the analytical results and the efficiency of the proposed system design guidance have been verified through extensive simulations. Our future work will focus on extending the results to the scenario of multiple chargers, in which besides *scheduling* the charging requests, their *assignment* among the chargers also has to be addressed.

Acknowledgement

This work was supported in part by Singapore-MIT International Design Center IDG31000101, SUTD-ZJU/RES/03/2011, NSERC Canada, NSF grant CNS-1217791, and the Singapore National Research Foundation under its IDM Futures Funding Initiative and administered by the Interactive & Digital Media Programme Office, Media Development Authority.

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