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Department of Mathematics and Computing Science

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by

V. Ciancio

and

G.A. Kluitenberg

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Department of Mathematics and Computing Science
Eindhoven University of Technology
P.O. Box 513
5600 MB Eindhoven
The Netherlands

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ON ELECTROMAGNETIC WAVES IN ISOTROPIC MEDIA WITH DIELECTRIC RELAXATION (*)

Dedicated to Academician István Gyarmati
on the occasion of his 60-th birthday

Vincenzo CIANCIO
Department of Mathematics University of Calabria
Arcavacata di Rende (CS)
Italy

Gerrit Alfred KLUITENBERG
Department of Mathematics and Computing Science
Eindhoven University of Technology
Eindhoven
The Netherlands

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Summary

In some previous papers one of us discussed dielectric relaxation phenomena from the point of view of nonequilibrium thermodynamics. If the theory is linearized one may derive a dynamical constitutive equation (relaxation equation) which has the form of a linear relation among the electric field **E**, the polarization **P**, the first derivatives with respect to time of **E** and **P** and the second derivative with respect to time of **P**. The Debye equation for dielectric relaxation in polar liquids and the De Groot-Mazur equation (obtained by these authors with the aid of methods which are also based on nonequilibrium thermodynamics) are special cases of the more general equation of which the structure has been described above. It is the purpose of the present paper to investigate the propagation and damping of electromagnetic waves. We consider the case in which the dielectric relaxation may be described by the above mentioned relaxation equation derived by one of us, the case in which the Debye equation may be used and the case in which one may apply the De Groot-Mazur equation. We derive solutions of the relaxation equations which also satisfy Maxwell's equations. We limit ourselves to plane waves of a single frequency in isotropic homogeneous linear media with vanishing electric conductivity. It is also assumed that the media are at rest. From thermodynamic arguments several inequalities are derived for the coefficients which occur in the relaxation equations. Explicit expressions are given for the complex wave vector, the complex dielectric permeability and for the phase velocity of the waves. All these quantities are functions of the frequency ω of the waves. For $\omega \rightarrow 0$ the complex permeability $\varepsilon_{(compl)} \rightarrow \varepsilon_{(eq)}$, where $\varepsilon_{(eq)}$ is the equilibrium value of the permeability for static fields. If $\omega \rightarrow \infty$ we find $\varepsilon_{(compl)} \rightarrow 1$ except for the case of the Debye equation. This is due to the fact that a part of the polarization changes in a reversible way in media for which the Debye equation holds.

1. Introduction

Nonequilibrium thermodynamics is a useful tool in investigations of irreversible phenomena such as, heat conduction, diffusion and electric conduction. A detailed and clear discussion of this theory is given in the well-known book by Gyarmati [1].

In the references 2-7 dielectric and magnetic relaxation phenomena are discussed with the aid of the general methods of nonequilibrium thermodynamics. In particular it was shown in reference 3 that a vectorial internal variable which influences the polarization gives rise to dielectric relaxation phenomena. Furthermore, with the aid of such a variable one can split up the polarization $\mathbf{P}$ into two parts

$$\mathbf{P} = \mathbf{P}^{(0)} + \mathbf{P}^{(1)}. \tag{1.1}$$

$\mathbf{P}^{(0)}$ and $\mathbf{P}^{(1)}$ may be called partial polarization vectors and changes in both these vectors are irreversible phenomena.

If one linearizes this theory and if one neglects cross effects as, for instance, the influence of electric conduction, heat conductions and (mechanical) viscosity on dielectric relaxation, the following relaxation equation may be derived (see ref. 4)

$$\chi^{(0)}_{(EP)}\,\mathbf{E} + \frac{d\,\mathbf{E}}{d\,t} = \chi^{(0)}_{(PE)}\,\mathbf{P} + \chi^{(1)}_{(PE)}\,\frac{d\,\mathbf{P}}{d\,t} + \chi^{(2)}_{(PE)}\,\frac{d^2\,\mathbf{P}}{d\,t^2}\,, \tag{1.2}$$

where $\mathbf{E}$ is the electric field and $\chi^{(0)}_{(EP)}$, $\chi^{(0)}_{(PE)}$, $\chi^{(1)}_{(PE)}$ and $\chi^{(2)}_{(PE)}$ are algebraic functions of the coefficients occurring in the phenomenological equations (describing the irreversible processes) and in the equations of state.

Moreover, in ref. 4, it was shown that the Debye equation for dielectric relaxation in polar liquids (see ref. 8) and the equation derived by De Groot and Mazur (see ref. 9) can be considered as degeneracies of equation (1.2). More detailed discussions are given in sections 3,4 and 5 of this paper.

Very recently, in ref. 6, the equation (1.2) has been generalized by including the abovementioned cross effects and it was shown that, in this case, the dynamical equation has the form of a linear relation among $\mathbf{E}$, $\mathbf{P}$ and T^{-1} grad T (T is the absolute temperature), the first derivatives with respect to time of these vectors and the second derivative with respect to time of $\mathbf{P}$.

Another generalization of (1.2) is derived in ref. 7, where the physical assumption is introduced that the polarization $\mathbf{P}$ is additively composed of $n + 1$ partial polarization vectors which play the role of thermodynamic internal variables in the Gibbs relation.

In the present paper we shall consider the equation (1.2) and we shall investigate the propagation of linear electromagnetic waves in an arbitrary direction. The positive x_1-axis is chosen so that it coalesces with this direction. It is our aim to elucidate the role played by the coefficients which occur in the equations of state and in the phenomenological equations describing the irreversible process which are considered in the theory discussed in refs. 2-7.

More precisely, in sects. 2 and 3, we recall the basic equations of the theory. In sect. 4 some inequalities are derived, which are connected with stability and with the nonnegative character of the entropy production. These inequalities play an important role in the theory of wave propagation.

In sects. 5 and 6 it is shown that the equation (1.2) generalizes the Debye and the De Groot-Mazur equations for dielectric relaxation phenomena in continuous media.

In sects. 7 and 8 we investigate linear electromagnetic waves. An explicit form for the complex dielectric permeability is derived. The dispersion law, the expression for the phase velocity and the law for the damping of the waves are obtained and the limiting cases of low and high frequencies are discussed.

Finally, in sects. 9 and 10, we consider the propagation of electromagnetic waves in media for which the Debye equation holds and in media for which the De Groot-Mazur equation holds and in section 11 we consider the limiting case of nondissipative media.

2. Vectorial internal variables and dielectric relaxation phenomena

Maxwell's equations read

$$\mathrm{rot}\,\mathbf{H} - \frac{1}{c}\,\frac{\partial \mathbf{D}}{\partial t} = \frac{1}{c}\,\mathbf{j}^{(el)}\,, \tag{2.1}$$

$$\mathrm{div}\,\mathbf{D} = \rho^{(el)}\,, \tag{2.2}$$

$$\mathrm{rot}\,\mathbf{E} + \frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}\,, \tag{2.3}$$

$$\mathrm{div}\,\mathbf{B} = 0\,, \tag{2.4}$$

where **E** and **B** are the electric and magnetic field strengths, **D** is the electric displacement field, **H** is the magnetic displacement field, $\mathbf{j}^{(el)}$ is the density of the electric current and $\rho^{(el)}$ is the electric charge density.

The polarization **P** and the magnetization **M** are defined by

$$\mathbf{P} = \mathbf{D} - \mathbf{E}\,, \tag{2.5}$$

$$\mathbf{M} = \mathbf{B} - \mathbf{H}. \tag{2.6}$$

We shall also use the specific polarization **p** defined by

$$\mathbf{p} = v\,\mathbf{P}\,, \tag{2.7}$$

where v is the specific volume related to the mass density ρ by

$$v = \frac{1}{\rho}. \tag{2.8}$$

In an analogous way the specific magnetization **m** is defined by

$$\mathbf{m} = v\,\mathbf{M}. \tag{2.9}$$

In previous papers (as noted in sect. 1) some types of dielectric (and magnetic) relaxation phenomena were discussed from the point of view of the thermodynamics of irreversible processes. It was assumed that there is a vectorial internal thermodynamic variable **Z** which influences the polarization. It has been shown [4] that this leads to the possibility to write the polarization in the form (see (1.1))

$$\mathbf{P} = \mathbf{P}^{(0)} + \mathbf{P}^{(1)}\,, \tag{2.10}$$

where $\mathbf{P}^{(1)}$ is a function of **Z** only (see (3.7) and (6.8) of ref. 4) and may replace **Z** as internal variable. In general changes both in $\mathbf{P}^{(0)}$ and $\mathbf{P}^{(1)}$ are irreversible phenomena. It has been shown (see ref. 4) that in the limiting case, where changes in $\mathbf{P}^{(0)}$ are reversible, the Debye equation for dielectric relaxation may be obtained from the developed formalism. Furthermore, if no internal variable occurs, the formalism reduces to a formalism proposed by De Groot and Mazur [9]. For magnetic relaxation phenomena analogous phenomenological considerations may be given.

The specific polarizations $\mathbf{p}^{(0)}$ and $\mathbf{p}^{(1)}$ are defined by

$$\mathbf{p}^{(0)} = v\,\mathbf{P}^{(0)}, \quad \mathbf{p}^{(1)} = v\,\mathbf{P}^{(1)}. \tag{2.11}$$

It will be assumed that the specific entropy s is a function of the specific internal energy u, the tensor of total strain $\varepsilon_{\alpha\beta}$ (or for a fluid the specific volume v), the polarizations $\mathbf{p}$ and $\mathbf{p}^{(1)}$ and the magnetization $\mathbf{m}$ (cf. eq. (3.7) of ref. 2 and eq. (3.19) of ref. 4). Hence,

$$s = s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}). \tag{2.12}$$

The absolute temperature T is given by

$$T^{-1} = \frac{\partial}{\partial u} s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}). \tag{2.13}$$

Furthermore, we define the tensor field $\tau_{\alpha\beta}^{(eq)}$ and the vector fields $\mathbf{E}^{(eq)}$ and $\mathbf{E}^{(1)}$ by

$$\tau_{\alpha\beta}^{(eq)} = -\rho\,T \frac{\partial}{\partial \varepsilon_{\alpha\beta}} s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}), \tag{2.14}$$

$$\mathbf{E}^{(eq)} = -T \frac{\partial}{\partial \mathbf{p}} s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}), \tag{2.15}$$

and

$$\mathbf{E}^{(1)} = T \frac{\partial}{\partial \mathbf{p}^{(1)}} s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}). \tag{2.16}$$

Finally, we assume that the magnetic field strength $\mathbf{B}$ which occurs in Maxwell's equations (2.3) and (2.4) satisfies the relation

$$\mathbf{B} = -T \frac{\partial}{\partial \mathbf{m}} s(u, \varepsilon_{\alpha\beta}, \mathbf{p}, \mathbf{p}^{(1)}, \mathbf{m}). \tag{2.17}$$

The latter relation follows from eq. (3.11) of ref. 2 if magnetic relaxation phenomena are left out of consideration. From (2.11)-(2.16) one has

$$T\,d\,s = d\,u - v \sum_{\alpha,\beta=1}^{3} \tau_{\alpha\beta}^{(eq)}\,d\,\varepsilon_{\alpha\beta} - \mathbf{E}^{(eq)} \cdot d\,\mathbf{p} + \mathbf{E}^{(1)} \cdot d\,\mathbf{p}^{(1)} - \mathbf{B} \cdot d\,\mathbf{m}. \tag{2.18}$$

It may be shown that if (2.17) is satisfied magnetic phenomena do not contribute to the entropy production. It appears that the entropy production is due to dissipative mechanical processes, dielectric relaxation, heat conduction and electric conduction. We shall neglect possible cross effects among dielectric relaxation and the other irreversible phenomena just mentioned. If, moreover, one assumes that the mass density is constant, one obtains the following phenomenological equations for dielectric relaxation in isotropic media (see the equations (6.12), (6.13) and (6.14) of ref. 4).

$$\mathbf{E} - \mathbf{E}^{(eq)} = L_{(P)}^{(0,0)} \frac{d\,\mathbf{P}}{d\,t} + L_{(P)}^{(0,1)}\, \mathbf{E}^{(1)} \tag{2.19}$$

and

$$\frac{d\,\mathbf{P}^{(1)}}{d\,t} = L_{(P)}^{(1,0)} \frac{d\,\mathbf{P}}{d\,t} + L_{(P)}^{(1,1)}\, \mathbf{E}^{(1)}. \tag{2.20}$$

In these relations $L_{(P)}^{(0,0)}$, $L_{(P)}^{(0,1)}$, $L_{(P)}^{(1,0)}$ and $L_{(P)}^{(1,1)}$ are phenomenological coefficients. We shall assume that these coefficients are constants. The coefficients $L_{(P)}^{(0,1)}$ and $L_{(P)}^{(1,0)}$ are connected with possible cross effects which may occur between the two types of dielectric relaxation phenomena described by (2.19) and (2.20). These coefficients satisfy the Onsager-Casimir reciprocal relations

$$L_{(P)}^{(0,1)} = -L_{(P)}^{(1,0)}. \tag{2.21}$$

Finally, in (2.17) and (2.18) $d\,/\,d\,t$ is the substantial derivative with respect to time defined by

$$\frac{d}{d\,t} = \frac{\partial}{\partial\,t} + \sum_{\gamma=1}^{3} v_\gamma \frac{\partial}{\partial\,x_\gamma}, \tag{2.22}$$

where x_γ is the γ-component of the position vector $\mathbf{x}$ with respect to an orthogonal Cartesian frame of axes which is fixed in space and v_γ is the γ-component of the velocity field $\mathbf{v}$ of the matter.

3. Linear equations of state and linear equations for dielectric relaxation

The specific free energy is defined by

$$f = u - T\, s. \tag{3.1}$$

Using (2.18) one has

$$d\,f = -s\, d\, T + v \sum_{\alpha,\beta=1}^{3} \tau_{\alpha\beta}^{(eq)}\, d\, \varepsilon_{\alpha\beta} + \mathbf{E}^{(eq)} \cdot d\,\mathbf{p} - \mathbf{E}^{(1)} \cdot d\,\mathbf{p}^{(1)} + \mathbf{B} \cdot d\,\mathbf{m}. \tag{3.2}$$

Hence,

$$\mathbf{E}^{(eq)} = \frac{\partial}{\partial \mathbf{p}} f(T\,,\, \varepsilon_{\alpha\beta}\,,\, \mathbf{p}\,,\, \mathbf{p}^{(1)}\,,\, \mathbf{m})\,, \tag{3.3}$$

$$\mathbf{E}^{(1)} = -\frac{\partial}{\partial \mathbf{p}^{(1)}} f(T\,,\, \varepsilon_{\alpha\beta}\,,\, \mathbf{p}\,,\, \mathbf{p}^{(1)}\,,\, \mathbf{m})\,, \tag{3.4}$$

and

$$\mathbf{B} = \frac{\partial}{\partial \mathbf{m}} f(T\,,\, \varepsilon_{\alpha\beta}\,,\, \mathbf{p}\,,\, \mathbf{p}^{(1)}\,,\, \mathbf{m}). \tag{3.5}$$

Similar equations hold for s and $\tau_{\alpha\beta}^{(eq)}$.

We shall suppose that one has in a first approximation for isotropic media

$$f = f^{(1)} + f^{(2)} + f^{(3)}\,, \tag{3.6}$$

where

$$f^{(1)} = f^{(1)}\,(T\,,\, \varepsilon_{\alpha\beta})\,, \tag{3.7}$$

$$f^{(2)} = \tfrac{1}{2}\, \rho\{a_{(P)}^{(0,0)}\, \mathbf{p} \cdot (\mathbf{p} - 2\,\mathbf{p}^{(1)}) + a_{(P)}^{(1,1)}\, (\mathbf{p}^{(1)})^2\} \tag{3.8}$$

and

$$f^{(3)} = \tfrac{1}{2}\, \rho\, \frac{\mu}{\mu - 1}\, \mathbf{m}^2. \tag{3.9}$$

In (3.8) and (3.9) $a_{(P)}^{(0,0)}$, $a_{(P)}^{(1,1)}$ and μ are scalar constants. It will be seen that μ is the magnetic permeability. Since

$$\mathbf{p} = \mathbf{p}^{(0)} + \mathbf{p}^{(1)} \tag{3.10}$$

(see (2.10), (2.7) and (2.11)) we have from (3.8)

$$f^{(2)} = \tfrac{1}{2}\, \rho\{a_{(P)}^{(0,0)}\, (\mathbf{p}^{(0)})^2 + (a_{(P)}^{(1,1)} - a_{(P)}^{(0,0)})\, (\mathbf{p}^{(1)})^2\}. \tag{3.11}$$

From (3.3)-(3.9) one obtains the linear equations of state

$$\mathbf{E}^{(eq)} = a^{(0,0)}_{(P)} (\mathbf{P} - \mathbf{P}^{(1)}) = a^{(0,0)}_{(P)} \mathbf{P}^{(0)} , \tag{3.12}$$

$$\mathbf{E}^{(1)} = a^{(0,0)}_{(P)} \mathbf{P} - a^{(1,1)}_{(P)} \mathbf{P}^{(1)} \tag{3.13}$$

and

$$\mathbf{B} = \frac{\mu}{\mu - 1} \mathbf{M} , \tag{3.14}$$

where we also used (2.7)-(2.11) (see eqs. (6.10) and (6.11) of ref. 4).

By eliminating $\mathbf{P}^{(0)}$, $\mathbf{P}^{(1)}$, $\mathbf{E}^{(eq)}$ and $\mathbf{E}^{(1)}$ from (2.19), (2.20), (3.12) and (3.13) one obtains the linear relaxation equation

$$\chi^{(0)}_{(EP)} \mathbf{E} + \frac{d\mathbf{E}}{dt} = \chi^{(0)}_{(PE)} \mathbf{P} + \chi^{(1)}_{(PE)} \frac{d\mathbf{P}}{dt} + \chi^{(2)}_{(PE)} \frac{d^2\mathbf{P}}{dt^2} , \tag{3.15}$$

where

$$\chi^{(0)}_{(EP)} = a^{(1,1)}_{(P)} L^{(1,1)}_{(P)} , \tag{3.16}$$

$$\chi^{(0)}_{(PE)} = a^{(0,0)}_{(P)} (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)}) L^{(1,1)}_{(P)} , \tag{3.17}$$

$$\chi^{(1)}_{(PE)} = a^{(0,0)}_{(P)} (1 + L^{(0,1)}_{(P)} - L^{(1,0)}_{(P)}) + a^{(1,1)}_{(P)} (L^{(0,0)}_{(P)} L^{(1,1)}_{(P)} - L^{(0,1)}_{(P)} L^{(1,0)}_{(P)}) , \tag{3.18}$$

$$\chi^{(2)}_{(PE)} = L^{(0,0)}_{(P)}. \tag{3.19}$$

See section 7 of ref. 4 for details concerning the derivation. In particular we note that in the derivation of (3.15) it is assumed that the phenomenological coefficients $L^{(0,0)}_{(P)}$, $L^{(1,1)}_{(P)}$, $L^{(0,1)}_{(P)}$ and $L^{(1,0)}_{(P)}$, which occur in (2.19) and (2.20), are constants. Using the Onsager-Casimir reciprocal relations (2.21) the expression (3.18) for $\chi^{(1)}_{(PE)}$ becomes

$$\chi^{(1)}_{(PE)} = a^{(0,0)}_{(P)} (1 + 2 L^{(0,1)}_{(P)}) + a^{(1,1)}_{(P)} \{L^{(0,0)}_{(P)} L^{(1,1)}_{(P)} + (L^{(0,1)}_{(P)})^2\}. \tag{3.20}$$

If we replace $\mathbf{P}$ by $\mathbf{D} - \mathbf{E}$ (see (2.5)) the relaxation equation (3.15) may be written in the form

$$\chi^{(0)}_{(ED)} \mathbf{E} + \chi^{(1)}_{(ED)} \frac{d\mathbf{E}}{dt} + \chi^{(2)} \frac{d^2\mathbf{E}}{dt^2} = \chi^{(0)}_{(DE)} \mathbf{D} + \chi^{(1)}_{(DE)} \frac{d\mathbf{D}}{dt} + \chi^{(2)} \frac{d^2\mathbf{D}}{dt^2} , \tag{3.21}$$

where

$$\chi^{(0)}_{(ED)} = \chi^{(0)}_{(EP)} + \chi^{(0)}_{(PE)} = \{a^{(1,1)}_{(P)} + a^{(0,0)}_{(P)} a^{(1,1)}_{(P)} - (a^{(0,0)}_{(P)})^2\} L^{(1,1)}_{(P)} , \tag{3.22}$$

$$\chi^{(1)}_{(ED)} = 1 + \chi^{(1)}_{(PE)} = 1 + a^{(0,0)}_{(P)} (1 + 2L^{(0,1)}_{(P)}) + a^{(1,1)}_{(P)} \{L^{(0,0)}_{(P)} L^{(1,1)}_{(P)} + (L^{(0,1)}_{(P)})^2\} , \tag{3.23}$$

$$\chi^{(0)}_{(DE)} = \chi^{(0)}_{(PE)} = a^{(0,0)}_{(P)} (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)}) L^{(1,1)}_{(P)} , \tag{3.24}$$

$$\chi^{(1)}_{(DE)} = \chi^{(1)}_{(PE)} = a^{(0,0)}_{(P)} \; (1 + 2\, L^{(0,1)}_{(P)}) + a^{(1,1)}_{(P)} \; \{L^{(0,0)}_{(P)} \, L^{(1,1)}_{(P)} + (L^{(0,1)}_{(P)})^2\} \, , \tag{3.25}$$

$$\chi^{(2)} = \chi^{(2)}_{(PE)} = L^{(0,0)}_{(P)} . \tag{3.26}$$

See also (3.16)-(3.19).

Finally, we multiply both sides of (3.14) by $\mu - 1$ and we replace $\mathbf{M}$ by $\mathbf{B} - \mathbf{H}$ (see (2.6)). We then obtain

$$\mathbf{B} = \mu \, \mathbf{H} . \tag{3.27}$$

From this equation it is seen that indeed μ is the magnetic permeability. It is obvious that (3.27) implies that magnetic relaxation phenomena are left out of consideration.

In the static case where all time derivatives vanish, we have from (3.21)

$$\mathbf{D} = \varepsilon_{(eq)} \, \mathbf{E} \quad \text{(static case)} \, , \tag{3.28}$$

where

$$\varepsilon_{(eq)} = \frac{\chi^{(0)}_{(ED)}}{\chi^{(0)}_{(DE)}} = 1 + \frac{a^{(1,1)}_{(P)}}{a^{(0,0)}_{(P)} \; (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})} . \tag{3.29}$$

See also (3.22) and (3.24). One may consider $\varepsilon_{(eq)}$ as the equilibrium dielectric permeability.

4. Some inequalities

In section 5 of ref. 4 we have seen that the entropy production is a nonnegative quantity provided

$$L^{(0,0)}_{(P)} \geq 0\,, \quad L^{(1,1)}_{(P)} \geq 0. \tag{4.1}$$

See eq. (5.27) of ref. 4. Furthermore, we shall assume that

$$a^{(1,1)}_{(P)} \; > \; a^{(0,0)}_{(P)} \; > \; 0. \tag{4.2}$$

It follows from the inequalities (4.2) that $f^{(2)} = 0$ if and only if both $\mathbf{p}^{(0)}$ and $\mathbf{p}^{(1)}$ vanish (see (3.11)). In all other cases we have $f^{(2)} > 0$. This means that energy is stored in the dielectric if it is polarized. Finally, we shall assume that

$$\mu \; > \; 0. \tag{4.3}$$

IF $0 < \mu < 1$ we have a diamagnetic medium and if $\mu > 1$ the medium is paramagnetic.

From (4.1)-(4.3) some other inequalities may be derived which play an important role in the theory of wave propagation. By virtue of (4.2) we have from (3.29) for the equilibrium dielectric permeability the well-known inequality

$$\varepsilon_{(eq)} \; > \; 1. \tag{4.4}$$

Using (4.1) and (4.2) it is seen from (3.16), (3.17) and (3.19) that

$$\chi^{(0)}_{(EP)} \geq 0\,, \quad \chi^{(0)}_{(PE)} \geq 0\,, \quad \chi^{(2)}_{(PE)} \geq 0. \tag{4.5}$$

Furthermore, we have from (3.20), (3.16) and (3.19) the identity

$$\chi^{(1)}_{(PE)} - \chi^{(0)}_{(EP)}\, \chi^{(2)}_{(PE)} = (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\, (L^{(0,1)}_{(P)})^2 + a^{(0,0)}_{(P)}\; (1 + L^{(0,1)}_{(P)})^2. \tag{4.6}$$

Hence, using (4.2), we get from this relation

$$\chi^{(1)}_{(PE)} - \chi^{(0)}_{(EP)}\, \chi^{(2)}_{(PE)} > 0 \tag{4.7}$$

and, because of (4.5), we have from (4.7) the inequality

$$\chi^{(1)}_{(PE)} \; > \; 0. \tag{4.8}$$

From (4.5) and (4.8) it is seen that none of the coefficients which occur in the relaxation equation (3.15) is negative. With the aid of (3.16), (3.17) and (3.20) we find

$$\chi^{(1)}_{(PE)}\, \chi^{(0)}_{(EP)} - \chi^{(0)}_{(PE)} = \{(a^{(1,1)}_{(p)}\; L^{(0,1)}_{(P)} + a^{(0,0)}_{(P)})^2 + (a^{(1,1)}_{(P)})^2\, L^{(0,0)}_{(P)}\; L^{(1,1)}_{(P)}\}\;\; L^{(1,1)}_{(P)} \tag{4.9}$$

and it follows from this relation that

$$\chi^{(1)}_{(PE)}\, \chi^{(0)}_{(EP)} - \chi^{(0)}_{(PE)} \geq 0\,, \tag{4.10}$$

where we also used (4.1).

Next, we consider the coefficients which occur in the form (3.21) for the relaxation equation. From (3.22)-(3.26), (4.5) and (4.8) we obtain

$$\chi^{(0)}_{(ED)} \geq 0, \quad \chi^{(0)}_{(DE)} \geq 0, \quad \chi^{(2)} \geq 0, \tag{4.11}$$

$$\chi^{(1)}_{(DE)} > 0, \quad \chi^{(1)}_{(ED)} > 1. \tag{4.12}$$

With the help of (3.22)-(3.25) we find the identity

$$\chi^{(0)}_{(ED)}\,\chi^{(1)}_{(DE)} - \chi^{(0)}_{(DE)}\,\chi^{(1)}_{(ED)} = \chi^{(1)}_{(PE)}\,\chi^{(0)}_{(EP)} - \chi^{(0)}_{(PE)} \tag{4.13}$$

and hence, from (4.10) we have the inequality

$$\chi^{(0)}_{(ED)}\,\chi^{(1)}_{(DE)} - \chi^{(0)}_{(DE)}\,\chi^{(1)}_{(ED)} \geq 0. \tag{4.14}$$

In the discussion of electromagnetic waves the quantity β, defined by

$$\beta = \chi^{(1)}_{(ED)}\,\chi^{(1)}_{(DE)} - \chi^{(2)}\,(\chi^{(0)}_{(ED)} + \chi^{(0)}_{(DE)}). \tag{4.15}$$

also plays an important role.

We wish to show that the right-hand side of (4.15) satisfies the inequality

$$\chi^{(1)}_{(ED)}\,\chi^{(1)}_{(DE)} - \chi^{(2)}\,(\chi^{(0)}_{(ED)} + \chi^{(0)}_{(DE)}) > 0. \tag{4.16}$$

Using (3.22)-(3.25) we can also write for (4.15)

$$\beta = (\chi^{(1)}_{(PE)})^2 + \chi^{(1)}_{(PE)} - \chi^{(2)}\,(\chi^{(0)}_{(EP)} + 2\,\chi^{(0)}_{(PE)}) \tag{4.17}$$

or, with the help of (3.16), (3.17), (3.20) and (3.26):

$$\begin{aligned}\beta = {} & [a^{(0,0)}_{(P)}\,(1+2\,L^{(0,1)}_{(P)}) + a^{(1,1)}_{(P)}\,\{L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)} + (L^{(0,1)}_{(P)})^2\}\,]^2 + \\ & + a^{(0,0)}_{(P)}\,(1+2\,L^{(0,1)}_{(P)} + a^{(1,1)}_{(P)}\,\{L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)} + (L^{(0,1)}_{(P)})^2\} - \\ & - \{a^{(1,1)}_{(P)} + 2\,a^{(0,0)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,\}\,L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)}\,,\end{aligned} \tag{4.18}$$

or

$$\begin{aligned}\beta = {} & [a^{(0,0)}_{(P)}\,(1+L^{(0,1)}_{(P)})^2 + a^{(1,1)}_{(P)}\,L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)} + (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,(L^{(0,1)}_{(P)})^2\,]^2 + \\ & + a^{(0,0)}_{(P)}\,(1+L^{(0,1)}_{(P)})^2 + (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,(L^{(0,1)}_{(P)})^2 - \\ & - 2\,a^{(0,0)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)}.\end{aligned} \tag{4.19}$$

Hence, we find

$$\begin{aligned}\beta = {} & (a^{(0,0)}_{(P)})^2\,(1+L^{(0,1)}_{(P)})^4 + (a^{(1,1)}_{(P)})^2\,(L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)})^2 + (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})^2\,(L^{(0,1)}_{(P)})^4 + \\ & + 2\,a^{(0,0)}_{(P)}\,a^{(1,1)}_{(P)}\,(1+L^{(0,1)}_{(P)})^2\,L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)} + \\ & + 2\,a^{(1,1)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,(L^{(0,1)}_{(P)})^2\,L^{(0,0)}_{(P)}\,L^{(1,1)}_{(P)} + \\ & + 2\,a^{(0,0)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,(1+L^{(0,1)}_{(P)})^2\,(L^{(0,1)}_{(P)})^2 +\end{aligned}$$

$$+ a^{(0,0)}_{\langle P\rangle} (1+L^{(0,1)}_{\langle P\rangle})^2 + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) (L^{(0,1)}_{\langle P\rangle})^2 -$$
$$- 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle}. \tag{4.20}$$

One may also write for the latter relation

$$\begin{aligned}
\beta = {} & (a^{(0,0)}_{\langle P\rangle})^2 (1+L^{(0,1)}_{\langle P\rangle})^4 + \\
& + \{(a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 + (a^{(0,0)}_{\langle P\rangle})^2 + 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})\} (L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle})^2 + \\
& + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^4 + \\
& + 2\, \{a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) + (a^{(0,0)}_{\langle P\rangle})^2\} (1+L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + 2\, \{(a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 + a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})\} (L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) (1+L^{(0,1)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^2 + \\
& + a^{(0,0)}_{\langle P\rangle} (1+L^{(0,1)}_{\langle P\rangle})^2 + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) (L^{(0,1)}_{\langle P\rangle})^2 - \\
& - 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle}
\end{aligned}$$

$$\begin{aligned}
= {} & (a^{(0,0)}_{\langle P\rangle})^2 (1+L^{(0,1)}_{\langle P\rangle})^4 + \{(a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 + (a^{(0,0)}_{\langle P\rangle})^2\} (L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle})^2 + \\
& + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^4 + 2\, (a^{(0,0)}_{\langle P\rangle})^2 (1+L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + 2\, (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + a^{(0,0)}_{\langle P\rangle} (1+L^{(0,1)}_{\langle P\rangle})^2 + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) (L^{(0,1)}_{\langle P\rangle})^2 + \\
& + 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) \{(L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle})^2 + (1+L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + (L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + (1+L^{(0,1)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^2 - L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle}\}\,,
\end{aligned} \tag{4.21}$$

or

$$\begin{aligned}
\beta = {} & (a^{(0,0)}_{\langle P\rangle})^2 (1+L^{(0,1)}_{\langle P\rangle})^4 + \{(a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 + (a^{(0,0)}_{\langle P\rangle})^2\} (L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle})^2 + \\
& + 2\, (a^{(0,0)}_{\langle P\rangle})^2 (1+L^{(0,1)}_{\langle P\rangle})^2 L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + \\
& + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle})^2 (L^{(0,1)}_{\langle P\rangle})^2 \{2\, L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + (L^{(0,1)}_{\langle P\rangle})^2\} + \\
& + a^{(0,0)}_{\langle P\rangle} (1+L^{(0,1)}_{\langle P\rangle})^2 + (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) (L^{(0,1)}_{\langle P\rangle})^2 + \\
& + 2\, a^{(0,0)}_{\langle P\rangle} (a^{(1,1)}_{\langle P\rangle} - a^{(0,0)}_{\langle P\rangle}) \{L^{(0,0)}_{\langle P\rangle} L^{(1,1)}_{\langle P\rangle} + (1+L^{(0,1)}_{\langle P\rangle}) L^{(0,1)}_{\langle P\rangle}\}^2.
\end{aligned} \tag{4.22}$$

By virtue of (4.1) and (4.2) one gets from (4.22) $\beta > 0$ and, thus, the inequality (4.16) follows from (4.15).

5. The Debye equation

If the phenomenological coefficients $L^{(0,0)}_{(P)}$ and $L^{(0,1)}_{(P)} = -L^{(1,0)}_{(P)}$ (see (2.21)) vanish, it follows from (2.19) that $\mathbf{E} = \mathbf{E}^{(eq)}$ and, hence, it is seen from (3.12) that

$$\mathbf{E} = a^{(0,0)}_{(P)}\, \mathbf{P}^{(0)}. \tag{5.1}$$

Furthermore, (2.20) reduces to

$$\frac{d\,\mathbf{P}^{(1)}}{d\,t} = L^{(1,1)}_{(P)}\, \mathbf{E}^{(1)}. \tag{5.2}$$

The expressions (3.16)-(3.19) for the coefficients which occur in (3.15) become

$$\chi^{(0)}_{(EP)} = a^{(1,1)}_{(P)}\, L^{(1,1)}_{(P)}\ , \tag{5.3}$$

$$\chi^{(0)}_{(PE)} = a^{(0,0)}_{(P)}\, (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\, L^{(1,1)}_{(P)} \tag{5.4}$$

$$\chi^{(1)}_{(PE)} = a^{(0,0)}_{(P)}\ , \tag{5.5}$$

$$\chi^{(2)}_{(PE)} = 0 \tag{5.6}$$

and, hence, (3.15) reduces to

$$\chi^{(0)}_{(EP)}\, \mathbf{E} + \frac{d\,\mathbf{E}}{dt} = \chi^{(0)}_{(PE)}\, \mathbf{P} + \chi^{(1)}_{(PE)}\, \frac{d\,\mathbf{P}}{dt}. \tag{5.7}$$

For the coefficients (3.22)-(3.26) we get

$$\chi^{(0)}_{(ED)} = \{a^{(1,1)}_{(P)} + a^{(0,0)}_{(P)}\, a^{(1,1)}_{(P)} - (a^{(0,0)}_{(P)})^2\}\, L^{(1,1)}_{(P)}\ , \tag{5.8}$$

$$\chi^{(1)}_{(ED)} = 1 + a^{(0,0)}_{(P)}\ , \tag{5.9}$$

$$\chi^{(0)}_{(DE)} = a^{(0,0)}_{(P)}\, (a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\, L^{(1,1)}_{(P)}\ , \tag{5.10}$$

$$\chi^{(1)}_{(DE)} = a^{(0,0)}_{(P)}\ , \tag{5.11}$$

$$\chi^{(2)} = 0. \tag{5.12}$$

Hence, one obtains for (3.21)

$$\chi^{(0)}_{(ED)}\, \mathbf{E} + \chi^{(1)}_{(ED)}\, \frac{d\,\mathbf{E}}{dt} = \chi^{(0)}_{(DE)}\, \mathbf{D} + \chi^{(1)}_{(DE)}\, \frac{d\,\mathbf{D}}{dt}. \tag{5.13}$$

In this case the contribution $\sigma^{(s)}_{(P)}$ of polarization phenomena to the entropy production is given by

$$\sigma^{(s)}_{(P)} = T^{-1}\, L^{(1,1)}_{(P)}\, (\mathbf{E}^{(1)})^2\ , \tag{5.14}$$

as may be seen from equation (5.25) of ref. 4. With the help of (5.2) one obtains

$$\sigma^{(s)}_{(P)} = (T\, L^{(1,1)}_{(P)})^{-1} \left[\frac{d\, \mathbf{P}^{(1)}}{dt} \right]^2 . \tag{5.15}$$

Hence, only changes in $\mathbf{P}^{(1)}$ contribute to the entropy production. This includes that changes in $\mathbf{P}^{(0)}$ are reversible. This may also be seen from (5.1). The latter equation expresses that $\mathbf{P}^{(0)}$ is in equilibrium with the electric field **E**. Thus, $\mathbf{P}^{(0)}$ does not contribute to the dielectric relaxation.

An equation of the type (5.7) has been derived by Debye for polar liquids [8]. In the Debye theory $\mathbf{P}^{(0)}$ is the polarization of the liquid due to the elastic deformation of the molecules. It is assumed that changes in these elastic deformations are reversible phenomena. $\mathbf{P}^{(1)}$ is the polarization due to the rotation of the molecules and these rotations are supposed to be irreversible processes.

It is seen from (5.2) that sudden changes in $\mathbf{P}^{(1)}$ are impossible. On the other hand, by virtue of (5.1) a sudden change $\Delta\, \mathbf{E}$ in **E** is associated with a sudden change $\Delta\, \mathbf{P}^{(0)}$ in $\mathbf{P}^{(0)}$. Therefore $\mathbf{P}^{(0)}$ may be called the elastic part of the polarization and $\mathbf{P}^{(1)}$ the inelastic part. Hence, we have

$$\Delta\, \mathbf{E} = a^{(0,0)}_{(P)}\, \Delta\, \mathbf{P} = a^{(0,0)}_{(P)}\, \Delta\, \mathbf{P}^{(0)} , \tag{5.16}$$

or

$$\Delta\, \mathbf{P} = \Delta\, \mathbf{P}^{(0)} = \chi_{(\Delta \mathbf{E})}\, \Delta\, \mathbf{E} , \tag{5.17}$$

where

$$\chi_{(\Delta \mathbf{E})} = 1 \,/\, a^{(0,0)}_{(P)} . \tag{5.18}$$

If we add $\Delta\, \mathbf{E}$ to both sides of (5.17) we get with the aid of (2.5)

$$\Delta\, \mathbf{D} = \varepsilon_{(\Delta \mathbf{E})}\, \Delta\, \mathbf{E} , \tag{5.19}$$

where

$$\varepsilon_{(\Delta \mathbf{E})} = 1 + \chi_{(\Delta \mathbf{E})} . \tag{5.20}$$

One may call $\varepsilon_{(\Delta \mathbf{E})}$ the dielectric jump-permeability and $\chi_{(\Delta \mathbf{E})}$ the dielectric jump-susceptibility. Using the second of the inequalities (4.2) it follows from (5.18) and (5.20) that

$$\chi_{(\Delta \mathbf{E})} > 0 , \quad \varepsilon_{(\Delta \mathbf{E})} > 1 . \tag{5.21}$$

The possibility of sudden changes in $\mathbf{P}^{(0)}$ is connected with the fact that if $L^{(0,0)}_{(P)}$, $L^{(0,1)}_{(P)}$ and $L^{(1,0)}_{(P)}$ vanish changes in $\mathbf{P}^{(0)}$ are reversible processes.

6. The De Groot-Mazur equation

In this section we consider the case in which $L^{(1,1)}_{(P)}$, $L^{(0,1)}_{(P)}$ and $L^{(1,0)}_{(P)}$ vanish. One then has from (2.19)

$$\mathbf{E} = \mathbf{E}^{(eq)} + L^{(0,0)}_{(P)} \frac{d\,\mathbf{P}}{dt}. \tag{6.1}$$

From (2.20) we find that $d\,\mathbf{P}^{(1)} / dt$ vanishes and we shall assume that

$$\mathbf{P}^{(1)} = \mathbf{0}. \tag{6.2}$$

Hence, from (2.10)

$$\mathbf{P} = \mathbf{P}^{(0)}. \tag{6.3}$$

From the linear equation of state (3.12) one then gets

$$\mathbf{E}^{(eq)} = a^{(0,0)}_{(P)}\, \mathbf{P}. \tag{6.4}$$

Using (6.4) the relation (6.1) becomes

$$\mathbf{E} = a^{(0,0)}_{(P)}\, \mathbf{P} + L^{(0,0)}_{(P)} \frac{d\,\mathbf{P}}{dt}, \tag{6.5}$$

or, if $\mathbf{P}$ is replaces by $\mathbf{D} - \mathbf{E}$,

$$(1 + a^{(0,0)}_{(P)})\, \mathbf{E} + L^{(0,0)}_{(P)} \frac{d\,\mathbf{E}}{dt} = a^{(0,0)}_{(P)}\, \mathbf{D} + L^{(0,0)}_{(P)} \frac{d\,\mathbf{D}}{dt}. \tag{6.6}$$

An equation of the type (6.5) has been derived by De Groot and Mazur [9]. In this case there is no splitting up of the polarization into two parts (see (2.10)). However, changes in the polarization are irreversible processes.

Since we assume in this section that the coefficients $L^{(1,1)}_{(P)}$, $L^{(0,1)}_{(P)}$ and $L^{(1,0)}_{(P)}$ vanish, the coefficients in the relaxation equation (3.15), which are given by (3.16)-(3.19), reduce to

$$\chi^{(0)}_{(EP)} = 0, \quad \chi^{(0)}_{(PE)} = 0, \quad \chi^{(1)}_{(PE)} = a^{(0,0)}_{(P)}, \quad \chi^{(2)}_{(PE)} = L^{(0,0)}_{(P)}. \tag{6.7}$$

Hence, (3.15) becomes

$$\frac{d\,\mathbf{E}}{dt} = \chi^{(1)}_{(PE)} \frac{d\,\mathbf{P}}{dt} + \chi^{(2)}_{(PE)} \frac{d^2\,\mathbf{P}}{dt^2} = a^{(0,0)}_{(P)} \frac{d\,\mathbf{P}}{dt} + L^{(0,0)}_{(P)} \frac{d^2\,\mathbf{P}}{dt^2}. \tag{6.8}$$

It is seen that (6.8) is also obtained if one takes the substantial derivative with respect to time of both sides of (6.5).

The coefficients in (3.21), given by (3.22)-(3.26), reduce to

$$\chi^{(0)}_{(ED)} = 0, \quad \chi^{(0)}_{(DE)} = 0, \quad \chi^{(2)} = L^{(0,0)}_{(P)}, \tag{6.9}$$

$$\chi^{(1)}_{(DE)} = a^{(0,0)}_{(P)}, \quad \chi^{(1)}_{(ED)} = 1 + a^{(0,0)}_{(P)}. \tag{6.10}$$

It is seen that if one takes the substantial derivative with respect to time of both sides of (6.6) the same relation is obtained as the relation (3.21) with the expressions (6.9) and (6.10) for the coefficients.

It is seen from (6.6) that in a static state where all time derivatives vanish

$$\mathbf{D} = \varepsilon_{(eq)}\,\mathbf{E} \qquad \text{(static case)}, \tag{6.11}$$

where the static dielectric permeability (or equilibrium dielectric permeability) $\varepsilon_{(eq)}$ is given by

$$\varepsilon_{(eq)} = \left[1 + \frac{1}{a^{(0,0)}_{(P)}}\right] = \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} \tag{6.12}$$

(see also (6.10)).

7. Electromagnetic waves

In the case that electric currents and electric charges are neglected, the equations (2.1)-(2.4) read

$$\text{rot } \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = \mathbf{0} , \tag{7.1}$$

$$\text{rot } \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} , \tag{7.2}$$

$$\text{div } \mathbf{D} = 0 , \tag{7.3}$$

$$\text{div } \mathbf{B} = 0. \tag{7.4}$$

Moreover, we recall that according to (3.27) we suppose that

$$\mathbf{B} = \mu \, \mathbf{H}. \tag{7.5}$$

Hence, we neglect magnetic relaxation phenomena.

If we assume that the medium is at rest the substantial derivative with respect to time $d / d t$ may be replaced by the local derivative with respect to time $\partial / \partial t$ (see (2.22)) and the equation (3.21) for dielectric relaxation then reduces to

$$\chi^{(0)}_{(ED)} \, \mathbf{E} + \chi^{(1)}_{(ED)} \frac{\partial \mathbf{E}}{\partial t} + \chi^{(2)} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \chi^{(0)}_{(DE)} \, \mathbf{D} + \chi^{(1)}_{(DE)} \frac{\partial \mathbf{D}}{\partial t} + \chi^{(2)} \frac{\partial^2 \mathbf{D}}{\partial t^2}. \tag{7.6}$$

It is the purpose of this section to show that equations (7.1)-(7.6) have solutions which only depend on

$$\xi = k \, x_1 - \omega \, t , \tag{7.7}$$

where k is the complex wave number and ω is the real angular frequency, i.e. we consider plane waves which propagate in the direction of the x_1-axis.

We assume that a generic vector $\mathbf{V}$ of the field $(\mathbf{H}, \mathbf{E}, \mathbf{D}, \mathbf{B})$ has the form

$$\mathbf{V} = \mathbf{V}^{(0)} \exp (i \, \xi) \quad \text{where } \mathbf{V}^{(0)} = (0, \mathbf{V}^{(0)}_2, \mathbf{V}^{(0)}_3) , \tag{7.8}$$

$i^2 = -1$ and $\mathbf{V}^{(0)}$ is a constant which may be complex.

Utilizing (7.7) and (7.8) we have the following relations

$$\frac{\partial \mathbf{V}}{\partial t} = - i \, \omega \, \mathbf{V} , \tag{7.9}$$

$$\frac{\partial \mathbf{V}}{\partial x_1} = i \, k \, \mathbf{V} , \; \frac{\partial \mathbf{V}}{\partial x_2} = \frac{\partial \mathbf{V}}{\partial x_3} = \mathbf{0} , \tag{7.10}$$

$$\frac{\partial^2 \mathbf{V}}{\partial t^2} = - \omega^2 \, \mathbf{V}. \tag{7.11}$$

By virtue of (7.8) one has

$$E_1 = D_1 = B_1 = H_1 = 0, \tag{7.12}$$

where we have indicated with subscript 1 the components of the vectors **E**, **D**, **B** and **H** with respect to the x_1-axis. Hence, we consider transverse waves, as may be seen from (7.7), (7.8) and (7.12). Using (7.10) and (7.12) it is seen that the Maxwell equations (7.3) and (7.4) are satisfied.

With the aid of (7.9), (7.10) and (7.12), the equation (7.1) leads to

$$D_2 = \frac{c\,k}{\omega} H_3 , \tag{7.13}$$

$$D_3 = -\frac{c\,k}{\omega} H_2 , \tag{7.14}$$

where the subscripts 2 and 3 indicate the components of the vectors with respect to the x_2- and x_3-axes, respectively.

In a similar way we have from (7.2)

$$B_2 = -\frac{c\,k}{\omega} E_3 , \tag{7.15}$$

$$B_3 = \frac{c\,k}{\omega} E_2 . \tag{7.16}$$

By virtue of (7.15) and (7.16) the equation (7.5) gives

$$H_2 = -\frac{c\,k}{\omega\,\mu} E_3 , \tag{7.17}$$

$$H_3 = \frac{c\,k}{\omega\,\mu} E_2 , \tag{7.18}$$

and so, utilizing the latter expressions, (7.13) and (7.14) become

$$D_2 = \frac{c^2 k^2}{\mu\,\omega^2} E_2 , \tag{7.19}$$

$$D_3 = \frac{c^2 k^2}{\mu\,\omega^2} E_3 , \tag{7.20}$$

respectively.

Thus, because of (7.12) and (7.15)-(7.20) the vectors of the electromagnetic field have the following forms

$$\mathbf{E} = (0, E_2, E_3) , \tag{7.21}$$

$$\mathbf{D} = (0, \frac{c^2 k^2}{\mu\,\omega^2} E_2 , \frac{c^2 k^2}{\mu\,\omega^2} E_3) , \tag{7.22}$$

$$\mathbf{B} = (0\,,\; -\frac{c\,k}{\omega}\,E_3\,,\; \frac{c\,k}{\omega}\,E_2)\,, \tag{7.23}$$

$$\mathbf{H} = (0\,,\; -\frac{c\,k}{\mu\,\omega}\,E_3\,,\; \frac{c\,k}{\mu\,\omega}\,E_2). \tag{7.24}$$

From (7.6) it is seen that the components of **E** and **D** must satisfy the relation

$$\chi^{(0)}_{(ED)}\,E_\alpha + \chi^{(1)}_{(ED)}\,\frac{\partial E_\alpha}{\partial t} + \chi^{(2)}\,\frac{\partial^2 E_\alpha}{\partial t^2} = \chi^{(0)}_{(DE)}\,D_\alpha + \chi^{(1)}_{(DE)}\,\frac{\partial D_\alpha}{\partial t} + \chi^{(2)}\,\frac{\partial^2 D_\alpha}{\partial t^2}$$

$$(\alpha = 1\,,\,2\,,\,3). \tag{7.25}$$

Since $E_1 = D_1 = 0$ (see (7.21) and (7.22)) the equation is satisfied for $\alpha = 1$. From (7.25) we obtain with the help of (7.9), (7.11), (7.19) and (7.20)

$$\left\{ (\chi^{(0)}_{(ED)} - i\,\omega\,\chi^{(1)}_{(ED)} - \omega^2\,\chi^{(2)}) - (\chi^{(0)}_{(DE)} - i\,\omega\,\chi^{(1)}_{(DE)} - \omega^2\,\chi^{(2)})\,\frac{c^2\,k^2}{\mu\,\omega^2} \right\} E_\alpha = 0$$

$$(\alpha = 2\,,\,3). \tag{7.26}$$

If we suppose that

$$E_\alpha \neq 0 \quad (\alpha = 2\,,\,3) \tag{7.27}$$

one has from (7.26)

$$k^2 = \frac{\mu\,\omega^2}{c^2} \left\{ \frac{(\chi^{(0)}_{(ED)} - \omega^2\,\chi^{(2)}) - i\,\omega\,\chi^{(1)}_{(ED)}}{(\chi^{(0)}_{(DE)} - \omega^2\,\chi^{(2)}) - i\,\omega\,\chi^{(1)}_{(DE)}} \right\}. \tag{7.28}$$

From (7.21)-(7.24) we find

$$\mathbf{E}\cdot\mathbf{B} = \mathbf{E}\cdot\mathbf{H} = 0\,, \quad \mathbf{D}\cdot\mathbf{B} = \mathbf{D}\cdot\mathbf{H} = 0 \tag{7.29}$$

and

$$\mathbf{D} = \frac{c^2\,k^2}{\mu\,\omega^2}\,\mathbf{E}\,, \tag{7.30}$$

or

$$\mathbf{D} = \varepsilon_{(compl)}\,\mathbf{E}\,, \tag{7.31}$$

where

$$\varepsilon_{(compl)} = \frac{c^2\,k^2}{\mu\,\omega^2} \tag{7.32}$$

is the complex dielectric permeability.

Using (7.28) one obtains from (7.32)

$$\varepsilon_{(compl)} = \frac{\chi^{(0)}_{(ED)} - \omega^2 \chi^{(2)} - i\,\omega\,\chi^{(1)}_{(ED)}}{\chi^{(0)}_{(DE)} - \omega^2 \chi^{(2)} - i\,\omega\,\chi^{(1)}_{(DE)}}. \tag{7.33}$$

Hence, for $\omega \to \infty$ we have

$$\varepsilon_{(\infty)} = \lim_{\omega\to\infty} \varepsilon_{(compl)} = 1. \tag{7.34}$$

and if $\omega \to 0$, using (3.29), we have

$$\varepsilon_{(0)} = \lim_{\omega\to 0} \varepsilon_{(compl)} = \frac{\chi^{(0)}_{(ED)}}{\chi^{(0)}_{(DE)}} = \varepsilon_{(eq)}\,, \tag{7.35}$$

where $\varepsilon_{(eq)}$ is the static dielectric permeability (see (3.28) and (3.29)).

Next, we wish to derive the velocity of propagation and the attenuation law of the electromagnetic waves. Let k_1 be the real part of the complex wave number k and k_2 the imaginary part. Hence,

$$k = k_1 + i\,k_2\,, \tag{7.36}$$

where k_1 and k_2 are real numbers.

It is our purpose to find k_1 and k_2 as function of ω, i.e. we wish to find the dispersion law.

To this end we note that we obtain from (7.28)

$$k_1^2 - k_2^2 = \frac{\mu\,\omega^2}{c^2}\left\{ \frac{\omega^4(\chi^{(2)})^2 + \omega^2\,[\chi^{(1)}_{(ED)}\,\chi^{(1)}_{(DE)} - \chi_{(2)}(\chi^{(0)}_{(ED)} + \chi^{(0)}_{(DE)})\,] + \chi^{(0)}_{(ED)}\,\chi^{(0)}_{(DE)}}{(\chi^{(0)}_{(DE)} - \omega^2\,\chi^{(2)})^2 + \omega^2(\chi^{(1)}_{(DE)})^2} \right\}, \tag{7.37}$$

and

$$k_1\,k_2 = \frac{\mu\,\omega^3}{2\,c^2}\left\{ \frac{\omega^2\,\chi^{(2)}\,(\chi^{(1)}_{(ED)} - \chi^{(1)}_{(DE)}) + (\chi^{(1)}_{(DE)}\,\chi^{(0)}_{(ED)} - \chi^{(1)}_{(ED)}\,\chi^{(0)}_{(DE)})}{(\chi^{(0)}_{(DE)} - \omega^2\,\chi^{(2)})^2 + \omega^2\,(\chi^{(1)}_{(DE)})^2} \right\}. \tag{7.38}$$

Utilizing the relations (3.23) and (3.25) we have

$$\chi^{(1)}_{(ED)} - \chi^{(1)}_{(DE)} = 1\,, \tag{7.39}$$

and the relation (7.38) becomes

$$k_1\,k_2 = \frac{\mu\,\omega^3}{2\,c^2}\left\{ \frac{\omega^2\,\chi^{(2)} + (\chi^{(1)}_{(DE)}\,\chi^{(0)}_{(ED)} - \chi^{(1)}_{(ED)}\,\chi^{(0)}_{(DE)})}{(\chi^{(0)}_{(DE)} - \omega^2\,\chi^{(2)})^2 + \omega^2\,(\chi^{(1)}_{(DE)})^2} \right\}. \tag{7.40}$$

By virtue of the inequalities (4.11) (4.14) and (4.16), one obtains from (7.37)

$$|\,k_1\,| \geq |\,k_2\,|\,, \tag{7.41}$$

and from (7.40)

$$k_1\, k_2 \geq 0\,, \quad \text{if } \omega > 0. \tag{7.42}$$

In (7.41) $|\, k_i \,|$ is the absolute value of $k_i (i = 1\,,\, 2)$.

Hence, if we consider waves travelling in the direction of the positive x_1-axis (with $\omega > 0$ and $k_1 > 0$, see (7.7) and (7.8)) then $k_2 \geq 0$ as may be seen from (7.42) i.e. there is attenuation of the amplitude (unless $k_2 = 0$).

Let us introduce the abbreviations

$$\Phi(\omega) = \frac{c^2}{2\,\mu\,\omega^2}\,(k_1^2 - k_2^2) =$$

$$= \frac{1}{2}\left\{ \frac{\omega^4 (\chi^{(2)})^2 + \omega^2\, [\chi^{(1)}_{(ED)}\, \chi^{(1)}_{(DE)} - \chi^{(2)}\, (\chi^{(0)}_{(ED)} + \chi^{(0)}_{(DE)})\,] + \chi^{(0)}_{(ED)}\, \chi^{(0)}_{(DE)}}{(\chi^{(0)}_{(DE)} - \omega^2\, \chi^{(2)})^2 + \omega^2 (\chi^{(1)}_{(DE)})^2} \right\}, \tag{7.43}$$

and

$$\Psi(\omega) = \frac{4 (k_1\, k_2)^2}{(k_1^2 - k_2^2)^2} =$$

$$= \frac{\omega^2\, \{\omega^2\, \chi^{(2)} + (\chi^{(1)}_{(DE)}\, \chi^{(0)}_{(ED)} - \chi^{(1)}_{(ED)}\, \chi^{(0)}_{(DE)})\}^2}{\{\omega^4 (\chi^{(2)})^2 + \omega^2\, [\chi^{(1)}_{(ED)}\, \chi^{(1)}_{(DE)} - \chi^{(2)}\, (\chi^{(0)}_{(ED)} + \chi^{(0)}_{(DE)})\,] + \chi^{(0)}_{(ED)}\, \chi^{(0)}_{(DE)}\}^2}, \tag{7.44}$$

where we used (7.37) and (7.40).

Using the inequalities (4.3) and (7.41) it is seen from (7.43) and (7.44) that

$$\Phi\,(\omega) \geq 0\,, \tag{7.45}$$

$$\Psi\,(\omega) \geq 0. \tag{7.46}$$

If we solve (7.37) and (7.40) we obtain with the help of (7.43) and (7.44) for the components of the wave number the following expressions

$$k_1 = \frac{\omega}{c}\, \sqrt{\mu} \left\{ \Phi(\omega) \left[\sqrt{1 + \Psi(\omega)} + 1 \right] \right\}^{1/2}, \tag{7.47}$$

$$k_2 = \frac{\omega}{c}\, \sqrt{\mu} \left\{ \Phi(\omega) \left[\sqrt{1 + \Psi(\omega)} - 1 \right] \right\}^{1/2}. \tag{7.48}$$

From (7.45) and (7.46) it is seen that k_1 and k_2 are real, positive if $\omega > 0$ and satisfy the inequalities (7.41) and (7.42).

Finally, we note that $v_{(ph)}$, the phase velocity of the waves, is given by

$$v_{(ph)} = \frac{\omega}{k_1}\,, \tag{7.49}$$

while

$$\exp(-k_2 x_1) \tag{7.50}$$

is a factor which is responsible for the attenuation of the amplitude.

8. Low and high frequencies

If ω is sufficiently small one obtains from (7.43), (7.44), (7.42) and (7.48)

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu \frac{\chi^{(0)}_{(ED)}}{\chi^{(0)}_{(DE)}}} \quad \text{(low freq.)}, \tag{8.1}$$

$$k_2 \cong 0 \qquad \text{(low freq.)}. \tag{8.2}$$

Using (7.35) one obtains from (8.1)

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu \, \varepsilon_{(0)}} \quad \text{(low freq.)}, \tag{8.3}$$

or

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu \, \varepsilon_{(eq)}} \quad \text{(low freq.)}. \tag{8.4}$$

For the phase velocity we have from (7.49), (8.3) or (8.4)

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu \, \varepsilon_{(0)}}} \quad \text{(low freq.)}, \tag{8.5}$$

or

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu \, \varepsilon_{(eq)}}} \quad \text{(low freq.)}. \tag{8.6}$$

By virtue of (7.44), (8.2), (8.5) (or (8.6)) it is seen that in a first approximation at low frequencies the dielectric relaxation phenomena do not influence the propagation of electromagnetic waves.

Next, we consider the case of high frequencies. From (7.43), (7.44), (7.47) and (7.48) one then finds

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu} \quad \text{(high freq.)}, \tag{8.7}$$

and

$$k_2 \cong 0 \qquad \text{(high freq.)}. \tag{8.8}$$

By virtue of (7.34) the relation (8.7) can be written in the following form

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu \, \varepsilon_{(\infty)}} \quad \text{(high freq.)}. \tag{8.9}$$

For the phase velocity of the waves we obtain from (7.49) and (8.9)

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu \, \varepsilon_{(\infty)}}} \quad \text{(high freq.)}. \tag{8.10}$$

From (7.50), (8.8) and (8.10) it is seen that also at high frequencies the dielectric relaxation phenomena do not influence the wave propagation.

In particular, using (7.31) and (7.34), we note that at high frequencies one has

$$\mathbf{D} \cong \mathbf{E} \quad \text{(high freq.)}\,, \tag{8.11}$$

and consequently from (2.5) we have

$$\mathbf{P} \cong 0 \quad \text{(high freq.)}. \tag{8.12}$$

The relation (8.12) means that at high frequencies the polarization cannot follow the electromagnetic field and changes in the polarization are impossible. Hence, damping of the waves due to polarization does not occur. If the frequencies are very low the polarization can follow the field instantaneously and changes in the polarization are quasistatic processes also without damping of the waves.

9. Wave propagation in media with dielectric relaxation phenomena described by the Debye equation

In sect. 5 we have shown that if changes in $\mathbf{P}^{(0)}$ are reversible the dynamical equation (7.6) reduces to the Debye equation. As noted in the first paragraph of sect. 5 the Debye equation is obtained if one assumes that $L^{(0,0)}_{(P)}$ and $L^{(0,1)}_{(P)} = -L^{(1,0)}_{(P)}$ vanish. In this case the expressions (3.22)-(3.26) become

$$\chi^{(2)} = 0\,, \tag{9.1}$$

$$\chi^{(0)}_{(ED)} = \{a^{(1,1)}_{(P)} + a^{(0,0)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\}\,L^{(1,1)}_{(P)} \geq 0\,, \tag{9.2}$$

$$\chi^{(1)}_{(ED)} = (1 + a^{(0,0)}_{(P)}) \geq 1\,, \tag{9.3}$$

$$\chi^{(0)}_{(DE)} = a^{(0,0)}_{(P)}\,(a^{(1,1)}_{(P)} - a^{(0,0)}_{(P)})\,L^{(1,1)}_{(P)} \geq 0\,, \tag{9.4}$$

$$\chi^{(1)}_{(DE)} = a^{(0,0)}_{(P)} \geq 0. \tag{9.5}$$

Using (9.1) one has from (7.43), (7.44) and (7.33)

$$\Phi(\omega) = \frac{1}{2}\left\{\frac{\omega^2\,\chi^{(1)}_{(ED)}\,\chi^{(1)}_{(DE)} + \chi^{(0)}_{(ED)}\,\chi^{(0)}_{(DE)}}{(\chi^{(0)}_{(DE)})^2 + \omega^2\,(\chi^{(1)}_{(DE)})^2}\right\}, \tag{9.6}$$

$$\Psi(\omega) = \frac{\omega^2\,(\chi^{(1)}_{(DE)}\,\chi^{(0)}_{(ED)} - \chi^{(1)}_{(ED)}\,\chi^{(0)}_{(DE)})^2}{(\omega^2\,\chi^{(1)}_{(ED)}\,\chi^{(1)}_{(DE)} + \chi^{(0)}_{(ED)}\,\chi^{(0)}_{(DE)})^2} \tag{9.7}$$

and

$$\varepsilon_{(compl)} = \frac{\chi^{(0)}_{(ED)} - i\,\omega\,\chi^{(1)}_{(ED)}}{\chi^{(0)}_{(DE)} - i\,\omega\,\chi^{(1)}_{(DE)}}\,, \tag{9.8}$$

where $\chi^{(0)}_{(DE)}$, $\chi^{(0)}_{(DE)}$, $\chi^{(1)}_{(ED)}$ and $\chi^{(1)}_{(DE)}$ are given by (9.2)-(9.5).

If ω is sufficiently small one obtains from (7.42), (7.48), (9.6), (9.7), (9.8) and (3.29)

$$k_1 \cong \frac{\omega}{c}\sqrt{\mu\,\frac{\chi^{(0)}_{(ED)}}{\chi^{(0)}_{(DE)}}} \quad \text{(low freq.)}\,, \tag{9.9}$$

$$k_2 \cong 0 \qquad \text{(low freq.)}\,, \tag{9.10}$$

and

$$\varepsilon_{(0)} = \lim_{\omega\to 0}\varepsilon_{(compl)} = \frac{\chi^{(0)}_{(ED)}}{\chi^{(0)}_{(DE)}} = \varepsilon_{(eq)}. \tag{9.11}$$

Using (7.49), (9.9) and (9.11) one has

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu\,\varepsilon_{(0)}}} = \frac{c}{\sqrt{\mu\,\varepsilon_{(eq)}}} \quad \text{(low freq.)}. \tag{9.12}$$

We note that for low freqencies we have the same results which were obtained in sect. 8 (see (8.2) and (8.5) or (8.6)).

For high frequency waves we have from (7.47), (7.48), (9.6), (9.7) and (9.8)

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}}} \quad \text{(high freq.)}, \tag{9.13}$$

$$k_2 \cong 0 \qquad \text{(high freq.)} \tag{9.14}$$

and

$$\varepsilon_{(\infty)} = \lim_{\omega \to \infty} \varepsilon_{(compl)} = \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} \quad \text{(high freq.)}. \tag{9.15}$$

Using (7.49), (9.13) and (9.15) we obtain

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu\, \varepsilon_{(\infty)}}} \quad \text{(high freq.)}. \tag{9.16}$$

We observe that, by virtue of (9.3), (9.5) and (5.18) the relation (9.13) becomes

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu\, (1+\chi_{(\Delta E)})} \quad \text{(high freq.)}, \tag{9.17}$$

where $\chi_{(\Delta E)}$ is the dielectric jump-susceptibility which was introduced in sect. 5 of this paper. Consequently, by virtue of (9.17), (7.49) and (5.20) we have

$$v_{(ph)} \cong \frac{c}{\sqrt{\mu\, \varepsilon_{(\Delta E)}}} \quad \text{(high freq.)}, \tag{9.18}$$

i.e. at high freqencies the phase velocity of electromagnetic waves which propagate in polar liquids depends on the magnetic permeability, μ, and the dielectric jump-permeability, $\varepsilon_{(\Delta E)}$ (see (5.20)).

10. Wave propagation in media with dielectric relaxation phenomena described by the De Groot-Mazur equation

In sect. 6 we have shown that if the polarization does not split up into two parts, we have (see (6.9) and (6.10))

$$\chi^{(2)} = L^{(0,0)}_{(P)} \geq 0 , \tag{10.1}$$

$$\chi^{(0)}_{(ED)} = \chi^{(0)}_{(DE)} = 0 , \tag{10.2}$$

$$\chi^{(1)}_{(ED)} = 1 + a^{(0,0)}_{(P)} > 1 , \tag{10.3}$$

$$\chi^{(1)}_{(DE)} = a^{(0,0)}_{(P)} > 0 \tag{10.4}$$

and the relaxation equation (7.6) reduces to the De Groot-Mazur equation (see sect. 6).

In this case, using (10.2) one has from (7.43), (7.44) and (7.33)

$$\Phi(\omega) = \frac{1}{2} \left\{ \frac{\omega^2 (\chi^{(2)})^2 + \chi^{(1)}_{(ED)}\, \chi^{(1)}_{(DE)}}{\omega^2 (\chi^{(2)})^2 + (\chi^{(1)}_{(DE)})^2} \right\} , \tag{10.5}$$

$$\Psi(\omega) = \frac{\omega^2\, (\chi^{(2)})^2}{\{\omega^2 (\chi^{(2)})^2 + \chi^{(1)}_{(ED)}\, \chi^{(1)}_{(DE)}\}^2} \tag{10.6}$$

and

$$\varepsilon_{(compl)} = \frac{\omega\, \chi^{(2)} + i\, \chi^{(1)}_{(ED)}}{\omega\, \chi^{(2)} + i\, \chi^{(1)}_{(DE)}} , \tag{10.7}$$

where $\chi^{(2)}$, $\chi^{(1)}_{(ED)}$ and $\chi^{(1)}_{(DE)}$ are given by (10.1), (10.3) and (10.4), respectively.

If ω is sufficiently small one obtains from (6.12), (7.47), (7.48), (10.5), (10.6) and (10.7)

$$k_1 \cong \frac{\omega}{c} \sqrt{\mu\, \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}}} = \frac{\omega}{c} \sqrt{\mu\, \varepsilon_{(eq)}} \quad \text{(low freq.)} , \tag{10.8}$$

$$k_2 \cong 0 \qquad \text{(low freq.)} , \tag{10.9}$$

$$\varepsilon_{(0)} = \lim_{\omega \to 0} \varepsilon_{(compl)} = \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} = 1 + \frac{1}{a^{(0,0)}_{(P)}} = \varepsilon_{(eq)} \quad \text{(low freq.)} \tag{10.10}$$

and

$$v_{(ph)} = \frac{\omega}{k_1} \cong \frac{c}{\sqrt{\mu\, \varepsilon_{(eq)}}} \quad \text{(low freq.).} \tag{10.11}$$

We observe that these results are the same as those obtained in the case of wave propagation at high freqencies in the Debye theory (see (9.13), (9.14) and (9.15)).

For waves at high frequencies we have from (7.47), (7.48), (10.5), (10.6) and (10.7)

$$k_1 \cong \frac{\omega}{c}\sqrt{\mu} \quad \text{(high freq.)}\,, \tag{10.12}$$

$$k_2 \cong 0 \quad \text{(high freq.)}\,, \tag{10.13}$$

$$\varepsilon_{(\infty)} = \lim_{\omega \to \infty} \varepsilon_{(compl)} = 1 \quad \text{(high freq.)} \tag{10.14}$$

and

$$v_{(ph)} = \frac{\omega}{k_1} \cong \frac{c}{\sqrt{\mu}} = \frac{c}{\sqrt{\mu\, \varepsilon_{(\infty)}}} \quad \text{(high freq.)}. \tag{10.15}$$

From (10.12), (10.13), (10.14) and (10.15) it is seen that at high frequencies the wave propagation in the De Groot-Mazur theory for media with dielectric relaxation has the same behaviour as described in the theory which was reviewed in the sections 2, 3, 4, 7 and 8 of this paper.

11. Wave propagation in media without dielectric relaxation

Finally, we consider media without dielectric relaxation phenomena and we will show that in this case our formalism reduces to the well-known theory for nondissipative media. We obtain the equations for a nondissipative medium if we assume that in the De Groot-Mazur equation

$$\chi^{(2)} = L^{(0,0)}_{(P)} = 0 \tag{11.1}$$

(see (10.1)). The equations (10.5), (10.6) and (10.7) then reduce to

$$\Phi = \frac{1}{2} \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} , \tag{11.2}$$

$$\Psi = 0 , \tag{11.3}$$

$$\varepsilon_{(compl)} = \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} . \tag{11.4}$$

It is seen that in this case $\varepsilon_{(compl)}$ is real and does not depend on ω.

Hence, we have (see (10.10), (10.14) and (11.4))

$$\varepsilon_{(0)} = \varepsilon_{(\infty)} = \varepsilon_{(compl)} = 1 + \frac{1}{a^{(0,0)}_{(P)}} = \frac{\chi^{(1)}_{(ED)}}{\chi^{(1)}_{(DE)}} = \varepsilon_{(eq)} . \tag{11.5}$$

Furthermore, we obtain from (7.47), (7.48), (11.2), (11.3) and (11.5)

$$k_1 = \frac{\omega}{c} \sqrt{\mu\, \varepsilon_{(eq)}} , \quad k_2 = 0 \tag{11.6}$$

and by virtue of (7.49)

$$v_{(ph)} = \frac{\omega}{k_1} = \frac{c}{\sqrt{\mu\, \varepsilon_{(eq)}}} . \tag{11.7}$$

Since k_2 vanishes there is no damping of the waves (see (7.50)).

Finally, we note that (11.7) is the well-known formula for the phase velocity of electromagnetic waves in nondissipative media.

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