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## ANNOUNCEMENTS OF NEW RESULTS

### ON ESTIMATING THE DIFFUSION COEFFICIENT

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Consider the diffusion process  $\xi$  defined on  $(\Omega, \mathcal{F}, P)$  by  

$$d\xi_t = a(\xi_t, \vartheta)dt + b(\xi_t, \vartheta)dW_t, \quad \xi_0 = x_0, \quad t \in [0, T],$$
 $\vartheta \in \Theta$ , where  $\Theta$  is an open subset of real line,  $\{W_t, t \in [0, T]\}$  is a standard Wiener process. Suppose that  $a(x, \vartheta), b(x, \vartheta)$  are real-valued functions, continuous on  $R \times \Theta$ ,  $b(x, \vartheta) > 0$  for all  $(x, \vartheta) \in R \times \Theta$  and such that  $a', a'', \dot{a}, \dot{a}', b', b'', b''', \dot{b}, \dot{b}', \dot{b}''$  are continuous on  $R \times \Theta$  (here the stroke and the dot denote derivative with respect to  $x$  and  $\vartheta$  respectively). Denote  $g(x, \vartheta) = \dot{b}(x, \vartheta)/b(x, \vartheta)$ .

The chain  $\{X_k\}_{k=0}^n$  of observations of the process  $\xi_t$  at discrete sampling points  $0=t_0 < t_1 < \dots < t_n=T$  is the Markov chain which generates on  $(\Omega^n, \mathcal{G}(X_1, \dots, X_n))$  the probability measure  $P_{\vartheta_0}^n$ . Local asymptotic mixed normality (LAMN). The families  $\{P_{\vartheta_0}^n, \vartheta_0 \in \Theta\}_{n \geq 1}$  satisfy the LAMN condition in some  $\vartheta_0 \in \Theta$ .

The minimax theorem. For any sequence  $\{T_n\}_{n \geq 1}$  of estimators based on  $X_k, k=0, 1, \dots, n$ , of unknown parameter  $\vartheta_0$  holds

$$\lim_{h \rightarrow \infty} \limsup_{n \rightarrow \infty} \sup_{|h| < \varepsilon} E_{\vartheta_0}^n (1(\sqrt{n}(T_n - \vartheta_{n,h}))) \geq \frac{1}{\sqrt{2\pi}} \int 1\left(\frac{z}{\sqrt{w}}\right) e^{-\frac{1}{2}z^2} dz dG(w),$$

where  $\vartheta_{n,h} = \vartheta_0 + h/\sqrt{n}$ ,  $l(x)$  is a loss function and  $G(w)$  is the distribution function of  $\Gamma(\vartheta_0) = \frac{2}{T} \int_0^T g^2(\xi_t, \vartheta) dt$ .

The lower bound is obtained only if

$$(T_n - \vartheta_0) \rightarrow \left[ \sum_{k=0}^{m-1} g(X_k, \vartheta_0) (n(\sigma W_k)^2 - 1) \right] \cdot \left[ 2 \sum_{k=0}^{m-1} g^2(X_k, \vartheta_0) \right]^{-1}$$

in  $P_{\vartheta_0}^n$ -probability as  $n \rightarrow \infty$ , where  $\sigma W_k = W_{k+1} - W_k$ .

In particular, if  $l(x) = x^2$ , then for any  $\varepsilon > 0$  and for sufficiently large  $T$

$$\lim_{\varepsilon \rightarrow \infty} \limsup_{n \rightarrow \infty} \sup_{|h| < \varepsilon} n E_{\vartheta_0}^n (T_n - \vartheta_{n,h})^2 \geq [\mu(g^2)] - \varepsilon,$$

where  $\mu$  is the invariant measure of  $\xi$ .

### ON A CLASS OF WEAK ASPLUND SPACES WHICH HAS SOME PERMANENCE PROPERTIES

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Real Banach spaces,  $X, Y, \dots$  are considered. The set of all