Gejza Dohnal

On estimating the diffusion coefficient

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ANNOUNCEMENTS OF NEW RESULTS

ON ESTIMATING THE DIFFUSION COEFFICIENT

Geiza Dohnal (Fakulta strojní ČVUT Praha, Karlovo nám. 13, 12135 Praha 2, Československo), received 14.10. 1985.

Consider the diffusion process ξ defined on $(\Omega, \mathcal{F}, P)$ by

$$d\xi_t = a(\xi_t, \vartheta)dt + b(\xi_t, \vartheta)dW_t, \quad \xi_0 = x_0, \; t \in [0,T],$$

$\vartheta \in \Theta$, where Θ is an open subset of real line, $\{W_t, t \in [0,T]\}$ is a standard Wiener process. Suppose that $a(x,\vartheta), b(x,\vartheta)$ are real-valued functions, continuous on $R \times \Theta$, $b(x,\vartheta) > 0$ for all $(x,\vartheta) \in R \times \Theta$ and such that $a', a'', \dot{a}, \dot{a}', b', b'', b''', \dot{b}, \dot{b}', \dot{b}''$ are continuous on $R \times \Theta$ (here the stroke and the dot denote derivative with respect to x and ϑ respectively). Denote $g(x,\vartheta) = \dot{b}(x,\vartheta)/b(x,\vartheta)$.

The chain $\{X_k\}_{k=0}^n$ of observations of the process ξ_t at discrete sampling points $0=t_0 < t_1 < \ldots < t_n = T$ is the Markov chain which generates on $(\Omega^n, \sigma(X_1,\ldots,X_n))$ the probability measure P^n_ϑ.

Local asymptotic mixed normality (LAMN). The families $\{P^n_\vartheta, \vartheta \in \Theta\}_{n\ge 1}$ satisfy the LAMN condition in some $\vartheta_0 \in \Theta$.

The minimax theorem. For any sequence $\{T_n\}_{n\ge 1}$ of estimators based on X_k, $k=0,1,\ldots,n$, of unknown parameter ϑ_0 holds

$$\lim_{b\to\infty} \varliminf_{n\to\infty} \sup_{|h|<b} E^n_{\vartheta_{n,h}}(l(\sqrt{n}(T_n - \vartheta_{n,h}))) \ge \frac{1}{\sqrt{2\pi}} \int l\left(\frac{z}{\sqrt{w}}\right) e^{-\frac{1}{2}z^2} dz dG(w),$$

where $\vartheta_{n,h} = \vartheta_0 + h/\sqrt{n}$, $l(x)$ is a loss function and $G(w)$ is the distribution function of $\Gamma(\vartheta_0) = \frac{2}{T}\int_0^T g^2(\xi_t, \vartheta)dt$.

The lower bound is obtained only if

$$(T_n - \vartheta_0) \longrightarrow \Big[\sum_{k=0}^{n-1} g(X_k,\vartheta_0)(n(\delta W_k)^2 - 1)\Big] \cdot \Big[2\sum_{k=0}^{n-1} g^2(X_k,\vartheta_0)\Big]^{-1}$$

in $P^n_{\vartheta_0}$-probability as $n \to \infty$, where $\delta W_k = W_{k+1} - W_k$.

In particular, if $l(x) = x^2$, then for any $\varepsilon > 0$ and for sufficiently large T

$$\lim_{b\to\infty} \varliminf_{n\to\infty} \sup_{|h|<b} nE^n_{\vartheta_{n,h}}(T_n - \vartheta_{n,h})^2 \ge [\mu(g^2)] - \varepsilon,$$

where μ is the invariant measure of ξ.

ON A CLASS OF WEAK ASPLUND SPACES WHICH HAS SOME PERMANENCE PROPERTIES

Luděk Jokl (ČVUT Praha, Thákurova 7, 16629 Praha 6, Československo), received 9.1. 1986.

Real Banach spaces, X, Y,... are considered. The set of all