ON EVALUATING A COEFFICIENT OF PARTIAL CORRELATION

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It is to be shown here that when the multiple correlation coefficient $R_{n;12\cdots(n-1)}$ is found by the method of Horst[1] the partial correlation coefficient $R_{n(n-1);12\cdots(n-2)}$ can be found in terms of the β's. If we are interested only in evaluating a partial correlation between two variables, we may also employ the method which will be given here.

Without loss of generality the dependent variables may be chosen to be the nth and $(n-1)$st. The coefficient of partial correlation as given by Rietz[2] may be expressed in the following form:

$$R_{n(n-1);12\cdots(n-2)} = \sqrt{\frac{\dfrac{R_{(n-1)(n-1)}}{R_{(n-1)(n-1)nn}} - \dfrac{R}{R_{nn}}}{\dfrac{R_{(n-1)(n-1)}}{R_{(n-1)(n-1)nn}}}}. \tag{1}$$

$R_{(n-1)(n-1)}$ may be treated as a new determinant R'. Regarding its elements as the coefficients of a set of normal equations ($n - 1$ in all) whose constant terms are zero, we may follow through the Doolittle elimination process. For the case where $n = 4$ we have the table given below.

In comparing this outline with the one illustrating the Doolittle elimination process for R when $n = 4$ we see that

$$\gamma'_{11} = \gamma_{11} = \frac{A_{11}}{R^2},$$

$$\gamma'_{22} = \gamma_{22} = \frac{rA_{1122}}{R^2A_{11}},$$

$$\gamma'_{33} = \alpha'_{33} - \sum_2^3 \beta'_{i3} = \alpha_{44} - \sum_2^3 \beta_{i4}.$$

Therefore, we have

$$R' = \frac{A_{11}}{R^2}\cdot\frac{rA_{1122}}{R^2A_{11}}\cdot\left(\alpha_{44} - \sum_2^3 \beta_{i4}\right)$$

$$= \prod_1^2 \gamma_{ii}\left(\alpha_{44} - \sum_2^3 \beta_{i4}\right).$$

[1] Horst Paul, *A Short Method for Solving for a Coefficient of Multiple Correlation*, Annals of Mathematical Statistics, Vol. III, No. 1, Feb. 1932, pp. 40–44.

[2] Rietz, H. L., *Mathematical Statistics*, p. 101.

Reciprocal	1	2	3	α	β	γ	δ
$-\frac{R^2}{A_{11}}$	$\frac{A_{11}}{R^2}$	$\frac{A_{12}}{R^2}$	$\frac{A_{14}}{R^2}$	α'_1		γ'_1	
	-1	$-\frac{A_{12}}{A_{11}}$	$-\frac{A_{14}}{A_{11}}$				δ'_1
		$\frac{A_{22}}{R^2}$	$\frac{A_{24}}{R^2}$	α'_2			
		$-\frac{A_{12}^2}{R^2A_{11}}$	$-\frac{A_{12}A_{14}}{R^2A_{11}}$		β'_{22}		
$-\frac{R^2A_{11}}{AA_{1122}}$		$\frac{AA_{1122}}{R^2A_{11}}$	$\frac{AA_{1124}}{R^2A_{11}}$			γ'_2	
		-1	$-\frac{A_{1124}}{A_{1122}}$				δ'_2
			$\frac{A_{44}}{R^2}$	α'_3			
			$-\frac{A_{14}^2}{R^2A_{11}}$		β'_{23}		
			$-\frac{AA_{1124}^2}{R^2A_{11}A_{1122}}$		β'_{33}		
			$\alpha'_3-\sum_{2}^{3}\beta'_{i3}$			γ'_3	
			-1				δ'_3

In the general case:

$$\gamma'_{11} = \gamma_{11},$$

$$\gamma'_{22} = \gamma_{22},$$

$$\vdots$$

$$\gamma'_{(n-2)(n-2)} = \gamma_{(n-2)(n-2)},$$

$$\gamma'_{(n-1)(n-1)} = \alpha_{nn} - \sum_{2}^{n-1} \beta_{in}.$$

Hence

$$R_{(n-1)(n-1)} = R' = \prod_{1}^{n-2} \gamma_{ii}\left(\alpha_{nn} - \sum_{2}^{n-1} \beta_{in}\right).$$

Since $R = \prod_{1}^{n} \gamma_{ii}$, then $R_{(n-1)(n-1)nn} = \prod_{1}^{n-2} \gamma_{ii}$, from which we see that

$$\frac{R_{(n-1)(n-1)}}{R_{(n-1)(n-1)nn}} = \frac{\prod_{1}^{n-2} \gamma_{ii}\left(\alpha_{nn} - \sum_{2}^{n-1} \beta_{in}\right)}{\prod_{1}^{n-2} \gamma_{ii}} = \alpha_{nn} - \sum_{2}^{n-1} \beta_{in}.$$

But since $\alpha_{nn} = 1$, then

$$\frac{R_{(n-1)(n-1)}}{R_{(n-1)(n-1)nn}} = 1 - \sum_{2}^{n-1} \beta_{in}.$$

It has been shown that

$$\frac{R}{R_{nn}} = 1 - \sum_{2}^{n} \beta_{in}.$$

Substituting the above values for $\frac{R_{(n-1)(n-1)}}{R_{(n-1)(n-1)nn}}$ and $\frac{R}{R_{nn}}$ in equation (1), we have

$$R_{n(n-1);12\cdots(n-2)} = \sqrt{\frac{1 - \sum_{2}^{n-1} \beta_{in} - \left(1 - \sum_{2}^{n} \beta_{in}\right)}{1 - \sum_{2}^{n-1} \beta_{in}}},$$

or

$$R_{n(n-1);12\cdots(n-2)} = \sqrt{\frac{\beta_{nn}}{1 - \sum_{2}^{n-1} \beta_{in}}}.$$

Hence it is seen that when the β's given by Horst (page 42) are calculated, it is an easy matter to solve for the partial correlation $R_{n(n-1);12\cdots(n-2)}$.

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