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On Exponential Trichotomy of Cocycles over Semiflows

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Abstract. The paper considers a concept of exponential trichotomy for cocycles over semiflows in Banach spaces and as a particular case the corresponding dichotomy concept. Our main objective is to give a characterization of exponential trichotomy for a cocycle over a semiflow in terms of exponential dichotomy of two associated cocycles over the same semiflow.

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1 Introduction

One of the most important asymptotic properties for the solutions of evolution equations is the exponential trichotomy, studied in the last years from various perspectives.
The main idea in the definition of the trichotomy property for evolution equations is to obtain at any moment, the decomposition of the state space into three closed subspaces: the stable subspace, the unstable subspace and the neutral one. In the particular case when all neutral subspaces contain only the null vector, one obtains the dichotomy property.
The notion of a cocycle over a semiflow comes naturally when one considers the linearization along an invariant manifold of a dynamical system (see for

example [2], Chapter 6 and [10]).
Important results in the study of asymptotical behaviors of the systems described by cocycles have been obtained by S. N. Chow and H. Leiva ([3], [4]).
The concept of trichotomy was identified for the first time in literature in the famous work of R. J. Sacker and J. R. Sell in [9]. They described trichotomy of linear differential systems by linear skew-product flows.
The first studies devoted to the trichotomy of differential equations were initiated by S. E. Elaydi and O. Hajek in [5].
In the last years, a substantial part of the asymptotic theory of dynamical systems was devoted to the extension of the methods used in dichotomy theory to the trichotomy case.
Characterizations of the exponential trichotomy for linear skew-product semiflows were obtained by M. Megan, C. Stoica, L. Buliga in [7] and C. Stoica and M. Megan in [13]. The case of nonuniform exponential trichotomy was considered by L. Barreira and C. Valls in [1].
A new perspective on the input-output techniques in the trichotomic behaviors of nonautonomous dynamical systems is considered by B. Sasu and L. Sasu in [11].
This paper considers a concept of exponential trichotomy for a cocycle over a semiflow and as a particular case the corresponding dichotomy concept. It is proved that the exponential trichotomy property of a cocycle over a semiflow is equivalent with exponential dichotomy of two associated cocycles over the same semiflow.

2 Cocycles over semiflows

Let us denote by X a metric space, by V a Banach space and by $\mathcal{B}(V)$ the Banach algebra of all bounded linear operators on V. The norm on V and on $\mathcal{B}(V)$ will be denoted by $\|\cdot\|$. Let I be the identity operator on V and we also shall denote by $\mathbb{R}_+$ the set of nonnegative real numbers and $Y = X \times V$.

Definition 2.1. *A mapping $\varphi : \mathbb{R}_+ \times X \to X$ is called* a semiflow *on X if*
(s_1) $\varphi(0, x) = x$, *for each* $x \in X$;
(s_2) $\varphi(t, \varphi(s, x)) = \varphi(t + s, x)$, *for all* $(t, s, x) \in \mathbb{R}_+^2 \times X$.

A trivial example of a semiflow is given in

Example 2.1. For every metric space X, the mapping

$$\varphi_0 : \mathbb{R}_+ \times X \to X \quad \text{defined by} \quad \varphi_0(t, x) = x,$$

for all $(t, x) \in \mathbb{R}_+ \times X$, is a semiflow on X.

Example 2.2. If $X = \mathbb{R}_+$ (endowed with the euclidian metric) then

$$\varphi : \mathbb{R}_+ \times X \to X, \quad \varphi(t,x) = t + x,$$

is a semiflow on $\mathbb{R}_+$.

Example 2.3. Let C be the metric space of all continuous functions $f : \mathbb{R}_+ \to \mathbb{R}_+$ endowed with the topology of uniform convergence on compact subsets of $\mathbb{R}_+$ and let $u : \mathbb{R}_+ \to [1,\infty)$ be a decreasing function with the property that $\lim\limits_{t\to\infty} u(t) = \alpha$. If $f \in C$ and $t \in \mathbb{R}_+$ then we denote by f_t the function

$$f_t : \mathbb{R}_+ \to \mathbb{R}_+, \quad f_t(s) = f(t+s), \quad \text{for each } s \in \mathbb{R}_+.$$

If X is the closure in C of the set $\{u_t, t \in \mathbb{R}_+\}$ then X is a metric space and

$$\varphi : \mathbb{R}_+ \times X \to X, \quad \varphi(t,x) = x_t$$

is a semiflow on X.

Definition 2.2. *A mapping* $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ *is called a* cocycle over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$ *on the space* $Y = X \times V$ *if*

(c_1) $\Phi(0,x) = x$*, for each* $x \in X$ *and*

(c_2) $\Phi(t,\varphi(s,x))\Phi(s,x) = \Phi(t+s,x)$*, for all* $(t,s,x) \in \mathbb{R}_+^2 \times X$.

Example 2.4. If X is a metric space and $S : \mathbb{R}_+ \to \mathcal{B}(V)$ is a semigroup on the Banach space V (i.e S(0)=I and $S(t+s) = S(t)S(s)$ for all $(t,s) \in \mathbb{R}_+^2$) then $\Phi_S : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ defined by

$$\Phi_S(t,x) = S(t), \quad \text{for every} \quad (t,x) \in \mathbb{R}_+ \times X$$

is a cocycle over every semiflow $\varphi : \mathbb{R}_+ \times X \to X$ on the space $Y = X \times V$.

Example 2.5. Let $E : \Delta = \{(t,s) \in \mathbb{R}_+^2 : t \geq s\} \to \mathcal{B}(V)$ be an evolution operator on the Banach space V (i.e. $E(t,t) = I$ and $E(t,s)E(s,t_0) = E(t,t_0)$, for all $t \geq s \geq t_0 \geq 0$).
If $X = \mathbb{R}_+$ then the mapping $\Phi_E : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ defined by

$$\Phi_E(t,x) = E(t+x,x), \quad \text{for all} \quad (t,x) \in \mathbb{R}_+^2 \times X$$

is a cocycle over the semiflow φ defined in Example 2.2 on $Y = \mathbb{R}_+ \times V$.

Example 2.6. Let X be a locally compact metric space, φ a semiflow on X, V a Banach space and $A : X \to \mathcal{B}(V)$ a continuous mapping. If $y(t, x, v)$ is the solution of the abstract Cauchy problem

$$\begin{cases} y'(t) = A(\varphi(t, x))y(t), & t \geq 0 \\ y(0) = v \end{cases}$$

then the mapping $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ defined by $\Phi(t, x)v = y(t, x, v)$ is a cocycle over the semiflow φ on $Y = X \times V$.

Example 2.7. Let X be the metric space and let φ be the semiflow defined in Example 2.3. We consider the Banach space $V = \mathbb{R}^3$ with the norm $\|(v_1, v_2, v_3)\| = |v_1| + |v_2| + |v_3|$.
Then the mapping $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ defined by

$$\Phi(t, x)(v_1, v_2, v_3) = (v_1 e^{-3tu(0)+\int_0^t x(s)ds}, v_2 e^{3tu(0)-\int_0^t x(s)ds}, v_3 e^{-tu(0)+2\int_0^t x(s)ds})$$

is a cocycle over the semiflow φ on $Y = X \times V$.

Example 2.8. If $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ is a cocycle over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$ on $Y = X \times V$ then for every $c \in \mathbb{R}$ the mapping

$$\Phi_c : \mathbb{R}_+ \times X \to \mathcal{B}(V), \quad \Phi_c(t, x) = e^{ct}\Phi(t, x)$$

is also a cocycle over φ, which is called *the c-shifted cocycle* of Φ.

3 Exponential trichotomy of cocycles over semiflows

Let $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ be a cocycle over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$ on $Y = X \times V$.

Definition 3.1. *A mapping $P : X \to \mathcal{B}(V)$ is called a* family of projectors *on the Banach space V if*

$$P(x)^2 = P(x), \quad \textit{for all} \quad x \in X.$$

The family of projectors $P : X \to \mathcal{B}(V)$ is called invariant *for the cocycle $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$, if*

$$\Phi(t, x)P(x) = P(\varphi(t, x))\Phi(t, x), \quad \textit{for all} \quad (t, x) \in \mathbb{R}_+ \times X.$$

Definition 3.2. *Two families of projectors* $P_1, P_2 : X \to \mathcal{B}(V)$ *are called* complementary *if*

$$P_1(x) + P_2(x) = I, \quad \textit{for each} \quad x \in X.$$

Remark 3.1. If P_1 and P_2 are complementary then

$$P_1(x)P_2(x) = P_2(x)P_1(x) = 0, \quad \text{for every} \quad x \in X.$$

Definition 3.3. *Three families of projectors* $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ *are called* supplementary, *if*
(i) $P_1(x) + P_2(x) + P_3(x) = I$
(ii) $P_i(x)P_j(x) = 0$
(iii) $\|P_i(x)v + P_j(x)v\|^2 = \|P_i(x)v\|^2 + \|P_j(x)v\|^2$,
for all $(x, v) \in Y$ *and all* $i, j \in \{1, 2, 3\}$ *with* $i \neq j$.

Remark 3.2. If V is a Hilbert space then in the above definition we have that *(ii)* $\Rightarrow$ *(iii)*.

Definition 3.4. *Let* Φ *be a cocycle over the semiflow* φ *on* Y. *We say that* Φ *is* exponentially trichotomic, *if there are three supplementary families of projectors* $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ *invariant for* Φ *and three constants* $N \geq 1$, $a > 0$, $b \in (0, a)$ *such that*
(t_1) $e^{at}\|\Phi(t, x)P_1(x)v\| \leq N\|P_1(x)v\|$
(t_2) $e^{at}\|P_2(x)v\| \leq N\|\Phi(t, x)P_2(x)v\|$
(t_3) $\|\Phi(t, x)P_3(x)v\| \leq Ne^{bt}\|P_3(x)v\|$
(t_4) $\|P_3(x)v\| \leq Ne^{bt}\|\Phi(t, x)P_3(x)v\|$,
for all $(t, x, v) \in \mathbb{R}_+ \times Y$.

The families P_1, P_2, P_3 are called *trichotomic projectors* and the constants $a > b > 0$ are called *trichotomic constants*.
If in the previous definition we have that $P_3(x) = 0$ for every $x \in X$ then Φ is called *exponentially dichotomic* with the *dichotomic projectors* P_1, P_2 and the *dichotomic constant* $a > 0$.

Example 3.1. Let Φ be the cocycle defined in Example 2.7 and let $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ be the families of projectors defined by

$$P_1(x)(v_1, v_2, v_3) = (v_1, 0, 0), \quad P_2(x)(v_1, v_2, v_3) = (0, v_2, 0)$$

and

$$P_3(x)(v_1, v_2, v_3) = (0, 0, v_3)$$

which are supplementary and invariant for Φ. Moreover,

$\|\Phi(t,x)P_1(x)v\| \le |v_1|e^{-2tu(0)} = e^{-2tu(0)}\|P_1(x)v\|$
$\|\Phi(t,x)P_2(x)v\| \ge |v_2|e^{2tu(0)} = e^{2tu(0)}\|P_2(x)v\|$
$\|\Phi(t,x)P_3(x)v\| \le |v_3|e^{tu(0)} = e^{tu(0)}\|P_3(x)v\|$
$\|\Phi(t,x)P_3(x)v\| \ge |v_3|e^{2\alpha t-tu(0)} \ge |v_3|e^{-tu(0)} = |v_3|e^{-tu(0)}\|P_3(x)v\|$,
for all $(t,x) \in \mathbb{R}_+ \times X$ and all $v = (v_1, v_2, v_3) \in V$.
Finally, it results that Φ is exponentially trichotomic.

4 Trichotomy implies dichotomy

Let $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ be a cocycle over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$.

Proposition 4.1. *If $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ are three families of supplementary projectors and invariant for the cocycle Φ then the mappings $Q_1, Q_2, R_1, R_2 : X \to \mathcal{B}(V)$ defined by*

$$Q_1 = P_1, \quad Q_2 = P_2 + P_3$$

and respectively

$$R_1 = P_1 + P_3, \quad R_2 = P_2$$

are families of projectors with the following properties:
(p_1) *Q_1 and Q_2 are complementary and invariant for Φ*
(p_2) *R_1 and R_2 are complementary and invariant for Φ*
(p_3) $R_1 - Q_1 = Q_2 - R_2 = R_1Q_2 = Q_2R_1$
(p_4) $Q_1R_2 = R_2Q_1 = 0$
(p_5) $Q_1R_1 = R_1Q_1 = Q_1$
(p_6) $\|R_1(x)v - Q_1(x)v\|^2 = \|R_1(x)v\|^2 - \|Q_1(x)v\|^2$
(p_7) $\|Q_1(x)v + R_2(x)v\|^2 = \|Q_1(x)v\|^2 + \|R_2(x)v\|^2$
(p_8) $\|Q_2(x)v - R_2(x)v\|^2 = \|Q_2(x)v\|^2 - \|R_2(x)v\|^2$,
for all $(x,v) \in Y$.

Proof. It is a simple verification.

□

For every $c > 0$ we associate to the cocycle Φ the following two cocycles $\Phi_c^+, \Phi_c^- : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ defined by

$$\Phi_c^+ = e^{ct}\Phi(t,x) \quad \text{and respectively} \quad \Phi_c^- = e^{-ct}\Phi(t,x),$$

for all $(t,x) \in \mathbb{R}_+ \times X$.

Theorem 4.2. *If the cocycle* $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ *over the semiflow* $\varphi : \mathbb{R}_+ \times X \to X$ *is exponentially trichotomic with the trichotomic constants* $a > b > 0$ *then there exists* $c \in (b, a)$ *such that the cocycles* Φ_c^+ *and* Φ_c^- *are exponentially dichotomic with a dichotomic constant* $d \in (0, c)$.

Proof. Suppose that Φ is exponentially trichotomic with respect to the supplementary projectors $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ and the trichotomic constants $a > b > 0$.
We consider a constant $c \in (b, \frac{a+b}{2})$. Let $Q_1, Q_2 : X \to \mathcal{B}(V)$ be the families of projectors given by Proposition 4.1.
1.) We shall prove that Φ_c^+ is exponentially dichotomic with respect to the families of projectors Q_1, Q_2 and the dichotomic constant $d = c - b > 0$.
(d_1^+) Using the condition (t_1) from Definition 3.4 and the observation that $c + d - a < 0$ we obtain that there exists $N \geq 1$ such that

$$\begin{aligned} & e^{dt}\|\Phi_c^+(t,x)Q_1(x)v\| = e^{(c+d)t}\|\Phi(t,x)P_1(x)v\| \leq \\ & \leq Ne^{(c+d-a)t}\|P_1(x)v\| \leq N\|Q_1(x)v\|, \end{aligned}$$

for all $(t, x, v) \in \mathbb{R}_+ \times Y$.
(d_2^+) Similarly, using the conditions (t_2) and (t_4) from Definition 3.4 and the invariance of P_2, P_3 and Q_2 for Φ it results that there is $N \geq 1$ such that

$$\begin{aligned} & e^{2dt}\|Q_2(x)v\|^2 = e^{2dt}(\|P_2(x)v + P_3(x)v\|^2) = \\ & = e^{2dt}(\|P_2(x)v\|^2 + \|P_3(x)v\|^2) \leq \\ & \leq N^2e^{2dt}(e^{-2at}\|\Phi(t,x)P_2(x)v\|^2 + e^{2bt}\|\Phi(t,x)P_3(x)v\|^2) \leq \\ & \leq N^2e^{2(b+d)t}(\|P_2(\varphi(t,x))\Phi(t,x)v\|^2 + \|P_3(\varphi(t,x))\Phi(t,x)v\|^2) = \\ & = N^2e^{2(b+d)t}\|[P_2(\varphi(t,x)) + P_3(\varphi(t,x))]\Phi(t,x)v\|^2 = \\ & = N^2e^{2(b+d)t}\|Q_2(\varphi(t,x))\Phi(t,x)v\|^2 = \\ & = N^2e^{2(b+d)t}\|\Phi(t,x)Q_2(x)v\|^2 = \\ & = N^2\|\Phi_c^+(t,x)Q_2(x)v\|^2 \end{aligned}$$

and hence

$$e^{dt}\|Q_2(x)v\| \leq N\|\Phi_c^+(t,x)Q_2(x)v\|,$$

for all $(t, x, v) \in \mathbb{R}_+ \times Y$.
2.) It remains to prove that the cocycle Φ_c^- is exponentially trichotomic with respect to the families of projectors $R_1 = P_1 + P_3, R_2 = P_2$ and the dichotomic constant $d = c - b \in (0, c)$.

(d_1^-) Similarly as in the proof of (d_2^+) we obtain

$$\begin{aligned}
&e^{2dt}\|\Phi_c^-(t,x)R_1(x)v\|^2 = e^{2(d-c)t}\|\Phi(t,x)[P_1(x)v + P_3(x)v)]\|^2 = \\
&= e^{2(d-c)t}\|[P_1(\varphi(t,x)) + P_3(\varphi(t,x))]\Phi(t,x)v\|^2 = \\
&= e^{2(d-c)t}[\|\Phi(t,x)P_1(x)v\|^2 + \|\Phi(t,x)P_3(x)v\|^2] \leq \\
&\leq N^2 e^{2(d-c)t}[e^{-2at}\|P_1(x)v\|^2 + e^{2bt}\|P_3(x)v\|^2] \leq \\
&\leq N^2 e^{2(d-c+b)t}[\|P_1(x)v\|^2 + \|P_3(x)v\|^2] = \\
&= N^2\|P_1(x)v + P_3(x)v\|^2 = \\
&= N^2\|R_1(x)v\|^2
\end{aligned}$$

and hence

$$e^{dt}\|\Phi_c^-(t,x)R_1(x)v\| \leq N\|R_1(x)v\|,$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$.

(d_2^-) From the definition of R_2 and the condition (t_2) we have that there exists $N \geq 1$ with

$$\begin{aligned}
&e^{dt}\|R_2(x)v\| = e^{dt}\|P_2(x)v\| \leq Ne^{(d-a)t}\|\Phi(t,x)P_2(x)v\| = \\
&= Ne^{(c+d-a)t}\|\Phi_c^-(t,x)P_2(x)v\| \leq N\|\Phi_c^-(t,x)R_2(x)v\|
\end{aligned}$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$. □

5 Dichotomy implies trichotomy

Firstly, we prove

Proposition 5.1. *Let $Q_1, Q_2, R_1, R_2 : X \to \mathcal{B}(V)$ be four families of projectors which are invariant for the cocycle $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ with the properties $(p_1) - (p_8)$ from Proposition 4.1. Then $P_1, P_2, P_3 : X \to \mathcal{B}(V)$ defined by $P_1 = Q_1, P_2 = R_2$ and respectively $P_3 = R_1 - Q_1$ are three supplementary families of projectors which are invariant for Φ.*

Proof. It results from Proposition 4.1 using the observations

$$P_1 + P_2 = Q_1 + R_2, \quad P_1 + P_3 = R_1 \quad \text{and} \quad P_2 + P_3 = Q_2.$$

□

Let $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ be a cocycle over the semiflow $\varphi : \mathbb{R}_+ \times X \to X$ and let Φ_c^+, Φ_c^- be the associated cocycles defined in Example 2.8 for $c > 0$. The main result of this section is

Theorem 5.2. *If there exists $c > 0$ such that the cocycles Φ_c^+ and Φ_c^- are exponentially dichotomic with a dichotomic constant $d \in (0,c)$ then the cocycle Φ is exponentially trichotomic with the trichotomic constants $a = d + c > c - d = b > 0$.*

Proof. Suppose that there exists $c > 0$ such that
(i) Φ_c^+ is exponentially dichotomic with respect to the complementary projectors Q_1, Q_2 and the dichotomic constant $d \in (0,c)$
and
(ii) Φ_c^- is exponentially dichotomic with respect to the complementary projectors R_1, R_2 and the dichotomic constant $d \in (0,c)$. Consider the supplementary projectors P_1, P_2, P_3 given by Proposition 5.1. If we denote by $a = c + d$ and $b = c - d$ then $a > b > 0$.
We shall prove that Φ is exponentially trichotomic with respect to the supplementary projectors P_1, P_2, P_3 and the trichotomic constants $a > b > 0$.
(t_1) By (d_1^+) we obtain

$$\begin{aligned} e^{at}\|\Phi(t,x)P_1(x)v\| &= e^{(a-c)t}\|\Phi_c^+(t,x)Q_1(x)v\| \leq \\ &\leq Ne^{(a-c-d)t}\|Q_1(x)v\| = N\|Q_1(x)v\| \end{aligned}$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$.
(t_2) By (d_2^+) it results that there is $N \geq 1$ with

$$\begin{aligned} e^{at}\|P_2(x)v\| &= e^{at}\|R_2(x)v\| \leq Ne^{(a-d)t}\|\Phi_c^-(t,x)R_2(x)v\| = \\ &= Ne^{(a-c-d)t}\|\Phi(t,x)P_2(x)v\| = N\|\Phi(t,x)P_2(x)v\| \end{aligned}$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$.
(t_3) Similarly, using (p_3) and (d_1^-) it follows that there exists $N \geq 1$ such that

$$\begin{aligned} \|\Phi(t,x)P_3(x)v\| &= \|\Phi(t,x)R_1(x)Q_2(x)v\| = \\ &= e^{ct}\|\Phi_c^-(t,x)R_1(x)Q_2(x)v\| \leq Ne^{(c-d)t}\|R_1(x)Q_2(x)v\| = \\ &= Ne^{bt}\|P_3(x)v\|, \end{aligned}$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$.
(t_4) Because $P_3 = Q_2R_1$, the condition (d_2^-) implies that there is $N \geq 1$ with

$$\begin{aligned} \|P_3(x)v\| &= \|Q_2(x)R_1(x)v\| \leq Ne^{dt}\|\Phi_c^-(t,x)Q_2(x)R_1(x)v\| = \\ &= Ne^{(d-c)t}\|\Phi(t,x)P_3(x)v\| = Ne^{bt}\|\Phi(t,x)P_3(x)v\|, \end{aligned}$$

for all $(t,x,v) \in \mathbb{R}_+ \times Y$.
Finally, we obtain that Φ is exponentially trichotomic. □

6 The main result

As a consequence of the Theorems 4.2 and 5.2 we obtain *the main result* of this paper given by

Theorem 6.1. *The cocycle* $\Phi : \mathbb{R}_+ \times X \to \mathcal{B}(V)$ *is exponentially trichotomic with the trichotomic constants* $a > b > 0$ *if and only if there exist* $c \in (b, a)$ *and* $d \in (0, c)$ *such that the cocycles* Φ_c^+ *and* Φ_c^- *are exponentially dichotomic with the dichotomic constant* d.

In particular, for the case of a C_0 semigroup $S : \mathbb{R}_+ \to \mathcal{B}(V)$ with the infinitesimal generator A, using

Definition 6.1. *The operator* A *is* exponentially trichotomic, *if the cocycle generated by* S *(see Example 2.4) is exponentially trichotomic.*

we obtain

Corollary 6.2. *The operator* A *is exponentially trichotomic if and only if there exists* $c > 0$ *such that the operators* $(A + cI)$ *and* $(A - cI)$ *are exponentially dichotomic.*

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