# On $\phi$-Recurrent Kenmotsu Manifolds 

Uday Chand De, Ahmet Yıldız, A. Funda Yalınız


#### Abstract

The object of this paper is to study $\phi$-recurrent Kenmotsu manifolds. Also three-dimensional locally $\phi$ recurrent Kenmotsu manifolds have been considered. Among others it is proved that a locally $\phi$-recurrent Kenmotsu spacetime is the Robertson-Walker spacetime. Finally we give a concrete example of a threedimensional Kenmotsu manifold.


Key Words: Kenmotsu manifolds, $\phi$-recurrent Kenmotsu manifolds, locally $\phi$-recurrent Kenmotsu manifolds.

## 1. Introduction

The notion of local symmetry of a Riemannian manifold has been weakend by many authors in several ways to a different extent. As a weaker version of local symmetry, T. Takahashi [16] introduced the notion of locally $\phi$-symmetry on a Sasakian manifold. Generalizing the notion of $\phi$-symmetry, one of the authors, De, [7] introduced the notion of $\phi$-recurrent Sasakian manifold. In the context of contact geometry the notion of $\phi$-symmetry is introduced and studied by Boeckx, Buecken and Vanhecke [3] with several examples.

On the other hand Kenmotsu [11] defined a type of contact metric manifold which is nowadays called Kenmotsu manifold. It may be mentioned that a Kenmotsu manifold is not a Sasakian manifold. Also, a Kenmotsu manifold is not compact because of $\operatorname{div} \xi=2 n$. In [11], Kenmotsu showed that locally a Kenmotsu manifold is a warped product $I \times{ }_{f} N$ of an interval $I$ and a Kahler manifold $N$ with warping function $f(t)=s e^{t}$, where $s$ is a nonzero constant.

The present paper is organized as follows: Section 2 is devoted to preliminaries. In section 3, we prove that a $\phi$-recurrent Kenmotsu manifold is an Einstein manifold and a locally $\phi$-recurrent Kenmotsu manifold is locally a hyperbolic space. In the next section, it is proved that a three-dimensional locally $\phi$-recurrent Kenmotsu manifold is a manifold of constant curvature. In section 5, we prove that a locally $\phi$-recurrent Kenmotsu spacetime is the Robertson-Walker spacetime. In the last section, we construct an example of a three-dimensional Kenmotsu manifold.

[^0]```
DE, YILDIZ, YALINIZ
```


## 2. Preliminaries

Let $M^{2 n+1}(\phi, \xi, \eta, g)$ be an almost contact Riemannian manifold, where $\phi$ is a $(1,1)$ tensor field, $\xi$ is the structure vector field, $\eta$ is a 1 -form and $g$ is the Riemannian metric. It is well known that $(\phi, \xi, \eta, g)$ satisfy

$$
\begin{gather*}
\phi \xi=0, \quad \eta(\phi X)=0, \quad \eta(\xi)=1,  \tag{2.1}\\
\phi^{2} X=-X+\eta(X) \xi  \tag{2.2}\\
g(X, \xi)=\eta(X)  \tag{2.3}\\
g(\phi X, \phi Y)=g(X, Y)-\eta(X) \eta(Y) \tag{2.4}
\end{gather*}
$$

for any vector fields $X$ and $Y$ on $M$ [1], [2].
If, moreover,

$$
\begin{gather*}
\left(\nabla_{X} \phi\right) Y=-\eta(Y) \phi X-g(X, \phi Y) \xi, \quad X, Y \in \chi(M)  \tag{2.5}\\
\nabla_{X} \xi=X-\eta(X) \xi \tag{2.6}
\end{gather*}
$$

where $\nabla$ denotes the Riemannian connection of $g$, then $(M, \phi, \xi, \eta, g)$ is called an almost Kenmotsu manifold [11].

Kenmotsu manifolds have been studied by many authors such as Binh, Tamassy, De and Tarafdar [4], Pitiş [15], De and Pathak [5], Jun, De and Pathak [10], Ozgür [13], Ozgür and De [14], Dileo and Pastore [8] and many others.

In a Kenmotsu manifold the following relations hold: [11] .

$$
\begin{gather*}
\left(\nabla_{X} \eta\right)(Y)=g(X, Y)-\eta(X) \eta(Y)  \tag{2.7}\\
\eta(R(X, Y) Z)=g(X, Z) \eta(Y)-g(Y, Z) \eta(X)  \tag{2.8}\\
R(X, Y) \xi=\eta(X) Y-\eta(Y) X,  \tag{2.9}\\
R(\xi, X) Y=\eta(Y) X-g(X, Y) \xi  \tag{2.10}\\
S(X, \xi)=-2 n \eta(X)  \tag{2.11}\\
\left(\nabla_{Z} R\right)(X, Y) \xi=g(X, Z) Y-g(Z, Y) X-R(X, Y) Z \tag{2.12}
\end{gather*}
$$

for any vector fields $X, Y, Z$, where $R$ is the Riemannian curvature tensor and $S$ is the Ricci tensor.

Definition 1 A Kenmotsu manifold is said to be a locally $\phi$-symmetric manifold if

$$
\begin{equation*}
\phi^{2}\left(\left(\nabla_{W} R\right)(X, Y) Z\right)=0 \tag{2.13}
\end{equation*}
$$

for all vector fields $X, Y, Z, W$ orthogonal to $\xi$.
This notion was introduced for Sasakian manifolds by Takahashi [16].

Definition 2 A Kenmotsu manifold is said to be a $\phi$-recurrent manifold if there exists a non-zero 1-form $A$ such that

$$
\begin{equation*}
\phi^{2}\left(\left(\nabla_{W} R\right)(X, Y) Z\right)=A(W) R(X, Y) Z \tag{2.14}
\end{equation*}
$$

for arbitrary vector fields $X, Y, Z, W$.
If $X, Y, Z, W$ are orthogonal to $\xi$, then the manifold is called locally $\phi$-recurrent manifold.
If the 1 -form $A$ vanishes, then the manifold reduces to a $\phi$-symmetric manifold.

## 3. $\phi$-Recurrent Kenmotsu Manifolds

To prove the main theorem of the paper we first prove the following lemma.
Lemma 1 In a $\phi$-recurrent Kenmotsu manifold $\left(M^{2 n+1}, g\right), n>1$, the characteristic vector field $\xi$ and the vector field $\rho$ associated to the 1 -form $A$ are co-directional and the 1 -form $A$ is given by

$$
A(W)=\eta(\rho) \eta(W)
$$

Proof. Two vector fields $P$ and $Q$ are said to be co-directional if $P=f Q$ where $f$ is a non-zero scalar. That is,

$$
\begin{equation*}
g(P, X)=f g(Q, X) \text { for all } X \tag{3.15}
\end{equation*}
$$

Let us consider a $\phi$-recurrent Kenmotsu manifold. Then by virtue of (2.2) and (2.14), we have

$$
\begin{equation*}
\left(\nabla_{W} R\right)(X, Y) Z=\eta\left(\left(\nabla_{W} R\right)(X, Y) Z\right) \xi-A(W) R(X, Y) Z \tag{3.16}
\end{equation*}
$$

From (3.16) and the Bianchi identity, we get

$$
\begin{equation*}
A(W) \eta(R(X, Y) Z)+A(X) \eta(R(Y, W) Z)+A(Y) \eta(R(W, X) Z)=0 \tag{3.17}
\end{equation*}
$$

Let $\left\{e_{i}\right\}, i=1,2,3, \ldots, 2 n+1$, be an orthonormal basis of the tangent space at any point of the manifold. Putting $Y=Z=e_{i}$ in (3.17) and taking summation over $i, 1 \leq i \leq 2 n+1$, we get by virtue of (2.8)

$$
\begin{equation*}
A(W) \eta(X)=A(X) \eta(W) \tag{3.18}
\end{equation*}
$$

for all vector fields $X, W$. Replacing $X$ by $\xi$ in (3.18), it follows that

$$
\begin{equation*}
A(W)=\eta(\rho) \eta(W) \tag{3.19}
\end{equation*}
$$

where $A(X)=g(X, \rho)$ and $\rho$ is the vector field associated to the 1 -form $A$. From (3.15) and (3.19) it is clear that $\xi$ and $\rho$ are co-directional.

Theorem $1 A \phi$-recurrent Kenmotsu manifold is an Einstein manifold.
Proof. From (3.16), we have

$$
\begin{equation*}
\left.-g\left(\nabla_{W} R\right)(X, Y) Z, U\right)+\eta\left(\left(\nabla_{W} R\right)(X, Y) Z\right) \eta(U)=A(W) g(R(X, Y) Z, U) \tag{3.20}
\end{equation*}
$$

Putting $X=U=e_{i}$ in (3.20) and taking summation over $i, 1 \leq i \leq 2 n+1$, we get

$$
\begin{equation*}
-\left(\nabla_{W} S\right)(Y, Z)+\sum_{i=1}^{2 n+1} \eta\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) Z\right) \eta\left(e_{i}\right)=A(W) S(Y, Z) \tag{3.21}
\end{equation*}
$$

The second term of (3.21) by putting $Z=\xi$ takes the form

$$
\begin{equation*}
\eta\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi\right) \eta\left(e_{i}\right)=g\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi, \xi\right) g\left(e_{i}, \xi\right) \tag{3.22}
\end{equation*}
$$

which is denoted by $E$. In this case $E$ vanishes. Namely, we have

$$
\begin{align*}
g\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi, \xi\right)= & g\left(\nabla_{W} R\left(e_{i}, Y\right) \xi, \xi\right)-g\left(R\left(\nabla_{W} e_{i}, Y\right) \xi, \xi\right)  \tag{3.23}\\
& -g\left(R\left(e_{i}, \nabla_{W} Y\right) \xi, \xi\right)-g\left(R\left(e_{i}, Y\right) \nabla_{W} \xi, \xi\right)
\end{align*}
$$

at $p \in M$. In local coordinates $\nabla_{X} e_{i}=X^{j} \Gamma_{j i}^{h} e_{h}$, where $\Gamma_{j i}^{h}$ are the Christoffel symbols. Since $\left\{e_{i}\right\}$ is an orthonormal basis, the metric tensor $g_{i j}=\delta_{i j}$, where $\delta_{i j}$ is the Kronecker delta and hence the Christoffel symbols are zero. Therefore, $\nabla_{X} e_{i}=0$. Also we have

$$
\begin{equation*}
g\left(R\left(e_{i}, \nabla_{W} Y\right) \xi, \xi\right)=0 \tag{3.24}
\end{equation*}
$$

since $R$ is skew-symmetric. Using (3.24) and $\nabla_{X} e_{i}=0$ in (3.23), we obtain

$$
g\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi, \xi\right)=g\left(\nabla_{W} R\left(e_{i}, Y\right) \xi, \xi\right)-g\left(R\left(e_{i}, Y\right) \nabla_{W} \xi, \xi\right)
$$

By virtue of $g\left(R\left(e_{i}, Y\right) \xi, \xi\right)=-g\left(R(\xi, \xi) Y, e_{i}\right)=0$, we have

$$
\begin{equation*}
g\left(\nabla_{W} R\left(e_{i}, Y\right) \xi, \xi\right)+g\left(R\left(e_{i}, Y\right) \xi, \nabla_{W} \xi\right)=0 \tag{3.25}
\end{equation*}
$$

which implies

$$
g\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi, \xi\right)=-g\left(R\left(e_{i}, Y\right) \xi, \nabla_{W} \xi\right)-g\left(R\left(e_{i}, Y\right) \nabla_{W} \xi, \xi\right)
$$

Since $R$ is skew-symmetric

$$
\begin{equation*}
g\left(\left(\nabla_{W} R\right)\left(e_{i}, Y\right) \xi, \xi\right)=0 \tag{3.26}
\end{equation*}
$$

Using (3.26) from (3.21), we get

$$
\begin{equation*}
\left(\nabla_{W} S\right)(Y, \xi)=-A(W) S(Y, \xi) \tag{3.27}
\end{equation*}
$$

We know that

$$
\left(\nabla_{W} S\right)(Y, \xi)=\nabla_{W} S(Y, \xi)-S\left(\nabla_{W} Y, \xi\right)-S\left(Y, \nabla_{W} \xi\right)
$$

Again using (2.6), (2.7) and (2.11), we get

$$
\begin{equation*}
\left(\nabla_{W} S\right)(Y, \xi)=-2 n g(Y, W)-S(Y, W) \tag{3.28}
\end{equation*}
$$

Now using (3.28) in (3.27), we obtain

$$
\begin{equation*}
S(Y, W)=-2 n A(W) \eta(Y)-2 n g(Y, W) \tag{3.29}
\end{equation*}
$$

Applying Lemma 1, equation (3.29) reduces to

$$
S(Y, W)=-2 n g(Y, W)-2 n \eta(\rho) \eta(Y) \eta(W)
$$

which implies that the manifold is an $\eta$-Einstein manifold.

In Corollary 9 of Proposition 8 of [11], it is proved that if a Kenmotsu manifold is an $\eta$-Einstein manifold of type $S=a g+b \eta \otimes \eta$ and if $b=$ constant (or $a=$ constant) then $M$ is an Einstein manifold. Hence by the above result a $\phi$-recurrent Kenmotsu manifold is an Einstein manifold.

Theorem 2 A locally $\phi$-recurrent Kenmotsu manifold $\left(M^{2 n+1}, g\right), n>1$, is a manifold of constant curvature -1 , i.e., it is locally a hyperbolic space.
Proof. From (2.12), we have

$$
\begin{equation*}
\left(\nabla_{W} R\right)(X, Y) \xi=g(W, X) Y-g(W, Y) X-R(X, Y) W \tag{3.30}
\end{equation*}
$$

By virtue of (2.8), it follows from (3.30) that

$$
\begin{equation*}
\eta\left(\left(\nabla_{W} R\right)(X, Y) \xi\right)=0 \tag{3.31}
\end{equation*}
$$

In view of (3.30) and (3.31), we obtain from (3.16)

$$
\begin{equation*}
-\left(\nabla_{W} R\right)(X, Y) \xi=A(W) R(X, Y) \xi \tag{3.32}
\end{equation*}
$$

from which by using (2.12), it follows that

$$
-g(X, W) Y+g(Y, W) X+R(X, Y) W=A(W) R(X, Y) \xi
$$

Hence if $X$ and $Y$ are orthogonal to $\xi$, then we get from (2.9)

$$
R(X, Y) \xi=0
$$

Thus, we obtain

$$
R(X, Y) W=-[g(Y, W) X-g(X, W) Y]
$$

for all $X, Y, W$.
Remark. It may be mentioned that a semi-symmetric $(R(X, Y) \cdot R=0)$ Kenmotsu manifold and a conformally flat Kenmotsu manifold of dimension $>3$ are of constant sectional curvature [11]. Also De and Pathak [5] proved that three dimensional Ricci semi-symmetric $(R(X, Y) \cdot S=0)$ Kenmotsu manifold is of constant sectional curvature.

## 4. Three-Dimensional Kenmotsu Manifolds

It is known that in a three-dimensional Kenmotsu manifold the curvature tensor has the following form [5]

$$
\begin{align*}
R(X, Y) Z= & \left(\frac{r+4}{2}\right)[g(Y, Z) X-g(X, Z) Y] \\
& -\left(\frac{r+6}{2}\right)[g(Y, Z) \eta(X) \xi-g(X, Z) \eta(Y) \xi+\eta(Y) \eta(Z) X-\eta(X) \eta(Z) Y] \tag{4.33}
\end{align*}
$$

## DE, YILDIZ, YALINIZ

Taking the covariant differentiation of the equation (4.33), we have

$$
\begin{align*}
\left(\nabla_{W} R\right)(X, Y) Z= & \frac{d r(W)}{2}[g(Y, Z) X-g(X, Z) Y]-\frac{d r(W)}{2}[g(Y, Z) \eta(X) \xi-g(X, Z) \eta(Y) \xi \\
& +\eta(Y) \eta(Z) X-\eta(X) \eta(Z) Y]-\left(\frac{r+6}{2}\right)\left[g(Y, Z)\left(\nabla_{W} \eta\right)(X) \xi+g(Y, Z) \eta(X) \nabla_{W} \xi\right.  \tag{4.34}\\
& -g(X, Z)\left(\nabla_{W} \eta\right)(Y) \xi-g(X, Z) \eta(Y) \nabla_{W} \xi+\left(\nabla_{W} \eta\right)(Y) \eta(Z) X+\eta(Y)\left(\nabla_{W} \eta\right)(Z) X \\
& \left.-\left(\nabla_{W} \eta\right)(X) \eta(Z) Y-\eta(X)\left(\nabla_{W} \eta\right)(Z) Y\right]
\end{align*}
$$

Now applying $\phi^{2}$ to the both sides of (4.34), we obtain

$$
\begin{align*}
\phi^{2}\left(\nabla_{W} R\right)(X, Y) Z= & \frac{-d r(W)}{2}[g(Y, Z) X-g(X, Z) Y-g(Y, Z) \eta(X) \xi+g(X, Z) \eta(Y) \xi+\eta(X) \eta(Z) Y \\
& -\eta(Y) \eta(Z) X]+\left(\frac{r+6}{2}\right)\left[\left(\nabla_{W} \eta\right)(Y) \eta(Z) X+\eta(Y)\left(\nabla_{W} \eta\right)(Z) X-\left(\nabla_{W} \eta\right)(X) \eta(Z) Y\right. \\
& \left.-\eta(X)\left(\nabla_{W} \eta\right)(Z) Y-\left(\nabla_{W} \eta\right)(Y) \eta(Z) \eta(X) \xi+\left(\nabla_{W} \eta\right)(X) \eta(Z) \eta(Y) \xi\right] \tag{4.35}
\end{align*}
$$

Taking $X, Y, Z, W$ orthogonal to $\xi$ and using (2.14), we finally get from (4.35)

$$
\begin{equation*}
A(W) R(X, Y) Z=\frac{-d r(W)}{2}[g(Y, Z) X-g(X, Z) Y] \tag{4.36}
\end{equation*}
$$

Putting $W=\left\{e_{i}\right\}$ in (4.36), where $\left\{e_{i}\right\}, i=1,2,3$, is an orthonormal basis of the tangent space at any point of the manifold and taking summation over $i, 1 \leq i \leq 3$, we obtain

$$
R(X, Y) Z=\lambda[g(Y, Z) X-g(X, Z) Y]
$$

where $\lambda=\frac{-d r\left(e_{i}\right)}{2 A\left(e_{i}\right)}$ is a scalar, since $A$ is a non-zero 1 -form. Then by Schur's theorem $\lambda$ will be a constant on the manifold. Therefore, $M^{3}$ is of constant curvature $\lambda$. Thus we get the following theorem.

Theorem 3 A three-dimensional locally $\phi$-recurrent Kenmotsu manifold is of constant curvature.

## 5. Locally $\phi$-Recurrent Kenmotsu Spacetime

In this section we consider locally $\phi$-recurrent Kenmotsu spacetime. By a spacetime, we mean a 4dimensional semi-Riemannian manifold endowed with Lorentzain metric of signature $(-+++)$. In a recent paper one of the authors De and Pathak [6] prove that the characteristic vector field $\xi$ in a Kenmotsu manifold is a concircular vector field [18]. Also from Theorem 2, we can easily prove that a locally $\phi$-recurrent Kenmotsu manifold is conformally flat. Hence $\operatorname{div} C=0$, where $C$ denotes the conformal curvature tensor and "div" denotes divergence.

Hence, we have

$$
\begin{equation*}
\left(\nabla_{X} S\right)(Y, Z)-\left(\nabla_{Y} S\right)(X, Z)=\frac{1}{2(n-1)}[g(Y, Z) d r(X)-g(X, Z) d r(Y)] \tag{5.37}
\end{equation*}
$$

## DE, YILDIZ, YALINIZ

Yano [17], prove that, in order that a Riemannian space admits a concircular vector field, it is necessary and sufficient that there exists a coordinate system with respect to which the fundamental quadratic differential form may be written in the form

$$
d s^{2}=\left(d x^{1}\right)^{2}+e^{q} g_{\alpha \beta}^{*} d x^{\alpha} d x^{\beta}
$$

where $g_{\alpha \beta}^{*}=g_{\alpha \beta}^{*}\left(x^{r}\right)$ are the functions of $x^{r}$ only $(\alpha, \beta, r=2,3, \ldots, n)$ and $q=q\left(x^{\prime}\right) \neq$ constant is a function of $x^{1}$ only. In the semi-Riemannian space, we can prove that

$$
d s^{2}=-\left(d x^{1}\right)^{2}+e^{q} g_{\alpha \beta}^{*} d x^{\alpha} d x^{\beta}
$$

Thus a Kenmotsu spacetime can be expressed as a warped product $-I \times_{e^{q}} M^{*}$, where $M^{*}$ is a three-dimensional Riemannian manifold. But Gebarowski [9] prove that warped product $-I \times_{e^{q}} M^{*}$ satisfies (5.37) if and only if $M^{*}$ is an Einstein manifold. Thus a locally $\phi$-recurrent Kenmotsu spacetime must be warped product $-I \times_{e^{q}} M^{*}$, where $M^{*}$ is an Einstein manifold. Since we consider a 4 -dimensional manifold, $M^{*}$ is a threedimensional Einstein manifold. It is known that a three- dimensional Einstein manifold is a manifold of constant curvature. Hence a locally $\phi$-recurrent Kenmotsu spacetime is the warped product $-I \times_{e^{q}} M^{*}$, where $M^{*}$ is a manifold of constant curvature. But such a warped product is the Robertson-Walker spacetime [12].

Thus we have the following theorem.
Theorem 4 A locally $\phi$-recurrent Kenmotsu spacetime is the Robertson-Walker spacetime.

## 6. Example of a Three-Dimensional Kenmotsu Manifold

We consider the three-dimensional manifold $M=\left\{(x, y, z) \in \mathbb{R}^{3}\right\}, z \neq 0$ where $(x, y, z)$ are the standard coordinates of $\mathbb{R}^{3}$. The vector fields

$$
e_{1}=z \frac{\partial}{\partial x}, \quad e_{2}=z \frac{\partial}{\partial y}, \quad e_{3}=-z \frac{\partial}{\partial z}
$$

are linearly indepent at each point of $M$. Let $g$ be the Riemannian metric defined by

$$
\begin{aligned}
& g\left(e_{1}, e_{3}\right)=g\left(e_{1}, e_{2}\right)=g\left(e_{2}, e_{3}\right)=0 \\
& g\left(e_{1}, e_{1}\right)=g\left(e_{2}, e_{2}\right)=g\left(e_{3}, e_{3}\right)=1
\end{aligned}
$$

That is, the form of the metric becomes

$$
g=\frac{\left(d x^{2}+d y^{2}+d z^{2}\right)}{z^{2}}
$$

Let $\eta$ be the 1 -form defined by $\eta(Z)=g\left(Z, e_{3}\right)$ for any $Z \in \chi(M)$. Let $\phi$ be the $(1,1)$-tensor field defined by

$$
\phi\left(e_{1}\right)=-e_{2}, \quad \phi\left(e_{2}\right)=e_{1}, \quad \phi\left(e_{3}\right)=0
$$

Then using the linearity of $\phi$ and $g$, we have

$$
\begin{aligned}
\eta\left(e_{3}\right) & =1 \\
\phi^{2} Z & =-Z+\eta(Z) e_{3} \\
g(\phi Z, \phi W) & =g(Z, W)-\eta(Z) \eta(W)
\end{aligned}
$$

for any $Z, W \in \chi(M)$. Then for $e_{3}=\xi$, the structure $(\phi, \xi, \eta, g)$ defines an almost contact metric structure on M.

Let $\nabla$ be the Levi-Civita connection with respect to the metric $g$. Then we have

$$
\left[e_{1}, e_{2}\right]=0, \quad\left[e_{1}, e_{3}\right]=e_{1}, \quad\left[e_{2}, e_{3}\right]=e_{2}
$$

The Riemannian connection $\nabla$ of the metric $g$ is given by

$$
2 g\left(\nabla_{X} Y, Z\right)=X g(Y, Z)+Y g(Z, X)-Z g(X, Y)-g(X,[Y, Z])-g(Y,[X, Z])+g(Z,[X, Y])
$$

which is known as Koszul's formula. Using this formula we obtain

$$
\begin{array}{lll}
\nabla_{e_{1}} e_{3}=e_{1}, & \nabla_{e_{2}} e_{3}=e_{2} \\
\nabla_{e_{1}} e_{2}=0, & \nabla_{e_{2}} e_{2}=-e_{3} \\
\nabla_{e_{2}} e_{2}=-e_{3}, & \nabla_{e_{2}} e_{2}=-e_{3} \\
\nabla_{e_{1}} e_{1}=-e_{3}, & \nabla_{e_{2}} e_{1}=0 \\
\nabla_{e_{3}} e_{1}=0, & \nabla_{e_{3}} e_{2}=0 \\
\nabla_{e_{3}} e_{3}=0 . &
\end{array}
$$

Thus (2.6) is satisfied. It is straightforward computation to verify that the manifold under consideration is a three-dimensional Kenmotsu manifold.

## Acknowledgement

The authors are thankfull to the referee for valuable suggestions towards the improvement of this paper.

## References

[1] Blair D. E., Riemannian geometry of contact and symplectic manifolds, Progress Mathematics, 203, Birkhauser, Boston-Basel-Berlin, (2002) .
[2] Blair D. E., Contact manifolds in Riemannian geometry, Lecture Notes in Math. 509, Berlin-Heidelberg-New York, (1976).
[3] Boeckx E., Buecken P. and Vanhacke L., $\phi$ - symmetric contact metric spaces, Glasgow Math. J. 41, 409-416, (1999).
[4] Binh, T.Q., Tamassy L., De U. C. and Tarafdar M., Some remarks on almost Kenmotsu manifolds, Mathematica Pannonica,13, 31-39, (2002).
[5] De U. C. and Pathak G., On 3-dimensional Kenmotsu manifolds, Indian J. Pure Applied Math., 35, 159-165, (2004).
[6] De U. C. and Pathak G., Torseforming vector field in a Kenmotsu manifold, Ann. Ştiint. Ale Univ., Al. I. Cuza, Iaşi, tomul XLIX, s. I.a, Matematica, 257-264, (2003).
[7] De U. C, Shaikh A. A. and Biswas S., On $\phi$ - recurrent Sasakian manifolds, Novi Sad J. Math., 33, 13-48, (2003).
[8] Dileo G. and Pastore A. M., Almost Kenmotsu manifolds and local symmetry, Bull. Belg. Math. Soc. Simon Stevin, 14, 2, 343-354, (2007).
[9] Gebarowaski A., Nearly conformally symmetric warped product manifolds, Bulletin of The Institute of Mathematics Academia Sinica, 4, 359-371, (1992).
[10] Jun J-B., De. U. C. and Pathak G., On Kenmotsu manifolds, J. Korean Math. Soc., 42, 435-445, (2005).
[11] Kenmotsu K., A class of almost contact Riemannian manifolds, Tohoku Math. J., 24, 93-103, (1972).
[12] O’Neill B., Semi-Riemannian Geometry, Academic Press. Inc., (1983).
[13] Özgür C., On weakly symmetric Kenmotsu manifolds, Differ. Geom. Dyn. Syst., 8, 204-209, (2006).
[14] Özgür C. and De U. C., On the quasi-conformal curvature tensor of a Kenmotsu manifold, Mathematica Pannonica, 17/2, 221-228, (2006).
[15] Pitiş G., A remark on Kenmotsu manifolds, Bul. Univ. Brasov, Ser. C, 30, 31-32, (1988).
[16] Takahashi T., Sasakian $\phi$ - symmetric spaces,Tohoku Math. J., 29, 91-113, (1977).
[17] Yano K., On the torseforming direction in Riemannian spaces, Proc. Imp. Acad., Tokyo, 20, 340-345, (1994).
[18] Yano K., Concircular geometry, I, Proc. Imp. Acad., Tokyo, 16, 195-200, (1940).

Uday Chand DE
Department of Mathematics
University of Kalyani
Kalyani-741235
West Bengal-INDIA
e-mail: uc_de@yahoo.com
Ahmet YILDIZ
Art and Science Faculty
Department of Mathematics
Dumlupinar University
Kütahya, TURKEY
e-mail: ahmetyildiz@dumlupinar.edu.tr
A. Funda YALINIZ

Art and Science Faculty
Department of Mathematics
Dumlupınar University
Kütahya, TURKEY
e-mail: fyaliniz@dumlupinar.edu.tr


[^0]:    1991 AMS Mathematics Subject Classification: 53C15, 53C40

