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ON FLUID INERTIA EFFECTS IN INFINITELY  
WIDE FOIL BEARINGS

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**ABSTRACT**

Equations for a foil over a lubricating film in which the effects of fluid inertia are taken into account are derived. Approximate solutions showing the effect of inertia and fluid compressibility are obtained. The effect of inertia is to increase the fluid film thickness.

NOMENCLATURE

C	Compressibility Parameter $\frac{p_a r_o}{T}$
C*	Normalized Compressibility Parameter $(1 + C)H^*$
f	Shape Factor for Velocity Profile
h	Radial Clearance
H	Dimensionless Clearance = $\frac{h}{r_o} \epsilon^{-2/3}$
$\bar{H}$	Normalized Clearance = $H/H^*$
I	Inertia Parameter $1/2 \rho_a U^2 / (T/U_o)$
I*	Normalized Inertia Parameter $IH^*$
k, m, n	Exponential Measures
L	Distance of Guide from Point of Tangency
p	Pressure under the Foil
$p_a$	Atmospheric Pressure
r	Polar Angle
$r_o$	Spindle Radius
R	Local Radius of Curvature
t	Temperature
$t_a$	Ambient Temperature
T	Tension per Unit Width of Foil
$v_e$	Velocity Component in the Angular Direction
$v_r$	Velocity Component in the Radial Direction
$\hat{v}$	Dimensionless Radial Component of Velocity

NOMENCLATURE (Cont)

$\hat{u}$	Dimensionless Angular Component of Velocity
$U$	Speed of Foil or Shaft
$\epsilon$	Dimensionless Parameter $\frac{6\mu U}{T}$
$\varphi$	Small Perturbation of Clearance
$\mu$	Air Viscosity
$\rho$	Air Density at $(p, t_a)$
$\rho_a$	Air Density at Ambient Conditions $(p_a, t_a)$
$\pi$	Dimensionless Pressure = $(p - p_a)/(T/r_o)$
$\theta$	Polar Angle
$\Theta$	Wrap Angle
$\xi$	Dimensionless Polar Angle
$\bar{\xi}$	Normalized Polar Angle
$\eta$	Dimensionless Radial Distance from Spindle



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ON FLUID INERTIA EFFECTS IN  
INFINITELY WIDE FOIL BEARINGS

1.0 INTRODUCTION

Recent applications of foil bearings (Fig. 1) in high speed tape transports and in high speed rotor supports necessitate the study of fluid inertia effects. This report is devoted to an approximate treatment of these effects.

2.0 ANALYSIS

We start with the steady, two-dimensional, compressible Navier-Stokes equations in polar coordinates:

$$\rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} \right] = - \frac{\partial p}{\partial r} + \mu \left[ 2 \frac{\partial^2 v_r}{\partial r^2} - \frac{2}{3} \left\{ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial r v_r}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right) \right\} + \right. \\ \left. + \frac{1}{r} \frac{\partial}{\partial \theta} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} + \frac{2}{r} \left( \frac{\partial v_r}{\partial r} - \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r} \right) \right] \quad (1)$$

$$\rho \left[ v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} \right] = - \frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[ \frac{2}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right) - \frac{2}{3r} \frac{\partial}{\partial \theta} \left\{ \frac{1}{r} \frac{\partial r v_r}{\partial r} + \right. \right. \\ \left. \left. + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right\} + \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} + \frac{2}{r} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} \right] \quad (2)$$

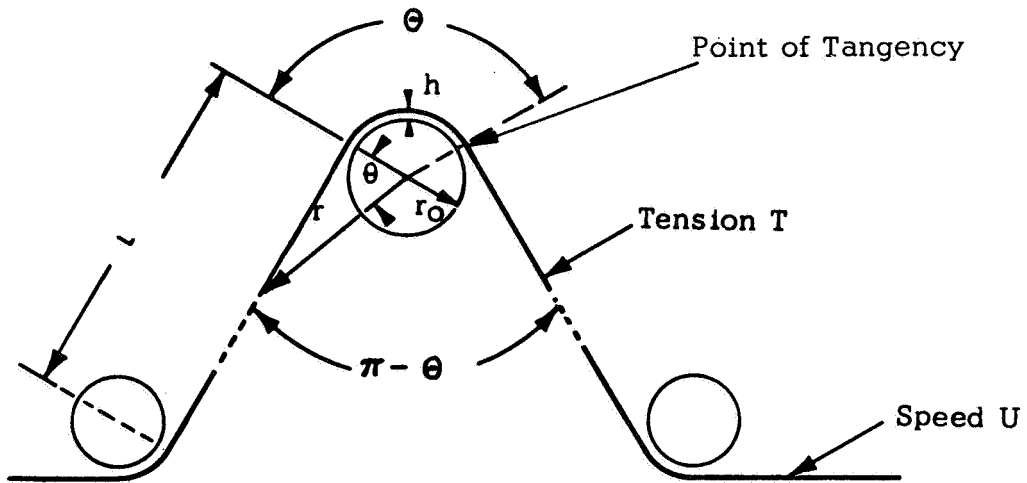


Fig. 1 Schematic View of Problem under Consideration

The continuity equation is

$$\frac{\partial(\rho U_r r)}{\partial r} + \frac{\partial(\rho U_\theta)}{\partial \theta} = 0 \quad (3)$$

The perfectly flexible foil equation is

$$p - p_a = \frac{T}{R} \quad (4)$$

The isothermal equation of state is

$$\frac{p}{p_a} = \frac{P}{P_a} \quad (5)$$

The curvature in polar coordinates is

$$\frac{1}{R} = \frac{2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2 r}{d\theta^2} + r^2}{\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]^{3/2}} \quad (6)$$

and the clearance

$$h = r - r_0 \quad (7)$$

In previous analyses of foil bearings [ 1 ], a small parameter expansion was used to formally discard terms of higher order. In the following, the same expansion is used for the Navier-Stokes equations. This will yield a consistent set of equations, including the effect of inertia. This reduction may not be correct in the large gap inlet and exit regions of the flow since the expansion is incomplete in terms of boundary conditions. It is expected, however, that for small inertia parameters, the effect of the boundary conditions will not penetrate into the small gap regions. Due to this rather heuristic character of the derivation, the results must be viewed as estimates only.

We introduce the dimensionless parameters

$$E = \frac{6\mu U}{T} \quad \text{foil bearing number} \quad (8)$$

$$C = \frac{p_a}{T/r_0} \quad \text{compressibility parameter} \quad (9)$$

$$I = \frac{\frac{1}{2} p_a U^2}{T/r_0} \quad \text{inertia parameter} \quad (10)$$

and the dimensionless variables

$$\hat{U} = \frac{U_r}{U} E^{-k} \quad (11)$$

$$\hat{u} = \frac{U}{U_0} \tag{12}$$

$$\pi = \frac{p - p_a}{T/r_0} \tag{13}$$

$$\xi = \theta e^{-m} \tag{14}$$

$$\eta = \frac{r}{r_0} e^{-n} \tag{15}$$

$$H = \frac{r}{r_0} e^{-n} \tag{16}$$

where k, m, n are constants to be determined.

In accordance with conventional foil bearing theory, the following requirements are imposed on the equations for the purpose of the determination of k, m, n: (a) The expansion of equation (6) has to retain the capability of satisfying the condition that the tape becomes flat at infinity. (b) The purpose of this work is to study cases in which inertia effects are significant. It will be required, therefore, that at least some inertia term will be of the order of some viscous term. (c) Finally, it is required that the lateral and longitudinal terms in the continuity equations will be of the same order. These three requirements result in the conditions

$$n = \frac{2}{3} \quad ; \quad m = \frac{1}{3} \quad ; \quad k = \frac{1}{3} \tag{17}$$

With these results, the equations can be shown to degenerate, to the zeroth approximation, into Prandtl's boundary layer equations.

$$I \left(1 + \frac{\pi}{c}\right) \left[ \hat{u} \frac{\partial \hat{u}}{\partial \xi} + \hat{v} \frac{\partial \hat{u}}{\partial \eta} \right] = -\frac{1}{2} \frac{d\pi}{d\xi} + \frac{1}{12} \frac{\partial^2 \hat{u}}{\partial \eta^2} \quad (18)$$

$$\frac{\partial}{\partial \eta} \left\{ (c + \pi) \hat{v} \right\} + \frac{\partial}{\partial \xi} \left\{ (c + \pi) \hat{u} \right\} = 0 \quad (19)$$

$$\pi = 1 - H'' \quad (20)$$

We assume that the velocity profile is parabolic. The profile which satisfies the boundary conditions

$$\hat{u} = 1 \quad \eta = 0 \quad (21)$$

$$\hat{u} = 0 \quad \eta = H \quad (22)$$

is

$$\hat{u} = -f \left( \frac{\eta}{H} \right)^2 + (f-1) \left( \frac{\eta}{H} \right) + 1 \quad (23)$$



where  $f$  is to be determined.

The three unknown functions  $\pi$ ,  $H$ ,  $f$  will be determined from the momentum and continuity integrals between  $\eta = 0$  and  $\eta = H$  and from the elastic equation.

The momentum integral becomes

$$I \left\{ \left( f + \frac{5}{2} \right) \frac{df}{d\xi} - \frac{1}{(\pi+c)H} \frac{d[(\pi+c)H]}{d\xi} \left( \frac{f-5}{2} \right) \right\} = -15 \frac{d\pi}{d\xi} + \frac{5f}{H^2} \quad (24)$$

The continuity integral is

$$\frac{d}{d\xi} \left[ (\pi+c) H \left( \frac{1}{2} + \frac{f}{6} \right) \right] = 0 \quad (25)$$

Equations (20), (24), and (25) constitute the approximate formulation of the problem. The first integral of Eq. (25) is

$$(\pi+c) H \left( 1 + \frac{f}{3} \right) = (1+c) H^* \quad (26)$$

where  $(1+c) H^*$  denotes the integration constant. The physical interpretation of  $H^*$  is the value of  $H$  in the central or uniformity region of the bearing. It is also a measure of the flow rate through the bearing.

From previous analyses [ 1], the boundary conditions representing the straight asymptotic behavior of the foil far from the spindle are:

$$H \sim \xi^2/2 + \dots \tag{27}$$

$$H' \sim \xi + \dots \tag{28}$$

as  $\xi \rightarrow \pm \infty$

The formulation is, thus, a function of three parameters: I, C, H\*. It is convenient to eliminate H\* from Eq. (26) For this purpose, the transformation

$$\frac{H}{H^*} = \bar{H} \quad ; \quad \frac{\xi}{H^*} = \bar{\xi} \quad ; \quad (1+C)H^* = C^* \quad ; \quad IH^* = I^* \tag{29}$$

is used. The equations become

$$f = \frac{3C^*}{(C^* - \bar{H}''')\bar{H}} - 3 \tag{30}$$

$$\bar{H}''' = \frac{\frac{5f}{\bar{H}^2} - I^* \frac{f^2 + 6f + 5}{\bar{H}} \bar{H}'}{15 - \frac{I^* (f^2 + 6f + 5)(f+3)}{3C^*}} \quad (31)$$

with

$$\begin{aligned} \bar{H}' &\sim H^* \bar{S} \\ \bar{H}'' &\sim H^* \end{aligned} \quad (32)$$

as  $\bar{S} \rightarrow \infty$

Following the procedure of [1], the solution is started from the central region towards the exit with a matched  $H^*$  at the central region. This is done by a linearized approach

$$\begin{aligned} \bar{H} &= 1 + \varphi \\ \bar{H}' &= \varphi' \\ \bar{H}'' &= \varphi'' \end{aligned} \quad (33)$$

where

$$\varphi \ll 1$$

The linearized continuity and momentum equations become

$$f = 3 \left( \frac{\varphi''}{c^*} - \varphi \right) \tag{34}$$

$$\varphi''' \left( 1 - \frac{\Gamma^*}{3c^*} \right) - \frac{\varphi''}{c^*} + \frac{\Gamma^* \varphi'}{3} \varphi = 0 \tag{35}$$

The roots of the characteristic equation are graphically shown in Fig. 2.

### 3. ASYMPTOTIC BEHAVIOR

It can be observed that as

$$\begin{aligned} \bar{H} \rightarrow \infty & \quad f \rightarrow -3 \\ \bar{H}''' \sim -\frac{4}{15} \frac{H'}{H} &= -\frac{4}{15} \frac{d(\ln \bar{H})}{d\bar{x}} \end{aligned} \tag{36}$$

Thus, the boundary conditions cannot be satisfied since  $\bar{H}''$  does not approach a constant as required by Eq. (32). It must be observed, however, that in the region of interest  $\bar{H}'''$  is fairly small.  $\bar{H}$  is, therefore, nearly parabolic, until finally the logarithmic contribution becomes overwhelming. Thus, one may consider that the boundary condition of an asymptotically straight tape is approximately satisfied. \*

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\*This interpretation of the results is due to Prof. H. G. Elrod Jr.

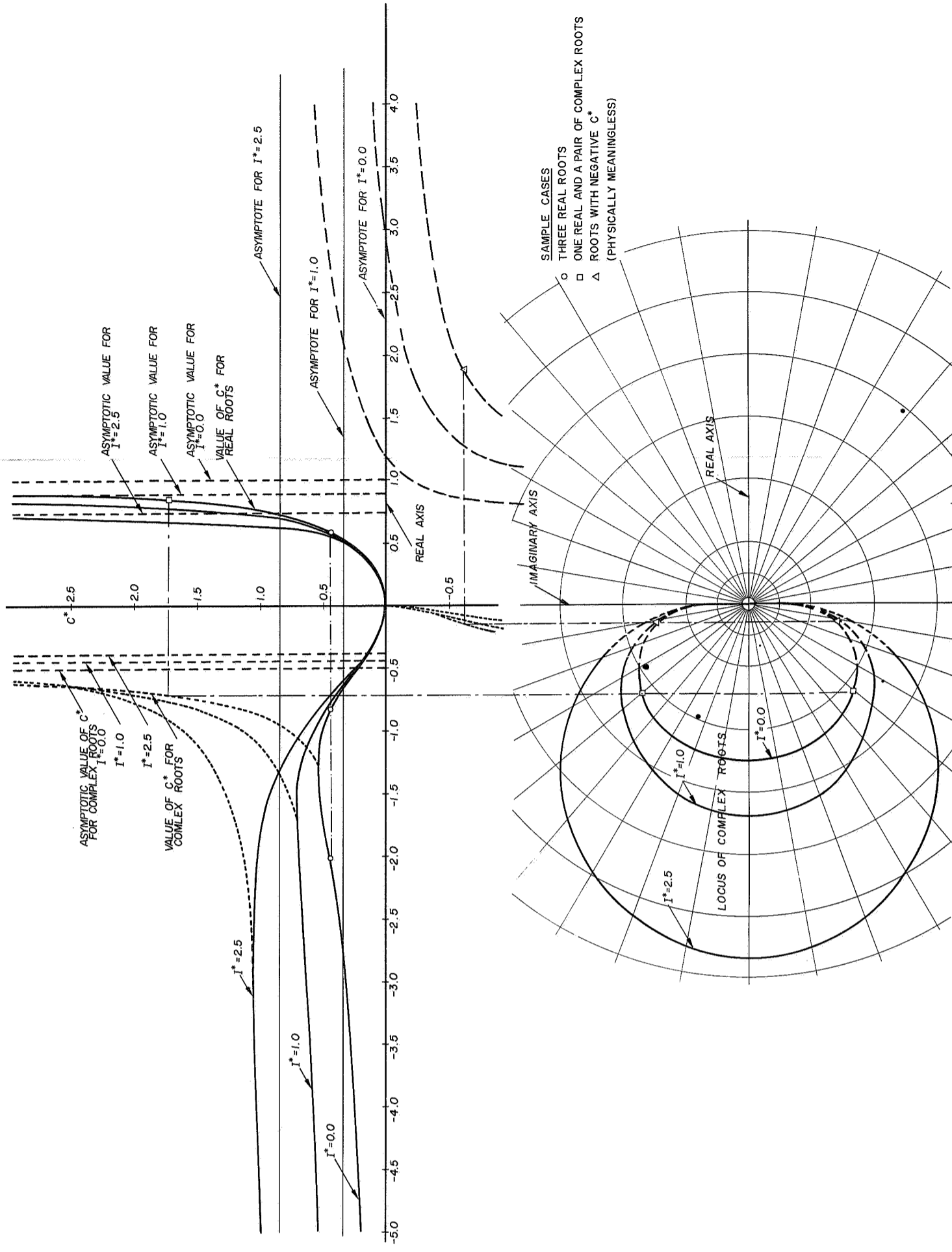


Fig. 2 Roots of Characteristic Equation

Due to the imperfect satisfaction of the boundary conditions, a degree of uncertainty is involved which increases with the inertia parameter  $I$  and vanishes for  $I \rightarrow 0$ . In the actual computation work the point at which the logarithmic behavior takes over was assumed to lie between  $\bar{H} = 20$  and  $\bar{H} = 1000$ . The value of  $H^*$  taken as "correct" was chosen as that corresponding to  $\bar{H} = 100$ .

Figure 3 shows the corresponding range of uncertainty in  $H^*$ . The main result of this work is given in Fig. 4. Here, the values of  $H^*$  as a function of the compressibility and inertia parameters is given. A similar chart, where  $H^*$  is expressed in terms of  $C$  and  $I^*$  which naturally appear in the equations, is given in Fig. 5.

For the purpose of practical calculations, an analytical formula was fitted to the results of Fig. 4:

$$H^* = 0.643 + 0.286 I + 1.905 I^2 - \frac{0.183}{C}$$

The formula approximates the results well in the region

$$0 < \frac{I}{C} < 1$$

$$0 < I < 0.5$$

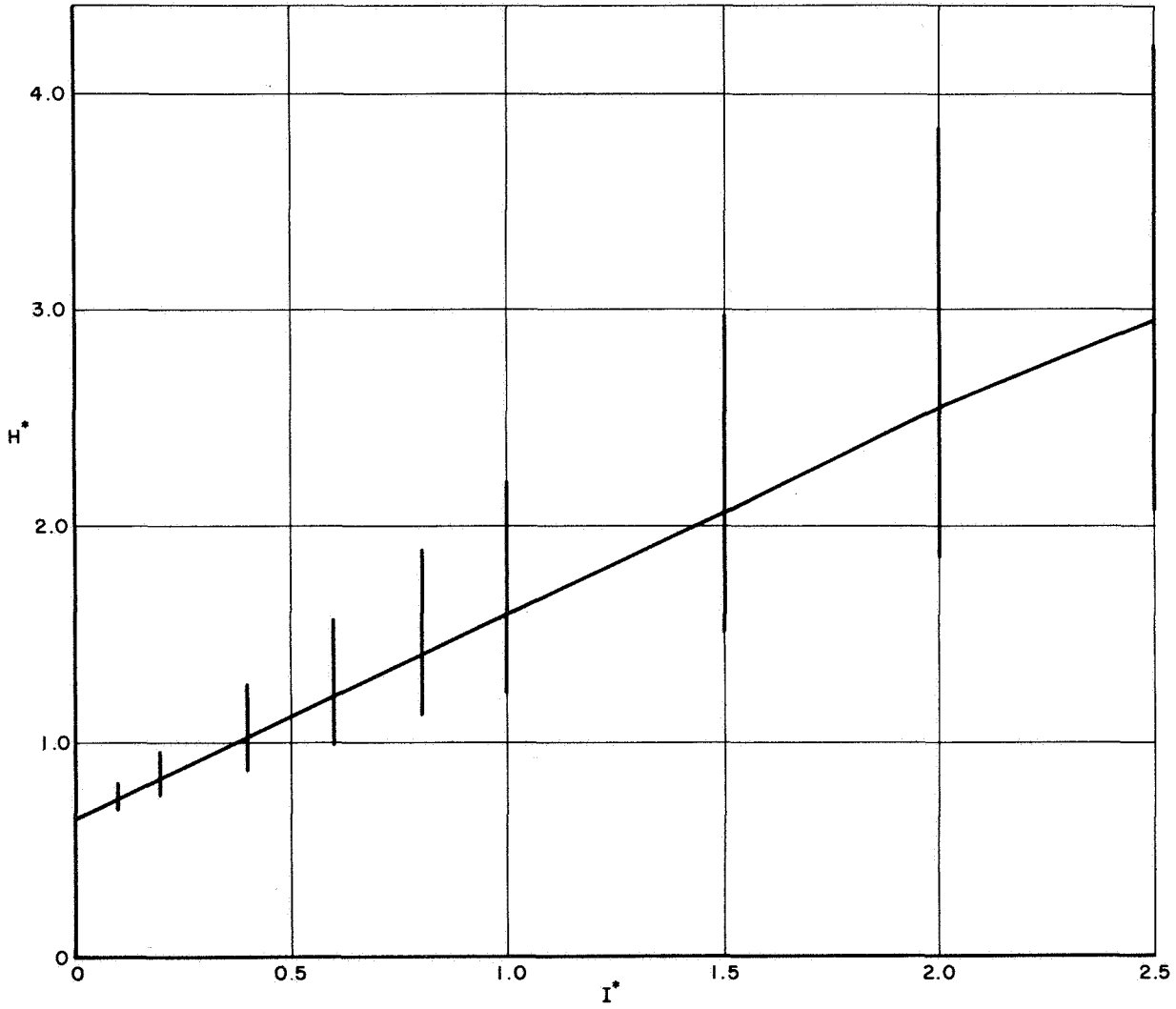


Fig. 3. Bounds of Uncertainty in the Evaluation of  $H^*$   
 ( $C^* = \infty$ )

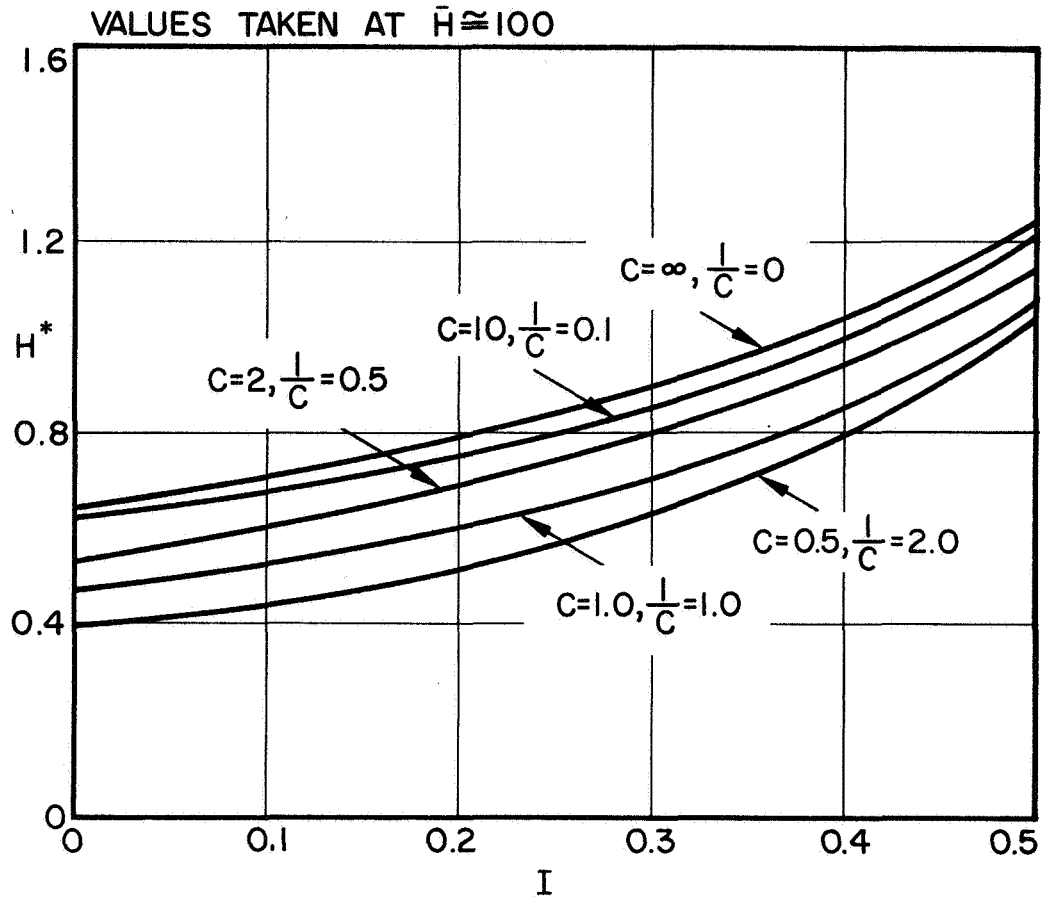


Fig. 4 Nominal Clearance  $H^*$  as a Function of Compressibility (C) and Inertia (I) Parameters



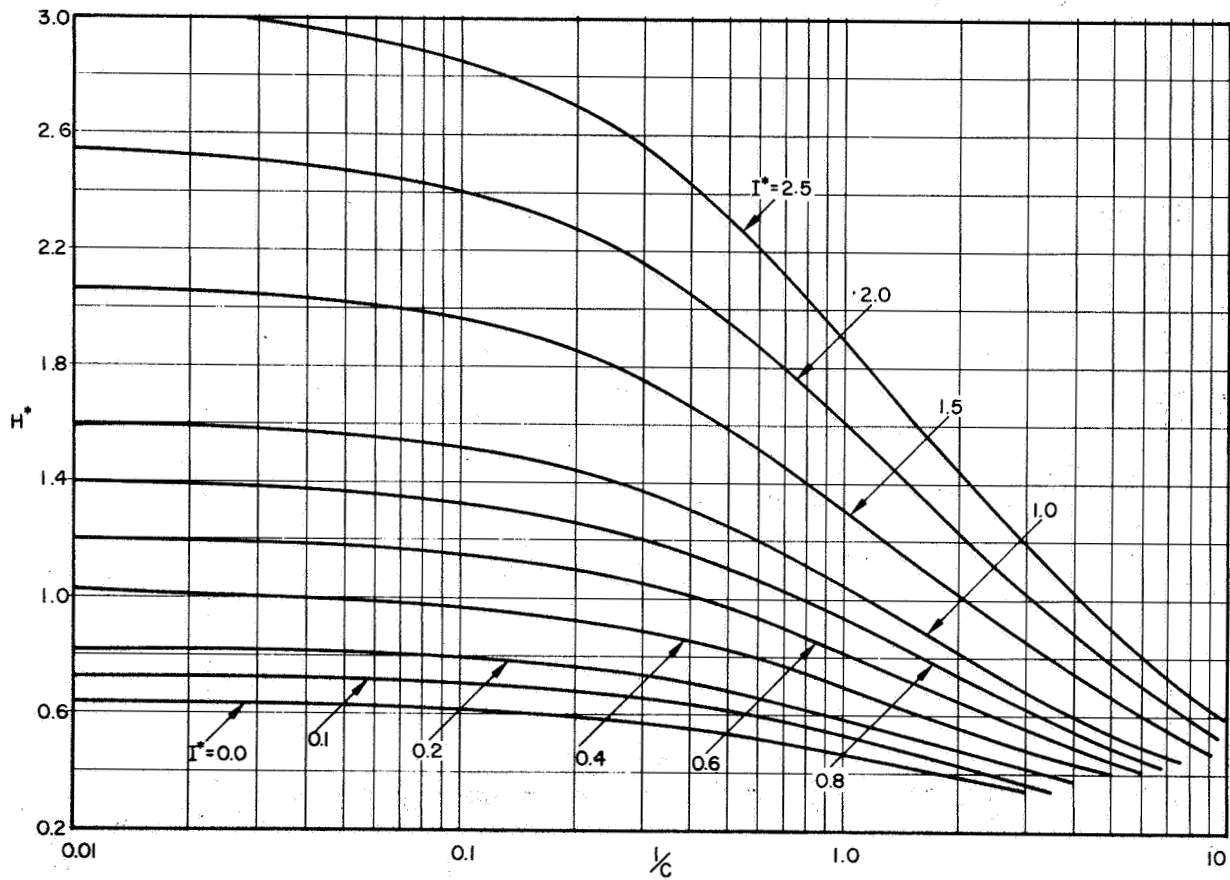


Fig. 5 Nominal Clearance  $H^*$  as a Function of Compressibility ( $C$ ) and the Normalized Inertia ( $I^*$ ) Parameters

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3. REPORT TITLE  On Fluid Inertia Effects in Infinitely Wide Foil Bearings		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report		
5. AUTHOR(S) (Last name, first name, initial) Eshel, A.		
6. REPORT DATE July 15, 1968	7a. TOTAL NO. OF PAGES 25	7b. NO. OF REFS 1
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11. SUPPLEMENTARY NOTES  None	12. SPONSORING MILITARY ACTIVITY Department of the Navy Office of Naval Research Washington, D. C. 20360	
13. ABSTRACT  Equations for a foil over a lubricating film in which the effects of fluid inertia are taken into account are derived. Approximate solutions showing the effect of inertia and fluid compressibility are obtained. The effect of inertia is to increase the fluid film thickness.		

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14. KEY WORDS	LINK A		LINK B		LINK C	
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