On Fuzzy Contra $g^{*}\alpha$-Continuous Functions

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ABSTRACT
In this paper we introduce and study the new class of functions called fuzzy contra $g^{*}\alpha$-continuous and fuzzy almost contra $g^{*}\alpha$-continuous mappings on fuzzy topological spaces. We investigate some of their properties. Also we provide the relation between fuzzy contra $g^{*}\alpha$-continuous mappings and fuzzy almost contra $g^{*}\alpha$-continuous mappings.

General Terms Fuzzy topology, fuzzy generalized closed set, fuzzy $g\alpha$-closed, fuzzy $g^{*}\alpha$-closed set, fuzzy contra α-continuous function fuzzy $g^{*}\alpha$-continuous.

Keywords
Fuzzy contra $g^{*}\alpha$-continuous function, fuzzy Contra $g^{*}\alpha$-irresolute function, fuzzy almost contra $g^{*}\alpha$-continuous functions.

1. INTRODUCTION
The fuzzy α-open and fuzzy α-continuous mappings were introduced and generalized by Bin Shahana [4]. N. Levine [8] introduced the concepts of generalized closed sets in general topology in the year 1970. Veera kumar [13] introduced and study the concept of g^{*}-closed set and g^{*}-continuity in topological space. In 2006, Eradal and Etienne [5] introduced the notation of fuzzy contra continuous mapping. S. S. Benchalli and G. P. Siddapur [3] introduced the notation of generalized pre-closed sets in fuzzy topological space in 2011. Recently M. Shukla introduced the concept of fuzzy contra $g^{*}p$-continuous [11] and fuzzy contra $g^{*}s$- continuous [12] in fuzzy topological space.

In this paper we introduce and study the new class of mappings called fuzzy contra $g^{*}\alpha$-continuous and fuzzy almost contra $g^{*}\alpha$-continuous functions in fuzzy topological spaces. Also we define the relation between of fuzzy contra $g^{*}\alpha$ -continuous and fuzzy almost contra $g^{*}\alpha$-continuous spaces and study some of their properties.

2. PRELIMINARY
Let X be a non empty set. A collection τ of fuzzy sets in X is called a fuzzy topology on X if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of τ and τ is closed with respect to any union and finite intersection. The members of τ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set.

The **closure** of a fuzzy set λ (denoted by $cl(\lambda)$) is the intersection of all fuzzy closed which contains λ. The **interior** of a fuzzy set λ (denoted by $int(\lambda)$) is the union of all fuzzy open subsets of λ. A fuzzy set λ in X is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

(i) semi-open fuzzy set [1] if $\lambda \leq cl(int(\lambda))$ and semi-closed fuzzy set if $int(cl(\lambda)) \leq \lambda$.
(ii) pre-open fuzzy set [4] if $\lambda \leq int(cl(\lambda))$ and pre-closed fuzzy set if $cl(int(\lambda)) \leq \lambda$.
(iii) α-open fuzzy set [4] if $\lambda \leq int\left(cl(int(\lambda))\right)$ and α-closed fuzzy set if $cl\left(int(cl(\lambda))\right) \leq \lambda$.
(iv) regular open fuzzy set [1] if $\lambda = int(cl(\lambda))$ and regular closed fuzzy set if $\lambda = cl(int(\lambda))$.

The α-closure (resp. semi-closure, pre-closuer) of a fuzzy set λ in fuzzy topological space (X, τ) is intersection of all α-closed (resp. semi-closed, preclosed) fuzzy sets in X containing λ and is denoted by $\alpha - cl(\lambda)$ (resp. $scl(\lambda), pcl(\lambda)$).

Definition 2.2: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

(i) generalized closed fuzzy set (g-closed) fuzzy set [2] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in(X, τ).
(ii) generalized α-closed fuzzy set ($g\alpha$-closed) fuzzy set [2] if $\alpha cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in(X, τ).
(iii) g^{*}- closed fuzzy set (g^{*}-closed) fuzzy set [8] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in(X, τ).
(iv) g^{*}-preclosed fuzzy set ($g^{*}p$-closed) fuzzy set [3] if $pcl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in(X, τ).
(v) g^{*}-semiclosed fuzzy set (g^{*}s-closed) fuzzy set [3] if $scl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in (X, τ).
(vi) g^{*}-alphaclosed fuzzy set ($g^{*}\alpha$-closed) fuzzy set [3] if $\alpha cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in(X, τ).

The complement of g-closed (resp. gp-closed, g^{*}-closed and $g^{*}p$-closed, $g^{*}s$-closed, $g^{*}\alpha$-closed) fuzzy sets are called fuzzy g-open (resp. gp-open, g^{*}-open and $g^{*}p$-open, $g^{*}s$-open, $g^{*}\alpha$-open) sets in fuzzy topological spaces.

Definition 2.3: A fuzzy topological space (X, τ) is called T_{α}^{*} -space [6] if every $g^{*}\alpha$-closed fuzzy set is a closed fuzzy set in X.

Definition 2.4: A function f from a fuzzy topological space (X, τ) to fuzzy topological space (Y, σ) is called:

(i) fuzzy-contra continuous if $f^{-1}(\lambda)$ is fuzzy closed in X for every fuzzy open set λ of Y [5].

(ii) fuzzy contra pre-continuous (fuzzy contra α-continuous [7], fuzzy contra semi-continuous) if $f^{-1}(\lambda)$ is fuzzy pre-closed (fuzzy α-closed, fuzzy semi-closed resp.) in X for every fuzzy open set λ of Y .

(iii) fuzzy g-continuous if $f^{-1}(\lambda)$ is fuzzy g-closed in X for every fuzzy closed set λ of Y [2].

(iv) fuzzy g pre- continuous (fuzzy $g\alpha$-continuous, fuzzy $g$semi-contonuous) if $f^{-1}(\lambda)$ is fuzzy gp-closed (fuzzy $g\alpha$-closed, fuzzy gs-closed resp.) in X for every fuzzy closed set λ of Y [6,].

(v) fuzzy g^*- continuous if $f^{-1}(\lambda)$ is fuzzy g^*-open in X for every fuzzy open set λ of Y [8].

(vi) fuzzy g^*p-continuous (fuzzy $g^*\alpha$-continuous, fuzzy g^*semi-continuous) if $f^{-1}(\lambda)$ is fuzzy g^*p –open (fuzzy $g\alpha^*$-open, fuzzy g^*s-open) in X for every fuzzy open set λ of Y [3,].

(vii) A function $f: X \to Y$ is called fuzzy contra g^*p-continuous (fuzzy contra g^*s-continuous) if $f^{-1}(\lambda)$ is fuzzy g^*p-closed (fuzzy g^*s-closed) set in X for every open set λ in Y [11].

(viii) fuzzy almost continuous if $f^{-1}(\lambda)$ is fuzzy open in X for every fuzzy regular open set λ of Y [1].

3. FUZZY CONTRA $g^*\alpha$-CONTINUOUS FUNCTION

Definition 3.1. A function $f: X \to Y$ is called **fuzzy contra $g^*\alpha$-continuous** if $f^{-1}(\lambda)$ is fuzzy $g^*\alpha$-closed set in X for every open set λ in Y.

Theorem 3.2. Every fuzzy contra continuous function is fuzzy contra $g^*\alpha$-continuous function.

Proof: It follows from the fact that every fuzzy closed set is $g^*\alpha$-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and μ, λ be fuzzy sets in X and Y, defined as $\mu(x_1) = 0.5$, $\mu(x_2) = 0.5$, $\lambda(y_1) = 0.4$, and $\lambda(y_2) = 0.3$. Let $\tau = \{0, \mu, 1\}$ and $\tau' = \{0,\lambda,1\}$ be fuzzy topologies on sets X and Y respectively. We see that map $f : X \to Y$ defined as $f(x_1) = y_1$ and $f(x_2) = y_2$ Then f is fuzzy contra $g^*\alpha$-continuous but not fuzzy contra continuous.

Theorem 3.4. Every fuzzy contra α-continuous mapping is fuzzy contra $g^*\alpha$-continuous function.

Proof. Straight forward and follows from the definitions.

The converse of the above theorem need not be true as seen from Example 3.2.

Theorem 3.5. Every fuzzy contra $g^*\alpha$ -continuous mapping is fuzzy contra g^*s-continuous function as well as fuzzy g^*p-continuous.

Proof: Straight forward and follows from the definitions.

The converse of the above theorem need not be true as seen from Example 3.6 and Example 3.7.

Example 3.6: Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$. Let η_1, η_2, η_3 be fuzzy sets in X, μ be a fuzzy set in Y, defined as $\eta_1(x_1) = 0.2$, $\eta_1(x_2) = 0.3$, $\eta_2(x_1) = 0.3$, $\eta_2(x_2) = 0.5$, $\eta_3(x_1) = 0.6$, $\eta_3(x_2) = 0.7$, $\mu(y_1) = 0.5$, and $\mu(y_2) = 0.6$. Let $\tau_X = \{0, \eta_1, \eta_2, \eta_3, 1\}$ and $\tau_Y = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. Let $f: X \to Y$ defined as $f(x_i) = y_i$, i = 1, 2. Then f is fuzzy contra g^*s-continuous but not fuzzy contra $g^*\alpha$-continuous.

Example 3.7: Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$. Let η_1 an, η_3 be fuzzy sets in X, μ be a fuzzy set in Y, defined as $\eta_1(x_1) = 0.1$, $\eta_1(x_2) = 0.2$, $\eta_2(x_1) = 0.4$, $\eta_2(x_2) = 0.6$, $\eta_3(x_1) = 0.7$, $\eta_3(x_2) = 0.7$, $\mu(y_1) = 0.2$ and $\mu(y_2) = 0.5$. Let $\tau_X = \{0, \eta_1, \eta_2, \eta_3, 1\}$ and $\tau_Y = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. Let $f: X \to Y$ defined as $f(x_i) = y_i$, i = 1, 2. Then f is fuzzy contra g^*p-continuous but not fuzzy contra $g^*\alpha$-continuous.

Theorem 3.8. If a function $f: (X, \tau) \to (Y, \sigma)$ is fuzzy contra $g^*\alpha$ –continuous and (X, τ) is fuzzy T^*_α-space, than f is fuzzy contra continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is $g^*\alpha$-closed fuzzy set in X. Since X is fuzzy T^*_α -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. Thus f is fuzzy contra continuous function.

Theorem 3.9. If a function $f: (X, \tau) \to (Y, \sigma)$ is fuzzy contra α -continuous and (X, τ) is fuzzy T^*_α-space, than f is fuzzy contra $g^*\alpha$-continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is α -closed fuzzy set in X. Since X is fuzzy T^*_α -space. $f^{-1}(\lambda)$ is $g^*\alpha$-closed fuzzy set in X. Thus f is fuzzy contra $g^*\alpha$-continuous function.

Theorem 3.10. If a function $f: (X, \tau) \to (Y, \sigma)$ is fuzzy contra $g^*\alpha$ –continuous and (X, τ) is fuzzy T^*_α-space, than f is fuzzy contra α -continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is $g^*\alpha$-closed fuzzy set in X. Since X is fuzzy T^*_α -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. And every closed fuzzy set isα -closed fuzzy set. Thus f is fuzzy contra α -continuous function.

Theorem 3.11 Let (X, τ) and (Y, σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \to Y$.

(i) f is fuzzy contra $g^*\alpha$- continuous.

(ii) $f^{-1}(\lambda)$ is $g^*\alpha$-open fuzzy set in X for each closed fuzzy set λ in Y.

(iii) for each $x \in X$ and each closed fuzzy set λ in Y containing $f(x)$. there exist a $g^*\alpha$-open fuzzy set η in X containing x such that $f(\eta) \le \lambda$.

(iv) for each $x \in X$ and open fuzzy set μ in Y non-containing $f(x)$, there exists a $g^*\alpha$-closed fuzzy set ϑ in X non-containing x such that $f^{-1}(\mu) \le \vartheta$.

Proof. (i)⇒ (ii). Let λ be a closed fuzzy set in (Y, σ). Then 1- λ is fuzzy open. By (i), $f^{-1}(1-\lambda) = 1 - f^{-1}(\lambda)$ is $g^*\alpha$-closed fuzzy set in X. So $f^{-1}(\lambda)$ is $g^*\alpha$ –open fuzzy set in X.

(ii) ⇒ (i). proof as above.

(ii) ⇒ (iii). Let λ be any closed fuzzy set in Y containing $f(x)$. By (ii). $f^{-1}(\lambda)$ is $g^*\alpha$-open fuzzy set in (X, τ) and $x \in f^{-1}(\lambda)$. Take $\eta = f^{-1}(\lambda)$. Then $f(\eta) \le \lambda$.

(iii) ⇒ (ii). Let λ be a closed fuzzy set in Y and $x \in f^{-1}(\lambda)$. From (iii), there exists a $g^*\alpha$-open fuzzy set η in X containing x such that $\eta \le f^{-1}(\lambda)$. We have $f^{-1}(\lambda) = \cup_{x \in f^{-1}(\lambda)} \eta$. Thus $f^{-1}(\lambda)$ is $g^*\alpha$-open fuzzy set in (X, τ).

(iii) ⇒ (iv). Let μ be any open fuzzy set in Y non-containing $f(x)$. Then 1-μ is a closed fuzzy set containing $f(x)$. By (iii) there exists a $g^*\alpha$-open fuzzy set η in X containing x such that $f(\eta) \le 1 - \mu$. Hence $\eta \le f^{-1}(1-\mu) \le 1 - f^{-1}(\mu)$ and then $f^{-1}(\mu) \le 1 - \eta$.

Take $\vartheta = 1 - \eta$. We obtain that ϑ is a $g^*\alpha$-closed fuzzy set in X non-containing x.
The converse can be shown easily.

Definition 3.12. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy Contra $g^*\alpha$–irresolute** if $f^{-1}(\lambda)$ is $g^*\alpha$-closed fuzzy set in X for every $g^*\alpha$-open fuzzy set λ in Y.

Theorem 3.13. A function $f:(X,\tau) \to (Y,\sigma)$is fuzzy contra $g^*\alpha$-continuous if and only if $f^{-1}(\lambda)$ is $g^*\alpha$-open fuzzy set in X for every $g^*\alpha$-closed fuzzy set λ in Y.

Theorem 3.14. Every fuzzy contra $g^*\alpha$-irresolute mapping is fuzzy contra $g^*\alpha$-continuous.

Proof. Let $f: X \to Y$ is fuzzy contra $g^*\alpha$-irresolute function. Let λ be a fuzzy open set in Y. Then λ is $g^*\alpha$-open fuzzy set in Y. Since f is fuzzy contra $g^*\alpha$-irresolute. $f^{-1}(\lambda)$ is $g^*\alpha$- fuzzy closed set in X. Hence f is fuzzy contra $g^*\alpha$-continuous function.

Theorem 3.15. Let $f: X \to Y$, $g: Y \to Z$ be two functions then

(i) $gof: X \to Z$ is fuzzy contra $g^*\alpha$-continuous, if f is fuzzy contra $g^*\alpha$-continuous and g are fuzzy continuous.
(ii) $gof: X \to Z$ is fuzzy contra $g^*\alpha$-continuous if f is fuzzy contra $g^*\alpha$-irresolute and g is fuzzy $g^*\alpha$-continuous.

4. FUZZY ALMOST CONTRA $g^*\alpha$-CONTINUOUS FUNCTION

Definition 4.1. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy almost contra $g^*\alpha$–Continuous** if $f^{-1}(\lambda)$ is fuzzy $g^*\alpha$-closed set in X for every regular open set λ in Y.

Theorem 4.2. Every fuzzy contra $g^*\alpha$-continuous function is fuzzy almost contra $g^*\alpha$-continuous.

Proof. Since every regular fuzzy open set is open fuzzy set, such that every fuzzy contra $g^*\alpha$-continuous mappings is fuzzy almost contra $g^*\alpha$-continuous.
The converse of the above theorem need not be true as seen from the following example.

Example 4.3: Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ λ, and μ be a fuzzy set in X and Y defined as $\lambda(x_1) = 0.5$, $\lambda(x_2) = 0.5$, $\mu(y_1) = 0.3$, $\mu(y_2) = 0.5$. Let $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. The map $f:(X,\tau) \to (Y,\tau')$ defined as $f(x_i) = y_i, i = 1,2$ is fuzzy almost contra $g^*\alpha$-continuous map but not fuzzy contra $g^*\alpha$-continuous.

Definition 4.4. A function $f: X \to Y$ is said to be fuzzy regular set connected if $f^{-1}(\lambda)$ is fuzzy clopen in X for every fuzzy regular open set λ of Y.

Theorem 4.5. If a function $f: X \to Y$ is fuzzy almost contra $g^*\alpha$-continuous and fuzzy almost continuous, then f is fuzzy regular set connected.

Proof. Let λ be a fuzzy regular open set in (Y,σ). Since f is fuzzy almost contra $g^*\alpha$-continuous and fuzzy almost continuous, $f^{-1}(\lambda)$ is fuzzy $g^*\alpha$-closed and open. Hence $f^{-1}(\lambda)$ is fuzzy clopen. Therefore f is fuzzy regular set connected.

Definition 4.6. A fuzzy topological spaces (X,τ) is called fuzzy $g^*\alpha$-connected if X cannot be written as the disjoint union of two non-empty fuzzy $g^*\alpha$-open sets.

Theorem 4.7. Let (X,τ) and (Y,σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \to Y$.

(i) f is fuzzy almost contra $g^*\alpha$- continuous.
(ii) $f^{-1}(\lambda)$ is fuzzy $g^*\alpha$-open set in X for every regular closed set λ in Y.
(iii) for each $x \in X$ and each fuzzy regular closed set λ in Y containing $f(x)$. there exist a fuzzy $g^*\alpha$-open set η in X containing x such that $f(\eta) \leq \lambda$.
(iv) for each $x \in X$ and fuzzy regular open set μ in Y non-containing $f(x)$, there exists a fuzzy $g^*\alpha$-closed set ϑ in X non-containing x such that $f^{-1}(\mu) \leq \vartheta$.

Proof. As theorem 3.8.

Theorem 4.8: Let X, Y and Z be fuzzy topological spaces and let $f: X \to Y$ and $g: Y \to Z$ be maps. If f is fuzzy contra $g^*\alpha$-continuous and g is fuzzy almost continuous then $gof: X \to Z$ is fuzzy almost contra $g^*\alpha$-continuous.

5. CONCLUSION

(i) We have introduced and studied new kind of map fuzzy contra $g * \alpha$ -continuous maps on fuzzy topological spaces.
(ii) We defined the relation between fuzzy contra αcontinuous and fuzzy contra $g * \alpha$-continuous map. We investigated some of their properties.
(iii) We proved that every fuzzy contra $g * \alpha$ -continuous map is fuzzy contra $g * p$ –precontinuos as well as fuzzy $g * s$-continuous mapping but converse may not be true by use of example.
(iv) We have established some significant properties of fuzzy contra $g * \alpha$- continuous maps.
(v) We introduce and study new kind of fuzzy almost contra $g * \alpha$-continuous map.

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