On Fuzzy Contra g^* Semi-Continuous Functions

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Abstract

In this paper we introduce and study the new class of functions called fuzzy contra g^*semi-continuous and almost fuzzy contra g^* semi-continuous mappings on fuzzy topological spaces. We investigate some of their properties. Also we provide the relation between fuzzy contra g^*semi-continuous mappings and fuzzy almost contra g^*semi-continuous mappings.

Key words: Fuzzy topology, fuzzy generalized closed set, fuzzy g^*s-closed set, fuzzy contra semi-continuous function, fuzzy g^*s-continuous function, fuzzy almost contra continuous functions.

1. Introduction

The fuzzy semi-open and fuzzy semi-continuous mappings were introduced and generalized by Bin Shahana [4]. N. Levine [9] introduced the concepts of generalized closed sets in general topology in the year 1970. T. Fukutake, R.K. Saraf, M. Caldas and S. Mishra [6] introduced the notation of generalized semi-closed sets in fuzzy topological space. S. S. Benchalli and G. P. Siddapur [3] introduced and investigate g^*s-continuous maps in fuzzy topological spaces. In 2011, P.G. Patil T.D. Rayanagoudar and Mahesh K. Bhat [13] introduced and studied the concepts of contra g^*s-continuous and almost contra g^*s-continuous mappings in general topological spaces.

In this paper we introduce and study the new class of mappings called fuzzy contra g^*s-continuous and fuzzy almost contra g^*s-continuous functions in fuzzy topological spaces. Also we define the relation between of fuzzy contra g^*s -continuous and fuzzy almost contra g^*s-continuous spaces and study some of their properties.

2. Preliminaries

Let X be a non empty set. A collection τ of fuzzy sets in X is called a fuzzy topology on X if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of τ and τ is closed with respect to any union and finite intersection. The members of τ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The **closure** of a fuzzy set λ (denoted by (λ)) is the intersection of all fuzzy closed which contains λ. The **interior** of a fuzzy set λ (denoted by $int(\lambda)$) is the union of all fuzzy open subsets of λ. A fuzzy set λ in X is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

(i) semi-open fuzzy set [1] if $\lambda \leq cl(int(\lambda))$ and semi-closed fuzzy set if $int(cl(\lambda)) \leq \lambda$.

(ii) pre-open fuzzy set [4] if $\lambda \leq int(cl(\lambda))$ and pre-closed fuzzy set if $cl(int(\lambda)) \leq \lambda$.

(iii) semi-preopen fuzzy set [13] (=β set) if $\lambda \leq cl(int(cl(\lambda)))$ and semi-preclosed fuzzy set if $int\left(cl(int(\lambda))\right) \leq \lambda$.

(iv) regular open fuzzy set [1] if $\lambda = int(cl(\lambda))$ and regular closed fuzzy set if $\lambda = cl(int(\lambda))$.

The semi-closure (resp. pre-closure, semi-preopen) of a fuzzy set λ in fuzzy topological space (X, τ) is intersection of all semi-closed (resp. pre-closed, semi-preclosed) fuzzy sets in X containing λ and is denoted by $scl(\lambda)$ (resp. $pcl(\lambda), spcl(\lambda)$).

Definition 2.2: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

(i) generalized closed fuzzy set (g-closed) fuzzy set [2] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in (X, τ).

(ii) generalized semi-closed fuzzy set (gs-closed) fuzzy set [2] if $scl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in (X, τ).

(iii) g^*- closed fuzzy set (g^*-closed) fuzzy set [8] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in (X, τ).

(iv) g^*-semiclosed fuzzy set (g^*s-closed) fuzzy set [11] if $scl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in (X, τ).

The complement of g-closed (resp. gs-closed, g^*-closed and g^*s-closed) fuzzy sets are called fuzzy g-open (resp. gp-open, g^*-open and g^*s-open) sets in fuzzy topological spaces.

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Definition 2.3: A fuzzy topological space (X, τ) is called T_p^* -space [6] if every g^*s-closed fuzzy set is a closed fuzzy set in X.

Definition 2.4: A function f from a fuzzy topological space (X, τ) to fuzzy topological space (Y, σ) is called:

(i) fuzzy-contra continuous if $f^{-1}(\lambda)$ is fuzzy closed in X for every fuzzy open set λ of Y [5].

(ii) fuzzy contra semicontinuous if $f^{-1}(\lambda)$ is fuzzy semiclosed in X for every fuzzy open set λ of Y [8].

(iii) fuzzy g-continuous if $f^{-1}(\lambda)$ is fuzzy g-closed in X for every fuzzy closed set λ of Y [2].

(iv) fuzzy gs- continuous if $f^{-1}(\lambda)$ is fuzzy gs-closed in X for every fuzzy closed set λ of Y [2].

(v) fuzzy g^*- continuous if $f^{-1}(\lambda)$ is fuzzy g^*-open in X for every fuzzy open set λ of Y [8].

(vi) fuzzy g^*s-continuous if $f^{-1}(\lambda)$ is fuzzy g^*s -open in X for every fuzzy open set λ of Y [].

(vii) fuzzy almost continuous if $f^{-1}(\lambda)$ is fuzzy open in X for every fuzzy regular open set λ of Y [1].

3. Fuzzy Contra g^*s-Continuous Functions

Definition 3.1. A function $f: X \to Y$ is called **fuzzy contra g^*s-continuous** if $f^{-1}(\lambda)$ is fuzzy g^*s-closed set in X for every open set λ in Y.

Theorem 3.2. Every fuzzy contra continuous function is fuzzy contra g^*s-continuous function. .

Proof: It follows from the fact that every fuzzy closed set is g^*s-closed set.[]

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = \{ x_1, x_2\}, Y = \{ y_{1,} y_2 \}$ λ, and μ be a fuzzy set in X and Y defined as $\lambda(x_1) = 0.4$, $\lambda(x_2) = 0.6$, $\mu(y_1) = 0.3$, $\mu(y_2) = 0.7$. Let $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. The map $f: (X, \tau) \to (Y, \tau')$ defined as $f(x_i) = y_i, i = 1,2$ is fuzzy contra g^*s-continuous map but not fuzzy contra continuous.

Theorem 3.4. Every fuzzy contra semi-continuous mapping is fuzzy contra g^*s-continuous function.

Proof. Straight forward and follows from the definitions.

The converse of the above theorem need not be true as seen from Example 3.3.

Theorem 3.5. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy contra g^*s –continuous and (X,τ) is fuzzy T_p^*-space, than f is fuzzy contra continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is g^*s-closed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. Thus f is fuzzy contra continuous function.

Theorem 3.6. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy contra semi-continuous and (X,τ) is fuzzy T_p^*-space, than f is fuzzy contra g^*s-continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is semi-closed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is g^*s-closed fuzzy set in X. Thus f is fuzzy contra g^*s-continuous function.

Theorem 3.7. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy contra g^*s –continuous and (X,τ) is fuzzy T_p^*-space, than f is fuzzy contra semi-continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is g^*s-closed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. And every closed fuzzy set is semi-closed fuzzy set. Thus f is fuzzy contra semi-continuous function.

Theorem 3.8. Let (X,τ) and (Y,σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \rightarrow Y$.

1. f is fuzzy contra g^*s- continuous.
2. $f^{-1}(\lambda)$ is g^*s-open fuzzy set in X for each closed fuzzy set λ in Y.
3. for each $x \in X$ and each closed fuzzy set λ in Y containing $f(x)$. there exist a g^*s-open fuzzy set η in X containing x such that $f(\eta) \leq \lambda$.
4. for each $x \in X$ and open fuzzy set μ in Y non-containing $f(x)$, there exists a g^*s-closed fuzzy set ϑ in X non-containing x such that $f^{-1}(\mu) \leq \vartheta$.

Proof. (1)$\Rightarrow$ (2). Let λ be a closed fuzzy set in (Y,σ). Then 1- λ is fuzzy open. By (1), $f^{-1}(1-\lambda) = 1 - f^{-1}(\lambda)$ is g^*s-closed fuzzy set in X. So $f^{-1}(\lambda)$ is g^*s –open fuzzy set in X.

(2) $\Rightarrow$ (1). proof as above.

(2) $\Rightarrow$ (3). Let λ be any closed fuzzy set in Y containing $f(x)$. By (2). $f^{-1}(\lambda)$ is g^*s-open fuzzy set in (X,τ) and $x \in f^{-1}(\lambda)$. Take $\eta = f^{-1}(\lambda)$. Then $f(\eta) \leq \lambda$.

(3) $\Rightarrow$ (2). Let λ be a closed fuzzy set in Y and $x \in f^{-1}(\lambda)$. From (3), there exists a g^*s-open fuzzy set η in X containing x such that $\eta \leq f^{-1}(\lambda)$. We have $f^{-1}(\lambda) = \cup_{x \in f^{-1}(\lambda)} \eta$. Thus $f^{-1}(\lambda)$ is g^*s-open fuzzy set in (X,τ).

(3) $\Rightarrow$ (4). Let μ be any open fuzzy set in Y non-containing $f(x)$. Then 1-μ is a closed fuzzy set containing $f(x)$. By (3) there exists a g^*s-open fuzzy set η in X containing x such that $f(\eta) \leq 1-\mu$. Hence $\eta \leq f^{-1}(1-\mu) \leq 1 - f^{-1}(\mu)$ and then $f^{-1}(\mu) \leq 1-\eta$. Take $\vartheta = 1-\eta$. We obtain that ϑ is a g^*s-closed fuzzy set in X non-containing x.

The converse can be shown easily.

Definition 3.9. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy Contra g^*s –irresolute** if $f^{-1}(\lambda)$ is g^*s-closed fuzzy set in X for every g^*s-open fuzzy set λ in Y.

Theorem 3.10. A function $f:(X,\tau) \to (Y,\sigma)$ is fuzzy contra g^*s-continuous if and only if $f^{-1}(\lambda)$ is g^*s-open fuzzy set in X for every g^*s-closed fuzzy set λ in Y.

Theorem 3.11. Every fuzzy contra g^*s-irresolute mapping is fuzzy contra g^*s-continuous.

Proof. Let $f: X \to Y$ is fuzzy contra g^*s-irresolute function. Let λ be a fuzzy open set in Y. Then λ is g^*s-open fuzzy set in Y. Since f is fuzzy contra g^*s-irresolute. $f^{-1}(\lambda)$ is g^*s-fuzzy closed set in X. Hence f is fuzzy contra g^*s-continuous function.

Theorem 3.13. Let $f: X \to Y$, $g: Y \to Z$ be two functions then

(i) $gof: X \to Z$ is fuzzy contra g^*s-continuous, if f is fuzzy contra g^*s-continuous and g are fuzzy continuous.

(ii) $gof: X \to Z$ is fuzzy contra g^*s-continuous if f is fuzzy contra g^*s-irresolute and g is fuzzy g^*s-continuous.

4. Fuzzy Almost Contra g^*s-Continuous Function

Definition 4.1. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy almost contra g^*s –Continuous** if $f^{-1}(\lambda)$ is fuzzy g^*s-closed set in X for every regular open set λ in Y.

Theorem 4.2. Every fuzzy contra g^*s-continuous function is fuzzy almost contra g^*s-continuous.

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Proof. Since every regular fuzzy open set is open fuzzy set, such that every fuzzy contra g^*s-continuous mappings is fuzzy almost contra g^*s-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.3: Let $X = \{ x_1, x_2\}, Y = \{ y_1, y_2 \}$ λ, and μ be a fuzzy set in X and Y defined as $\lambda(x_1) = 0.3$, $\lambda(x_2) = 0.5$, $\mu(y_1) = 0.3$, $\mu(y_2) = 0.4$. Let $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. The map $f: (X, \tau) \to (Y, \tau')$ defined as $f(x_i) = y_i, i = 1,2$ is fuzzy almost contra g^*s-continuous map but not fuzzy contra g^*s-continuous.

Definition 4.4. A function $f: X \to Y$ is said to be fuzzy regular set connected [] if $f^{-1}(\lambda)$ is fuzzy clopen in X for every fuzzy regular open set λ of Y.

Theorem 4.5. If a function $f: X \to Y$ is fuzzy almost contra g^*s-continuous and almost continuous, then f is fuzzy regular set connected.

Proof. Let λ be a fuzzy regular open set in (Y, σ). Since f is fuzzy almost contra g^*s-continuous and fuzzy almost continuous, $f^{-1}(\lambda)$ is fuzzy g^*s-closed and open. Hence $f^{-1}(\lambda)$ is fuzzy clopen. Therefore f is fuzzy regular set connected.

Definition 4.6. A fuzzy topological spaces (X, τ) is called fuzzy g^*s-connected if X cannot be written as the disjoint union of two non-empty fuzzy g^*s-open sets.

Theorem 4.7. Let (X, τ) and (Y, σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \to Y$.

1. f is fuzzy almost contra g^*s- continuous.
2. $f^{-1}(\lambda)$ is fuzzy g^*s-open set in X for every regular closed set λ in Y.
3. for each $x \in X$ and each fuzzy regular closed set λ in Y containing $f(x)$. there exist a fuzzy g^*s-open set η in X containing x such that $f(\eta) \leq \lambda$.
4. for each $x \in X$ and fuzzy regular open set μ in Y non-containing $f(x)$, there exists a fuzzy g^*s-closed set ϑ in X non-containing x such that $f^{-1}(\mu) \leq \vartheta$.

Proof. As theorem 3.8.

Theorem 4.8: Let X, Y and Z be fuzzy topological spaces and let $f: X \to Y$ and $g: Y \to Z$ be maps. If f is fuzzy contra g^*s-continuous and g is fuzzy almost continuous then $gof: X \to Z$ is fuzzy almost contra g^*s-continuous.

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