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On fuzzy D-Baire spaces

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Abstract. In this paper the concepts of fuzzy D-Baire spaces are introduced. Several characterizations of fuzzy D-Baire spaces are studied. Some results concerning functions that preserve fuzzy D-Baire spaces in the context of images and pre-images are obtained. Several examples are given to illustrate the concepts introduced in this paper.

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1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A.ZADEH in his classical paper [19] in the year 1965. Thereafter the paper of C.L.CHANG [4] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. TANG [11] has used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS. Fuzzy set theory finds its applications for Modeling [5] , uncertainty and vagueness in various fields of Science and Engineering such as Nonlinear Dynamical Systems, Control of Chaos, Quantum Physics.

The concepts of Baire spaces have been studied extensively in classical topology in [9], [10], [6] and [7]. The concept of Baire space in fizzy setting was introduced and studied by G. THANGARAJ and S. ANJALMOSE in [14] . In this paper we introduce the concepts of fuzzy D-Baire spaces. Also we discuss several characterizations of fuzzy D-Baire spaces. In this paper some results concerning functions that preserve fuzzy D-Baire spaces in the context of images and pre-images are obtained. Several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG [4].

Definition 2.1. Let λ and μ be any two fuzzy sets in (X,T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows : $(\lambda \vee \mu)(x) = max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0,1]$ as follows : $(\lambda \wedge \mu)(x) = min\{\lambda(x), \mu(x)\}$.

Definition 2.2 ([2])**.** Let(X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). we define $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$. For any fuzzy set λ in a fuzzy topological space (X,T), it is easy to see that $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$.

Definition 2.3 ([18])**.** A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$.

Definition 2.4 ([14])**.** A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int \ \ cl(\lambda) = 0$.

Definition 2.5 ([3])**.** A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy F_σ set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.6 ([3])**.** A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy G_δ set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2.7 ([18])**.** A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.8 ([14])**.** Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $1 - \lambda$ is called a fuzzy residual set in (X,T).

Definition 2.9. [18] A fuzzy topological space (X,T) is called fuzzy first category if $1 = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy nowhere dense sets in (X,T). A topological space which is not of fuzzy first category, is said to be of fuzzy second category.

Lemma 2.10 ([2])**.** *For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.*

Definition 2.11 ([17])**.** A fuzzy topological space (X,T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X,T) such that $cl(1-\lambda) = 1$. Otherwise (X,T) is called a fuzzy irresolvable space.

Definition 2.12 ([16])**.** A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is, every non-zero fuzzy G_δ set in (X,T), is fuzzy open in (X,T).

3. Fuzzy Baire spaces

Definition 3.1 ([14]). Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy nowhere dense sets in (X,T).

Theorem 3.2 ([14]). *Let (X,T) be a fuzzy topological space. Then the following are equivalent:*
(1). (X,T) is a fuzzy Baire space.
(2). $int(\lambda) = 0$ for every fuzzy first category set λ in (X,T).
(3). $cl(\mu) = 1$ for every fuzzy residual set μ in (X,T).

If every fuzzy first category set in a fuzzy topological space is a fuzzy nowhere dense set, then the fuzzy topological space is a fuzzy Baire space. For, consider the following proposition.

Proposition 3.3. *If λ is a fuzzy first category set in a fuzzy topological space (X,T) such that $int\ \ cl(\lambda) = 0$, then (X,T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now $int\ \ cl(\lambda) = 0$ and $int(\lambda) \leq int\ \ cl(\lambda)$, implies that $int(\lambda) = 0$. Hence $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy nowhere dense sets in (X,T). Therefore (X,T) is a fuzzy Baire space. □

If a fuzzy first category set is a fuzzy closed set in a fuzzy Baire space, then it must be a fuzzy nowhere dense set. For, consider the following proposition.

Proposition 3.4. *If a fuzzy first category set λ in a fuzzy Baire space (X,T) is a fuzzy closed set, then $int\ \ cl(\lambda) = 0$ in (X,T).*

Proof. Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Since (X,T) is fuzzy Baire space, by theorem 3.2, $int(\lambda) = 0$. Now λ is a fuzzy closed set in (X,T) implies that $cl(\lambda) = \lambda$. Hence $int\ \ cl(\lambda) = int(\lambda) = 0$. Therefore $int\ \ cl(\lambda) = 0$ in (X,T). □

4. Fuzzy D-Baire spaces

Motivated by the classical concept introduced in [8] we shall now define:

Definition 4.1. A fuzzy topological space (X,T) is called a fuzzy D-Baire space if every fuzzy first category set in (X,T) is a fuzzy nowhere dense set in (X,T). That is, (X,T) is a fuzzy D-Baire space if $intcl(\lambda) = 0$ for each fuzzy first category set λ in (X,T).

Example 4.2. Let $X = \{a, b, c\}$ and λ, μ, γ be the fuzzy sets defined on X as follows:
$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 0.7; \lambda(b) = 0.6; \lambda(c) = 0.2$.
$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.8$.
$\gamma : X \to [0, 1]$ defined as $\gamma(a) = 0.5; \gamma(b) = 0.3; \gamma(c) = 0.6$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \vee \gamma), (\lambda \wedge \mu), (\lambda \wedge \gamma), \gamma \vee (\lambda \wedge \mu), 1\}$ is a fuzzy topology on X. The fuzzy nowhere dense sets in (X,T) are $(1 - \lambda), 1 - (\lambda \vee \mu)$, $1 - (\lambda \vee \gamma)$ and the fuzzy first category sets $(1 - \lambda), (1 - (\lambda \vee \mu))$ in (X,T) are fuzzy nowhere dense sets in (X,T) and hence (X,T) is a fuzzy D-Baire space.

Example 4.3. Let $X = \{a, b, c\}$ and λ, μ, γ be the fuzzy sets defined on X as follows:
$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 1; \lambda(b) = 0.2; \lambda(c) = 0.7$.
$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.3; \mu(b) = 1; \mu(c) = 0.2$.
$\gamma : X \to [0, 1]$ defined as $\gamma(a) = 0.7; \gamma(b) = 0.4; \gamma(c) = 1$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), (\mu \vee \gamma), (\mu \wedge \gamma), \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \wedge (\lambda \vee \mu), 1\}$ is a fuzzy topology on X, and $(1 - \lambda), (1 - \mu), (1 - \gamma), (1 - \lambda \vee \mu), (1 - \lambda \vee \gamma), (1 - \mu \vee \gamma), 1 - \lambda \vee (\mu \wedge \gamma), 1 - \mu \vee (\lambda \wedge \gamma)$ are the fuzzy nowhere dense sets in (X,T) and $(1 - (\lambda \wedge \mu))$ is a fuzzy first category set in (X,T). $intcl(1 - (\lambda \wedge \mu) \neq 0$ and hence (X,T) is not a fuzzy D-Baire space.

Clearly every fuzzy D-Baire space is a fuzzy Baire space. For, consider the following proposition.

Proposition 4.4. *If (X,T) is a fuzzy D-Baire space, then (X,T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy first category set in a fuzzy D-Baire space (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets and λ is a fuzzy nowhere dense set in (X,T). Then we have $intcl(\lambda) = 0$ and $int(\lambda) \leq intcl(\lambda)$, implies that $int(\lambda) = 0$. Hence $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy nowhere dense sets in (X,T). Therefore (X,T) is a fuzzy Baire space. □

Remark 4.5. A fuzzy Baire space need not be a fuzzy D-Baire space. For, consider the following example:

Example 4.6. Let $X = \{a, b, c\}$ and λ, μ, γ be the fuzzy sets defined on X as follows:
$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 0.8; \lambda(b) = 0.6; \lambda(c) = 0.7$.
$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.6; \mu(b) = 0.9; \mu(c) = 0.8$.
$\gamma : X \to [0, 1]$ defined as $\gamma(a) = 0.7; \gamma(b) = 0.5; \gamma(c) = 0.9$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), (\mu \vee \gamma), (\mu \wedge \gamma), \lambda \wedge (\mu \vee \gamma), \lambda \vee (\mu \wedge \gamma), \mu \wedge (\lambda \vee \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \wedge (\lambda \vee \mu), \gamma \vee (\lambda \wedge \mu), (\lambda \wedge \mu \wedge \gamma), (\lambda \vee \mu \vee \gamma), 1\}$ is a fuzzy topology on X. Define the fuzzy set Ω defined on X as follows: Ω: $X \to [0, 1]$ is defined as $\Omega(a) = 0.4; \Omega(b) = 0.5; \Omega(c) = 0.3$. Now $\Omega = \{(1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma) \vee (1 - (\lambda \vee \mu)) \vee (1 - (\lambda \vee \gamma)) \vee (1 - (\mu \vee \gamma)) \vee (1 - (\lambda \wedge \mu)) \vee (1 - (\lambda \wedge \gamma)) \vee (1 - \mu \wedge \gamma)\}$ is a fuzzy first category set in (X,T), where $(1 - \lambda), (1 - \mu), (1 - \gamma), (1 - (\lambda \vee \mu)), (1 - (\lambda \vee \gamma)), (1 - (\mu \vee \gamma)), (1 - (\lambda \wedge \mu)), (1 - (\lambda \wedge \gamma)), (1 - (\mu \wedge \gamma))$ are fuzzy nowhere dense sets in (X,T) and $intcl(\Omega) \neq 0$ and hence (X,T) is not a fuzzy D-Baire space whereas (X,T) is a fuzzy Baire space, since for every fuzzy first category set δ in (X,T), $int(\delta) = 0$.

A fuzzy Baire space in which every fuzzy first category set is a fuzzy closed set, is a fuzzy D-Baire space. For, consider the following proposition.

Proposition 4.7. *If λ is any fuzzy first category set in a fuzzy Baire space (X,T) such that $cl(\lambda) = \lambda$, then (X,T) is a fuzzy D-Baire space.*

Proof. The proof follows from the proof of proposition 3.4. □

Proposition 4.8. *If the fuzzy topological space (X,T) is a fuzzy first category space, then (X,T) is not a fuzzy D-Baire space.*

Proof. Let (X,T) be a fuzzy first category space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now for the fuzzy first category set 1_X, we have $intcl(1_X) \neq 0$. Hence (X,T) is not a fuzzy D-Baire space. □

Theorem 4.9 ([12])**.** *If λ is a fuzzy dense and fuzzy G_δ set in a fuzzy topological space (X,T), then $1-\lambda$ is a fuzzy first category set in (X,T).*

Proposition 4.10. *If a fuzzy topological space (X,T) has a fuzzy dense and fuzzy G_δ set, then (X,T) is not a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in (X,T). Then by theorem 4.9, $1-\lambda$ is a fuzzy first category set in (X,T). Now $intcl(1-\lambda) = 1 - clint(\lambda) > 1 - cl(\lambda) > 1 - 1 = 0$(since $cl(\lambda) = 1$). Hence we have $intcl(1-\lambda) \neq 0$ for a fuzzy first category set $1-\lambda$ in (X,T). Therefore (X,T) is not a fuzzy D-Baire space. □

Proposition 4.11. *If $cl\ int(\lambda) = 1$ for every fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X,T), then (X,T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X,T). Then by theorem 4.9, $1-\lambda$ is a fuzzy first category set in (X,T). Now $clint(\lambda) = 1$ implies that $1 - clint(\lambda) = 0$. Then we have $intcl(1-\lambda) = 0$. Hence, for a fuzzy first category set $1-\lambda$ in (X,T) we have $intcl(1-\lambda) = 0$. Therefore (X,T) is a fuzzy D-Baire space. □

Proposition 4.12. *If $cl\ int(\mu) = 1$ for every fuzzy residual set μ in a fuzzy topological space, then (X,T) is a fuzzy D-Baire space.*

Proof. Let μ be a fuzzy residual set in (X,T). Then $1-\mu$ is a fuzzy first category set in (X,T). Now $clint(\mu) = 1$, implies that $1-clint(\mu) = 0$. Then we have $intcl(1-\mu) = 0$. Hence for a fuzzy first category set $1-\mu$ in (X,T), we have $intcl(1-\mu) = 0$. Therefore (X,T) is a fuzzy D-Baire space. □

Definition 4.13. A fuzzy topological space (X,T) is called a fuzzy nodec space if every fuzzy nowhere dense set in (X,T) is a fuzzy closed set in (X,T).

Proposition 4.14. *If the fuzzy topological space (X,T) is a fuzzy nodec space, then (X,T) is not a fuzzy D-Baire space.*

Proof. Let λ_i's be fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy nodec space, λ_i's are fuzzy closed sets in (X,T). That is, $cl(\lambda_i) = \lambda_i$. Then $\vee_{i=1}^{\infty}(\lambda_i) = \vee_{i=1}^{\infty}cl(\lambda_i)$. Now $int(\vee_{i=1}^{\infty}(\lambda_i)) = int(\vee_{i=1}^{\infty}cl(\lambda_i)) > \vee_{i=1}^{\infty}intcl(\lambda_i) = 0$. (since λ_i's are fuzzy nowhere dense sets in (X,T), $intcl(\lambda_i) = 0$). Hence we have $int(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$. Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$. Then λ is a fuzzy first category set in (X,T) such that $int(\lambda) \neq 0$. Now we claim that $intcl(\lambda) \neq 0$. For, if $intcl(\lambda) = 0$, then $int(\lambda) \leq intcl(\lambda)$ implies that $int(\lambda) = 0$, a contradiction. Hence for a fuzzy first category set λ in (X,T), we have $intcl(\lambda) \neq 0$. Therefore (X,T) is not a fuzzy D-Baire space. □

Proposition 4.15. *If the fuzzy topological space (X,T) is a fuzzy D-Baire space, then no non-zero fuzzy dense set is a fuzzy first category set in (X,T).*

Proof. Suppose that λ is a non-zero fuzzy first category and fuzzy dense set in (X,T). Since (X,T) is a fuzzy D-Baire space and λ is a fuzzy first category set, we have $intcl(\lambda) = 0$............(1). Our assumption that $cl(\lambda) = 1$, implies that $intcl(\lambda) = int(1) = 1 \neq 0$, which is a contradiction to (1). Hence we must have $cl(\lambda) \neq 1$. Therefore no non-zero fuzzy dense set is a fuzzy first category set in a fuzzy D-Baire space. □

Definition 4.16 ([15])**.** A fuzzy topological space (X,T) is called a D/fuzzy space if for $\lambda(\neq 0) \in T$, there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. equivalently $cl(\lambda) = 1$ for all $\lambda(\neq 0) \in T$ in (X,T).

Definition 4.17. A fuzzy topological space (X,T) is called a fuzzy almost P-space if for every non-zero fuzzy G_δ-set λ in (X,T), $int(\lambda) \neq 0$ in (X,T).

Proposition 4.18. *If the fuzzy topological space (X,T) is a D/fuzzy and fuzzy almost P-space, then (X,T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Now $1 - cl(\lambda_i)$ is a fuzzy open set in (X,T). Then $\mu = \wedge_{i=1}^{\infty}(1 - cl(\lambda_i))$ is a fuzzy G_δ set in (X,T). Now $1 - \vee_{i=1}^{\infty}(cl(\lambda_i)) \leq 1 - \vee_{i=1}^{\infty}(\lambda_i) = 1 - \lambda$ implies that $int(1 - \lambda) \neq 0$. Then there exists a fuzzy open set δ in (X,T) such that $\delta < 1 - \lambda$. Then $\lambda < 1 - \delta$ implies that $intcl(\lambda) < intcl(1 - \delta) = int(1 - \delta) = 1 - cl(\delta)$ (since $1 - \delta$ is a fuzzy closed set in (X,T), $cl(1 - \delta) = 1 - \delta$). since (X,T) is a D/fuzzy space, $cl(\delta) = 1$. Hence we have $intcl(\lambda) < 1 - 1 = 0$. That is, $intcl(\lambda) = 0$. Hence for the fuzzy first category λ in (X,T), we have $intcl(\lambda) = 0$. Therefore (X,T) is a fuzzy D-Baire space. □

Remark 4.19. In view of the above proposition 4.18 and since every fuzzy D-Baire space is a fuzzy Baire space, a D/fuzzy and fuzzy almost P-space, is a fuzzy Baire space.

Proposition 4.20. *If every fuzzy residual set in a fuzzy P-space (X,T) contains a non-zero fuzzy dense and fuzzy G_δ set in (X,T), then (X,T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in (X,T). Then $(1 - \lambda)$ is a fuzzy residual set in (X,T). As in the proof of 4.18, there exists a fuzzy G_δ set μ in (X,T) such that $\mu \leq (1 - \lambda)$. Then $\lambda \leq (1 - \mu)$. Now $intcl(\lambda) \leq intcl(1 - \mu,)$ implies that $intcl(\lambda) \leq 1 - clint(\mu)$. Since (X,T) is a fuzzy P-space, the fuzzy G_δ set μ is fuzzy open in (X,T) and $int(\mu) = \mu$. Therefore we have $intcl(\lambda) \leq 1 - cl(\mu) = 1 - 1 = 0$ (since μ is fuzzy dense). Then we have $intcl(\lambda) = 0$ and hence λ is a fuzzy nowhere dense set in (X,T). Therefore (X,T) is a fuzzy D-Baire space. □

Definition 4.21 ([17])**.** A fuzzy topological space (X,T) is called a fuzzy open hereditarily irresolvable space if $intcl(\lambda) \neq 0$, implies that $int(\lambda) \neq 0$ for any non-zero fuzzy set λ in (X,T).

Theorem 4.22 ([17])**.** *If (X,T) is a fuzzy open hereditarily irresolvable space, any non-zero fuzzy set λ with $int(\lambda) = 0$, is a fuzzy nowhere dense set in (X,T).*

Proposition 4.23. *If the fuzzy topological space (X,T) is a fuzzy Baire and fuzzy open hereditarily irresolvable space, then (X,T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in (X,T). Since (X,T) is a fuzzy Baire space, by theorem 3.2, $int(\lambda) = 0$. Since (X,T) is fuzzy open hereditarily irresolvable, by theorem 4.22 λ is a fuzzy nowhere dense set in (X,T). Hence for any fuzzy first category set λ in (X,T), we have $intcl(\lambda) = 0$. Therefore (X,T) is a fuzzy D-Baire space. □

5. Fuzzy D-Baire spaces and functions

Let f be a function from the fuzzy topological space (X,T) to the fuzzy topological space (Y,S). Underwhat conditions on " f " may we assert that if (X,T) is a fuzzy D-Baire space, then (Y,S) is a fuzzy D-Baire space?

It may be noticed that fuzzy continuous image of a fuzzy D-Baire space may fail to be a fuzzy D-Baire space. For consider the following example:

Example 5.1. Let $X = \{a, b\}$. The fuzzy sets λ, μ, and γ are defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5; \lambda(b) = 0.7$
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8; \mu(b) = 0.4$.
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.2; \gamma(b) = 0.6$.
Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\mu \vee \gamma), (\mu \wedge \gamma), 1\}$ and $S = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ are fuzzy topologies on X.Now the fuzzy nowhere dense sets in (X,T) are $1 - \lambda$ and $1 - (\lambda \vee \mu)$ and $((1 - \lambda) \vee (1 - \lambda \vee \mu)) = 1 - \lambda$ and $1 - \lambda$ is a fuzzy first category set in (X,T) and $intcl(1 - \lambda) = 0$. Hence (X,T) is a fuzzy D-Baire space.

The fuzzy nowhere dense sets in (X,S) are $1 - \lambda, 1 - \mu$ and $1 - (\lambda \vee \mu)$ and $((1-\lambda) \vee (1-\mu) \vee (1-(\lambda \vee \mu)) = 1-(\lambda \wedge \mu)$ and $1-(\lambda \wedge \mu)$ is a fuzzy first category set in (Y,S) such that $intcl(1 - (\lambda \wedge \mu)) = (\lambda \wedge \mu) \neq 0$. Hence (X,S) is not a fuzzy D-Baire space. Now define a function $f : (X, T) \to (Y, S)$ by $f(a) = a$ and $f(b) = b$. Clearly f is a fuzzy continuous function from the fuzzy topological space (X,T) to the topological space (Y,S). Hence the fuzzy continuous image of a fuzzy D-Baire space fails to be a fuzzy D-Baire space.

It may also be noticed that the image of a fuzzy D-Baire space under a fuzzy open function may fail to be a fuzzy D-Baire space, for consider the following example:

Example 5.2. Let $X = \{a, b\}$. The fuzzy sets λ, μ, and γ are defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.6; \lambda(b) = 0.7$
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.9; \mu(b) = 0.5$.
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.3; \gamma(b) = 0.6$.
Then $T = \{0, \lambda, \gamma, 1\}$ and $S = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, \mu \vee \gamma, \lambda \wedge (\mu \vee \gamma), 1\}$ are fuzzy topologies on X.Now the fuzzy nowhere dense sets in (X,S) are $1-\lambda, 1-\mu, 1-\gamma, 1-(\lambda \vee \mu), 1 - (\mu \vee \gamma), 1 - (\lambda \wedge (\mu \vee \gamma))$ and $((1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma) \vee (1 - \lambda \vee \mu) \vee (1 - (\mu \vee \gamma)) \vee (1 - (\lambda \wedge (\mu \vee \gamma)))) = 1 - (\mu \wedge \gamma)$ and $1 - (\mu \wedge \gamma)$ is a fuzzy first category set in (X,S) such that $intcl(1 - (\mu \wedge \gamma) = (\lambda \wedge \mu) \neq 0$. Hence (X,S) is not a fuzzy D-Baire space. Now the fuzzy nowhere dense sets in (X,T) are $1 - \lambda, 1 - \gamma$ and $(1 - \lambda) \vee (1 - \gamma) = 1 - \gamma$ and $1 - \gamma$ is a fuzzy first category set in (X,T) such that $intcl(1 - \gamma) = 0$. Hence (X,T) is a fuzzy D-Baire space. Now define a function $f : (X, T) \to (X, S)$ by $f(a) = a$ and $f(b) = b$. Clearly f is a fuzzy open function

from the fuzzy topological space (X,T) to the fuzzy topological space (Y,S). Hence the fuzzy open image of a fuzzy D-Baire space fails to be a fuzzy D-Baire space.

Theorem 5.3. [13] *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function, then for any fuzzy set λ in (X,T), $f(int(cl(\lambda))) = int(cl(f(\lambda)))$.*

Theorem 5.4 ([13]). *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function, then for any fuzzy set λ in (X,T), λ is a fuzzy nowhere dense set in (X,T) if and only if $f(\lambda)$ is a fuzzy nowhere dense set in (Y,S).*

Lemma 5.5 ([1]). *Let $f : X \to Y$ be a fuzzy open and fuzzy continuous mapping from a fuzzy topological space (X,T) to a fuzzy topological space (Y,S). Then for each fuzzy set η in Y, $int(f^{-1}(\eta)) = f^{-1}(int(\eta))$.*

Remark 5.6. From the above lemma 5.5, if $f : X \to Y$ is a fuzzy open and fuzzy continuous mapping from a fuzzy topological space (X,T) to a fuzzy topological space (Y,S), for each fuzzy set η in Y, $cl(f^{-1}(\eta)) = f^{-1}(cl(\eta))$.

Proposition 5.7. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function, then a fuzzy set λ in (Y,S) is a fuzzy nowhere dense set if and only if $f^{-1}(\lambda)$ is a fuzzy nowhere dense set in (X,T).*

Proof. For a fuzzy set $cl(\lambda)$ in (Y,S), by lemma 5.5 $int(f^{-1}(cl(\lambda))) = f^{-1}(intcl(\lambda))$. Since λ is a fuzzy nowhere dense set in (Y,S) $intcl(\lambda) = 0$. Then $int(f^{-1}(cl(\lambda))) = f^{-1}[0] = 0$. Now $cl(f^{-1}(\lambda)) = f^{-1}(cl(\lambda))$ implies that

$$intcl(f^{-1}(\lambda)) = int(f^{-1}(cl(\lambda))) = 0.$$

That is, $intcl(f^{-1}(\lambda)) = 0$. Hence $f^{-1}(\lambda)$ is a fuzzy nowhere dense set in (X,T). □

Proposition 5.8. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function, then a fuzzy set λ in (X,T), λ is a fuzzy first category set if and only if $f(\lambda)$ is a fuzzy first category set in (Y,S).*

Proof. Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy nowhere dense sets in (X,T). Then $f(\lambda) = f(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty} f(\lambda_i)$. Since λ_i is a fuzzy nowhere dense set in (X,T), by theorem 5.4 $f(\lambda_i)$ is a fuzzy nowhere dense set in (Y,S). Hence $f(\lambda)$ is a fuzzy first category set in (Y,S).

Conversely, let $f(\lambda)$ be a fuzzy first category set in (Y,S). Then we have $f(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where δ_i's are fuzzy nowhere dense sets in (Y,S).

Now $f^{-1}f(\lambda) = f^{-1}(\vee_{i=1}^{\infty}(\delta_i)) = \vee_{i=1}^{\infty}(f^{-1}(\delta_i))$. Since f is 1-1, $f^{-1}f(\lambda) = \lambda$. Hence we have $\lambda = \vee_{i=1}^{\infty}(f^{-1}(\delta_i))$. By proposition 5.7, $f^{-1}(\delta_i)$ is a fuzzy nowhere dense set in (X,T) and $\lambda = \vee_{i=1}^{\infty} f^{-1}(\delta_i)$, where $f^{-1}(\delta_i)$'s are fuzzy nowhere dense sets in (X,T), implies that λ is a fuzzy first category set in a fuzzy topological space (X,T). □

Proposition 5.9. *If a function $f : (X,T) \rightarrow (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function and if (X,T) is a fuzzy D-Baire space, then (Y,S) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in (X,T). Since (X,T) is a fuzzy D-Baire space, $intcl(\lambda) = 0$. By proposition 5.8, $f(\lambda)$ is a fuzzy first category set in (Y,S). We have $f(int(cl(\lambda))) = int(cl(f(\lambda)))$ implies that $int(cl(f(\lambda)))f(0) = 0$. Hence for the fuzzy first category set $f(\lambda)$ in (Y,S), we have $int(cl(f(\lambda))) = 0$. This implies that (Y,S) is a fuzzy D-Baire space. □

Proposition 5.10. *If a function $f : (X,T) \rightarrow (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous, fuzzy open, 1-1 and onto function and if (Y,S) is a fuzzy D-Baire space, then (X,T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy first category set in (X,T). Then, by proposition 5.8, $f(\lambda)$ is a fuzzy first category set in (Y,S). Since (Y,S) is a fuzzy D-Baire space, $intcl(f(\lambda)) = 0$. But $f(int(cl(\lambda))) = int(cl(f(\lambda)))$, implies that $f(int(cl(\lambda))) = 0$. Then $f^{-1}f(int(cl(\lambda))) = f^{-1}(0) = 0$. Since f is 1-1, $f^{-1}f(int(cl(\lambda))) = 0$ implies that $int(cl(\lambda)) = 0$. Hence for the fuzzy first category set λ in (X,T), we have $int(cl(f(\lambda))) = 0$. Therefore (X,T) is a fuzzy D-Baire space. □

6. Conclusions

In this paper we first defined fuzzy D-Baire spaces. Inter relations between fuzzy D-Baire spaces, fuzzy Baire spaces, fuzzy first category spaces, fuzzy nodec spaces, D/fuzzy spaces, fuzzy open hereditarily irresolvable spaces and fuzzy almost P-spaces are studied. Finally some results concerning fuzzy continuous, fuzzy open functions that preserve D-Baire spaces in the context of images and pre-images are established.

References

[1] N. Ajmal and S. K. Azad, Fuzzy almost continuity and its pointwise charecterization by dual points and fuzzy nets, Fuzzy Sets and Systems 34 (1990) 81–101.

[2] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.

[3] G. Balasubramanian, Maximal fuzzy topologies, Kybernetika (Prague) 31(5) (1995) 459–464.

[4] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.

[5] S. DU, Q. Qin,Q. Wang and B. Li, Fuzzy description of topological relations I; a Unified fuzzy 9-intersection model. Lecture Notes in Comput. Sci. 3612 : 1261 : 1273, 2005.

[6] Z. Frolik, Baire spaces and some generalizations of complete metric spaces, Czechoslovak Math. J. 11(86) (1961) 237–248.

[7] Z. Frolik, Remarks concerning the invariance of Baire spaces under mappings, Czechoslovak Math. J. 11(86) (1961) 381–385.

[8] S. Garcia-Ferreira, A. Garcia-Maynez and M. Hrusak, Spaces in which every dense subset is Baire, Houston J. Math. 39(1) (2013) 247–263.

[9] G. Gruenhage and D. Lutzer, Baire and volterra spaces, Proc. Amer. Soc. 128 (2000) 3115–3124.

[10] T. Neubrunn, A note on mappings of Baire spaces, Math. Slovaca 27(2) (1977) 173–176.

[11] X. Tang, Spatial object modeling in fuzzy topological spaces with applications to land cover chang in China, Ph. D. Dissertation, University of Twente, Enschede, The Netherlands, 2004 ITC Dissertation No. 108.

[12] G. Thangaraj and S. Anjalmose, A note on fuzzy Baire spaces, Inter. J. Fuzzy Math. Systems 3(4) (2013) 269–274.
[13] G. Thangaraj and S. Anjalmose, Fuzzy Baire spaces and functions, Inter. J. Fuzzy Math. Systems 3(4) (2013) 285–293.
[14] G. Thangaraj and S. Anjalmose, On fuzzy Baire spaces, J. Fuzzy Math. 21(3) (2013) 667–676.
[15] G. Thangaraj and G. Balasubramanian, On dense fuzzy topologies, Bull. Calcutta. Math. Soc. 92(3) (2000) 167–174.
[16] G. Thangaraj and G. Balasubramanian, On fuzzy basically disconnected spaces, J. Fuzzy Math. 9(1) (2001) 103–110.
[17] G. Thangaraj and G. Balasubramanian, On fuzzy resolvable and fuzzy irresolvable spaces, Fuzzy sets, Rough sets and Multi-valued operations and Applications, 1(2) (July-Dec 2009) 173–180.
[18] G. Thangaraj and G. Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math. 11(3) (2003) 725–736.
[19] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

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