

## On Fuzzy Irresolute Functions

Erdal Ekici\* and Jin Han Park\*\*

\* Department of Mathematics, Canakkale Onsekiz Mart University

\*\* Division of Mathematical Sciences, Pukyong National University

### Abstract

As a generalization of the notions of fuzzy  $\alpha$ -irresolute, fuzzy preirresolute, fuzzy irresolute and fuzzy  $\beta$ -irresolute functions, we introduce the notion of fuzzy  $\beta\alpha$ -continuous functions and investigate the relationships between fuzzy  $\beta\alpha$ -continuous functions and fuzzy separation axioms.

**Key words :** fuzzy  $\beta$ -open set, fuzzy  $\alpha$ -open set, fuzzy  $\beta\alpha$ -continuity

### 1. Introduction and Preliminaries

The notions of fuzzy irresolute, fuzzy  $\alpha$ -irresolute and fuzzy preirresolute functions were introduced and studied by Mukherjee and Sinha [6], Thakur and Saraf [11] and Park and Park [8], respectively.

The aim of this paper is to introduce a new class of fuzzy functions which is called  $\beta\alpha$ -continuous functions including the classes of fuzzy irresolute, fuzzy  $\alpha$ -irresolute, fuzzy preirresolute and fuzzy  $\beta$ -irresolute functions. Furthermore, we obtain basic properties of  $\beta\alpha$ -continuous functions and investigate relationships between fuzzy  $\beta\alpha$ -continuity and fuzzy covering properties and fuzzy  $\beta\alpha$ -continuity and fuzzy separations axioms, respectively.

The class of fuzzy sets on a universe  $X$  will be denoted by  $I^X$  and fuzzy sets on  $X$  will be denoted by Greek letters as  $\mu, \rho, \eta$ , etc. A family  $\tau$  of fuzzy sets in  $X$  is called a fuzzy topology [3] for  $X$  iff (1)  $\emptyset, X \in \tau$  (2)  $\mu \wedge \rho \in \tau$  whenever  $\mu, \rho \in \tau$  and (3)  $\bigvee \{\mu_\alpha : \alpha \in I\} \in \tau$  whenever each  $\mu_\alpha \in \tau$  ( $\alpha \in I$ ). In this case, the pair  $(X, \tau)$  (or simply  $X$ ) is called a fuzzy topological space (for short, fuzzy space). Every member of  $\tau$  is called a fuzzy open set [7]. For a fuzzy set  $\mu$  in  $X$ ,  $\text{int}\mu$  and  $\text{cl}\mu$  will denote the interior and closure of  $\mu$ , respectively. A fuzzy set in  $X$  is called a fuzzy point iff it takes the value 0 for all  $y \in X$  except one, say,  $x \in X$ . If its value at  $x$  is  $\alpha$  ( $0 < \alpha \leq 1$ ), we denote this fuzzy point by  $x_\alpha$ , where the point  $x$  is called its support [7]. For any fuzzy point  $x_\epsilon$  and any fuzzy set  $\mu$ , we write  $x_\epsilon \in \mu$  iff  $\epsilon \leq \mu(x)$ .

**Definition 1.1.** A fuzzy set  $\mu$  in  $X$  is called:

- (1) fuzzy  $\alpha$ -open [10] if  $\mu \leq \text{int cl int}(\mu)$ ;
- (2) fuzzy semiopen [1] if  $\mu \leq \text{cl int}(\mu)$ ;
- (3) fuzzy preopen [10]  $\mu \leq \text{int cl}(\mu)$ ;
- (4) fuzzy  $\beta$ -open [5, 12] if  $\mu \leq \text{cl int cl}(\mu)$ .

The complement of a fuzzy  $\alpha$ -open (resp. fuzzy semiopen, fuzzy preopen, fuzzy  $\beta$ -open) set is called fuzzy  $\alpha$ -closed (resp. fuzzy semiclosed, fuzzy preclosed, fuzzy  $\beta$ -closed).

**Definition 1.2.** A fuzzy function  $f: X \rightarrow Y$  is said to be:

- (1) fuzzy open [3] (resp. always fuzzy  $\beta$ -open) if  $f(\rho)$  is fuzzy open (resp. fuzzy  $\beta$ -open) in  $Y$  for every fuzzy open (resp. fuzzy  $\beta$ -open) set  $\rho$  in  $X$ ;
- (2) fuzzy irresolute [6] if  $f^{-1}(\rho)$  is fuzzy semiopen in  $X$  for each fuzzy semiopen set  $\rho$  in  $Y$ ;
- (3) fuzzy  $\alpha$ -irresolute [11] if  $f^{-1}(\rho)$  is fuzzy  $\alpha$ -open in  $X$  for each fuzzy  $\alpha$ -open set  $\rho$  in  $Y$ ;
- (4) fuzzy preirresolute [8] if  $f^{-1}(\rho)$  is fuzzy preopen in  $X$  for each fuzzy preopen set  $\rho$  in  $Y$ ;
- (5) fuzzy  $\beta$ -irresolute if  $f^{-1}(\rho)$  is fuzzy  $\beta$ -open in  $X$  for each fuzzy  $\beta$ -open set  $\rho$  in  $Y$ .

### 2. Fuzzy $\beta\alpha$ -continuous functions

**Definition 2.1.** A fuzzy function  $f: X \rightarrow Y$  is said to be fuzzy  $\beta\alpha$ -continuous if for each fuzzy point  $x_\epsilon \in X$  and each fuzzy  $\alpha$ -open set  $\rho$  in  $Y$  containing  $f(x_\epsilon)$ , there exists a fuzzy  $\beta$ -open set  $\mu$  in  $X$  containing  $x_\epsilon$  such that  $f(\mu) \leq \rho$ .

**Theorem 2.2.** For a fuzzy function  $f: X \rightarrow Y$ , the following statements are equivalent:

- (1)  $f$  is fuzzy  $\beta\alpha$ -continuous;
- (2) for every fuzzy  $\alpha$ -open set  $\rho$  in  $Y$ ,  $f^{-1}(\rho)$  is fuzzy

$\beta$ -open;

(3) for every fuzzy  $\alpha$ -closed set  $\rho$  in  $Y$ ,  $f^{-1}(\rho)$  is fuzzy  $\beta$ -closed.

**Proof.** (1)  $\Rightarrow$  (2): Let  $\rho$  be a fuzzy  $\alpha$ -open set in  $Y$  and let  $x_\epsilon \in f^{-1}(\rho)$ . Since  $f(x_\epsilon) \in \rho$ , by (1), there exists a fuzzy  $\beta$ -open set  $\mu_{x_\epsilon}$  in  $X$  containing  $x_\epsilon$  such that  $\mu_{x_\epsilon} \leq f^{-1}(\rho)$ . We obtain that  $f^{-1}(\rho) = \bigvee_{x_\epsilon \in f^{-1}(\rho)} \mu_{x_\epsilon}$ . Thus,  $f^{-1}(\rho)$  is fuzzy  $\beta$ -open.

(2)  $\Rightarrow$  (1): Let  $\rho$  be a fuzzy  $\alpha$ -open set in  $Y$  and let  $f(x_\epsilon) \in \rho$ . We have  $x_\epsilon \in f^{-1}(\rho)$ . By (2),  $f^{-1}(\rho)$  is a fuzzy  $\beta$ -open set. Take  $\eta = f^{-1}(\rho)$ . Then  $f(\eta) \leq \rho$ . Thus,  $f$  is fuzzy  $\beta\alpha$ -continuous.

(2)  $\Rightarrow$  (3): Let  $\rho$  be a fuzzy  $\alpha$ -closed set in  $Y$ . Then  $Y \setminus \rho$  is fuzzy  $\alpha$ -open. By (2),  $f^{-1}(Y \setminus \rho) = X \setminus f^{-1}(\rho)$  is fuzzy  $\beta$ -open. Thus,  $f^{-1}(\rho)$  is fuzzy  $\beta$ -closed.

(3)  $\Rightarrow$  (2): Similar to (2)  $\Rightarrow$  (3).

**Definition 2.3.** A fuzzy filter base  $\Lambda$  is said to be fuzzy  $\beta$ -convergent (resp. fuzzy  $\alpha$ -convergent) to a fuzzy point  $x_\epsilon \in X$  if for any fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) set  $\rho$  in  $X$  containing  $x_\epsilon$ , there exists a fuzzy set  $\mu \in \Lambda$  such that  $\mu \leq \rho$ .

**Theorem 2.4.** If a fuzzy function  $f: X \rightarrow Y$  is fuzzy  $\beta\alpha$ -continuous, then for each fuzzy point  $x_\epsilon \in X$  and each fuzzy filter base  $\Lambda$  in  $X$  which is  $\beta$ -convergent to  $x_\epsilon$ , the fuzzy filter base  $f(\Lambda)$  is fuzzy  $\alpha$ -convergent to  $f(x_\epsilon)$ .

**Proof.** Let  $x_\epsilon \in X$  and  $\Lambda$  be any fuzzy filter base in  $X$  which is  $\beta$ -convergent to  $x_\epsilon$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous, then for any fuzzy  $\alpha$ -open set  $\lambda$  in  $Y$  containing  $f(x_\epsilon)$ , there exists a fuzzy  $\beta$ -open set  $\mu$  in  $X$  containing  $x_\epsilon$  such that  $f(\mu) \leq \lambda$ . Since  $\Lambda$  is fuzzy  $\beta$ -convergent to  $x_\epsilon$ , there exists a  $\rho \in \Lambda$  such that  $\rho \leq \mu$ . This means that  $f(\rho) \leq \lambda$  and therefore the fuzzy filter base  $f(\Lambda)$  is fuzzy  $\alpha$ -convergent to  $f(x_\epsilon)$ .

**Remark 2.5.** For  $f: X \rightarrow Y$ , the following diagram holds:

$$\begin{array}{c} \text{fuzzy } \alpha\text{-irresolute} \\ \Downarrow \\ \text{fuzzy } \beta\text{-irr.} \Rightarrow \text{fuzzy } \beta\alpha\text{-conti.} \Leftarrow \text{fuzzy preirr.} \\ \Uparrow \\ \text{fuzzy irresolute} \end{array}$$

The following examples show that these implications are not reversible.

**Example 2.6.** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and  $\lambda, \mu$  are fuzzy sets defined as follows:

$$\lambda(a) = 0.3, \lambda(b) = 0.6,$$

$$\mu(x) = 0.7, \mu(y) = 0.5.$$

Let  $\tau_1 = \{X, \emptyset, \lambda\}$  and  $\tau_2 = \{Y, \emptyset, \mu\}$ . Then the fuzzy function  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by  $f(a) = x$  and  $f(b) = y$  is fuzzy  $\beta\alpha$ -continuous but neither fuzzy  $\alpha$ -irresolute nor irresolute.

**Example 2.7.** In the above example, we take

$$\mu(x) = 0.7, \mu(y) = 0.6.$$

Then the fuzzy function  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by  $f(a) = x$  and  $f(b) = y$  is fuzzy  $\beta\alpha$ -continuous but neither fuzzy preirresolute nor  $\beta$ -irresolute.

**Theorem 2.8.** Let  $f: X \rightarrow Y$  be a fuzzy function and let  $g: X \rightarrow X \times Y$  be the fuzzy graph function of  $f$  [1], defined by  $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$  for each  $x_\epsilon \in X$ . If  $g$  is fuzzy  $\beta\alpha$ -continuous, then  $f$  is fuzzy  $\beta\alpha$ -continuous.

**Proof.** Let  $\rho$  be fuzzy  $\alpha$ -open set in  $Y$ . Then  $X \times \rho$  is fuzzy  $\alpha$ -open set in  $X \times Y$ . Since  $g$  is fuzzy  $\beta\alpha$ -continuous, then  $f^{-1}(\rho) = g^{-1}(X \times \rho)$  is fuzzy  $\beta$ -open in  $X$ . Thus,  $f$  is fuzzy  $\beta\alpha$ -continuous.

**Definition 2.9.** A fuzzy space  $X$  said to be:

- (1) fuzzy  $\beta$ -compact [4] (resp. fuzzy  $\alpha$ -compact [4, 11]) if every fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) cover of  $X$  has a finite subcover;
- (2) fuzzy countably  $\beta$ -compact (resp. fuzzy countably  $\alpha$ -compact) if every fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) countably cover of  $X$  has a finite subcover;
- (3) fuzzy  $\beta$ -Lindelof (resp. fuzzy  $\alpha$ -Lindelof) if every cover of  $X$  by fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) sets has a countable subcover.

**Theorem 2.10.** Let  $f: X \rightarrow Y$  be a fuzzy  $\beta\alpha$ -continuous surjection. Then the following statements hold:

- (1) If  $X$  is fuzzy  $\beta$ -compact, then  $Y$  is fuzzy  $\alpha$ -compact.
- (2) If  $X$  is fuzzy  $\beta$ -Lindelof, then  $Y$  is fuzzy  $\alpha$ -Lindelof.
- (3) If  $X$  is fuzzy countably  $\beta$ -compact, then  $Y$  is fuzzy countably  $\alpha$ -compact.

**Proof.** We prove only (1). Let  $\{\mu_\alpha: \alpha \in I\}$  be any fuzzy  $\alpha$ -open cover of  $Y$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous, then  $\{f^{-1}(\mu_\alpha): \alpha \in I\}$  is a fuzzy  $\beta$ -open cover of  $X$ . Since  $X$  is fuzzy  $\beta$ -compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \bigvee \{f^{-1}(\mu_\alpha): \alpha \in I_0\}$ . Then we have  $Y = \bigvee \{\mu_\alpha: \alpha \in I_0\}$  and thus  $Y$  is fuzzy  $\alpha$ -compact.

**Definition 2.11.** A fuzzy space  $X$  is said to be fuzzy  $\beta$ -

connected (resp. fuzzy connected [9]) if it cannot be expressed as the union of two nonempty, disjoint fuzzy  $\beta$ -open (resp. fuzzy open) sets.

**Theorem 2.12.** If  $f: X \rightarrow Y$  is fuzzy  $\beta\alpha$ -continuous surjective function and  $X$  is fuzzy  $\beta$ -connected space, then  $Y$  is fuzzy connected space.

**Proof.** Suppose that  $Y$  is not fuzzy connected space. Then there exists nonempty disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $Y = \beta \vee \mu$ . Hence,  $\beta$  and  $\mu$  are fuzzy  $\alpha$ -open sets in  $Y$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  are fuzzy  $\beta$ -closed and  $\beta$ -open. Moreover,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  are nonempty disjoint and  $X = f^{-1}(\beta) \vee f^{-1}(\mu)$ . This shows that  $X$  is not fuzzy  $\beta$ -connected. This is a contradiction. Therefore,  $Y$  is fuzzy connected.

**Definition 2.13.** A fuzzy space  $X$  is called hyperconnected [2] if every fuzzy open set is dense.

**Remark 2.14.** The following example shows that fuzzy  $\beta\alpha$ -continuous surjection do not necessarily preserve fuzzy hyperconnectedness.

**Example 2.15.** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and  $\lambda, \mu$  are fuzzy sets defined as follows:

$$\begin{aligned} \lambda(a) &= 0.3, \quad \lambda(b) = 0.6, \\ \mu(x) &= 0.5, \quad \mu(y) = 0.5. \end{aligned}$$

Let  $\tau_1 = \{X, \emptyset, \lambda\}$  and  $\tau_2 = \{Y, \emptyset, \mu\}$ . Then the fuzzy function  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by  $f(a) = x$  and  $f(b) = y$  is fuzzy  $\beta\alpha$ -continuous surjective and  $(X, \tau_1)$  is hyperconnected. But  $(Y, \tau_2)$  is not hyperconnected.

### 3. Several properties

In this section, we investigate the relationships between fuzzy  $\beta\alpha$ -continuous functions and separation axioms and those graphs.

**Definition 3.1.** A fuzzy space  $X$  is said to be fuzzy  $\beta$ - $T_1$  (resp. fuzzy  $\alpha$ - $T_1$ ) if for each pair of distinct fuzzy points  $x_\epsilon$  and  $y_\nu$  of  $X$ , there exist fuzzy  $\beta$ -open (resp.  $\alpha$ -open) sets  $\beta$  and  $\mu$  containing  $x_\epsilon$  and  $y_\nu$ , respectively, such that  $y_\nu \notin \beta$  and  $x_\epsilon \notin \mu$ .

**Theorem 3.2.** If  $f: X \rightarrow Y$  is a fuzzy  $\beta\alpha$ -continuous injection and  $Y$  is fuzzy  $\alpha$ - $T_1$ , then  $X$  is fuzzy  $\beta$ - $T_1$ .

**Proof.** Suppose that  $Y$  is fuzzy  $\alpha$ - $T_1$ . For any distinct fuzzy points  $x_\epsilon$  and  $y_\nu$  in  $X$ , there exist fuzzy  $\alpha$ -open sets  $\mu$  and  $\rho$  in  $Y$  such that  $f(x_\epsilon) \in \mu$ ,  $f(y_\nu) \notin \mu$ ,  $f(x_\epsilon) \notin \rho$

and  $f(y_\nu) \in \rho$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\mu)$  and  $f^{-1}(\rho)$  are  $\beta$ -open sets in  $X$  such that  $x_\epsilon \in f^{-1}(\mu)$ ,  $y_\nu \notin f^{-1}(\mu)$ ,  $x_\epsilon \notin f^{-1}(\rho)$  and  $y_\nu \in f^{-1}(\rho)$ . This shows that  $X$  is fuzzy  $\beta$ - $T_1$ .

**Definition 3.3.** A fuzzy space  $X$  is said to be fuzzy  $\beta$ - $T_2$  (resp. fuzzy  $\alpha$ - $T_2$ ) if for each pair of distinct fuzzy points  $x_\epsilon$  and  $y_\nu$  of  $X$ , there exist disjoint fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) sets  $\beta$  and  $\mu$  in  $X$  such that  $x_\epsilon \in \beta$  and  $y_\nu \in \mu$ .

**Theorem 3.4.** If  $f: X \rightarrow Y$  is a fuzzy  $\beta\alpha$ -continuous injection and  $Y$  is fuzzy  $\alpha$ - $T_2$ , then  $X$  is fuzzy  $\beta$ - $T_2$ .

**Proof.** For any pair of distinct fuzzy points  $x_\epsilon$  and  $y_\nu$  in  $X$ , there exist disjoint fuzzy  $\alpha$ -open sets  $\beta$  and  $\mu$  in  $Y$  such that  $f(x_\epsilon) \in \beta$  and  $f(y_\nu) \in \mu$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  is fuzzy  $\beta$ -open in  $X$  containing  $x_\epsilon$  and  $y_\nu$ , respectively. Then we obtain  $f^{-1}(\beta) \wedge f^{-1}(\mu) = \emptyset$ . This shows that  $X$  is fuzzy  $\beta$ - $T_2$ .

**Definition 3.5.** A fuzzy space  $X$  is said to be fuzzy strongly  $\alpha$ -regular (resp. fuzzy strongly  $\beta$ -regular) if for each fuzzy  $\alpha$ -closed (resp. fuzzy  $\beta$ -closed) set  $\eta$  and each fuzzy point  $x_\epsilon \notin \eta$ , there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\eta \leq \beta$  and  $x_\epsilon \in \mu$ .

**Definition 3.6.** A fuzzy space  $X$  is said to be fuzzy strongly  $\alpha$ -normal (resp. fuzzy strongly  $\beta$ -normal) if for every pair of disjoint fuzzy  $\alpha$ -closed (resp. fuzzy  $\beta$ -closed) sets  $\eta_1$  and  $\eta_2$  in  $X$ , there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\eta_1 \leq \beta$  and  $\eta_2 \leq \mu$ .

**Theorem 3.7.** If  $f: X \rightarrow Y$  is fuzzy  $\beta\alpha$ -continuous, fuzzy open bijection and  $X$  is a fuzzy strongly  $\beta$ -regular space, then  $Y$  is fuzzy strongly  $\alpha$ -regular.

**Proof.** Let  $\eta$  be fuzzy  $\alpha$ -closed set in  $Y$  and  $y_\nu \notin \eta$ . Take  $y_\epsilon = f(x_\epsilon)$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\eta)$  is a fuzzy  $\beta$ -closed set. Take  $\lambda = f^{-1}(\eta)$ . We have  $x_\epsilon \notin \lambda$ . Since  $X$  is fuzzy strongly  $\beta$ -regular, there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\lambda \leq \beta$  and  $x_\epsilon \in \mu$ . Thus, we obtain that  $\eta = f(\lambda) \leq f(\beta)$  and  $y_\epsilon = f(x_\epsilon) \in f(\mu)$  such that  $f(\beta)$  and  $f(\mu)$  are disjoint fuzzy open sets. This shows that  $Y$  is fuzzy strongly  $\alpha$ -regular.

**Theorem 3.8.** If  $f: X \rightarrow Y$  is fuzzy  $\beta\alpha$ -continuous, fuzzy open bijection and  $X$  is a fuzzy strongly  $\beta$ -normal space, then  $Y$  is fuzzy strongly  $\alpha$ -normal.

**Proof.** Let  $\eta_1$  and  $\eta_2$  be disjoint fuzzy  $\alpha$ -closed sets in

$Y$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\eta_1)$  and  $f^{-1}(\eta_2)$  are fuzzy  $\beta$ -closed sets. Take  $\beta = f^{-1}(\eta_1)$  and  $\mu = f^{-1}(\eta_2)$ . We have  $\beta \wedge \mu = \emptyset$ . Since  $X$  is fuzzy strongly  $\beta$ -normal, there exist disjoint fuzzy open sets  $\lambda$  and  $\rho$  such that  $\beta \leq \lambda$  and  $\mu \leq \rho$ . We obtain that  $\eta_1 = f(\beta) \leq f(\lambda)$  and  $\eta_2 = f(\mu) \leq f(\rho)$  such that  $f(\lambda)$  and  $f(\rho)$  are disjoint fuzzy open sets. Thus,  $Y$  is fuzzy strongly  $\alpha$ -normal.

Recall that for a fuzzy function  $f: X \rightarrow Y$ , the subset  $\{(x_\epsilon, f(x_\epsilon)) : x_\epsilon \in X\} \leq X \times Y$  is called the graph of  $f$  and is denoted by  $G(f)$ .

**Definition 3.9.** A graph  $G(f)$  of a fuzzy function  $f: X \rightarrow Y$  is said to be fuzzy  $\beta$ - $\alpha$ -closed if for each  $(x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f)$ , there exist a fuzzy  $\beta$ -open set  $\beta$  in  $X$  containing  $x_\epsilon$  and a fuzzy  $\alpha$ -open set  $\mu$  in  $Y$  containing  $y_\nu$  such that  $(\beta \times \mu) \wedge G(f) = \emptyset$ .

**Lemma 3.10.** A graph  $G(f)$  of  $f: X \rightarrow Y$  is fuzzy  $\beta$ - $\alpha$ -closed in  $X \times Y$  if and only if for each  $(x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f)$ , there exist a fuzzy  $\beta$ -open set  $\beta$  in  $X$  containing  $x_\epsilon$  and a fuzzy  $\alpha$ -open set  $\mu$  in  $Y$  containing  $y_\nu$  such that  $f(\beta) \wedge \mu = \emptyset$ .

**Theorem 3.11.** If  $f: X \rightarrow Y$  is fuzzy  $\beta\alpha$ -continuous and  $Y$  is fuzzy  $\alpha$ -Hausdorff, then  $G(f)$  is fuzzy  $\beta$ - $\alpha$ -closed in  $X \times Y$ .

**Proof.** Let  $(x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f)$ . Then  $f(x_\epsilon) \neq y_\nu$ . Since  $Y$  is fuzzy  $\alpha$ -Hausdorff, there exist disjoint fuzzy  $\alpha$ -open sets  $\beta$  and  $\mu$  in  $Y$  such that  $f(x_\epsilon) \in \beta$  and  $y_\nu \in \mu$ . Since  $f$  is fuzzy  $\beta\alpha$ -continuous, there exists a  $\beta$ -open set  $\rho$  in  $X$  containing  $x_\epsilon$  such that  $f(\rho) \leq \beta$ . Therefore, we obtain  $y_\nu \in \mu$  and  $f(\rho) \wedge \mu = \emptyset$ . This shows that  $G(f)$  is fuzzy  $\beta$ - $\alpha$ -closed.

**Theorem 3.12.** Let  $f: X \rightarrow Y$  has a fuzzy  $\beta$ - $\alpha$ -closed graph  $G(f)$ . If  $f$  is injective, then  $X$  is fuzzy  $\beta$ - $T_1$ .

**Proof.** Let  $x_\epsilon$  and  $y_\nu$  be any two distinct fuzzy points of  $X$ . Then,  $(x_\epsilon, f(y_\nu)) \in (X \times Y) \setminus G(f)$ . By definition of fuzzy  $\beta$ - $\alpha$ -closed graph, there exist a fuzzy  $\beta$ -open set  $\beta$  in  $X$  and a fuzzy  $\alpha$ -open set  $\mu$  in  $Y$  such that  $f(x_\epsilon) \in \beta$ ,  $y_\nu \in \mu$  and  $f(\rho) \wedge \mu = \emptyset$  and hence  $\beta \wedge f^{-1}(\mu) = \emptyset$ . Therefore, we have  $y_\nu \notin \beta$ . This implies that  $X$  is fuzzy  $\beta$ - $T_1$ .

**Theorem 3.13.** Let  $f: X \rightarrow Y$  has a fuzzy  $\beta$ - $\alpha$ -closed graph  $G(f)$ . If  $f$  is surjective always fuzzy  $\beta$ -open function, then  $Y$  is fuzzy  $\beta$ - $T_2$ .

**Proof.** Let  $y_\nu$  and  $y_\epsilon$  be any distinct points of  $Y$ . Since  $f$  is surjective,  $f(x_\nu) = y_\nu$  for some  $x_\nu \in X$  and  $(x_\mu, y_\epsilon) \in (X \times Y) \setminus G(f)$ . By the fuzzy  $\beta$ - $\alpha$ -closedness of graph  $G(f)$ , there exists a fuzzy  $\beta$ -open set  $\beta$  in  $X$  and a fuzzy  $\alpha$ -open set  $\mu$  in  $Y$  such that  $x_\nu \in \beta$ ,  $y_\epsilon \in \mu$  and  $(\beta \times \mu) \wedge G(f) = \emptyset$ . Then, we have  $f(\beta) \wedge \mu = \emptyset$ . Since  $f$  is always fuzzy  $\beta$ -open, then  $f(\beta)$  is fuzzy  $\beta$ -open such that  $f(x_\nu) = y_\nu \in f(\beta)$ . This implies that  $Y$  is fuzzy  $\beta$ - $T_2$ .

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**Erdal Ekici**

Present: Prof. in Department of Mathematics, Canakkale Onsekiz Mart University  
 Research interest: Fuzzy topology, Intuitionistic fuzzy metric, General topology

E-mail : eekici@comu.edu.tr

**Jin Han Park**

1994~present: Prof. in Division of Mathematical Sciences,  
Pukyong National University

Research interest: Fuzzy topology, Intuitionistic fuzzy metric,  
General topology

Phone : +82-51-620-6326

Fax : +82-51-611-6356

E-mail : jihpark@pknu.ac.kr