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Erratum to: On Galvin's theorem for compact Hausdorff right-topological semigroups with dense topological centers

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In the third paragraph on p. 2421 and in Theorem $1'$ of [3] it is wrongly assumed that "the multiplication on a topological semigroup T can be extended to a right continuous multiplication on βT"; in other words, we wrongly assume that "βT is a compact Hausdorff right-topological semigroup for any topological semigroup T". So using [3, Theorem 1] we can only obtain the following two theorems in place of [3, Theorem $1'$], where X' denotes the limit-point set of a space X and $J(E)$ denotes the set of all idempotents in a compact Hausdorff right-topological semigroup E.

Firstly by [3, Theorem 1] and [6] we can obtain the following instead of [3, Theorem $1'$].

Theorem $1'$ (See [1, Lemma 2.1] and [4, Theorem 3.3] for I and II). *Let T be a discrete semigroup. Then the following statements hold*:

I. *Let $v \in (\beta T)' \cap J(\beta T)$ and let $\{V_n\}_{n=1}^{\infty}$ be a sequence of neighborhoods of v in βT. Then there exists a* 1-1 *map $\tau_{\centerdot}\colon \mathbb{N} \to T$ such that $FP(\langle \tau_n \rangle_{n=k}^{\infty}) \subseteq V_k$ for all $k \geqslant 1$. In particular, any neighborhood of $v \in (\beta T)' \cap J(\beta T)$ contains an infinite IP-set in T.*

IIa. *For any IP-set $FP(\langle t_n \rangle_{n=1}^{\infty})$ in T, there exists a $v \in J(\beta T)$ such that $v \in \bigcap_{k=1}^{\infty} \mathrm{cls}_{\beta T} FP(\langle t_n \rangle_{n=k}^{\infty})$.*

IIb. *Let H be a subsemigroup of T such that H' is a non-empty subsemigroup of βT. Then for any infinite IP-set $A \subseteq H$, there exists an idempotent $v \in (\beta T)' \cap \mathrm{cls}_{\beta T} A$.*

III. *If T is weakly left cancelable, then $(\beta T)'$ is T-invariant and so $(\beta T)'$ is a left ideal in βT.*

Let G be a Hausdorff topological group with continuous multiplication

$$\lambda\colon G \times G \xrightarrow{(t,x)\mapsto tx} G$$

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and with identity $\mathbb{1}$. Let $\mathscr{U}_{\mathbb{1}}$ be the neighborhood system of $\mathbb{1}$ in G. Let $I = [0,1]$ be endowed with the usual topology. A function $\varphi\colon G \to I$ is *left uniformly continuous*, denoted by $F_{\mathrm{luc}}(G)$, if and only if given $\varepsilon > 0$ there exists a $U \in \mathscr{U}_{\mathbb{1}}$ such that $|\varphi(x) - \varphi(tx)| < \varepsilon$ for all $x \in G$ and $t \in U$. Define the evaluation map

$$e_{\mathrm{luc}}\colon G \xrightarrow{x \mapsto (\varphi(x))_{\varphi \in F_{\mathrm{luc}}(G)}} I^{F_{\mathrm{luc}}(G)},$$

and let $G^{\mathrm{LUC}} = \mathrm{cls}_{I^{F_{\mathrm{luc}}(G)}} e_{\mathrm{luc}}(G)$, which is a compact Hausdorff subspace of the compact product space $I^{F_{\mathrm{luc}}(G)}$.

G^{LUC} is called the LUC-*compactification* of G, which is a compact Hausdorff right-topological semigroup with respect to the canonical multiplication (see [2, 5, 7]). Then by [3, Theorem 1] we can obtain the following Theorem 1$''$.

Theorem 1$''$. *Let G be any Hausdorff topological group. Then*

(I) *Let $v \in (G^{\mathrm{LUC}})' \cap J(G^{\mathrm{LUC}})$ and let $\{V_n\}_{n=1}^{\infty}$ be a sequence of neighborhoods of v in G^{LUC}. Then there exists a 1-1 map $\tau_{\bullet}\colon \mathbb{N} \to G$ such that $FP(\langle \tau_n \rangle_{n=k}^{\infty}) \subseteq V_k$ for all $k \geqslant 1$. In particular, any neighborhood of $v \in (G^{\mathrm{LUC}})' \cap J(G^{\mathrm{LUC}})$ contains an infinite IP-set in G.*

(IIa) *For any IP-set $FP(\langle t_n \rangle_{n=1}^{\infty})$ in G, there exists a $v \in J(G^{\mathrm{LUC}})$ such that*

$$v \in \bigcap_{k=1}^{\infty} \mathrm{cls}_{G^{\mathrm{LUC}}} FP(\langle t_n \rangle_{n=k}^{\infty}).$$

(IIb) *Let H be a subsemigroup of G such that H' is a non-empty subsemigroup of G^{LUC}. Then for any infinite IP-set $A \subseteq H$, there exists an idempotent $v \in (G^{\mathrm{LUC}})' \cap \mathrm{cls}_{G^{\mathrm{LUC}}} A$.*

(III) *$(G^{\mathrm{LUC}})'$ is G-invariant and so $(G^{\mathrm{LUC}})'$ is a left ideal in G^{LUC}.*

References

1 Bergelson V, Hindman N. Some topological semicommutative van der Waerden type theorems and their combinatorial consequences. J Lond Math Soc (2), 1992, 45: 385–403

2 Berglund J F, Junghenn H D, Milnes P. Analysis on Semigroups. New York: Wiley, 1989

3 Dai X P, Liang H L. On Galvin's theorem for compact Hausdorff right-topological semigroups with dense topological centers. Sci China Math, 2017, 60: 2421–2428

4 Ellis D, Ellis R, Nerurkar M. The topological dynamics of semigroup actions. Trans Amer Math Soc, 2001, 353: 1279–1320

5 Flor P. Über eine Kompaktifizierung topologischer Gruppen. J Reine Angew Math, 1967, 228: 193–198

6 Hindman N, Strauss D. Algebra in the Stone-Čech Compactification: Theory and Applications. Berlin: Walter de Gruyter, 1998

7 Knapp A W. Decomposition theorem for bounded uniformly continuous functions on a group. Amer J Math, 1967, 88: 902–914

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