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Abstract

This paper considers the special class of cooperative sequencing games that arise from one-machine sequencing situations in which all jobs have equal processing times and the ready time of each job is a multiple of the processing time. By establishing relations between optimal orders of subcoalitions it is shown that each sequencing game within this class is convex.

KEYWORDS: Cooperative games, one-machine sequencing problems.

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1 Introduction

In one-machine sequencing situations each agent (player) has one job that has to be processed on a single machine. Each job is specified by its ready time, the earliest time that the processing of the job can begin, and its processing time, the time the machine takes to handle the job. We assume that the costs of a player depend linearly on the completion time of his job. Furthermore, there is an initial order on the jobs of the agents before the processing of the machine starts.

A group of agents (a coalition) can save costs by rearranging their jobs in a way that is admissible with respect to the various ready times and the initial order. By defining the value of a coalition as the maximum cost savings a coalition can make in this way, we obtain a cooperative sequencing game related to a one-machine sequencing situation. The formal model can be found in section 2.

The above game theoretic approach was introduced by *Curiel, Pederzoli* and *Tijs (1989)*. They showed convexity for all sequencing games arising from one-machine sequencing situations in which all jobs have equal ready times. *Potters, Curiel, Rajendra Prasad, Tijs* and *Veltman (1990)* introduced a class of balanced games that contains sequencing games corresponding to one-machine sequencing situations in which the ready times of all jobs are not necesarry equal. In section 4 it is shown that the convexity result of *Curiel et al. (1989)* can be generalized to the special class of sequencing games that arise from one-machine sequencing situations in which all jobs have equal processing times and the ready time of each job is a multiple of the processing time.

In section 3 we consider some properties of this special class of one-machine sequencing situations. By modifying an algorithm of *Rinnooy Kan (1976)* one easily determines the optimal order of any coalition in these sequencing situations. However, this algorithm does not immediately describe the relation between optimal orders of various subcoalitions. Some of these relations are provided in section 3.

2 Sequencing situations

In a one-machine sequencing situation there is a queue of agents, each with one job, before a machine (counter). Each agent (player) has to process his job on the machine. The finite set of agents is denoted by N and $|N| = n$.

By a bijection $\sigma : N \to \{1, ..., n\}$ we can describe the position of the agents in the queue. Specifically, $\sigma(i) = j$ means that player i is in position j.

The ready time r_i of the job of agent i is the earliest time the processing of this job can begin. The processing time p_i of the job of agent i is the time the machine takes to handle this job.

We assume that every agent has a affine cost function $c_i : [0, \infty) \to \mathbf{R}$ defined by $c_i(t) = \alpha_i t + \beta_i$ with $\alpha_i > 0, \beta_i \in \mathbf{R}$.

Further it is assumed that there is an initial order $\sigma_0 : N \to \{1, ..., n\}$ on the jobs of the players before the processing of the machine starts with the property that for all $i, j \in N$ with $\sigma_0(i) < \sigma_0(j)$ it holds that $r_i \leq r_j$. A sequencing situation as described above is denoted by (σ_0, r, p, α), where $\sigma_0 : N \to \{1, ..., n\}$, $r = (r_i)_{i \in N} \in [\mathbf{0}, \infty)^{\mathbf{N}}$, $p = (p_i)_{i \in N} \in \mathbf{R}_+^{\mathbf{N}}$ and $\alpha = (\alpha_i)_{i \in N} \in \mathbf{R}_+^{\mathbf{N}}$.

The vector $\beta = (\beta_i)_{i \in N} \in \mathbf{R}^{\mathbf{N}}$ is ommited in the description of a sequencing situation since it will not affect the values of the corresponding sequencing game.

For player $i \in N$ we define the following sets with respect to a bijection σ. The set of predecessors of player i is $P(\sigma, i) := \{j \mid \sigma(j) < \sigma(i)\}$ and the set of followers of player i is $F(\sigma, i) := \{j \mid \sigma(j) > \sigma(i)\}$. The *head* of i is $P(\sigma_0, i) \cup \{i\}$ and his *tail* is $F(\sigma_0, i) \cup \{i\}$. For notational convenience let $P(i) := P(\sigma_0, i)$ and $F(i) := F(\sigma_0, i)$.

The starting time of the job of agent i if processed according to a bijection $\sigma : N \to \{1, ..., n\}$ (in a semi-active way) is

$$t_{i,\sigma} := \begin{cases} \max(r_i, t_{j,\sigma} + p_j) & \text{if } \sigma(i) > 1 \\ r_i & \text{if } \sigma(i) = 1 \end{cases}$$

where $j \in N$ such that $\sigma(j) = \sigma(i) - 1$.

Hence, the completion time of the job of agent i is equal to $C(\sigma, i) := t_{i,\sigma} + p_i$. The total costs $c_\sigma(S)$ of a coalition $S \subset N$, is given by

$$c_\sigma(S) := \sum_{i \in S} \alpha_i (C(\sigma, i)) + \beta_i.$$

The (maximal) costsavings of a coalition S depend on the set of admissible rearrangements of this coalition. We call a bijection $\sigma : N \to \{1, ..., n\}$ admissible for S if it satisfies the following two conditions:

(i) the starting time of each agent outside the coalition S is equal to his starting time in the initial order: $t_{i,\sigma_0} = t_{i,\sigma}$ for all $i \in N \backslash S$.

(ii) the agents of S are not allowed to jump over players outside S:

$$P(i) \cap N \backslash S = P(\sigma, i) \cap N \backslash S \text{ for all } i \in S.$$

The set of admissible rearrangements for a coalition S is denoted by Σ_S .

Before formally introducing sequencing games we recall some well known facts concerning cooperatives games.

A *cooperative game* is a pair (N, v) where N is a finite set of players and v is a mapping $v : 2^N \to \mathbf{R}$ with $v(\emptyset) = 0$ and 2^N the collection of all subsets of N.

A game (N, v) is called *convex* if for all coalitions $S, T \in 2^N$ it holds that

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \tag{1}$$

or, equivalently, if for all coalitions $S, T \in 2^N$ and all $i \in N$ with

$S \subset T \subset N \backslash \{i\}$ it holds that

$$v(T \cup i) - v(T) \geq v(S \cup i) - v(S).$$

Cooperative game theory focusses on 'fair' and/or 'stable' division rules from the value $v(N)$ of the grand coalition. A core element $x = (x_i)_{i \in N} \in \mathbf{R^N}$ is such that no coalition has an incentive to split of, i.e.

$$\sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \in 2^N.$$

The core $C(v)$ consists of all core elements. A game is called balanced if its core is non-empty. We note that convex games are balanced.

Given a sequencing situation (σ_0, r, p, α) the corresponding sequencing game *(Curiel et al. (1989))* is defined in such a way that the the worth of a coalition S is equal to the maximal cost savings the coalition can achieve by means of an admissible rearrangement. Formally we have,

$$\begin{aligned} v(S) &= \max_{\sigma \in \Sigma_S} \{\sum_{i \in S} (\alpha_i C(\sigma_0, i) + \beta_i) - \sum_{i \in S} (\alpha_i C(\sigma, i) + \beta_i)\} \\ &= \max_{\sigma \in \Sigma_S} \{\sum_{i \in S} \alpha_i C(\sigma_0, i) - \sum_{i \in S} \alpha_i C(\sigma, i)\} \end{aligned} \tag{2}$$

A coalition S is called *connected* with respect to σ_0 if for all $i,j \in S$ and $k \in N$, $\sigma_0(i) < \sigma_0(k) < \sigma_0(j)$ implies $k \in S$. A connected coalition $S \subset T$ is a *component* of T if $S \cup \{i\}$ is not connected for every $i \in T \backslash S$. The components of T form a partition of T, denoted by T/σ_0. According to condition (ii) of an admissible rearrangement of a coalition S the players of S are not allowed to jump over players outside the coalition. This implies that an optimal rearrangement is such that the players in each component are rearranged optimally. Hence, for any coalition T,

$$v(T) = \sum_{S \in T/\sigma_0} v(S). \tag{3}$$

In the final part of this section we consider sequencing games that arise from sequencing situations with criteria equivalent to the weighted cost criterion that is used in this paper. Criteria are called equivalent if any optimal rearrangement with respect to one criterium is also an optimal rearrangement for the others. In several textbooks (cf.*Conway, Maxwell, Miller (1967)*) is shown that the weighted flowtime criterion, the weighted waiting time and the weighted lateness criterion are equivalent to the weighted cost criterion. With respect to the corresponding sequencing games one easily verifies

Proposition 1 *Each sequencing situation with a criterion equivalent to the weighted cost criterion generates the sequencing game decribed in* (2).

3 Optimal orders of different subcoalitions

In this section we concentrate on sequencing situations (σ_0, r, p, α) in which all jobs have equal processing time and the ready times of each job is a multiple of the processing time. W.l.o.g. we restrict attention to sequencing situations (σ_0, r, p, α) with $r_i \in \mathbf{N}$ and $p_i = 1$ for all $i \in N$. To calculate the worth of a coalition S in the corresponding sequencing game we need to find an optimal (=cost minimizing) rearrangement $\hat{\sigma}_S : N \to \{1, ..., n\}$ in the set of admissable rearrangements of the coalition S. For this we use the following algorithm due to *Rinnooy Kan (1976)* which generalizes the Smith rule (*Smith (1956)*), that gives an optimal order for sequencing situations with equal ready times. According to the Smith rule, players are processed in decreasing order of urgency, where the urgency of u_i of player i is defined by $u_i := \alpha_i p_i^{-1}$. To obtain the optimal order of the coalition N, if $p_i = 1$ for all $i \in N$ and if the ready times are not necessary equal but all integers, at each time $t \in \{0, 1, ...\}$ all jobs are considered that are available at moment t, i.e. the jobs that are not processed before t and that have a ready time smaller than or equal to t. The job with the highest urgency of all available jobs at t will be processed at that time. If there is more then one available job at t with the highest urgency, we pick the one with the smallest index number. Note that the set of available jobs at a given time can be empty. The algorithm stops when all jobs are assigned a position. A similar procedure can be applied to find an optimal rearrangement $\hat{\sigma}_S$ for an arbitrary coalition S with components $S_1, ..., S_r, r \geq 1$ such that $\min\{\sigma_0(j) \mid j \in S_1\} < ... < \min\{\sigma_0(j) \mid j \in S_r\}$. Obviously, for all players $j \in N \backslash S$ we have that $\hat{\sigma}_S(j) = \sigma_0(j)$. Then, inductively, one can determine the position of all players in S in the following way. If the position of each player in $N \backslash S_k \cup ... \cup S_r$ is determined , calculate the earliest possible starting time t_k of a job in the component $S_k, k \in \{1, ..., r\}$. Then all players in S_k are assigned positions by using the same principle as above, i.e. considering the urgency of the available jobs in S_k at the time moments $t \in \{t_k, t_k + 1, ...\}$. In the next step all players in the component S_{k+1} will be reordered by first calculating t_{k+1}. Repeating this procedure we have the optimal order of S.

To illustrate the algorithm we give the following example. For notational convenience we denote a bijection $\sigma : N \rightarrow \{1, ..., n\}$ by a n -dimensional vector $(i_1, ..., i_n)$ with $\{i_1, ..., i_n\} = N$ where i_k denotes the player that is assigned to position k.

Example 1 Let $N = \{1, 2, ..., 6\}, \sigma_0 = (1, 2, 3, 4, 5, 6), r = (0, 0, 1, 5, 6, 6)$, $p = (1, 1, 1, 1, 1, 1)$ and $\alpha = (1, 2, 3, 1, 5, 5)$. Note that the urgency of a player i coincides with α_i since $p_i = 1$. First we give the opimal order of N. Let A_t be the set of players whose job is available at time $t \in \{0, 1, 2, ...\}$. Then $A_0 = \{1, 2\}$ and since $\alpha_2 > \alpha_1$ we have that $\hat{\sigma}_N(2) = 1$. $A_1 = \{1, 3\}$ and since $\alpha_3 > \alpha_1$ we have $\hat{\sigma}_N(3) = 1$. Since $A_2 = \{1\}$ it follows that $\hat{\sigma}_N(1) = 3$. Then $A_3 = A_4 = \emptyset$ since no job is available at time $t = 3$ and $t = 4$. $A_5 = \{4\}$, hence $\hat{\sigma}_N(4) = 4$. $A_6 = \{5, 6\}$. Then $\hat{\sigma}_N(5) = 5$ since $\alpha_5 = \alpha_6$. Finally we have that $A_7 = \{6\}$ and hence $\hat{\sigma}_N(6) = 6$. Since all jobs are assigned a position the algorithm stops. Hence, $\hat{\sigma}_N = (2, 3, 1, 4, 5, 6)$.

Second we give the optimal order of $S = \{2, 3, 5, 6\}$. Obviously we have that $\hat{\sigma}_S(1) = 1$ and $\hat{\sigma}_S(4) = 4$ and it is sufficient to rearrange optimal $S_1 = \{2, 3\}$ and $S_2 = \{5, 6\}$.

Since $t_1 = 1$ we have that $A_1 = \{2,3\}$ and hence $\hat{\sigma}_S(3) = 2$ and consequently $\hat{\sigma}_S(2) = 3$. Since $t_2 = 6$ we have that $A_6 = \{5,6\}$ and hence $\hat{\sigma}_S(5) = 5$ and $\hat{\sigma}_S(6) = 6$. Hence, $q_S = (1,3,2,4,5,6)$.

One readily verifies

Lemma 1 *Let (σ_0, r, p, α) be a sequencing situation such that $r_i \in \mathbf{N}$ and $p_i = 1$ for all $i \in N$. Then for each coalition S the above algorithm generates a unique optimal rearrangement $\hat{\sigma}_S \in \Sigma_S$, i.e. $c_{\hat{\sigma}_S}(S) = \min_{\sigma \in \Sigma_S} c_\sigma(S)$.*

Note that if player i has the same position in the initial rearrangement σ_0 as player j in the optimal rearrangement $\hat{\sigma}_S$ of a coalition S, then the starting times of both players coincide, i.e. if $\sigma_0(i) = \hat{\sigma}_S(j)$, then $t_{i,\sigma_0} = t_{j,\hat{\sigma}_S}$.

In sequencing situations where all ready times are equal the optimal rearrangement of the grand coalition N also induces the optimal rearrangements $\hat{\sigma}_S$ of all other coalitions S,i.e. with $i,j \in S$ it holds that $\hat{\sigma}_S(i) < \hat{\sigma}_S(j)$ if and only if $\hat{\sigma}_N(i) < \hat{\sigma}_N(j)$. The following example shows that this need not be the case if the ready times are not equal.

Example 2 Let $N = \{1,2,3\}, \sigma_0 = (1,2,3), r = (0,0,1), p = (1,1,1)$ and $\alpha = (1,2,3)$. Then the optimal order of N is $\hat{\sigma}_N = (2,3,1)$ while $\hat{\sigma}_{\{2,3\}} = (1,3,2)$.

The following example shows that in case the requirement $p_i = 1$ for all $i \in N$ is dropped the above algorithm is not appropriate to obtain the optimal rearrangement.

Example 3 Let $N = \{1,2,3\}, \sigma_0 = (1,2,3), r = (0,0,1), p = (1,2,3)$ and $\alpha = (1,3,12)$. Then the costs w.r.t. the optimal rearrangement $(1,3,2)$ are 67 while the costs w.r.t. the rearrangement $(2,3,1)$ that is obtained by the algorithm are 72.

The following lemma shows that in the optimal rearrangement of N the position of player $n_0 := \sigma_0^{-1}(n)$, the last player according the initial order σ_0, is smaller than or equal to the position of player n_0 in the optimal rearrangement of any tail. This implies that player n_0 in the optimal rearrangement of N can not improve his position by joining another coalition S, since player n_0 is in a component of S which is a tail.

Lemma 2 *Let (σ_0, r, p, α) be a sequencing situation with $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$. Then $\hat{\sigma}_N(n_0) \leq \hat{\sigma}_{F(i)}(n_0)$ for all $i \in N$.*

PROOF:

Let $i \in N$. Suppose $\hat{\sigma}_N(n_0) > \hat{\sigma}_{F(i)}(n_0)$.

Choose $n_1 \in N \backslash \{n_0\}$ such that $\hat{\sigma}_N(n_1) = \hat{\sigma}_{F(i)}(n_0)$. Since $r_{n_0} \leq t_{n_0,\hat{\sigma}_{F(i)}} = t_{n_1,\hat{\sigma}_N}$ the optimality of $\hat{\sigma}_N$ implies that $\alpha_{n_1} \geq \alpha_{n_0}$.

(a:1) Suppose $n_1 \in P(i) \cup \{i\}$. Then $| P(\hat{\sigma}_N, n_1) \cap F(i) | \geq \hat{\sigma}_N(n_1) - i \geq 1$. For each $k \in F(i) \cap P(\hat{\sigma}_N, n_1)$ it holds that $\alpha_k > \alpha_{n_1}$ since $r_{n_1} \leq r_k$. This implies that $\alpha_k > \alpha_{n_0}$ and hence $k \in P(\hat{\sigma}_{F(i)}, n_0)$. However, this would imply that $\hat{\sigma}_{F(i)}(n_0) > i + | P(\hat{\sigma}_N, n_1) \cap F(i) | \geq \hat{\sigma}_N(n_1)$ which contradicts the fact that $\hat{\sigma}_N(n_1) = \hat{\sigma}_{F(i)}(n_0)$.

So we may assume that $n_1 \in F(i)$. Then, consequently, we have $n_1 \in P(\hat{\sigma}_{F(i)}, n_0)$. However, if $\hat{\sigma}_{F(i)}(n_0) = i + 1$ then $P(\hat{\sigma}_{F(i)}, n_0) \cap F(i) = \emptyset$ and we arrive at a contradiction. Hence, $\hat{\sigma}_{F(i)}(n_0) > i + 1$. Now choose $n_2 \in N \backslash \{n_0, n_1\}$ such that $\hat{\sigma}_N(n_2) = \hat{\sigma}_{F(i)}(n_1)$. The optimality of $\hat{\sigma}_{F(i)}$ implies $\alpha_{n_2} \geq \alpha_{n_1}$.

(a:2) Suppose $n_2 \in P(i) \cup \{i\}$. Then $| P(\hat{\sigma}_N, n_2) \cap F(i) | \geq \hat{\sigma}_N(n_2) - i \geq 1$. Moreover, for each $k \in F(i) \cap P(\hat{\sigma}_N, n_2)$ we have $\alpha_k > \alpha_{n_2}$ and thus $\alpha_k > \alpha_{n_1}$. This implies that $\hat{\sigma}_{F(i)}(n_1) > i + | P(\hat{\sigma}_N, n_2) \cap F(i) | \geq \hat{\sigma}_N(n_2)$. Contradiction.

So we may assume that $n_2 \in F(i)$. Then $n_2 \in P(\hat{\sigma}_{F(i)}, n_1)$ and we have a contradiction if $\hat{\sigma}_{F(i)}(n_0) = i + 2$. Hence, $\hat{\sigma}_{F(i)}(n_0) > i + 2$. Now choose $n_3 \in N \backslash \{n_0, n_1, n_2\}$ such that $\hat{\sigma}_N(n_3) = \hat{\sigma}_{F(i)}(n_2)$.

Using the same line of argument as in (a:1) and (a:2) we then find that $\hat{\sigma}_{F(i)}(n_0) > i + 3$. We may conclude that, if $\hat{\sigma}_{F(i)}(n_0) = i + k$ with $k \in \{1, 2, ..., n - i\}$, we arrive at a contradiction after k repetitions. □

In the following example we show that lemma 2 need not hold for an arbitrary sequencing situation.

Example 4 Let $N = \{1, 2, 3\}, \sigma_0 = (1, 2, 3), r = (0, 0, 4)$, $p = (4, 1, 2)$ and $\alpha = (4, 2, 5)$. Then the optimal rearrangement of N is $\sigma_N = (2, 1, 3)$ while the optimal rearrangement of $\{2, 3\}$ is $\sigma_{\{2,3\}} = (1, 3, 2)$. Hence $\sigma_N(3) > \sigma_{\{2,3\}}(3)$.

In the next lemma we show that in the optimal rearrangement of any tail the jobs from n_0 on are ordered in decreasing urgency.

Lemma 3 *Let (σ_0, r, p, α) be a sequencing situation with $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$. Let $i \in N$ and $k, l \in N$ be such that $\hat{\sigma}_{F(i) \cup \{i\}}(n_0) \leq \hat{\sigma}_{F(i) \cup \{i\}}(k) < \hat{\sigma}_{F(i) \cup \{i\}}(l)$. Then $\alpha_k \geq \alpha_l$.*

PROOF:

Suppose $\alpha_k < \alpha_l$. Since $r_l \leq r_{n_0} \leq t_{n_0, \hat{\sigma}_{F(i) \cup \{i\}}}$ player k and l can switch and decrease the total costs of $F(i) \cup \{i\}$. This contradicts the optimality of $\hat{\sigma}_{F(i)\cup\{i\}}$. $\square$

Note that lemma 3 can be generalized to sequencing situations with no restrictions on the ready time or processing time of a player. Then the players that follow n_0 in the optimal rearrangement of any tail are ordered in decreasing urgency.

The following lemma shows that the optimal rearrangement $\hat{\sigma}_{N\backslash\{n_0\}}$ of $N\backslash\{n_0\}$ is induced by the optimal rearrangement $\hat{\sigma}_N$ of N.

Lemma 4 *Let (σ_0, r, p, α) be a sequencing situation such that $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$. Then $\hat{\sigma}_{N\backslash\{n_0\}}(k) = \hat{\sigma}_N(k)$ if $\hat{\sigma}_N(k) < \hat{\sigma}_N(n_0)$ and $\hat{\sigma}_{N\backslash\{n_0\}}(k) = \hat{\sigma}_N(k) - 1$ if $\hat{\sigma}_N(k) > \hat{\sigma}_N(n_0)$.*

PROOF:

Let $(N\backslash\{n_0\})_t$ and N_t be the sets of players whose job is available at time t in determining the optimal rearrangement of $N\backslash\{n_0\}$ and N, respectively. For each $t \in \{1, ..., r_{n_0} - 1\}$ we have $(N\backslash\{n_0\})_t = N_t$ and consequently we have $\hat{\sigma}_{N\backslash\{n_0\}}(k) = \hat{\sigma}_N(k)$ for all k such that $t_{\hat{\sigma}_N,k} \leq r_{n_0} - 1$. For each $t \in \{r_{n_0}, ..., t_{\hat{\sigma}_N,n_0} - 1\}$ (a possibly empty set) we have $(N\backslash\{n_0\})_t \cup \{n_0\} = N_t$, but since n_0 is not chosen w.r.t N we have $\hat{\sigma}_{N\backslash\{n_0\}}(k) = \hat{\sigma}_N(k)$ for all k such that $r_{n_0} \leq t_{\hat{\sigma}_N,k} < t_{\hat{\sigma}_N,n_0}$. For $t \in \{t_{\hat{\sigma}_N,n_0}, ...\}$ we have $(N\backslash\{n_0\})_t = N_{t+1}$ it follows that $\hat{\sigma}_{N\backslash\{n_0\}}(k) = \hat{\sigma}_N(k) - 1$ for all k such that $t_{\hat{\sigma}_N,k} > t_{\hat{\sigma}_N,n_0}$. $\square$

The following propositon follows directly from lemma 4.

Proposition 2 *Let (σ_0, r, p, α) be a sequencing situation such that $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$. Let $i \in N$ be such that $\sigma_0(i) > \sigma_0(k)$ for all $k \in S$. Then for all $k \in S$ it holds that $\hat{\sigma}_S(k) = \hat{\sigma}_{S\cup\{i\}}(k)$ if $\hat{\sigma}_{S\cup\{i\}}(k) < \hat{\sigma}_{S\cup\{i\}}(i)$ and $\hat{\sigma}_S(k) = \hat{\sigma}_{S\cup\{i\}}(k) - 1$ if $\hat{\sigma}_{S\cup\{i\}}(k) > \hat{\sigma}_{S\cup\{i\}}(i)$.*

Note that this propositition gives another approach to obtain the optimal order of a coalition S.

The following example shows that the result of the last lemma need not hold for an arbitrary sequencing situation.

Example 5 Let $N = \{1,2,3\}, \sigma_0 = (1,2,3), r = (0,0,1), p = (1,2,3)$ and $\alpha = (1,3,12)$. Then the optimal rearrangement of $\{1,2\}$is $(2,1,3)$, but the optimal rearrangement of N is $(1,3,2)$.

The next lemma shows that, for any tail, the urgency of player k is larger than or equal to the urgency of player l if player k takes the same position in the optimal rearrangement of the tail as player l in the optimal rearrangement of N.

Lemma 5 *Let (σ_0, r, p, α) be a sequencing situation such that $r_i \in \mathbf{N}$ and $p_i = 1$ for all $i \in N$. Let $i \in N$, $k \in F(i)$ and $l \in N$ be such that $\hat{\sigma}_{F(i)}(k) = \hat{\sigma}_N(l)$. Then $\alpha_k \geq \alpha_l$.*

PROOF:

The proof is by induction on the number of players. For $|N| = 2$ the lemma is trivial. Assume that for $|N| < m$ the lemma holds. Let $|N| = m$. We distinguish between three cases.

(i) $\hat{\sigma}_{F(i)}(n_0) \geq \hat{\sigma}_{F(i)}(k), \hat{\sigma}_N(n_0) \leq \hat{\sigma}_N(l)$.

It follows that $r_{n_0} \leq t_{\hat{\sigma}_N, l} = t_{\hat{\sigma}_{F(i)}, k}$ and hence $\alpha_k \geq \alpha_{n_0}$. Since $\hat{\sigma}_N(n_0) \leq \hat{\sigma}_N(l)$ we have that $\alpha_{n_0} \geq \alpha_l$. Hence $\alpha_k \geq \alpha_l$.

(ii) $\hat{\sigma}_{F(i)}(n_0) \geq \hat{\sigma}_{F(i)}(k), \hat{\sigma}_N(n_0) > \hat{\sigma}_N(l)$.

Lemma 4 yields that $\hat{\sigma}_{N\setminus\{n_0\}}(l) = \hat{\sigma}_N(l) = \hat{\sigma}_{F(i)}(k) = \hat{\sigma}_{F(i)\setminus\{n_0\}}(k)$. From the induction hypothesis it follows that $\alpha_k \geq \alpha_l$.

(iii) $\hat{\sigma}_{F(i)}(n_0) < \hat{\sigma}_{F(i)}(k)$

Using lemma 2 we have $\hat{\sigma}_N(n_0) \leq \hat{\sigma}_{F(i)}(n_0) < \hat{\sigma}_{F(i)}(k) = \hat{\sigma}_N(l)$. By lemma 4 we have that $\hat{\sigma}_{N\setminus\{n_0\}}(l) = \hat{\sigma}_N(l) - 1 = \hat{\sigma}_{F(i)}(k) - 1 = \hat{\sigma}_{F(i)\setminus\{n_0\}}(k)$. From the induction hypothesis it follows that $\alpha_k \geq \alpha_l$. □

An immediate consequence of lemma 5 is

Corollary 1 *Let (σ_0, r, p, α) be a sequencing situation such that $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$. Let $i \in N$ then*

$$\sum_{k:\ \hat{\sigma}_{F(i)}(k) > \hat{\sigma}_{F(i)}(n_0)} \alpha_k \geq \sum_{l:\ \hat{\sigma}_N(l) > \hat{\sigma}_{F(i)}(n_0)} \alpha_l$$

4 On the convexity of sequencing games

In this section it is shown that sequencing games that correspond to sequencing situations (σ_0, r, p, α) with $r_i \in N$ and $p_i = 1$ for all $i \in N$ are convex. First, we will give another expression of the value of a coalition S. Let $\sigma_0 = (i_1, ..., i_n)$ and assume that σ_{i_k} is the optimal rearrangement of $S \cap (P(i_k) \cup \{i_k\})$ within the restrict player set $P(i_k) \cup \{i_k\}$.

According to propostion 2 the optimal order $\sigma_{i_{k+1}}$ is obtained by inserting player i_{k+1} somewhere in the order σ_{i_k}. Clearly, if $i_{k+1} \notin S$, then $\sigma_{i_{k+1}}(i_{k+1}) = k+1$ otherwise, $\sigma_{i_{k+1}}$ can be obtained by consecutive switches of i_{k+1} with his immediate predecessor. The costsavings contributed by a neighbour switch with a player m is equal to $\alpha_{i_{k+1}} - \alpha_m$. Since $\hat{\sigma}_S$ can be constructed by subsequently rearranging (part of) S in the restricted player set $P(i_k) \cup \{i_k\}$ for $k = 1, 2, ..., n$, one derives

$$v(S) = \sum_{i \in S} \sum_{k \in P(i) \cap F(\hat{\sigma}_S, i)} (\alpha_i - \alpha_k). \tag{4}$$

In the following lemma we give an expression for the difference of the values of a tail and the same tail where player n_0 is excluded. Let $\overline{F(i)} := F(i) \cup \{i\}$ be the tail of player i.

Lemma 6 *Let (σ_0, r, p, α) be a sequencing situation such that $r_j \in \mathbf{N}$ and $p_j = 1$ for all $j \in N$ and let (N, v) be the corresponding sequencing game. Let $i \in N$ then*

$$v(\overline{F(i)}) - v(\overline{F(i)} \backslash \{n_0\}) = \sum_{k \in F(\hat{\sigma}_{\overline{F(i)}}, n_0)} (\alpha_{n_0} - \alpha_k).$$

PROOF:

$$\begin{aligned}
& v(\overline{F(i)}) - v(\overline{F(i)} \backslash \{n_0\}) \\
&= \sum_{l \in \overline{F(i)}} \sum_{k \in P(l) \cap F(\hat{\sigma}_{\overline{F(i)}}, l)} (\alpha_l - \alpha_k) - \sum_{l \in \overline{F(i)} \backslash \{n_0\}} \sum_{k \in P(l) \cap F(\hat{\sigma}_{\overline{F(i)} \backslash \{n_0\}}, l)} (\alpha_l - \alpha_k) \\
&= \sum_{k \in P(n_0) \cap F(\hat{\sigma}_{\overline{F(i)}}, n_0)} (\alpha_{n_0} - \alpha_k) \\
&+ \sum_{l \in \overline{F(i)} \backslash \{n_0\}} \{ \sum_{k \in P(l) \cap F(\hat{\sigma}_{\overline{F(i)}}, l)} (\alpha_l - \alpha_k) - \sum_{k \in P(l) \cap F(\hat{\sigma}_{\overline{F(i)} \backslash \{n_0\}}, l)} (\alpha_l - \alpha_k) \} \\
&= \sum_{k \in F(\hat{\sigma}_{\overline{F(i)}}, n_0)} (\alpha_{n_0} - \alpha_k)
\end{aligned}$$

The first equality follows by (4) and the third by lemma 4. □

The restriction $(S, v_{|S})$ of the game (N, v) to the player set $S \subset N$ is defined by $v_{|S}(T) := v(T)$ for all $T \subset S$. The next lemma shows that the restriction $(S, v_{|S})$ of a sequencing game (N, v) arising from a sequencing situation with integer ready times and the processing times all equal to one is again such a sequencing game if the coalition S is connected. Since the proof of this lemma is straightforward it is omitted.

Lemma 7 *Let (σ_0, r, p, α) be a sequencing situation such that $r_i \in \mathbf{N}$ and $p_i = 1$ for all $i \in N$ and let (N, v) be the corresponding sequencing game. If the coalition S is connected with respect to σ_0, then the game $(S, v_{|S})$ is the corresponding sequencing game corresponding to the sequencing situation*

$(\overline{\sigma_0}, (\overline{r_i})_{i\in S}, (p_i)_{i\in S}, (\alpha_i)_{i\in S})$

where the bijection $\overline{\sigma_0} : S \to \{1, ..., | S |\}$ *is defined by*

$\overline{\sigma_0}(i) = \sigma_0(i) + 1 - \min_{j\in S} \sigma_0(j)$ *for all* $i \in S$ *and* $\overline{r}_i = \max\{r_i, \min_{j\in S} t_{\sigma_0,j}\}$ *for all* $i \in S$.

Now we can formulate

Theorem 1 *Let* (σ_0, r, p, α) *be a sequencing situation such that* $r_i \in \mathbf{N}$ *and* $p_i = 1$ *for all* $i \in N$ *and let* (N, v) *be the corresponding sequencing game. Then* (N, v) *is convex.*

PROOF:

The proof is by induction on the number of players. Obviously, if $| N |= 1$, v is convex. Assume that for $| N |< n$ the game v is convex. Let $| N |= n$.
Let $i \in N, S \in 2^N$ and $T \in 2^N$ be such that $S \subset T \subset N\backslash\{i\}$. We have to prove that

$$v(T \cup \{i\}) - v(T) \geq v(S \cup \{i\}) - v(S) \tag{5}$$

(a) Suppose there exists a player $j \in N, j \neq i$ such that $j \notin T$.
If $\{i\}$ is a component of $T\cup\{i\}$ then $\{i\}$ is also a component of $S\cup\{i\}$ and, consequently, $v(T \cup \{i\}) = v(T)$ and $v(S \cup \{i\}) = v(S)$. So in this case (5) is trivial.
If $\{i\}$ is not a component of $T \cup \{i\}$ there exist $T_1 \subset T$ and $T_2 \subset T$ such that $T_1 \neq \emptyset$ and $v(T \cup \{i\}) - v(T) = v(T_1 \cup \{i\} \cup T_2) - v(T_1 \cup T_2)$. Moreover, there are $S_1 \subset T_1$ and $S_2 \subset T_2$ such that $v(S \cup \{i\}) - v(S) = v(S_1 \cup \{i\} \cup S_2) - v(S_1 \cup S_2)$. Hence, it suffices to show that

$$v(T_1 \cup \{i\} \cup T_2) - v(T_1 \cup T_2) \geq v(S_1 \cup \{i\} \cup S_2) - v(S_1 \cup S_2) \tag{6}$$

Then, since $T_1 \cup \{i\} \cup T_2 \subset N\backslash\{j\}$, lemma 7 and the induction hypothesis imply that $(T_1 \cup \{i\} \cup T_2, v_{|T_1\cup\{i\}\cup T_2})$ is a convex sequencing game and therefore (6) is satisfied.

(b) Hence, we may assume that $T = N\backslash\{i\}$. Moreover, (a) implies also that it is sufficient to prove that

$$v(N) - v(N\backslash\{i\}) \geq v(N\backslash\{j\}) - v(N\backslash\{i,j\}) \text{ for all } i,j \in N$$

W.l.o.g. we assume that $\sigma_0(i) < \sigma_0(j)$. Then

$$\begin{aligned}&v(N) - v(N\backslash\{i\}) - v(N\backslash\{j\}) + v(N\backslash\{i,j\})\\&= v(N) - v(P(i)) - v(F(i)) - v(P(j)) - v(F(j))\\&+v(P(i)) + v(F(j)) + v(P(j)\backslash P(i+1))\end{aligned}$$

$$= v(N) - v(P(j)) + v(P(j)\backslash P(i+1)) - v(F(i))$$

Two cases are distinguished.

(i) $j = n_0$

Then we have

$$\begin{aligned}
& v(N) - v(N\backslash\{n_0\}) + v(F(i)\backslash\{n_0\}) - v(F(i)) \\
&= \sum_{l \in F(\hat{\sigma}_N, n_0)} (\alpha_{n_0} - \alpha_l) - \sum_{k \in F(\hat{\sigma}_{F(i)}, n_0)} (\alpha_{n_0} - \alpha_k) \\
&= \sum_{l:\ \hat{\sigma}_N(n_0) < \hat{\sigma}_N(l) \leq \hat{\sigma}_{F(i)}(n_0)} (\alpha_{n_0} - \alpha_l) \\
&\quad + \sum_{l:\ \hat{\sigma}_N(l) > \hat{\sigma}_{F(i)}(n_0)} (\alpha_{n_0} - \alpha_l) - \sum_{k:\ \hat{\sigma}_{F(i)}(k) > \hat{\sigma}_{F(i)}(n_0)} (\alpha_{n_0} - \alpha_k) \\
&= \sum_{l:\ \hat{\sigma}_N(n_0) < \hat{\sigma}_N(l) \leq \hat{\sigma}_{F(i)}(n_0)} (\alpha_{n_0} - \alpha_l) \\
&\quad - \sum_{l:\ \hat{\sigma}_N(l) > \hat{\sigma}_{F(i)}(n_0)} \alpha_l + \sum_{k:\ \hat{\sigma}_{F(i)}(k) > \hat{\sigma}_{F(i)}(n_0)} \alpha_k \geq 0
\end{aligned}$$

where the first equality follows from lemma 6 and the inequality follows from lemma 3 and corollary 1.

(ii) $j < n_0$

The induction hypotheses implies that the game $(N\backslash\{n_0\}, v_{|(N\backslash\{n_0\})})$ is convex. Then the convexity condition (1) with $S = F(i)\backslash\{n_0\}$ and $T = P(j)$ yields

$$v(N\backslash\{n_0\}) + v(P(j)\backslash P(i+1)) - v(F(i))\backslash\{n_0\}) - v(P(j)) \geq 0 \tag{7}$$

Then (7) and (i) imply

$$\begin{aligned}
& v(N) - v(P(j)) + v(P(j)\backslash P(i+1)\}) - v(F(i)) \\
&= [v(N\backslash\{n_0\}) + v(P(j)\backslash P(i+1)) - v(F(i))\backslash\{n_0\}) - v(P(j))] \\
&\quad + [v(N) - v(N\backslash\{n_0\}) + v(F(i)\backslash\{n_0\}) - v(F(i))] \ \geq \ 0
\end{aligned}$$

□

The following example shows that in general sequencing games are not necessary convex.

Example 6 Let $N = \{1,2,3\}, \sigma_0 = (1,2,3), r = (0,0,1), p = (1,2,3)$ and $\alpha = (1,3,12)$. The optimal rearrangement of N is $(1,3,2)$ and, consequently, $v(N) = 15$. Obviously the optimal rearrangement of $\{2,3\}$ is $(1,3,2)$ and so $v(\{2,3\}) = 15$. Since $v(\{1,2\}) = 1$ and $v(\{2\}) = 0$ we have that

$$v(N) - v(\{2,3\}) = 0 < 1 = v(\{1,2\}) - v(\{2\}).$$

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