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# On Generalized $\phi$-Recurrent LP-Sasakian Manifolds

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Abstract. In this paper we studied generalized $\phi$-recurrent and generalized concircular $\phi$-recurrent LP-Sasakian manifolds.

## 1. Introduction

In 1977, T. Takahashi[4], introduced the notion of locally $\phi$-symmetric Sasakian manifolds and obtained some interesting properties. Some authors like U.C. De and G. Pathak[8],Venkatesha and C.S. Bagewadi[9], A.A. Shaikh and U.C. De[10] have extended this notion to 3-dimentional Kenmotsu, trans-Sasakian, LP-Sasakian manifolds respectively.

In present paper we studied generalized $\phi$-recurrent and generalized concircular $\phi$-recurrent LP-Sasakian manifolds. The paper is organized as follows: In section 2, we give a brief account of LP-Sasakian manifolds. In section 3, we studied generalized $\phi$-recurrent LP-Sasakian manifolds. At first it is shown that a generalized $\phi$-recurrent LP-Sasakian manifold is an Einstein manifold. Then we have shown that in a generalized $\phi$-recurrent LP-Sasakian manifold the characteristic vector field $\xi$ and the vector field $\rho + \sigma$ associated with 1-form $\alpha + \beta$ are in opposite direction. In section 4, we studied generalized concircular $\phi$-recurrent LP-Sasakian manifolds. We first obtain the relation between the 1-forms $\alpha$ and $\beta$, then we prove that it is an Einstein manifold. Finally we have shown that the characteristic vector field $\xi$ and the vector fields $\rho$, $\sigma$ associated to the 1-forms $\alpha$, $\beta$ respectively are in opposite direction.

## 2. Preliminaries

An $n$-dimensional differentiable manifold $M_n$ is called an LP-Sasakian manifolds [1], [2] if it admitts a (1,1) tensor field $\phi$, a vector field $\xi$, a 1-form $\eta$ and a Lorentzian metric $g$ which satisfy

$$\eta(\xi) = -1, \phi^2(X) = X + \eta(X)\xi, \tag{1}$$

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$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2}$$

$$g(X, \xi) = \eta(X), \bigtriangledown_X \xi = \phi X, \tag{3}$$

$$(\bigtriangledown_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{4}$$

where $\bigtriangledown$ denotes the operator of covariant differentiation with respect to the Lorentzian metric $g$.

It can easily be seen that in an LP-Sasakian manifold, the following relations hold

$$\phi\xi = 0, \eta(\phi X) = 0, \tag{5}$$

$$rank(\phi) = n - 1. \tag{6}$$

Further in an LP-Sasakian manifold the following relations also hold[2],[3]

$$\begin{aligned} &(a) \quad R(X, Y, \xi) = \eta(Y)X - \eta(X)Y, \\ &(b) \quad R(\xi, X, Y) = g(X, Y)\xi - \eta(Y)X, \end{aligned} \tag{7}$$

$$S(X, \xi) = (n - 1)\eta(X), \tag{8}$$

$$R(X, \xi, \xi) = -X - \eta(X)\xi, \tag{9}$$

$$\eta(R(X, Y, Z)) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{10}$$

$$S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y), \tag{11}$$

$$(\bigtriangledown_X \eta)(Y) = g(X, \phi Y) = g(\phi X, Y), \tag{12}$$

for any vector fields $X$, $Y$, $Z$. where $R(X, Y, Z)$ is the curvature tensor, $S$ is the Ricci tensor.

## 3. Generalized $\phi$-recurrent LP-Sasakian Manifolds.

Analogous of consideration of generalized recurrent manifolds[5], we give the following definition.

**Definition 3.1.** An LP-Sasakian manifold is called generalized $\phi$-recurrent if its curvature tensor $R$ satisfies the condition

$$\phi^2((\bigtriangledown_W R)(X, Y, Z)) = \alpha(W)R(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y], \tag{13}$$

where $\alpha$ and $\beta$ are two 1-forms, $\beta$ is non-zero and these are defined by

$$\alpha(W) = g(W, \rho), \beta(W) = g(W, \sigma), \tag{14}$$

and $\rho$, $\sigma$ are vector fields associated with 1-forms $\alpha$, $\beta$ respectively.

If the 1-form $\beta$ in (13) becomes zero, then the manifold reduces to a $\phi$-recurrent LP-Sasakian manifold. $\phi$-recurrent LP-Sasakian manifolds have been studied by Al-Aqeel, De and Ghosh[13].

From (13) using (1), we have

$$\begin{aligned}(\nabla_W R)(X, Y, Z) + \eta((\nabla_W R)(X, Y, Z))\xi \\ = \alpha(W)R(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y],\end{aligned} \tag{15}$$

from which it follows that

$$\begin{aligned}&g((\nabla_W R)(X, Y, Z), U) + \eta((\nabla_W R)(X, Y, Z))\eta(U) \\ &= \alpha(W)g(R(X, Y, Z), U) + \beta(W)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)].\end{aligned} \tag{16}$$

Let $\{e_i\}$, $i = 1, 2, ....., n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (16) and taking summation over $i$, $1 \leqslant i \leqslant n$ , we get

$$\begin{aligned}(\nabla_W S)(Y, Z) + \sum_{i=1}^{n} \eta((\nabla_W R)(e_i, Y, Z))\eta(e_i) \\ = \alpha(W)S(Y, Z) + (n-1)\beta(W)g(Y, Z).\end{aligned} \tag{17}$$

The second term of L.H.S. in (17) by putting $Z = \xi$ assumes the form

$$g((\nabla_W R)(e_i, Y, \xi), \xi)g(e_i, \xi),$$

which is denoted by $E$. In this case $E$ vanishes. Namely we have

$$\begin{aligned}g((\nabla_W R)(e_i, Y, \xi), \xi) =& g(\nabla_W R(e_i, Y, \xi), \xi) - g(R(\nabla_W e_i, Y, \xi), \xi) \\ &- g(R(e_i, \nabla_W Y, \xi), \xi) - g(R(e_i, Y, \nabla_W \xi), \xi),\end{aligned}$$

at $p \in M$. Since $\{e_i\}$ is an orthonormal basis, so $\nabla_X e_i = 0$ at p. Using (7), we obtain

$$g(R(e_i, \nabla_W Y, \xi), \xi) = g(\nabla_W Y, \xi)g(e_i, \xi) - g(\xi, e_i)g(\nabla_W Y, \xi) = 0.$$

Thus we obtain

$$g((\nabla_W R)(e_i, Y, \xi), \xi) = g(\nabla_W R(e_i, Y, \xi), \xi) - g(R(e_i, Y, \nabla_W \xi), \xi).$$

In virtue of $g(R(e_i, Y, \xi), \xi) = g(R(\xi, \xi, Y), e_i) = 0$, we have

$$g(\nabla_W R(e_i, Y, \xi), \xi) + g(R(e_i, Y, \xi), \nabla_W \xi) = 0,$$

which implies

$$g((\bigtriangledown_W R)(e_i, Y, \xi), \xi) = -g(R(e_i, Y, \xi), \bigtriangledown_W \xi) - g(R(e_i, Y, \bigtriangledown_W \xi), \xi).$$

Hence we reach

$$\begin{aligned} E &= -\sum_{i=1}^{n}\{g(R(\phi W, \xi, Y), e_i)g(\xi, e_i) + g(R(\xi, \phi W, Y), e_i)g(\xi, e_i)\} \\ &= -\{g(R(\phi W, \xi, Y), \xi) + g(R(\xi, \phi W, Y), \xi)\} = 0. \end{aligned}$$

Replacing $Z$ by $\xi$ in (17) and using (8), we get

$$(\bigtriangledown_W S)(Y, \xi) = (n-1)\alpha(W)\eta(Y) + (n-1)\beta(W)\eta(Y). \tag{18}$$

Now, we have

$$(\bigtriangledown_W S)(Y, \xi) = \bigtriangledown_W S(Y, \xi) - S(\bigtriangledown_W Y, \xi) - S(Y, \bigtriangledown_W \xi).$$

Using (8) and (3) in the above relation, it follows that

$$(\bigtriangledown_W S)(Y, \xi) = (n-1)g(W, \phi Y) - S(Y, \phi W). \tag{19}$$

In view of (18) and (19), we obtain

$$(n-1)g(W, \phi Y) - S(Y, \phi W) = (n-1)\alpha(W)\eta(Y) + (n-1)\beta(W)\eta(Y). \tag{20}$$

Replacing $Y$ by $\phi Y$ in (20) and then using (1), (5) and (11), we get

$$S(Y, W) = (n-1)g(Y, W), \tag{21}$$

for all $Y, W$. This leads the following.

**Theorem 3.1.** *A generalized $\phi$-recurrent LP-Sasakian manifold is an Einstein manifold.*

Two vector fields $P$ and $Q$ are said to be codirectional if $P = fQ$, where $f$ is a non-zero scalar. That is $g(P, X) = fg(Q, X)$ for all $X$.

Now from (13), we have

$$\begin{aligned} (\bigtriangledown_W R)(X, Y, Z) = &- \eta((\bigtriangledown_W R)(X, Y, Z))\xi + \alpha(W)R(X, Y, Z) \\ &+ \beta(W)[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \tag{22}$$

Then by use of second Bianchi's identity and (22), we get

$$\begin{aligned} \alpha(W)\eta(R(X, Y, Z)) + \alpha(X)\eta(R(Y, W, Z)) &+ \alpha(Y)\eta(R(W, X, Z)) \\ &+ \beta(W)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\ &+ \beta(X)[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\ &+ \beta(Y)[g(X, Z)\eta(W) - g(W, Z)\eta(X)] = 0. \end{aligned} \tag{23}$$

By virtue of (10), we obtain from (23)

$$\begin{aligned}(24)\qquad &[\alpha(W)+\beta(W)][g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\\ &+[\alpha(X)+\beta(X)][g(W,Z)\eta(Y)-g(Y,Z)\eta(W)]\\ &+[\alpha(Y)+\beta(Y)][g(X,Z)\eta(W)-g(W,Z)\eta(X)]=0.\end{aligned}$$

Putting $Y=Z=e_i$ in (24) and taking summation over $i$, $1\leqslant i\leqslant n$, we get

$$(25)\qquad \{\alpha(W)+\beta(W)\}\eta(X)=\{\alpha(X)+\beta(X)\}\eta(W),$$

for all vector fields $X,W$.
Replacing $X$ by $\xi$ in (25), it follows that

$$(26)\qquad \alpha(W)+\beta(W)=-\eta(W)[\eta(\rho)+\eta(\sigma)],$$

for any vector field $W$, where $\alpha(\xi)=g(\xi,\rho)=\eta(\rho)$ and $\beta(\xi)=g(\xi,\sigma)=\eta(\sigma)$.

From (25) and (26) we can state that the following.

**Theorem 3.2.** *In a generalized $\phi$-recurrent LP-Sasakian manifold the characteristic vector field $\xi$ and the vector field $\rho+\sigma$ associated to the 1-form $\alpha+\beta$ are in opposite direction.*

In view of (3) and (7) it can be easily seen that in an LP-Sasakian manifold the following relation holds

$$(27)\qquad (\bigtriangledown_W R)(X,Y,\xi)=g(W,\phi Y)X-g(W,\phi X)Y-R(X,Y,\phi W).$$

By virtue of (10), it follows from (27), that

$$(28)\qquad \eta((\bigtriangledown_W R)(X,Y,\xi))=0.$$

Now assume that $X$, $Y$ and $Z$ are (local) vector fields such that $(\bigtriangledown X)_p=(\bigtriangledown Y)_p=(\bigtriangledown Z)_p=0$ for a fixed point $p$ of $M_n$. By Ricci identity for $\phi$[6]

$$-(R(X,Y).\phi Z)=(\bigtriangledown_X\bigtriangledown_Y\phi)Z-(\bigtriangledown_Y\bigtriangledown_X\phi)Z.$$

We have at the point $p$,

$$-R(X,Y,\phi Z)+\phi R(X,Y,Z)=\bigtriangledown_X((\bigtriangledown_Y\phi)Z)-\bigtriangledown_Y((\bigtriangledown_X\phi)Z).$$

Using (4), we have

$$\begin{aligned}-R(X,Y,\phi Z)&+\phi R(X,Y,Z)\\ &=\bigtriangledown_X\{g(Y,Z)\xi+\eta(Z)Y+2\eta(Y)\eta(Z)\xi\}\\ &\quad-\bigtriangledown_Y\{g(X,Z)\xi+\eta(Z)X+2\eta(X)\eta(Z)\xi\}\\ &=g(Y,Z)\bigtriangledown_X\xi+(\bigtriangledown_X\eta)(Z)Y+2(\bigtriangledown_X\eta)(Y)\eta(Z)\xi\\ &\quad+2(\bigtriangledown_X\eta)(Z)\eta(Y)\xi+2\eta(Y)\eta(Z)\bigtriangledown_X\xi\\ &\quad-g(X,Z)\bigtriangledown_Y\xi-(\bigtriangledown_Y\eta)(Z)X-2(\bigtriangledown_Y\eta)(X)\eta(Z)\xi\\ &\quad-2(\bigtriangledown_Y\eta)(Z)\eta(X)\xi-2\eta(X)\eta(Z)\bigtriangledown_Y\xi.\end{aligned}$$

In view of (3) and (12) the above equation becomes

$$R(X,Y,\phi Z) = -g(\phi X,Z)Y - g(Y,Z)\phi X + g(\phi Y,Z)X + g(X,Z)\phi Y - 2g(X,\phi Z)\eta(Y)\xi - 2\eta(Y)\eta(Z)\phi X + 2g(Y,\phi Z)\eta(X)\xi + 2\eta(X)\eta(Z)\phi Y + \phi R(X,Y,Z), \tag{29}$$

for any $X$, $Y$, $Z \in T_pM$. From (27) and (29), it follows that

$$(\bigtriangledown_W R)(X,Y,\xi) = g(Y,W)\phi X - g(X,W)\phi Y + 2g(X,\phi W)\eta(Y)\xi + 2\eta(Y)\eta(W)\phi X - 2g(Y,\phi W)\eta(X)\xi - 2\eta(X)\eta(W)\phi Y - \phi R(X,Y,W). \tag{30}$$

In view of (15) and (28), we obtain from (30)

$$\alpha(W)R(X,Y,\xi) + \beta(W)[g(Y,\xi)X - g(X,\xi)Y] = g(Y,W)\phi X - g(X,W)\phi Y + 2g(X,\phi W)\eta(Y)\xi + 2\eta(Y)\eta(W)\phi X - 2g(Y,\phi W)\eta(X)\xi - 2\eta(X)\eta(W)\phi Y - \phi R(X,Y,W).$$

In view of (7) and (26), the above equation becomes

$$-\eta(W)[\eta(\rho)+\eta(\sigma)][\eta(Y)X - \eta(X)Y] = g(Y,W)\phi X - g(X,W)\phi Y + 2g(X,\phi W)\eta(Y)\xi + 2\eta(Y)\eta(W)\phi X - 2g(Y,\phi W)\eta(X)\xi - 2\eta(X)\eta(W)\phi Y - \phi R(X,Y,W). \tag{31}$$

Hence if $X$ and $Y$ are orthogonal to $\xi$, then (31) reduces to

$$\phi R(X,Y,W) = g(Y,W)\phi X - g(X,W)\phi Y. \tag{32}$$

operating $\phi$ on both sides of (32) and using(1), we get

$$R(X,Y,W) = g(Y,W)X - g(X,W)Y.$$

This leads the following.

**Theorem 3.3.** *A generalized $\phi$-recurrent LP-Sasakian manifold is a space of constant curvature provided that $X$ and $Y$ are orthogonal to $\xi$.*

## 4. Generalized concircular $\phi$-recurrent LP-Sasakian manifolds.

Analogous of consideration of generalized recurrent manifolds[5], we give the following definition.

**Definition 4.1.** An LP- Sasakian manifold is called generalized concircular $\phi$-recurrent if its concircular curvature tensor $C$,

$$C(X,Y,Z) = R(X,Y,Z) - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] \tag{33}$$

satisfies the condition

$$\phi^2((\bigtriangledown_W C)(X,Y,Z)) = \alpha(W)C(X,Y,Z) + \beta(W)[g(Y,Z)X - g(X,Z)Y], \tag{34}$$

where $\alpha$ and $\beta$ are defined as (14) and $r$ is the scalar curvature.

If the 1-form $\beta$ in (34) becomes zero, then the manifold reduces to a concircular $\phi$-recurrent manifolds. Concircular $\phi$-recurrent LP-Sasakian manifolds has been studied by Venkatesha and Bagewadi[14].

Let us consider a generalized concircular $\phi$-recurrent LP-Sasakian manifold. Then by virtue of (1) and (34), we have

$$\begin{aligned}(\bigtriangledown_W C)(X,Y,Z) + \eta((\bigtriangledown_W C)(X,Y,Z))\xi \\ =\alpha(W)C(X,Y,Z) + \beta(W)[g(Y,Z)X - g(X,Z)Y],\end{aligned} \tag{35}$$

from which it follows that

$$\begin{aligned}g((\bigtriangledown_W C)(X,Y,Z),U) + \eta((\bigtriangledown_W C)(X,Y,Z))\eta(U) \\ =\alpha(W)g(C(X,Y,Z),U) + \beta(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)].\end{aligned} \tag{36}$$

Let $\{e_i\}$, $i = 1,2,.....,n$ be orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (36) and taking summation over $i$, $1 \leqslant i \leqslant n$, we get

$$\begin{aligned}(\bigtriangledown_W S)(X,U) - \frac{W(r)}{n}g(X,U) + (\bigtriangledown_W S)(X,\xi)\eta(U) - \frac{W(r)}{n}\eta(X)\eta(U) \\ = \alpha(W)[S(X,U) - \frac{r}{n}g(X,U)] + (n-1)\beta(W)g(X,U).\end{aligned} \tag{37}$$

Replacing $U$ by $\xi$ in (37) then using (1) and (8), we have

$$\alpha(W)[(n-1) - \frac{r}{n}]\eta(X) + (n-1)\beta(W)\eta(X) = 0. \tag{38}$$

Putting $X = \xi$ in (38), we obtain

$$[(n-1) - \frac{r}{n}]\alpha(W) + (n-1)\beta(W) = 0. \tag{39}$$

**Theorem 4.1.** *Let $(M_n, g)$ be a generalized concircular $\phi$-recurrent LP-Sasakian manifold then*

$$[(n-1) - \frac{r}{n}]\alpha(W) + (n-1)\beta(W) = 0.$$

Now putting $X = U = e_i$ in (36) and taking summation over $i$, $1 \leqslant i \leqslant n$, we get

$$(40) \qquad (\nabla_W S)(Y,Z) = -\sum_{i=1}^{n} g((\nabla_W R)(e_i,Y,Z),\xi)g(e_i,\xi) + \frac{W(r)}{n}g(Y,Z) - \frac{W(r)}{n(n-1)}[g(Y,Z)+\eta(Y)\eta(Z)] + \alpha(W)[S(Y,Z) - \frac{r}{n}g(Y,Z)] + (n-1)\beta(W)g(Y,Z).$$

Replacing $Z$ by $\xi$ in (40) and using (38), we have

$$(41) \qquad (\nabla_W S)(Y,\xi) = \frac{W(r)}{n}\eta(Y).$$

Now, we have

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi).$$

Using (3) and (8) in the above relation, it follows that

$$(42) \qquad (\nabla_W S)(Y,\xi) = (n-1)g(\phi Y,W) - S(\phi Y,W).$$

In view of (41) and (42)

$$(43) \qquad S(Y,\phi W) = (n-1)g(Y,\phi W) - \frac{W(r)}{n}\eta(Y).$$

Replacing $Y$ by $\phi Y$ in (43) then using (2),(5) and (11), we obtain

$$(44) \qquad S(Y,W) = (n-1)g(Y,W).$$

This leads to the folowing theorem.

**Theorem 4.2.** *A generalized concircular $\phi$-recurrent LP-Sasakian manifold is an Einstein manifold.*

Now from (35), we have

$$(45) \qquad (\nabla_W C)(X,Y,Z) = \alpha(W)C(X,Y,Z) + \beta(W)[g(Y,Z)X - g(X,Z)Y] - \eta((\nabla_W C)(X,Y,Z))\xi.$$

This implies

$$(46) \qquad (\nabla_W R)(X,Y,Z) = -\eta((\nabla_W R)(X,Y,Z))\xi + \alpha(W)R(X,Y,Z) + \frac{W(r)}{n(n-1)}[g(Y,Z)X - g(X,Z)Y + g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi] - [\frac{r}{n(n-1)}\alpha(W) - \beta(W)] [g(Y,Z)X - g(X,Z)Y].$$

Now from (46) and Bianchi's second identity, we have

$$\tag{47} \begin{aligned} &\alpha(W)\eta(R(X,Y,Z)) + \alpha(X)\eta(R(Y,W,Z)) + \alpha(Y)\eta(R(W,X,Z)) \\ &= -\{\beta(W) - \frac{r}{n(n-1)}\alpha(W)\}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ &\quad -\{\beta(X) - \frac{r}{n(n-1)}\alpha(X)\}[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ &\quad -\{\beta(Y) - \frac{r}{n(n-1)}\alpha(Y)\}[g(X,Z)\eta(W) - g(W,Z)\eta(X)]. \end{aligned}$$

By virtue of (10), we obtain from (47), that

$$\tag{48} \begin{aligned} &\alpha(W)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ &+\alpha(X)[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ &+\alpha(Y)[g(X,Z)\eta(W) - g(W,Z)\eta(X)] \\ &= -\{\beta(W) - \frac{r}{n(n-1)}\alpha(W)\}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ &\quad -\{\beta(X) - \frac{r}{n(n-1)}\alpha(X)\}[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ &\quad -\{\beta(Y) - \frac{r}{n(n-1)}\alpha(Y)\}[g(X,Z)\eta(W) - g(W,Z)\eta(X)]. \end{aligned}$$

Putting $Y = Z = e_i$ in (48) and taking summation over $i$, $1 \leqslant i \leqslant n$, we get

$$\tag{49} \begin{aligned} &(a)\quad \alpha(W)\eta(X) = \alpha(X)\eta(W), \\ &(b)\quad \beta(W)\eta(X) = \beta(X)\eta(W), \end{aligned}$$

for all vector fields $X$, $W$. Replacing $X$ by $\xi$ in (49), we get

$$\tag{50} \begin{aligned} &(a)\quad \alpha(W) = -\eta(W)\eta(\rho), \\ &(b)\quad \beta(W) = -\eta(W)\eta(\sigma), \end{aligned}$$

for all vector field $W$.
From (49) and (50), we can state the following.

**Theorem 4.3.** *In a generalized concircular $\phi$-recurrent LP-Saskian manifold the characteristic vector field $\xi$ and the vector fields $\rho$, $\sigma$ associated to the 1-forms $\alpha$, $\beta$ respectively are in opposite direction and the 1-forms $\alpha$, $\beta$ are given by* (50).

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