Mon. Not. R. astr. Soc. (1969) **146**, 423–444.

ON H II REGIONS AND PULSAR DISTANCES*

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(Received 1969 July 11)

SUMMARY

In this paper we first of all give a list of all clusters and associations within 1 kpc of the Sun (Table III) and all hot O-stars and supergiants which do not belong to clusters (Table IV). We also calculate the radius of the H II regions surrounding these objects. We then find statistically the distribution of stars of a given spectral class which have not been covered so far and we evaluate statistically the contributions from these stars to H II regions. We briefly discuss the level of ionization due to cosmic rays to be found in H I regions. We combine the results to derive from the observed dispersions the distances of pulsars from the Sun. In this evaluation we look first for the possibility that the line-of-sight to a pulsar passes through the H II region of one of the objects in Tables III and IV; this is the case certainly for six pulsars and possibly for another seven. Of the remaining 24 pulsars, we did not have dispersion data for one, and the others were discussed, using the expressions for the statistical contributions from H II and H I regions, taking into account the fall-off in density with height above the galactic plane. We find that, indeed, most pulsars lie within 1 kpc from the Sun. In fact, of the 36 pulsar distances given in our Table VII, 18 are less than 1 kpc, seven less than 2 kpc (of which two may well lie within 1 kpc), and of the remaining 11, eight may well lie within 1 kpc. The uncertainty in some of our distance determinations lies mainly in our lack of knowledge of the galactic structure at a height of more than about 500 pc from the galactic plane. It is felt that our discussion of possible contributions to dispersion of radio waves may also be useful for investigations other than pulsar distances. As a by-product of our calculations we found a value of 0·04 cm^{-3} as the median value of the electron density in the galactic plane in the neighbourhood of the Sun.

I. INTRODUCTION

It is well known that from the dispersion of the pulsar signals one obtains $\int n_e\, dl$ where the integration is along the line of sight from the pulsar to us. If, therefore, we knew n_e as a function of galactic position, the distances of the pulsars could be obtained. As a first estimate, many people have used a value of 0·1 cm^{-3} for n_e, and using this value one finds that most pulsars lie within about 1 kpc from the Sun. It should, therefore, be possible to do better than just use an average value for n_e as the positions of the pulsars in the sky are quite well known and we can use our knowledge about stars along the line of sight and about the general distribution of hydrogen in the neighbourhood of the Sun to evaluate to a fair approximation n_e along the line of sight and thus to estimate fairly accurately at what point the observed $\int n_e\, dl$ runs out of distance. It is clear that, on the one hand, for a line of sight passing through an H II region the average value of n_e will be much higher than 0·1 cm^{-3} while, on the other hand, the average value in H I regions will be considerably lower.

* A preliminary account of this work was published in *Nature* (Prentice & ter Haar 1969).

In the present paper we are aiming to use data about the local galactic structure to determine the distances of all those pulsars about which reliable values of $\int n_e \, dl$ are available. Our programme is the following one. In the next section we give an expression for the Strömgren radius of main sequence stars in terms of their temperature only and discuss what to do for other stars. In Section 3 we first of all consider the local distribution of O- and B-stars, paying particular attention to clusters and associations and also to supergiants. We give a list of all known clusters and associations within 1 kpc from us with an earliest member not later than B5. We also give a list of all O to B5 supergiants and O-type stars earlier than O9·5 which lie within 1 kpc from us and have not been included in the cluster list. Finally, we consider in Section 3 main sequence stars and giants cooler than O9 which are not members of clusters and associations. We use statistical arguments to consider these objects. In Section 4 we consider the contribution to $\int n_e \, dl$ from H I regions. We assume that the neutral hydrogen density falls off exponentially with height above the galactic plane and give reasons for that assumption. In Section 5 we consider in turn the three possible contributions to $\int n_e \, dl$: (i) the statistical contribution from the H II regions produced by all stars except the stars in clusters or associations and the hot supergiants and O-stars; (ii) the statistical contribution from H I regions; and (iii) the contribution from single hot stars or clusters. We then use all these data to estimate the distances from us of all known pulsars. These distances are given and it is found that the model discussed in Section 4 seems to fit the pulsar data satisfactorily. We briefly discuss the results obtained in Section 6.

2. STRÖMGREN SPHERES

Strömgren (1936) has shown that if we neglect re-radiation from ionized hydrogen, a star of effective temperature T (in °K) and radius R (in solar radii) will be surrounded by an H II region, that is, a sphere of ionized hydrogen, of radius S_o (in pc) where S_o is given by the equation (log is the logarithm to the base 10)

$$\log (S_o N^{2/3}) = -0{\cdot}44 - 4{\cdot}51 \, \frac{5040}{T} + \tfrac{1}{2} \log T + \tfrac{2}{3} \log R$$

where N (in cm^{-3}) is the hydrogen number density in the Strömgren sphere.

If M_v, M_{bol}, and $B.C.$ are, respectively, the absolute visual magnitude, the absolute bolometric magnitude and the bolometric correction, we have (Allen 1964, p. 190)

$$\begin{aligned} 5 \log R &= 96{\cdot}56 - 10 \log T - M_{\mathrm{bol}} \\ &= 96{\cdot}56 - 10 \log T - M_v + B.C., \end{aligned}$$

and hence

$$\log (S_o N^{2/3}) = 5{\cdot}21 - \frac{22730}{T} - \tfrac{5}{6} \log T + \tfrac{2}{15} B.C. - \tfrac{2}{15} M_v. \qquad (1)$$

To use equation (1), we need to know T, $B.C.$, and M_v for various stars. This can only be done if we have reliable models for stellar atmospheres to relate the luminosity and spectral class to T, $B.C.$, and M_v. We use the calculations by Morton & Adams (1968) and give their values of T and $B.C.$ in columns 2 and 3 of Table I. The columns 4 to 10 give the absolute magnitudes for the various luminosity classes as given in Landolt–Börnstein (1965, p. 301). We have used Becvar's data (1964)

TABLE I

Values of the effective temperature, T, bolometric correction B.C., and absolute visual magnitude M_v for various luminosity classes and spectral types

Sp	*T*(°K)	*B.C.*	M_v V	IV	III	II	Ib	Iab	Ia
O5	37500	3·23	−5·6						
O6	36500	3·17	−5·4						
O7	35700	3·12	−5·2						
O8	35000	3·07	−5·0						
O9	34300	3·01	−4·7	−5·3	−5·7	−6·0	−6·1	−6·2	−6·2
O9·5	32100	2·85	−4·5	−5·0	−5·5	−5·8	−6·0	−6·2	−6·2
B0	30900	2·76	−4·2	−4·8	−5·0	−5·4	−5·8	−6·2	−6·2
B0·5	26200	2·36	−3·5	−4·1	−4·3	−5·1	−5·8	−6·2	−6·4
B1	22600	2·00	−2·9	−3·5	−3·8	−4·9	−5·7	−6·3	−6·6
B2	20500	1·77	−2·5	−3·1	−3·6	−4·8	−5·7	−6·3	−6·8
B3	17900	1·48	−1·7	−2·5	−3·1	−4·6	−5·7	−6·3	−6·8
B4	16800	1·34	−1·4	−2·2	−2·7	−4·5	−5·7	−6·3	−6·9
B5	15600	1·19	−1·0	−1·8	−2·2	−4·4	−5·7	−6·3	−7·0
B6	14600	1·05	−0·7	−1·5	−1·9	−4·2	−5·6	−6·4	−7·0
B7	13600	0·91	−0·4	−1·2	−1·6	−4·0	−5·6	−6·4	−7·1
B8	12000	0·69	0·0	−0·7	−1·0		−5·6	−6·5	−7·2
B9	10700	0·47	+0·5	−0·2	−0·4		−5·5	−6·5	−7·0

TABLE II

Values of Strömgren radii S_0 for various luminosity classes and spectral types for the case where the electron number density $N = 1\ cm^{-3}$

Sp	S_0 *pc* V	IV	III	II	Ib	Iab	Ia
O5	94	—	—	—	—	—	—
O6	85	—	—	—	—	—	—
O7	78	—	—	—	—	—	—
O8	72	—	—	—	—	—	—
O9	63	76	86	94	97	100	100
O9·5	54	63	73	80	85	91	91
B0	46	56	59	67	75	85	85
B0·5	28	34	36	46	56	64	68
B1	17	21	23	32	40	48	53
B2	12	14	17	24	32	39	45
B3	6·6	8·5	10	16	23	27	32
B4	5·0	6·4	7·5	13	19	23	27
B5	3·6	4·6	5·2	10	15	18	22
B6	2·6	3·4	3·8	7·7	12	15	18
B7	1·9	2·4	2·7	5·6	9·2	12	15
B8	1·0	1·3	1·4	—	5·7	7·5	9·3
B9	0·5	0·7	0·7	—	3·4	4·6	5·4

and those from the Yale Catalogue (1964) to divide B1 (Mt Wilson) into two subclasses, B0·5 and B1.

As there do not exist any self-consistent model atmosphere calculations, such as those of Morton and Adams, beyond the main sequence, we are adopting the *Sp–T* calibration of Table I for giants and supergiants as well, although we are well aware that, for fixed *Sp*, *T* decreases slightly with stellar mass, due to a weakening of the gravitational field at the surface, but we feel that this is of secondary importance for our applications.

In Table II we give the results of substituting the values for T, $B.C.$, and M_v from Table I into equation (1); the values of S_0 given in the table are for $N = 1\ \text{cm}^{-3}$. We feel that in view of the criticism raised (Davidson & Terzian 1969) against the S_0-values of Murdin & Sharpless (1968) and of Rubin (1968) our values are probably more reliable than theirs.

To obtain in Section 4 the contribution to $\int n_e\, dl$ from the H II regions around main sequence stars, we need a relation between S_0 and the effective temperature T. We do this by first fitting a straight line in the $B.C. - \log T$ diagram, using the data from Table I. The result is

$$B.C. = 0{\cdot}25 + 5{\cdot}07 \log (T/10^4).$$

Secondly, we fit a straight line in the $M_v - \log T$ diagram for the main sequence stars (luminosity class V) and the result is

$$M_v\,(\text{main sequence}) = 0{\cdot}87 - 10{\cdot}41 \log (T/10^4).$$

Substituting these results into equation (1) we finally get—for the main sequence stars only—

$$\log S_0 N^{2/3} = -3{\cdot}126 - 22730/T + 1{\cdot}230 \log T. \qquad (2)$$

If we wish to use equation (2) for other stars, we must add to the right-hand side a term $-(2/15)[M_v - M_v\,(\text{main sequence})]$.

3. THE LOCAL DISTRIBUTION OF H II REGIONS

In order to find the contribution to $\int n_e\, dl$ from the H II regions around hot stars we need to consider what O and B stars occur in the immediate neighbourhood of the Sun—which for our present purposes we define as a sphere of radius 1 kpc. We shall first consider the very hot O-stars. On the one hand, these are so few in number that they cannot be treated statistically and, on the other hand, it was pointed out by Ambartzumian (1949) that they tend to occur in loose groupings, the so-called O-associations, inside which there occur sometimes so-called clusters of B-stars having an O-type star as earliest member. Of course, not all O-type stars belong to a recognizable association or cluster.

Secondly, we must consider the O to B5 supergiants which, although of lower temperature than the hot O-stars, have a large luminosity and therefore, have large H II regions around them (see Table II, luminosity classes Ia, Iab, and Ib).

Thirdly and lastly, there are the relatively numerous and (in longitude) randomly distributed main sequence stars and giants of spectral type later than O9. These we shall treat statistically, and they provide a background level of small H II regions.

We shall consider these three groups of objects separately.

3(a) *Associations and clusters*

For our purpose we need a catalogue of all clusters and associations with earliest members not later than B5 inside a sphere of 1 kpc around the Sun. This meant extending Blaauw's investigations (1964) to include B3–B5 stars. At the same time we have revised and supplemented some of Blaauw's entries using data which became available after Blaauw's paper was written. Where no direct spectral data were available, we have deduced them from the extensive *UBV* photometry work by

Hoag and collaborators (1961) using the tables of unreddened colours given in Landolt–Börnstein (1965, pp. 298, 299). To supplement Blaauw's list, we have used the compilations by Becker (1963), Johnson and co-workers (1961) and Trumpler (1930), while we used the recent accurate MK spectral types from the Yale Catalogue (1964) to revise Blaauw's original work (1946) on the Scorpio–Centaurus association.

The results are presented in Table III. We have used broader luminosity class divisions: *d* for class *V*, *g* for classes IV to II, and *c* for class I, rather than magnitude intervals. The Strömgren radii given have been computed using Strömgren's formula (1936) for clusters,

$$S_o^c = \left[\sum_i S_{oi}^3\right]^{1/3}, \tag{3}$$

where the S_{oi} are the Strömgren radii of the individual members of the cluster, for which we used Table II. We have also given the galactic coordinates (l^{II}, b^{II}, and R) of the centre of the Strömgren sphere, calculated giving proper weights, that is, S_{oi}^3, to each individual star. As footnotes to the table we give the source of the spectral-type-luminosity observational data used by us.

We believe that our survey is practically complete, bar possibly the data on I Cep where we agree with Blaauw that the data are complete only down to about B3. Luckily it follows from Table II that stars cooler than B3 make negligible contributions to S_o^c so that even if our data about the later type stars are incomplete, this would hardly affect our calculations.

We must also mention that there are a few O-type stars which, although just outside the boundaries of a given cluster or association, appear from proper motion and radial velocity analysis to have originally been members of the association. If such runaway stars were lying within the Strömgren sphere *with* $N = 2\ cm^{-3}$ of the parent group, we have included them in the association and indicated them within brackets in Table III. The three cases included are ξ Per in II Per, 29UW CMa in NGC 2362, and λ Ori in I Ori. However, stars such as ζ Oph and 9α Cam which have run away too far are not included in the stars in Table III.

3(b) *Supergiants and hot O-stars which are not members of associations or clusters*

We now consider the H II regions surrounding O to B5 supergiants and O-type stars earlier than O9·5 within 1 kpc which were not included in Table III.

We used Becvar's catalogue (1963), cross-checking in the Yale Catalogue (1954). It can be argued that measurements of parallaxes less than 0·002″ are somewhat questionable; however, by considering the distance modulus $m-M$ we can be fairly sure whether a star lies closer than 1 kpc. In fact, we have the relation

$$5 \log (r/10) = m - M - A(r), \tag{4}$$

where r is the distance in pc, m the apparent and M the absolute magnitude, while $A(r)$ is the stellar extinction which has been examined in great detail by Johnson (1968). As Becvar lists all stars with apparent magnitude less than 6·25, it follows from equation (4) that any star with absolute magnitude brighter than $-3{\cdot}75 - A(r)$ and lying within 1 kpc from the Sun should appear in his catalogue. As all stars considered by us satisfy this criterion, we feel that our list, given in Table IV, is practically complete.

TABLE III

List of associations and clusters within 1 kpc from the Sun, their distances R_c from the Sun, their positions l^{II}, b^{II}, and the Strömgren radii S_0^c associated with them

Object	LC	Number of Stars in each Spectral Class: O5	O6	O7	O8	O9	O9·5	B0	B0·5	B1	B2	B3	B4	B5	S_0^c (pc)	R_c (pc)	l^{II}	b^{II}	Ref.
III Cep	g	—	—	—	—	—	—	3	1	—	—	—	—	—	101	725	110°·3	3°·2	(1)
	d	1	—	—	2	—	—	2	4	10	5	2	—	6					
	c	—	—	—	—	—	1	1	1	—	—	—	—	—					
I Ori	g	—	—	—	—	1	1	—	—	—	2	—	—	—	212	460	206·6	−17·6	(2)
	d	—	1	—	(1)	—	2	2	—	15	26	35	—	22					
	c	—	—	—	—	—	—	—	1	—	1	—	—	—					
I Cep	g	—	—	—	—	1	—	2	1	1	1	1	—	—	154	830	100·9	3·6	(3)
	d	—	1	—	—	1	—	5	4	7	2	3	—	—					
II Mon	d	—	—	1	—	—	—	—	1	1	3	4	2	1	79	715	202·9	2·2	(4)
	c	—	—	—	—	—	—	—	—	1	—	—	—	—					
II Per	g	—	—	—	—	—	—	—	—	1	—	2	—	—	58	400	160·7	−16·9	(5)
	d	—	—	(1)	—	—	—	—	1	2	4	—	—	1					
NGC 2362	g	—	—	—	(1)	1	—	—	—	—	—	—	—	—	106	1000	238·0	−5·5	(6)
	d	—	—	—	—	—	—	—	—	1	4	4	—	3					
I Lac	g	—	—	—	—	—	—	—	—	—	5	2	—	—	54	600	96·7	−17·0	(7)
	d	—	—	—	—	1	—	—	—	3	9	2	—	—					
IC 2602	d	—	—	—	—	—	1	—	—	—	—	1	1	1	35	155	289·6	−4·9	(8)
NGC 1502	d	—	—	—	—	—	—	1	—	1	3	4	1	2	48	880	143·6	7·6	(9)
II Sco	g	—	—	—	—	—	—	—	—	1	2	—	—	—	72	170	350·6	18·7	(10)
	d	—	—	—	—	—	—	2	1	2	12	6	—	—					
Upper Centaurus	g	—	—	—	—	—	—	—	—	—	5	1	—	3	33	170	321·8	14·2	(10)
Lupus	d	—	—	—	—	—	—	—	—	—	4	5	—	3					
Lower Centaurus	g	—	—	—	—	—	—	—	1	—	2	6	2	2	43	160	302·5	3·2	(10)
Crux	d	—	—	—	—	—	—	—	—	1	2	4	2	3					
NGC 2169	d	—	—	—	—	—	—	—	—	1	1	2	2	—	23	930	195·6	−2·3	(11)
NGC 7160	d	—	—	—	—	—	—	—	—	—	1	—	2	1	13	705	104·0	6·5	(11)
IC 2395	g	—	—	—	—	—	—	—	—	—	1	—	1	1	27	960	266·6	−3·8	(12)
	d	—	—	—	—	—	—	—	—	—	2	—	3	2					
IC 4725	d	—	—	—	—	—	—	—	—	—	1	—	—	3	12	600	13·6	−4·5	(13)
α Persei	d	—	—	—	—	—	—	—	—	—	—	2	—	6	9	170	148	−4	(7)
NGC 2168	d	—	—	—	—	—	—	—	—	—	—	1	—	—	7	870	186·6	2·2	(11)
NGC 2422	d	—	—	—	—	—	—	—	—	—	—	1	—	—	7	480	231·0	3·1	(11)
IC 2391	d	—	—	—	—	—	—	—	—	—	—	2	—	5	10	150	270·4	−6·9	(12)
NGC 2516	d	—	—	—	—	—	—	—	—	—	—	1	—	—	7	370	274·0	−15·9	(14)
NGC 4755	c	—	—	—	—	—	—	—	—	—	—	—	—	1	21	830	303·2	2·5	(15)
	g	—	—	—	—	—	—	—	—	—	—	1	—	1					
NGC 6475	d	—	—	—	—	—	—	—	—	—	—	—	1	—	5	230	355·9	−4·5	(11)
NGC 6405	d	—	—	—	—	—	—	—	—	—	—	—	1	2	7	630	356·8	−0·5	(16)
M 54, Tr 10,	g	—	—	—	—	—	—	—	—	—	—	—	—	1	< 5	—	—	—	(14)
NGC's 2287, 2323, 6087, 7031	d	—	—	—	—	—	—	—	—	—	—	—	—	12					

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In Table IV we have given for about twenty stars their galactic position, spectral class, and corresponding Strömgren radius. The latter was taken from Table II, except for γ^2 Vel. The binary γ^2 Vel is, according to Smith (1968), the only Wolf–Rayet star within 1 kpc from the Sun. As Wolf–Rayet stars are extremely luminous in the ultraviolet, we expect a large Strömgren sphere. Indeed, taking $M = -6{\cdot}6$ from Smith's data (1968) and assuming that $T \approx 80\,000\,°\mathrm{K}$, we get from equation (2), including the extra correction term, $S_0 = 232$ pc, again taking $N = 1\ \mathrm{cm}^{-3}$.

TABLE IV

List of supergiants and O-stars within 1 kpc from the Sun which do not belong to associations and clusters, their galactic position, l^{II}, b^{II}, R, their spectral characteristics, and the Strömgren radii S_0 associated with them

Star	l^{II}	b^{II}	R(pc)	*Sp*	L.C.	S_0(*pc*)
AO Cas	117°·6	11°·1	1000	O9	III	86
15κ Cas	120·8	0·1	1000	B1	Ia	53
9 α Cam	144·1	14·0	1000	O9·5	Ia	91
HD 31327	168·1	−4·4	1000	B2	Ib	32
25 χ Aur	175·8	0·6	1000	B5	Iab	18
47 ρ Leo	234·9	52·8	830	B1	Ib	40
24 σ^2 CMa	235·6	−8·2	590	B3	Ia	32
31 η CMa	242·6	−6·5	400	B5	Ia	22
ζ Pup	256·0	−4·7	330	O5f	—	94
J Pup	262·1	−10·5	770	B1	Ib	40
γ^2 Vel	262·8	−7·7	460	WC8+O7	—	232
HD 150136	336·7	−1·6	700	O7?	—	78
μ Nor	339·4	2·5	1000	B0	Ia	85
HD 149404	340·5	3·0	800	O9	III	86
HD 154090	350·8	4·3	480	B1	Iab	48
ζ Oph	6·3	23·6	170	O9·5	V	54
67 Oph	29·7	12·6	700	B5	Ib	15
HD 193322	78·1	2·8	650	O8	—	72
55 Cyg	85·8	1·5	1000	B3	Ia	32
68 A Cyg	87·6	−3·9	750	O8	—	72
26 Cep	108·5	6·4	830	B0·5	Ib	56

3(c) *The Cool Stars*

The stars which we have not discussed so far will be taken into account statistically. In the present and the next sub-section we shall derive an expression giving us the spatial stellar density as a function of the stellar effective temperature. We want to obtain an expression for the number of stars per unit volume per unit temperature interval in the neighbourhood of the Sun. We first determine the density of stars of spectral type Sp and absolute magnitude between $M-\frac{1}{2}$ and $M+\frac{1}{2}$ at height z above the galactic plane, $\phi(M, Sp, z)$.

From the available data about stellar magnitudes (Landolt–Börnstein 1965, p. 613) it seems that to a very good approximation the magnitude of a star is a random variable distributed about the main sequence value

$$M \text{ (main sequence)} = \bar{M}(Sp),$$

provided we exclude supergiants. If we define

$$\mu = M(Sp) - \bar{M}(Sp), \tag{5}$$

we have thus

$$\phi(M, Sp, z) = \frac{\phi(Sp, z)}{\sqrt{2\pi}\sigma(Sp)} \exp\left[-\frac{\mu^2}{2\sigma^2(Sp)}\right] \tag{6}$$

where the standard deviation $\sigma(Sp) = 0{\cdot}8$ for most spectral classes. In fact, equation (6) closely represents the relative number of stars in each magnitude interval and each luminosity class V to II. The supergiants (class I), however, are many times more abundant, even though they are few in number, than is predicted by the Gaussian equation (6). Indeed, we have already recognized that they are a distinct group of objects in treating them separately in the preceding sub-section. The function $\phi(Sp, z)$ is the total number of stars other than supergiants of spectral type Sp per unit volume at galactic height z.

From equation (4) it follows that the largest radius $r(Sp)$ inside which all stars of spectral type Sp are listed by Becvar (1964) is given by

$$r(Sp) = 1{\cdot}93 - 0{\cdot}2\bar{M}(Sp), \tag{7}$$

where we have allowed a couple of standard deviations for sub-luminous stars. We are here faced with a slight dilemma. If we take r too small—in order to assure completeness—we have poor statistics and the standard error in the count will be large. If we take r too large, we may miss several stars. If there is a danger that counts become too small, one can compromise and take r slightly larger in such a way that at least all stars more luminous than $\bar{M}(Sp)$ have been counted and assume a symmetrical distribution, as given by equation (6), to correct for the less luminous stars. In Table V we have given the results of the counting in Becvar's catalogue (1964). For convenience, we have included all O5 to O9 stars, excluding supergiants, within 1 kpc, which have already been considered in earlier subsections. The first column lists the spectral class, the second the total number of stars with a parallax not less than the value Π given in the third column. The fourth column lists the appropriate effective temperature. The function $\phi_o(T)$ given in the last column will be introduced presently.

We must now consider the z-dependence of the distribution. Kurochkin (1958), in his study of the spatial distribution of early-type stars, concludes that

$$\phi(Sp, z) = \phi_o(Sp) \exp\left[-|z|/a(Sp)\right], \tag{8}$$

TABLE V

List of all stars except supergiants with a parallax not less than Π, their effective temperature T and distribution function ϕ_o. The quantity Σ is the total number of stars found in Becvar's catalogue (1964)

Sp	Σ	Π″	*T* (°K)	$10^{12}\ \phi_o\ (T)$ (°K^{-1} pc^{-3})
O5	2	0·001		
O6	2	0·001	36350	6·3
O7	4	0·001		
O8	6	0·001	34620	13
O9	7	0·001		
O9·5	5	0·002	31450	20
B0	6	0·002		
B0·5–B1	52	0·002	24400	64
B2	20	0·004	20500	340
B3	46	0·007	17790	2600
B4	5	0·007		
B5	66	0·007	15480	5300
B6	9	0·007		
B7	26	0·007	12210	9600
B8	176	0·007		
B9	324	0·007	10700	$3·5\times10^4$
A0–A1	89	0·018	9550	$1·6\times10^5$
A2–A5	—	—	8760	$6·4\times10^5$
F0–F5	—	—	7340	$1·2\times10^6$
F8–G2	—	—	6080	$2·1\times10^6$
G5	—	—	5560	$1·2\times10^7$
G8–K3	—	—	4970	$1·5\times10^7$

with

$$a(Sp) = \begin{Bmatrix} 76 \text{ pc for B0} \geqslant Sp \geqslant \text{B5}, \\ 102 \text{ pc for B6} \geqslant Sp \geqslant \text{A4}, \end{Bmatrix}. \tag{9}$$

As the Sun lies essentially in the galactic plane (Allen 1964), we can relate our numbers $\Sigma(Sp)$ to the density $\phi_o(Sp)$ in the galactic plane, and using equation (8) we find

$$\phi_o(Sp) = \Sigma(Sp, r)\chi(Sp, r), \tag{10}$$

where

$$\chi^{-1}(Sp, r) = 2\pi ar^2 \left[1-2\left(\frac{a}{r}\right)^2+2\frac{a}{r}\left(1+\frac{a}{r}\right)e^{-r/a}\right] \tag{11}$$

and where the argument *r* of Σ indicates that in Table V we use different values of Π for different spectral classes. If $\Delta T(Sp)$ is the temperature range corresponding to the spectral class *Sp*—which can be found from the values of *T* in Table I—we have for the density of main sequence and giant stars per unit volume and unit temperature interval

$$\phi_o(T) = \frac{\phi_o(Sp)}{\Delta T(Sp)}, \tag{12}$$

and this quantity is listed in the last column of Table V.

We have in Table V combined certain spectral classes, either where the numbers Σ were so small that we wanted to improve statistics, or where there exist spectral classes such as B4 and B6 which tend to be avoided by spectroscopists when stellar spectra are classified.

Although our studies were mainly restricted to O- and B-stars, as they contribute most to H II regions, we have extended Table V down to K-stars, by using the values of $\phi_0(Sp)$ listed in Landolt–Börnstein (1965, p. 613) for the range A–K. As Morton and Adams' calculations do not extend below G2, we followed their suggestion and adopted Popper's temperature scale (1959), in the range G8–K5, fitting the two scales at G5 with a value $T = 5560$°K.

3(d) *The density–temperature relation and the probability to belong to an association or cluster*

From the figures in Table V it is clear that we probably can safely use statistical arguments to find the influence of main sequence stars later than O9. To find an analytical expression for $\phi_0(T)$ as function of T we have plotted in Fig. 1 log ϕ_0

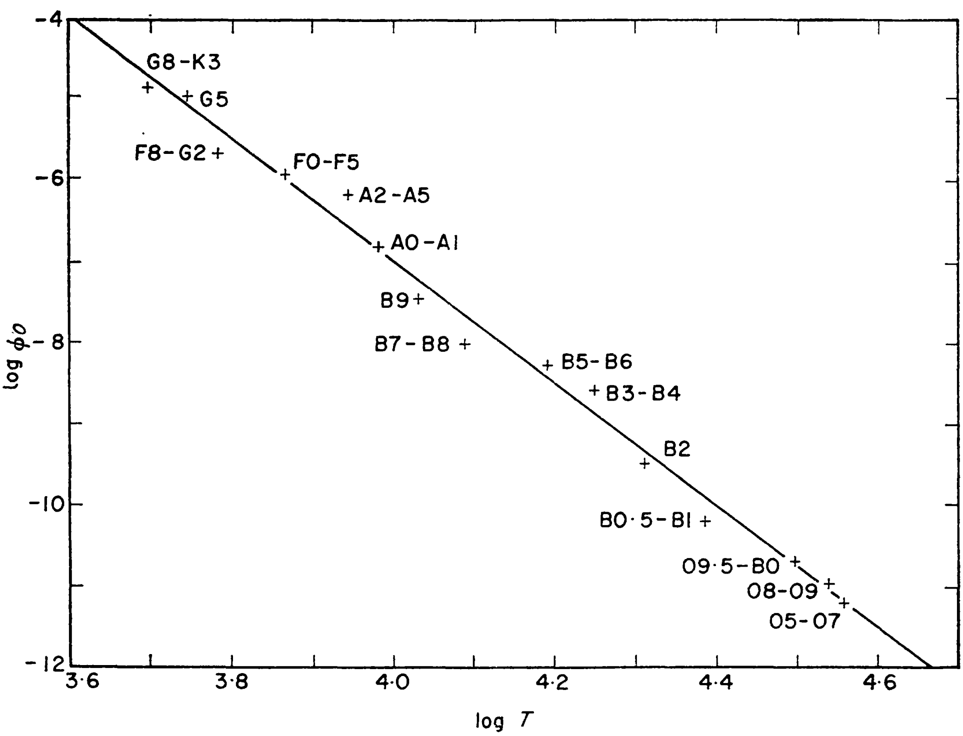

FIG. 1. *The distribution function $\phi_0(T)$ as function of T. The values plotted are taken from Table V.*

against log T. We see that a linear relation seems to exist. The best-fit line through the points is

$$\log \phi_0(T) = -\beta \log (T/10^4) + B, \tag{13}$$

with

$$\beta = 7{\cdot}5 \pm 0{\cdot}2, \quad B = -7{\cdot}01 \pm 0{\cdot}06. \tag{14}$$

So far we have not distinguished between stars which lie in clusters or associations and other stars, and in fact equation (13) refers to both groups. As we are going to distinguish between the two groups when considering the contribution from H II regions to $\int n_e \, dl$, we must find out how large a contribution to $\phi_0(T)$ is made by cluster members. As far as hot stars are concerned we have already collected the relevant data in earlier subsections.

As the volume occupied by associations and clusters is probably less than 2 per cent of the total volume of space, we can to a good approximation represent the background (field) star density by

$$\phi_f(T, z) = [1-p(T)]\phi_0(T, z), \tag{15}$$

where $p(T)$ is the probability that a star of effective temperature T is a member of an association or cluster.

TABLE VI

List of O- and B-stars (other than supergiants) in associations and clusters with parallax less than Π_c, their mean effective temperature $\bar{T}$, and the probability $p(T)$ for a star to belong to an association or cluster. The number Σ_c of stars within the distance determined by Π_c is taken from Table III

Sp	Σ_c	Π_c''	T (°K)	p	O-stars included in Σ_c with clusters in brackets
O5	1	0·001	36400	0·63	HD 217086 (III Cep)
O6	2	0·001			θ^1 Ori (I Ori), HD 206267 (I Cep)
O7	2	0·001			ξ Per (II Per), 15 S Mon (II Mon)
O8	4	0·001	34600	0·69	λ Ori (I Ori), 29 UW CMa (NGC 2362),
O9	5	0·001			HD 216532 (III Cep), HD 216898 (III Cep)
O9·5	4	0·002	31500	0·73	ι Ori (I Ori), 30 τ CMa (NGC 2362), 10 Lac (I Lac), HD 207198 and 14 Cep (I Cep)
B0	4	0·002			
B0·5, B1	25	0·002	24400	0·48	—
B2	97	0·001	20500	0·26	—
B3, B4	110	0·001	17800	0·03	—
B5	81	0·001	15600	0·02	—

In Table VI we have given the value of p for the spectral classes from O5 to B5. We have listed the total number Σ_c of stars in various spectral classes which are members of associations or clusters and which have parallaxes not less than a given value of Π_c. Note that the parallax limit in Table VI differs from that in Table V for the case of B2–B5 stars, as the data on B2–B5 stars in clusters are not very reliable, so that we felt it worthwhile to enlarge the sample to obtain better statistics. For stars later than B5, the data are even less reliable, and moreover are not needed for our present purpose. The function p was calculated from the formula

$$p(Sp) = \Sigma_c(Sp, r_c)\chi(Sp, r_c)/\Sigma(Sp, r)\chi(Sp, r), \tag{16}$$

where the difference between r_c and r arises from the difference between Π_c and Π in Tables V and VI. In Fig. 2 we have plotted p as a function of T, and at the same time a least-squares fitted curve for which we found the relation

$$p(T) = p_1 \left[\frac{T_1}{T} \exp\left(1-\frac{T_1}{T}\right)\right]^\nu, \tag{17}$$

with

$$p_1 = 0{\cdot}73 \pm 0{\cdot}09, \quad T_1 = 32\,100\,°\mathrm{K}, \quad \nu = 12 \pm 1. \tag{18}$$

We do not want to attach great importance to the analytic relation equation (17) —or even to the values of p in Table VI, but it is convenient for our later calculations to have an analytic expression for $p(T)$.

4. THE GALACTIC H I LAYER

We have mentioned earlier that appreciable densities of electrons of the order of 1 cm^{-3} or more are unlikely to be found anywhere except in the H II regions surrounding stars, and especially O- and B-stars. However, due to ionization by cosmic

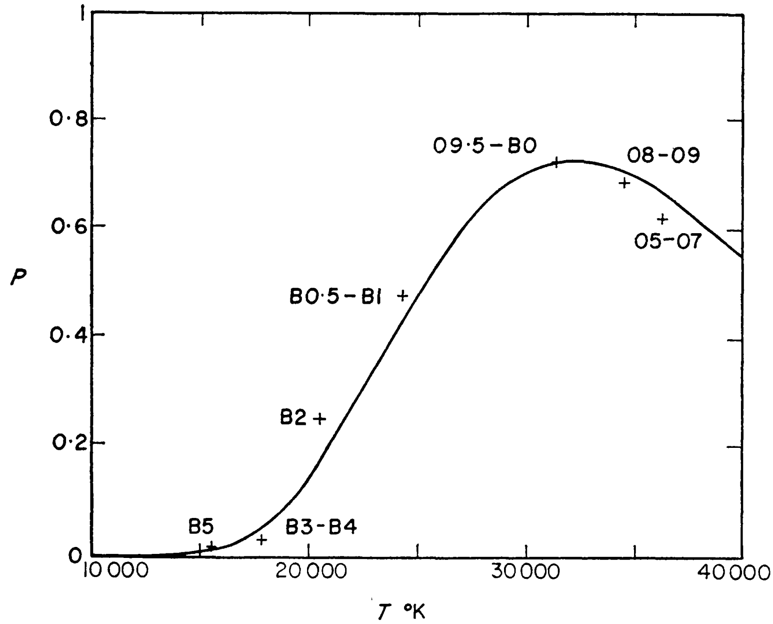

FIG. 2. *The probability $p(T)$ that a star with effective temperature T belongs to a cluster. The full-drawn curve corresponds to equation* (17).

rays even in H I regions there will be a non-vanishing n_e. In the present section we want to discuss the value of n_e to be expected in different regions of our galaxy.

We feel that it is too simple-minded to take the H I layer to be simply a uniform slab, symmetric with respect to the galactic plane with a thickness of about 200 pc. Instead, we feel that the H I density should be reflected by the distribution of early OB-stars. As we are restricting ourselves in the present paper to the neighbourhood of the Sun in the local arm, that is, within the Gould Belt, it is sufficient to consider only variations with galactic height z, and we can use the results from Section 3c, and hence we shall take for the density $n_{\mathrm{H\,I}}(z)$ the following expression

$$n_{\mathrm{H\,I}} = N_0 \exp(-|z|/76), \tag{19}$$

where z is in pc.

There is empirical support for equation (19) from the space distribution of H II regions found by Murdin & Sharpless (1968). We should expect that spatial distribution to coincide with equation (19), and indeed, fitting an exponential curve to the data in their Fig. 3, we find again an attenuation length of 76 pc, exactly the same value as in equation (19). The value of N_0 in equation (19) is immaterial to our discussion.

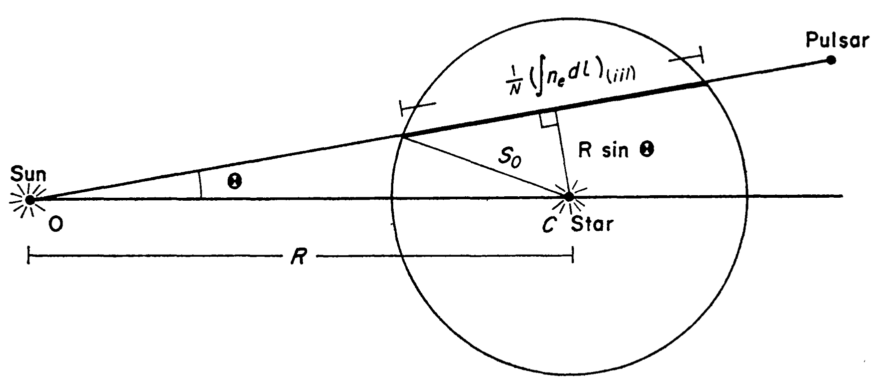

FIG. 3. *Line-of-sight intersecting a Strömgren sphere.*

We must now consider the level of ionization in the H I region due to cosmic rays. This is determined, according to Pottasch (1968; compare also Prentice & ter Haar 1969a), by a steady balance between the radiative recombination rate of ions and electrons and the cosmic-ray collisional ionization rate, or,

$$n_{\mathrm{H}^+} n_e = \alpha n_{\mathrm{H}}, \tag{20}$$

where α is a constant depending on the temperature of the gas and the cosmic-ray intensity, and where n_{H^+} and n_{H} are the densities of ionized and neutral hydrogen. Using the fact that $n_{\mathrm{H}^+} \approx n_e$ and combining equations (19) and (20), we find

$$n_e = n_0 \exp\left[-|z|/152\right], \tag{21}$$

which gives the interstellar electron density n_e at height z pc above the galactic plane. In the neighbourhood of the Sun, that is, within 1 kpc, we can probably safely consider n_0 to be a constant parameter. We shall see in the next section that this assumption seems to be well justified by the pulsar density data.

5. ELECTRON DISPERSION AND PULSAR DISTANCES

We mentioned in the Introduction that there are three main contributions to $\int n_e \, dl$ which we shall consider in turn: (i) a contribution from the H II regions around B-type main sequence stars and giants, which we shall consider statistically; (ii) a contribution from H I regions; and (iii) a contribution from the H II regions around one or more of the known hot O-stars, clusters, or associations which may intercept the line-of-sight to the pulsar.

5(a) *The statistical contribution from H* II *regions*

To find the contribution from H II regions of main-sequence and giant stars cooler than O9, we calculate what fraction of a line-of-sight in the galactic plane intercepts those H II regions. To find the fractional intercept for a different galactic latitude is then easy. The calculation is a statistical one, and details are given in the Appendix. We first find the fraction of stars $P_0(y)$ giving an intercept y. This function is given by the normalized expression

$$P_0(y) = \frac{\frac{1}{2}y \int_0^{T_0} \int_{-\infty}^{+\infty} \phi_f(T, \mu) \mathrm{H}[S_0(T, \mu) - \frac{1}{2}y] \, dT \, d\mu}{\int_0^{T_0} \int_{-\infty}^{+\infty} S_0^2 \phi_f(T, \mu) \, dT \, d\mu}, \tag{22}$$

where H(x) is the Heaviside unit function (Jeffreys & Jeffreys 1946)

$$\mathrm{H}(x) = 1, \quad x > 0; \qquad \mathrm{H}(x) = 0, \quad x < 0, \tag{23}$$

μ is the magnitude deviation (compare equations (5) and (6)), the cut-off in temperature corresponds to the temperature break between O9 and O9·5,

$$T_0 = 33\,200\,^\circ\mathrm{K}, \tag{24}$$

and the function $\phi_f(T, \mu)$ is given by equations (15) and (6), putting $z = 0$.

Given $P_0(y)$ we now find for the average intercept per unit length $\bar{I}$, assuming that the centres of the Strömgren intercepts y fall randomly along the line-of-sight, the expression

$$\bar{I} = 1 - e^{-\bar{y}/a}, \tag{25}$$

where

$$\bar{y} = \int_0^\infty y P_0(y)\, dy, \tag{26}$$

and a is the average spacing between the centres of adjacent intercepts; one can show that (for details, see Appendix)

$$\bar{y}/a = \frac{4\pi}{3} \iint S_0^3 \phi_f(T, \mu)\, d\mu\, dT. \tag{27}$$

As we have analytic expressions for S_0 (equation (2) with the correction mentioned there) and ϕ_f (equations (15) and (6) with p given by equation (17)), we can evaluate $\bar{y}$ and a, and the result is

$$\bar{y}/a = 0{\cdot}056 N^{-2}, \tag{28}$$

where N is again the hydrogen number density in the H II region in cm^{-3}. As $N \gtrsim 2$ cm^{-3} for all H II regions (Murdin & Sharpless 1968), we see that $\bar{y}/a \ll 1$ so that we can write

$$\bar{I} \approx \frac{\bar{y}}{a} = 0{\cdot}056 N^{-2}. \tag{29}$$

The dispersion per pc from this source is thus (the subscript (i) indicates that we are considering the first contribution mentioned at the beginning of Section 5 to this dispersion)

$$\frac{d}{dl}\left[\int n_e\, dl\right]_{(i)} = N\bar{I} = 0{\cdot}056 N^{-1}\ \mathrm{cm}^{-3}. \tag{30}$$

As we are using statistical arguments, we must find the average value of N^{-1}. From Murdin & Sharpless' data (1968) on 73 H II regions we find

$$\langle N^{-1} \rangle = 0{\cdot}069\ \mathrm{cm}^3, \tag{31}$$

and hence

$$\left[\int_0^l n_e\, dl\right]_{(i)} = 0{\cdot}0039 l\ \mathrm{cm}^{-3}\ \mathrm{pc}. \tag{32}$$

From the various statistical errors in the data we have used, we estimate that the coefficient in equation (32) can be written as $(3{\cdot}9 \pm 0{\cdot}5)10^{-3}$.

From equations (27), (29), (30) and (8), and bearing in mind that the main contribution to the dispersion will come from O9·5 to B6 stars (compare Table II), we see that if a line-of-sight makes an inclination b to the galactic plane, we would have instead of equation (30)

$$\frac{d}{dl}\left[\int_0^{l(b)} n_e\, dl\right]_{(i)} = 3{\cdot}9 \times 10^{-3} \exp\left[-l|\sin b|/76\right], \tag{33}$$

and hence—as we start from the galactic plane—

$$\left[\int_0^{l(b)} n_e\, dl\right]_{(i)} = 3{\cdot}9 \times 10^{-3} \frac{76}{|\sin b|}\left[1 - \exp\{-l|\sin b|/76\}\right]. \tag{34}$$

5(b) *The contribution from H* I *regions*

We found in Section 4 that the electron density in H I regions is given by equation (21), and we get thus

$$\left[\int_0^{l(b)} n_e\, dl\right]_{(ii)} = n_o \frac{152}{|\sin b|} [1 - \exp\{-l|\sin b|/152\}]. \tag{35}$$

5(c) *The contribution from single stars or clusters*

Let us consider a Strömgren sphere of radius $S_o(N) = S_o(1)N^{-2/3}$ with centre C (see Fig. 3) at a distance R from the Sun, and let Θ be the angle between OC and the line-of-sight. An intercept with the Strömgren sphere will occur only if

$$R \sin \Theta < S_o(N) \tag{36}$$

which will lead to the following contribution to the dispersion:

$$\left[\int n_e\, dl\right]_{(iii)} = 2S_o(1)N[N^{-4/3} - \theta^2]^{1/2} \equiv S_o(1)f(N, \theta), \tag{37}$$

where

$$\theta = \frac{R \sin \Theta}{S_o(1)}. \tag{38}$$

In Fig. 4 we have drawn $f(N, \theta)$ for various values of θ as function of N and we note that this function is not very sensitive to values of N over large ranges of that variable. The physical reason for this is that while increasing N decreases $S_o(N)$, at the same time each unit length makes a larger contribution to the dispersion. This insensitivity is rather fortunate, as the observational data on N are rather scarce and uncertain. We shall, therefore, average $f(N, \theta)$ over all N from $N_{\min}$, for which we take 2 cm^{-3}, using the data from Murdin and Sharpless' catalogue (1968), and $N_{\max}$ for which we take $\theta^{-3/2}$, as this is the value for which $f(N, \theta)$ vanishes. We then get for the contribution from a Strömgren sphere to the dispersion the

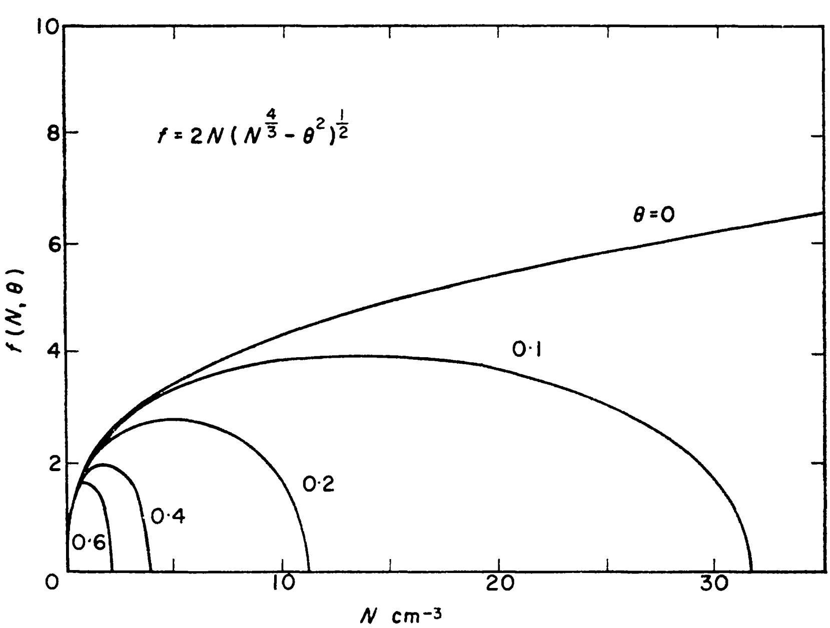

FIG. 4. *The function* $f(N, \theta) = 2N(N^{-4/3} - \theta^2)^{1/2}$.

expression

$$\left[\int n_e\, dl\right]_{(iii)} = S_0(1)\theta^{-1/2}\frac{[1-2^{4/3}\theta^2]^{3/2}}{1-2\theta^{3/2}}\,\mathrm{H}(2^{-2/3}-\theta), \tag{39}$$

where $\mathrm{H}(x)$ is the function given by equation (23), while for the average of $S_0(N)$ we find

$$\overline{S_0(N)} = 3S_0(1)\theta\,\frac{1-2^{1/3}\theta^{1/2}}{1-2\theta^{3/2}}. \tag{40}$$

5(d) *The pulsar distances*

We now have all data necessary to derive from the measured values of $\int n_e\, dl$ the likely values of pulsar distances. In Table VII we give a list of 37 pulsars, their galactic coordinates and the measured $\int n_e\, dl$ for all but one of them.

TABLE VII

List of all pulsars known to us with their galactic positions, l^{II}, and b^{II}, their measured dispersion, $\int n_e\, dl$, the electron density in the neighbourhood of the Sun derived by assuming $l|\sin b| \gg 152$, and the pulsar distance l

Pulsar	l^{II}	b^{II}	$\int n_e\, dl$ cm^{-3} pc	$10^2\, n_0(\infty)$ cm^{-3}	l pc
MP 0031	111°	−69°	12	7·2	>500
CP 0328	145	−1	27	0·27	500
MP 0450	217	−34	25	9·0	>500
NP 0527	184	−7	49	3·7	1600
NP 0532	185	−6	56	3·6	1700
PSR 0628−28	237	−17	10	1·7	200
MP 0736	254	−9	100	10·0	400
CP 0808	140	32	6	1·8	130
AP 0823+26	197	32	19	6·5	>900
PSR 0833−45	264	−3	63	1·8	400
CP 0834	220	26	13	3·5	400
MP 0835	260	0	120	0·5	400
PSR 0904+77	135	34	—	—	—
MP 0940	279	−3	145	4·0	500
PP 0943	228	42	17	7·3	>700
CP 0950	229	44	3	1·2	60
MP 0959	281	−1	90	0·6	$500<l<1700$
CP 1133	242	69	5	2·8	130
AP 1237	253	87	9	5·4	>500
MP 1240	302	−1	220	3·3	1100
MP 1426	313	−5	60	3·4	$200<l<1800$
MP 1449	315	−5	90	5·3	>1000
PSR 1451−68	314	−9	12	1·0	250
HP 1506	91	52	20	10·0	>600
MP 1530	326	2	20	0·3	400
AP 1541	18	46	35	16·3	>600
PSR 1642−03	14	26	33	9·3	160
MP 1727	341	−9	140	14·1	>1300
MP 1747	344	−11	40	4·7	$1300<l<1900$
PSR 1749−28	2	−1	51	0·4	1000
MP 1818	25	5	70	3·8	2000
CP 1919	56	4	13	0·3	250
PSR 1929+10	47	−4	8	0·2	150
JP 1933+16	52	−2	143	3·1	$2000<l<4000$
AP 2015+28	68	−4	14	0·5	300
PSR 2045−16	31	−33	11	3·9	400
PSR 2218+47	98	−8	44	3·7	1400

The first thing to do is to check whether the line-of-sight to a pulsar cuts across the Strömgren sphere of any of the hot objects listed in Tables III and IV. Inspection shows that this is the case for MP 0736, PSR 0833–45, MP 0835, MP 0940, MP 1240, PSR 1642–03, and possibly for MP 0450, MP 0959, MP 1426, MP 1449, MP 1727, MP 1747 and JP 1933.

Disregarding those thirteen pulsars for the moment, we conclude that for the remaining pulsars the measured dispersion is probably due to the contributions from the H I regions and from the later-type main sequence H II regions, or,

$$\left[\int n_e\, dl\right]_{\text{obs}} = \left[\int n_e\, dl\right]_{(i)} + \left[\int n_e\, dl\right]_{(ii)}$$

$$= \frac{152}{|\sin \mathrm{b}|}\,\{n_0(1 - e^{-l|\sin \mathrm{b}|/152}) + 0{\cdot}0020(1 - e^{-l|\sin \mathrm{b}|/76})\}. \qquad (41)$$

This equation could directly be solved for l, provided we knew n_0, that is, the electron density in the galactic plane near the Sun. We expect by using the first rough estimate of $\overline{n_e} \sim 0{\cdot}1$ cm^{-3} that the pulsar distances will be of the order of a few hundred pc so that we may expect that for several pulsars, especially those with high galactic latitude, the following inequality holds:

$$l\,|\sin b| > 152 \qquad (42)$$

which means that the pulsar lies essentially outside the galactic electron layer (see equation (21)). If equation (42) holds strongly equation (41) leads to

$$n_0 = n_0(\infty) \equiv \frac{|\sin b|}{152}\left[\int n_e\, dl\right]_{\text{obs}} - 0{\cdot}002. \qquad (43)$$

If equation (42) is not valid, $n_0(\infty)$ underestimates n_0, so that we can interpret n_0 as the upper limit of $n_0(\infty)$, attained by pulsars lying outside the galactic electron layer. The values of $n_0(\infty)$ derived from equation (43) are given in Table VII. If we take an average of $n_0(\infty)$ over those pulsars which have a high b^{II}, a largish dispersion, and which were not excluded, because their dispersion might have a contribution from hot objects, we find $n_0 \approx 0{\cdot}05$ cm^{-3}. It is interesting to note that we had obtained essentially the same value from the smaller sample of pulsars which was available when we wrote our earlier report (Prentice & ter Haar 1969). The pulsars used to obtain the n_0-value were: MP 0031, NP 0527, NP 0532, AP 0823+26, CP 0834, PP 0943, AP 1237, HP 1506, MP 1818, PSR 2045–16 and PSR 2218+47.

Now that we have found a value for n_0, we can use equation (41) to determine l for all pulsars, except the ones excluded earlier. The results are given in Table VII. As a check we calculated the galactic height of each pulsar used to determine n_0 and found that, indeed, they all satisfied inequality equation (42). For the case of pulsars for which $n_0(\infty) > n_0$, there is no real solution of equation (41), and we can only conclude that they lie beyond the galactic electron layer, or

$$l \gtrsim 3\,\frac{152}{|\sin b|}\ \text{pc}. \qquad (44)$$

This was the case for MP 0031, AP 0823+26, PP 0943, AP 1237, HP 1506, and AP 1541.

Let us note that we find for CP 0328 a distance of about 500 pc which agrees with the recent work by Guélin *et al.* (1969) who place it nearer than the Perseus arm, although de Jager *et al.* (1968) thought that it was lying at a distance of at least 4 kpc. We also note that our values of 1600 and 1700 for the distances of NP 0527 and NP 0532 which are thought to lie in the Crab nebula are in good agreement with Trimble's value (1968) of 2 kpc for the distance to the Crab nebula. We also note that our value of 1000 pc for l for PSR 1749−28 agrees with the upper limit for l of 1 kpc given by Guélin *et al.* (1969).

We now come to the thirteen pulsars excluded so far, which we shall discuss one by one. The method used is the following one. We first determine the distance l_0 between the Sun and the first point of contact with the Strömgren sphere with averaged radius $\bar{S}_0$, defined by equation (40), and evaluate the contribution to the dispersion from equation (41). One then uses equation (39) to find the contribution to the dispersion from the Strömgren sphere. If this does not use up the total observed dispersion, one applies equation (41) again, and so on, until the total observed dispersion is used up. If a line-of-sight just misses the $S_0(N = 2)$ Strömgren sphere, but lies within the $S_0(N = 1)$ sphere and the dispersion has not otherwise ' run-out ', we supply l with a lower bound, corresponding to the distance of the relevant hot object. We feel that this is justified in view of the uncertainty in the angular positions of the pulsars.

MP 0450: The line-of-sight to this pulsar just misses the $S_0(N = 2)$ Strömgren sphere of I Ori.

MP 0736, PSR 0833−45, MP 0835: The lines-of-sight to these pulsars hit the H II regions of ζ Pup and γ^2 Vel; PSR 0833−45 has been tentatively identified with the supernova remnant Vela X which is at a distance of about 500 pc.

MP 0940: The line-of-sight to this pulsar hits the H II region of γ^2 Vel.

MP 1240: The line-of-sight to this pulsar hits the H II regions of α^1 Cru and Lower Centaurus Crux (β Cru), while a B1 star (HD 110432) may also contribute to the dispersion.

PSR 1642−03: The line-of-sight to this pulsar hits the H II region of ζ Oph.

MP 0959: The line-of-sight to this pulsar just misses the H II regions of γ^2 Vel and IC 2602 which constitute the lower bound to l.

MP 1426: The line-of-sight to this pulsar skirts the H II regions of Lower Centaurus Crux which should account for most of the dispersion.

MP 1449: The line-of-sight to this pulsar just misses the H II regions of δ Cir (an O9V star) and HD 135591 (an O9Ib star).

MP 1727 and MP 1747: The lines-of-sight to these pulsars pass close to the H II region of I Scorpius at 1300 pc, which could account for most of the dispersion.

JP 1933+16: The line-of-sight for this pulsar passes within 3° of WR star No 93 (Smith 1968) which lies at 2 kpc.

CONCLUSION

Firstly, we see that from our results it follows that, indeed, most of the pulsars observed to date lie within 1 kpc of the Sun, which removes the necessity of studying the power output difficulties connected with a possible extra-galactic distance scale.

Secondly, we feel that, although distance determinations might be improved upon by using even more detailed data such as Hα emission surveys, our self-consistent empiric method provides sufficient accuracy for the present moment.

Thirdly, we have shown how one can use the pulsar data to provide an empiric estimate for the average electron density in the galactic plane. We found an average value of 0·05 cm^{-3}, while the median value turns out to be about 0·04 cm^{-3}, both considerably lower than the *ad hoc* value of 0·1 cm^{-3} used earlier.

We have not derived values for the distances of pulsars lying outside the galactic electron layer. The very high values of $n_0(\infty)$ found in a few cases might be due to either anomalously high electron concentrations in some directions, or to spurious observational data, or even to an extra-galactic plasma (Friedman *et al.* 1969). We want to emphasize that the results given in this paper can be applied also to other radio-astronomy problems involving electron-density dispersions.

ACKNOWLEDGMENTS

We express our gratitude to Wolfson College, Oxford and the Royal Commission for the Exhibition of 1851 for the support of one of us (A. J. R. P.) and to members of our department and the Oxford Astrophysics Department, especially L. M. Hall, M. F. Ingham, F. K. Lamb, and J. V. Peach, for helpful discussions.

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APPENDIX

To derive equations (25) and (27) we consider Fig. 5. From this figure we see that those stars lying at a distance r from the line-of-sight contribute to $P_0(y)$ if

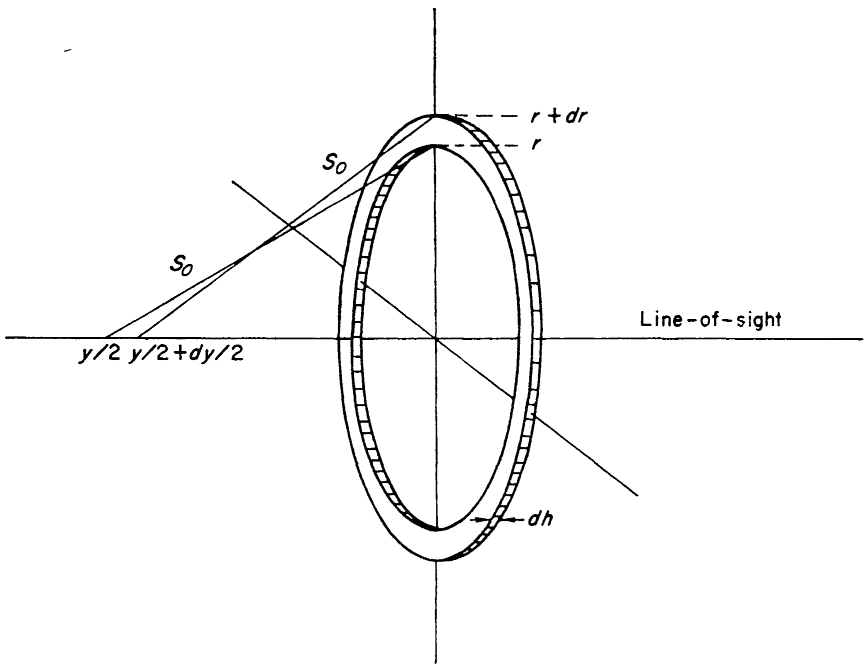

FIG. 5. *Intercepts of Strömgren spheres with line-of-sight.*

$r^2 = S_0^2 - \frac{1}{4}y^2$. Stars lying within a ring of thickness dh and width dr will contribute to $P_0(y)$ (we use the relation $r dr = \frac{1}{4} y dy$)

$$\tfrac{1}{2}\pi y \, dy \phi_f(T, \mu) \mathrm{H}(S_0 - \tfrac{1}{2}y) \, dh,$$

where the notation is the same as in Section 5. From this equation (22) follows immediately.

Given $P_0(y)$, what is $\bar{I}$? Consider a line of length L, and let x_i, $x_i + y_i$ be the coordinates of the i-th intercept. Let $c_i(x)$ be the function

$$c_i(x) = 1, \quad x_i \leqslant x \leqslant x_i + y_i; \quad c_i(x) = 0 \text{ elsewhere},$$

which indicates whether or not the point x is covered by the i-th intercept. We can then construct a covering function $c(x)$ as follows:

$$c(x) \equiv 1 - \prod_{i=1}^{n} [1 - c_i(x)] = 1, \text{ if } x \text{ is covered}$$
$$= 0, \text{ if } x \text{ is not covered},$$

where n is the total number of intercepts on the line.

The fractional length I_n occupied by the n intercepts is thus given by the equation

$$I_n = \frac{1}{L}\int_0^L c(x)\,dx$$

$$= 1 - \frac{1}{L}\int_0^L dx \sum_{r=0}^{n} (-1)^r \sum_{\{i_r\}} c_{i_1}(x) \ldots c_{i_r}(x).$$

To find $\bar{I}$, we must average over the positions x_i which we can take to be random variables, and let L and n go to infinity, while letting their ratio stay finite,

$$\lim_{\substack{n\to\infty \\ L\to\infty}} \frac{L}{n} = a,$$

where a is clearly the average distance between the centres of the intercepts. Moreover, we must take into account that the lengths are distributed according to the distribution function $P_0(y)$.

We then get, after straightforward but tedious calculations, first,

$$\bar{I}_n = 1 - \sum_{r=0}^{n} (-1)^r \int_0^L \frac{dx}{L^{r+1}} \sum_{\{i_r\}} \int \ldots \int dx_{i_1} \ldots dx_{i_r} c_{i_1}(x) \ldots c_{i_r}(x)$$

$$= 1 - \sum_{r=0}^{n} \frac{(-1)^r}{L^r} \sum_{\{i_r\}} y_{i_1} \ldots y_{i_r}$$

$$= 1 - \sum_{r=0}^{n} \frac{(-1)^r}{L^r} \binom{n}{r} \langle y_{i_1} \ldots y_{i_r} \rangle,$$

and as

$$\langle y_{i_1} \ldots y_{i_r} \rangle = \bar{y}^r$$

with

$$\bar{y} = \int_0^\infty y P_0(y)\,dy,$$

we get

$$\bar{I} = 1 - \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \left(\frac{n}{L}\right)^r \bar{y}^r$$

$$= 1 - e^{-\bar{y}/a}$$

which proves equation (25), where we have replaced $n!/(n-r)!$ by n^r which is correct for large n.

To prove equation (27) we first of all note that a^{-1} is equal to the number of stars per unit length of line-of-sight which produce a non-vanishing intercept which is given by the expression

$$a^{-1} = \pi \iint S_0^2 \phi_f(T, \mu)\,dT\,d\mu.$$

From the definition of y, equation (26) and the equation for a, we get after some manipulations equation (27).

From equations (2) and (15) we have

$$S_0(T, \mu) = CN^{-2/3} e^{-b/T} T^{\lambda} e^{-\alpha\mu},$$

and

$$\phi_f(T, \mu) = \frac{B'}{\sqrt{2\pi\sigma}} T^{-\beta} e^{-\mu^2/2\sigma^2} \left[1 - p_1 \left\{\frac{T_1}{T} e^{(T-T_1/T)}\right\}^{\nu}\right],$$

and hence

$$\int S_o{}^n d\phi_f = N^{-2n/3} B' C^n e^{\alpha^2\sigma^2 n^2/2} (nb)^{n\lambda-\beta+1}$$
$$\times \{\Gamma(\beta - n\lambda - 1, t_o) - p_1 (e\gamma_n)^{\nu} (1 + \nu\gamma_n)^{1+n\lambda-\beta-\nu}$$
$$\times \Gamma(\beta + \nu - n\lambda - 1, (1 + \nu\gamma_n) t_o)\}$$

where $t_o = nb/T_o$, $\gamma_n = T_1/nb$, and

$$\Gamma(a, z) = \int_z^{\infty} t^{a-1} e^{-t}\, dt.$$

With the values of the constants given in Sections (2) and (3) we get equation (28) as well as $a = 370 N^{4/3}$ pc, which is considerably lower than the estimate given by Davidson & Terzian (1969; compare also Rees & Sciama 1969).

NOTE ADDED IN PROOF

Since this paper was written, six more pulsars have been found, four by the Molonglo group (Large, M. I., Vaughan, A. E. and Wielebinski, R., 1969. *Nature*, **223**, 1249) and two by Jodrell Bank (announced at the 1969 October 11 meeting of the Royal Astronomical Society). Their measured dispersion and distance from the Sun, derived from it are as follows (in the same units as in Table VII): MP 0254: 10, >500; MP 1154: 270, 3000; MP 1706: 10, 220; MP 1911: 75, >2100; JP 2022: 15, 300; JP 2112: 110, 900. The line of sight to JP 2112 hits the HII region of 68A Cyg. As MP 1911 lies beyond the galactic layer, the uncertainty in the dispersion allows us only to quote a lower limit.