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ON I-ALEXANDROFF AND I_g-ALEXANDROFF IDEAL TOPOLOGICAL SPACES

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Abstract

In this paper, the notions of I-Alexandroff and I_g-Alexandroff ideal topological spaces are introduced and studied. Also, characterizations and properties of I-Alexandroff and I_g-Alexandroff ideal topological spaces are investigated.

1 Introduction and preliminaries

Alexandroff spaces were first studied by Alexandroff [2]. It is a topological space in which arbitrary intersection of open sets is open. Equivalently, each singleton has a minimal neighborhood base. Alexandroff spaces have important attentions because of their use in digital topology [10], [14]. In 1998, Arenas et al. [3] introduced and studied generalized Alexandroff topological spaces. Moreover, in 2000, Arenas et al. [4] studied some weak separation axioms related with Alexandroff topological spaces. It is known that any intersection of open sets is g-open in a generalized Alexandroff topological spaces [3]. It is shown in [3] that any $T_{\frac{1}{2}}$ g-Alexandroff space is locally path-connected, first countable, orthocompact and that in any $T_{\frac{1}{2}}$ g-Alexandroff space, the notions of path-connectedness, connectedness and chain-connectedness coincide. Furthermore, Arenas et al. [3] introduced that in digital topology, Khalimsky line [9, 11], various problems are related with generalized Alexandroff spaces. In this paper, the notions of I-Alexandroff and I_g-Alexandroff ideal topological spaces are introduced and studied. Moreover, characterizations and properties of I-Alexandroff and I_g-Alexandroff ideal topological spaces are discussed.

In this paper, (X,τ) or (Y,σ) denote a topological space with no separation properties assumed. $Cl(S)$ and $Int(S)$ denote the closure and interior of S in (X,τ), respectively for a subset S of a topological space (X,τ). An ideal I on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies

(1) $S \in I$ and $K \subset S$ implies $K \in I$,

(2) $S \in I$ and $K \in I$ implies $S \cup K \in I$ [12].

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For a topological space (X, τ) with an ideal I on X, if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \rightarrow P(X)$, said to be a local function [12] of $S \subset X$ with respect to τ and I is defined as follows:

$S^*(I, \tau) = \{x \in X : N \cap S \notin I$ for every $N \in \tau(x)\}$ where $\tau(x) = \{N \in \tau : x \in N\}$.

A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I, \tau)$, said to be the $\star$-topology and finer than τ, is defined by $Cl^*(S) = S \cup S^*(I, \tau)$ [8]. We will briefly write S^* for $S^*(I, \tau)$ and τ^* or $\tau^*(I)$ for $\tau^*(I, \tau)$. For an ideal I on X, (X, τ, I) is said to be an ideal topological space or briefly an ideal space. For an ideal topological space (X, τ, I), the collection $\{S \backslash N : S \in \tau$ and $N \in I\}$ is a basis for τ^* [8].

A subset S of a topological space (X, τ) is said to be g-closed in (X, τ) [13] if $Cl(S) \subset V$ whenever $S \subset V$ and V is open in (X, τ). A subset S of a topological space (X, τ) is called g-open in (X, τ) [13] if $X \backslash S$ is g-closed. A subset S of an ideal topological space (X, τ, I) is said to be $\star$-dense in itself [7] if $S \subset S^*$.

Definition 1. A subset S of an ideal topological space (X, τ, I) is said to be

(1) I_g-closed [6] in (X, τ, I) if $S^* \subset N$ whenever $S \subset N$ and N is open in (X, τ, I).

(2) I_g-open [6] in (X, τ, I) if $X \backslash S$ is I_g-closed.

Theorem 1. [15] *For a subset S of an ideal topological space (X, τ, I), S is I_g-open if and only if $N \subset Int^*(S)$ whenever $N \subset S$ and N is closed in X.*

Theorem 2. [15] *For an ideal topological space (X, τ, I) and $S \subset X$, the following properties are equivalent:*

(1) S is I_g-closed,

(2) $Cl^*(S) \subset N$ whenever $S \subset N$ and N is open in X.

Definition 2. A topological space (X, τ) is said to be

(1) Alexandroff [2] if any intersection of open sets is open.

(2) generalized Alexandroff [3] if any intersection of open sets is g-open.

2 I-Alexandroff and I_g-Alexandroff ideal spaces

Definition 3. An ideal topological space (X, τ, I) is said to be I-Alexandroff if any intersection of open sets is $\star$-open.

Theorem 3. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*

(1) (X, τ, I) is I-Alexandroff,

(2) any union of closed sets in (X, τ, I) is $\star$-closed.

Proof. It follows from the fact that the complement of a $\star$-open set is $\star$-closed. □

Definition 4. An ideal topological space (X, τ, I) is called I_g-Alexandroff if any intersection of open sets in (X, τ, I) is I_g-open.

Theorem 4. *Let (X, τ, I) be an ideal topological space. If there exists a point $x \in X$ such that x has only $\star$-neighborhood which is X itself, then (X, τ, I) is an I_g-Alexandroff ideal space.*

Proof. Suppose that there exists a point $x \in X$ such that x has only $\star$-neighborhood which is X itself.

Let $\{K_i : i \in I\}$ is a family of open sets in (X, τ, I) for each $i \in I$. We take $K = \bigcap_{i \in I} K_i$. Let $M \subset K$ and M be a closed set.

Suppose that $M = \emptyset$. Then we have $M \subset Int^*(K)$.

Suppose that $M \neq \emptyset$. If $M = X$, then $M \subset K = X$. Hence, $M \subset Int^*(K)$. If $M \neq X$, then $X \backslash M$ is an open set. It follows that $x \notin X \backslash M$ and then $x \in M$. Since $M \subset K$, then $x \in K_i$ for each $i \in I$. Since x has only $\star$-neighborhood which is X itself, then $K_i = X$ for each $i \in I$. Moreover, we have $K = X$ and then $M \subset Int^*(K)$. Hence, K is I_g-open.

Thus, (X, τ, I) is an I_g-Alexandroff ideal space. □

Theorem 5. *Let (X, τ, I) be an ideal topological space. If (X, τ, I) is an I-Alexandroff ideal space, then (X, τ, I) is I_g-Alexandroff.*

Proof. The proof follows from the fact that any $\star$-open set is I_g-open. □

Remark 1. The reverse implication of Theorem 5 is not true in general as shown in the following example:

Example 1. Suppose that R is the set of real numbers and $\tau = \{(-\frac{1}{n}, \frac{1}{n}) : n \in N \backslash \{0\}\} \cup \{R, \varnothing\}$ where N is the set of naturel numbers. Let $I = \{\varnothing, \{3\}\}$. Then the ideal topological space (R, τ, I) is I_g-Alexandroff by Theorem 4 but (R, τ, I) is not I-Alexandroff. Furthermore, suppose that $J = \{\varnothing\}$. Arenas et al. [3] show that the topological space (R, τ) is g-Alexandroff but (R, τ) is not Alexandroff. Therefore, the ideal topological space (R, τ, J) is I_g-Alexandroff but (R, τ, J) is not I-Alexandroff.

Definition 5. A subset S of an ideal topological space (X, τ, I) is called

(1) I_g^*-closed in (X, τ, I) if $Cl(S) \subset N$ whenever $S \subset N$ and N is $\star$-open in (X, τ, I).

(2) I_g^*-open in (X, τ, I) if $X \backslash S$ is I_g^*-closed.

Remark 2. Let (X, τ, I) be an ideal topological space. The following diagram holds for a subset S of X:

$$\begin{array}{ccccc} I_g^*\text{-open} & \longrightarrow & g\text{-open} & \longrightarrow & I_g\text{-open} \\ \uparrow & & & \nearrow & \\ \text{open} & \longrightarrow & \star\text{-open} & & \end{array}$$

None of these implications is reversible as shown in the following examples and in [8].

Example 2. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varnothing, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\varnothing, \{a\}, \{d\}, \{a, d\}\}$. Then the set $\{c\}$ is I_g^*-open but it is nether open nor $\star$-open. The set $\{b, c, d\}$ is $\star$-open but it is not g-open. The set $\{a, c\}$ is I_g-open but it is not $\star$-open.

Example 3. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varnothing, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$ and $I = \{\varnothing, \{a\}, \{d\}, \{a, d\}\}$. Then the set $\{a, b, d\}$ is g-open but it is not I_g^*-open.

Theorem 6. *For a subset S of an ideal topological space (X, τ, I), S is I_g^*-open if and only if $N \subset Int(S)$ whenever $N \subset S$ and N is $\star$-closed in (X, τ, I).*

Proof. Let S be an I_g^*-open set in (X, τ, I). Suppose that $N \subset S$ and N is $\star$-closed in (X, τ, I). It follows that $X \backslash S \subset X \backslash N$ and $X \backslash N$ is $\star$-open in (X, τ, I). Since $X \backslash S$ is I_g^*-closed, then $Cl(X \backslash S) \subset X \backslash N$. We have $Cl(X \backslash S) = X \backslash Int(S) \subset X \backslash N$. Thus, $N \subset Int(S)$. The converse is similar. □

Theorem 7. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*

(1) (X, τ, I) is an I_g-Alexandroff ideal space,

(2) Any intersection of I_g^*-open sets in (X, τ, I) is I_g-open.

Proof. (1) $\Rightarrow$ (2) : Let (X, τ, I) be an I_g-Alexandroff ideal space. Suppose that $\{S_i : i \in I\}$ is a family of I_g^*-open sets. We take $S = \underset{i \in I}{\cap} S_i$. Let $K \subset X$ be a closed set and $K \subset S$. We have $K \subset S_i$ for each $i \in I$. Since S_i is I_g^*-open set for every $i \in I$, then $K \subset Int(S_i)$ for each $i \in I$. We take $M = \underset{i \in I}{\cap} Int(S_i)$. Since (X, τ, I) is an I_g-Alexandroff ideal space, then $M = \underset{i \in I}{\cap} Int(S_i)$ is I_g-open. Since $M = \underset{i \in I}{\cap} Int(S_i)$ is I_g-open and $K \subset M$, then $K \subset Int^*(M)$. Hence, $Int^*(M) \subset Int^*(S)$ and thus, $K \subset Int^*(S)$. It follows that S is I_g-open.

(2) $\Rightarrow$ (1) : Suppose that any intersection of I_g^*-open sets in (X, τ, I) is I_g-open. Since every open set is I_g^*-open by Remark 2, it follows from (2) that any intersection of open sets in (X, τ, I) is I_g-open. Thus, (X, τ, I) is an I_g-Alexandroff ideal space. □

Theorem 8. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*

(1) (X, τ, I) is an I_g-Alexandroff ideal space,

(2) any union of I_g^*-closed sets in (X, τ, I) is I_g-closed.

Proof. It follows from Theorem 7. □

Theorem 9. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*

(1) (X, τ, I) is an I_g-Alexandroff ideal space,

(2) any union of closed sets in (X, τ, I) is I_g-closed.

Proof. It follows from the fact that the complement of an I_g-open set is I_g-closed. □

Theorem 10. *Let (X,τ,I) be an ideal topological space and $S \subset X$. If (X,τ,I) is an I-Alexandroff ideal space, then S is an I-Alexandroff ideal space.*

Proof. Let (X,τ,I) be an I-Alexandroff ideal space. Suppose that $\{K_i : i \in I\}$ is a family of open sets in (S,τ_S). We take $K = \underset{i\in I}{\cap} K_i$. It follows that $K_i = S \cap N_i$ where N_i is open in (X,τ,I) for each $i \in I$. Therefore, we have

$$K = \underset{i\in I}{\cap} K_i = \underset{i\in I}{\cap} (S \cap N_i) = S \cap (\underset{i\in I}{\cap} N_i).$$

Since (X,τ,I) is an I-Alexandroff ideal space, then $\underset{i\in I}{\cap} N_i$ is $\star$-open in (X,τ,I). It follows that $K = S \cap (\underset{i\in I}{\cap} N_i)$ is $\star$-open in S. Thus, S is an I-Alexandroff ideal space. □

Theorem 11. *Let (X,τ,I) be an ideal topological space. If (X,τ,I) is T_1 and an I_g-Alexandroff ideal space, then (X,τ,I) is a discrete ideal space with respect to τ^*.*

Proof. Let (X,τ,I) be T_1 and an I_g-Alexandroff ideal space. Suppose that $x \in X$. Since (X,τ,I) is a T_1 space, then for each $y \neq x$, there exists an open set S_y containing x such that $y \notin S_y$. It follows that $\{x\} = \underset{y\neq x}{\cap} S_y$. Since (X,τ,I) is a T_1-space, then $\{x\}$ is a closed set. Since (X,τ,I) is an I_g-Alexandroff ideal space, then $\{x\}$ is an I_g-open set. Since $\{x\} \subset \{x\}$, then we have $\{x\} \subset Int^*(\{x\})$. It follows that $\{x\}$ is $\star$-open in (X,τ,I). Thus, (X,τ,I) is a discrete ideal space with respect to τ^*. □

Theorem 12. *Let (X,τ,I) be an ideal topological space. If (X,τ,I) is a discrete ideal space with respect to τ^*, then (X,τ,I) is an I_g-Alexandroff ideal space.*

Proof. Let (X,τ,I) be a discrete ideal space with respect to τ^*. Suppose that $\{K_i : i \in I\}$ is a family of open sets in (X,τ,I). It follows that $\underset{i\in I}{\cap} K_i$ is $\star$-open in (X,τ,I). By Remark 2, $\underset{i\in I}{\cap} K_i$ is I_g-open. Hence, (X,τ,I) is an I_g-Alexandroff ideal space. □

Remark 3. The following example shows that the reverse of Theorem 11 is not true in general:

Example 4. Suppose that R is the set of real numbers and $\tau = \{(-\frac{1}{n}, \frac{1}{n}) : n \in N\backslash\{0\}\} \cup \{R, \varnothing\}$ where N is the set of naturel numbers. Let $I = P(X)$ which is the power set of X. Then (X,τ,I) is a discrete ideal space with respect to τ^* but (X,τ,I) is not a T_1-space.

Definition 6. [1] Let (X,τ,I) be an ideal topological space. (X,τ,I) is said to be an F^*-space if every open subset of (X,τ,I) is $\star$-closed.

Theorem 13. *Let (X,τ,I) be an ideal topological space. If (X,τ,I) is a T_1 and F^*-space, then (X,τ,I) is a discrete ideal space with respect to τ^*.*

Proof. Suppose that (X, τ, I) is a T_1 and F^*-space. Since (X, τ, I) is a T_1 space, then $\{x\}$ is a closed set for every $x \in X$. Since (X, τ, I) is an F^*-space, then $\{x\}$ is a $\star$-open set for every $x \in X$. It follows that (X, τ, I) is a discrete ideal space with respect to τ^*. □

Theorem 14. [15] *For an ideal topological space (X, τ, I), every subset of X is I_g-closed if and only if every open set is $\star$-closed.*

Theorem 15. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*
(1) (X, τ, I) is an F^*-space,
(2) every subset of (X, τ, I) is an I_g-closed set.

Proof. It follows by Theorem 14. □

Theorem 16. *Let (X, τ, I) be an ideal topological space. If (X, τ, I) is an F^*-space, then (X, τ, I) is an I_g-Alexandroff ideal space.*

Proof. Suppose that (X, τ, I) is an F^*-space. By Theorem 15, every subset of (X, τ, I) is an I_g-closed set. It follows that (X, τ, I) is an I_g-Alexandroff space. □

Definition 7. [5] A topological space (X, τ) is said to be an R_0-space if $Cl(\{x\}) \subset U$ for each $x \in X$ and each open set U with $x \in U$.

Theorem 17. *Let (X, τ, I) be an ideal topological space. If (X, τ, I) is an R_0 and I_g-Alexandroff ideal space, then (X, τ, I) is an F^*-space.*

Proof. Let (X, τ, I) be an R_0 and I_g-Alexandroff ideal space. Suppose that $S \subset X$ is an open set. Since (X, τ, I) is an R_0 space, then we have $Cl(\{x\}) \subset S$ for every $x \in S$. It follows that

$$S = \underset{x \in S}{\cup} Cl(\{x\}).$$

Since (X, τ, I) is an I_g-Alexandroff ideal space, then S is an I_g-closed set. Since $S \subset S$ and S is I_g-closed, then we have $Cl^*(S) \subset S$. It follows that S is $\star$-closed. Thus, (X, τ, I) is an F^*-space. □

Theorem 18. *Let (X, τ, I) be an ideal topological space and $M \subset X$. If (X, τ, I) is an I_g-Alexandroff ideal space and M is closed, then M is an I_g-Alexandroff ideal space.*

Proof. Let (X, τ, I) be an I_g-Alexandroff space and $M \subset X$ be a closed set. Suppose that $\{S_i : i \in I\}$ is a family of open sets in (M, τ_M). We take $S = \underset{i \in I}{\cap} S_i$. It follows that $S_i = M \cap K_i$ where K_i is an open set (X, τ, I) for each $i \in I$. Let $N \subset M$ be a closed set in (M, τ_M) and $N \subset S$. It follows that N is a closed set in (X, τ, I) and $N \subset \underset{i \in I}{\cap} K_i$. Since (X, τ, I) is an I_g-Alexandroff ideal space, then we have $N \subset Int^*(\underset{i \in I}{\cap} K_i)$. Also, we have

$$M \cap Int^*(\underset{i \in I}{\cap} K_i) \subset S.$$

Since $M \cap Int^*(\cap_{i\in I} K_i)$ is a $\star$-open set in M, then $N \subset Int^*_M(S)$. It follows that S is I_g-open in M. Hence, M is an I_g-Alexandroff ideal space. □

Remark 4. Arenas et al. [3] show that any subset of a generalized Alexandroff space (X, τ) need not be a generalized Alexandroff space. So, for the ideal $J = \{\varnothing\}$ and hence for any ideal I on X, any subset of an I_g-Alexandroff ideal space (X, τ, I) need not be an I_g-Alexandroff ideal space.

3 The relationships

Definition 8. A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be $\star$-closed if $f(A)$ is $\star$-closed in (Y, σ, J) for every $\star$-closed subset A of (X, τ, I).

Theorem 19. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ be a continuous and $\star$-closed surjective function. If (X, τ, I) is an I-Alexandroff ideal space, then (Y, σ, J) is an I-Alexandroff ideal space.*

Proof. Suppose that $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous and $\star$-closed function. Let (X, τ, I) be an I-Alexandroff ideal space. Suppose that $\{M_i : i \in I\}$ is a family of closed sets in (Y, σ, J). Since $f : (X, \tau, I) \to (Y, \sigma, J)$ is continuous, then $N = \underset{i\in I}{\cup} f^{-1}(M_i)$ is a $\star$-closed set in (X, τ, I). We take $M = \underset{i\in I}{\cup} M_i$. Since $f : (X, \tau, I) \to (Y, \sigma, J)$ is a $\star$-closed function, then

$$f(N) = f(\underset{i\in I}{\cup} f^{-1}(M_i)) = M$$

is $\star$-closed. It follows that (Y, σ, J) is an I-Alexandroff ideal space. □

Theorem 20. *Let (X, τ, I) be an ideal topological space and $S \subset X$ be I_g-closed. If $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous and $\star$-closed function, then $f(S)$ is an I_g-closed set in Y*

Proof. Suppose that $S \subset X$ is a I_g-closed set and $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous and $\star$-closed function. Let $f(S) \subset K$ where K is open in (Y, σ, J). It follows that $S \subset f^{-1}(K)$. Since $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous function and S is an I_g-closed set, then we have $Cl^*(S) \subset f^{-1}(K)$. Moreover, we have $f(Cl^*(S)) \subset f(f^{-1}(K)) \subset K$. Since f is a $\star$-closed function, then

$$Cl^*(f(S)) \subset Cl^*(f(Cl^*(S))) = f(Cl^*(S)) \subset K.$$

It follows that $Cl^*(f(S)) \subset K$ and hence $f(S)$ is an I_g-closed set in (Y, σ, J). □

Theorem 21. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ be a continuous and $\star$-closed surjective function. If (X, τ, I) is an I_g-Alexandroff ideal space, then (Y, σ, J) is an I_g-Alexandroff ideal space.*

Proof. Suppose that $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous and $\star$-closed surjective function. Let (X, τ, I) be an I_g-Alexandroff ideal space and $\{M_i : i \in I\}$ be a family of closed sets in (Y, σ, J). Since $f : (X, \tau, I) \to (Y, \sigma, J)$ is a continuous function, then $K = \bigcup_{i\in I} f^{-1}(M_i)$ is an I_g-closed set in (X, τ, I). We take $M = \bigcup_{i\in I} M_i$. It follows from Theorem 20 that, $f(K) = f(\bigcup_{i\in I} f^{-1}(M_i)) = M$ is an I_g-closed set. Thus, (Y, σ, J) is an I_g-Alexandroff ideal space. □

Theorem 22. [15] *Let (X, τ, I) be a T_1 ideal topological space and $A \subset X$. If A is an I_g-closed set in (X, τ, I), then A is $\star$-closed.*

Theorem 23. *For a T_1 ideal topological space (X, τ, I), the following properties are equivalent:*
(1) (X, τ, I) is an I-Alexandroff ideal space,
(2) (X, τ, I) is an I_g-Alexandroff ideal space.

Proof. Since any I_g-closed set is $\star$-closed in a T_1 ideal topological space (X, τ, I), then by Remark 2 and Theorem 22, (X, τ, I) is an I-Alexandroff ideal space if and only if (X, τ, I) is an I_g-Alexandroff ideal space. □

Theorem 24. [15] *Let (X, τ, I) be an ideal topological space and $A \subset X$. If A is $\star$-dense in itself and I_g-closed in (X, τ, I), then A is g-closed.*

Theorem 25. *Let (X, τ, I) be an ideal topological space. Suppose that every subset of (X, τ, I) is $\star$-dense in itself. Then the following properties are equivalent:*
(1) (X, τ, I) is an I_g-Alexandroff ideal space,
(2) (X, τ, I) is a generalized Alexandroff space.

Proof. Since every subset is $\star$-dense in itself, then by Remark 2 and Theorem 24, (X, τ, I) is an I_g-Alexandroff ideal space if and only if (X, τ, I) is a generalized Alexandroff space. □

Theorem 26. [6] *Let (X, τ, I) be an ideal topological space where $I = \{\oslash\}$ and $A \subset X$. Then A is I_g-closed if and only if A is g-closed.*

Theorem 27. *For an ideal topological space (X, τ, I) where $I = \{\oslash\}$, the following properties are equivalent:*
(1) (X, τ, I) is an I_g-Alexandroff ideal space,
(2) (X, τ, I) is a generalized Alexandroff space.

Proof. Let (X, τ, I) be an ideal topological space where $I = \{\oslash\}$ and $S \subset X$. Since S is an I_g-closed set if and only if S is a g-closed set by Theorem 26, then (X, τ, I) is an I_g-Alexandroff ideal space if and only if (X, τ, I) is a generalized Alexandroff space. □

Theorem 28. *Let (X, τ, I) be an ideal topological space and $I = \{\oslash\}$. Then the following properties are equivalent:*
(1) (X, τ, I) is an Alexandroff space,
(2) (X, τ, I) is an I-Alexandroff ideal space.

Proof. Since $I = \{\oslash\}$, then we have $\tau = \tau^*$. It follows that (X, τ, I) is an Alexandroff space if and only if (X, τ, I) is an I-Alexandroff ideal space. □

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