On Interdependent Supergames: Multimarket

Contact, Concavity, and Collusion[*]

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Following Bernheim and Whinston (1990), this paper addresses the effects of multimarket contact on firms' ability to collude. Real world imperfections tend to make firms' objective function strictly concave and market supergames "interdependent": firms' payoffs in each market depend on how they are doing in others. Then, multimarket contact always facilitates collusion. It may even make it sustainable in all markets when otherwise it would not be sustainable in any. The effects of conglomeration are discussed. "Multigame contact" is shown to facilitate cooperation in supergames other than oligopolies as long as agents' objective function is submodular in material payoffs. *Journal of Economic Literature* Classification Numbers: C72, D43, L13, L21.

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1. Introduction

The traditional view that multimarket contact facilitates collusion "in general" (e.g. Edwards [10]) has not been fully supported by Bernheim and Whinston's [4] game-theoretic analysis. These authors do identify a number of circumstances, typically implying asymmetries between firms or markets, in which multimarket contact facilitates collusion by optimizing the allocation of available enforcing power between markets. However, their benchmark is an *irrelevance* result: when firms and markets are identical and there are constant returns to scale, multimarket contact does not strengthen firms' ability to collude.

In this paper we identify an additional circumstance in which multimarket contact facilitates collusion, one that is independent from asymmetries and that brings some grist to the mill of the traditional view. We show that when real world imperfections make firms' static objective function strictly concave, multimarket contact *always* facilitates collusion. A strictly concave objective function makes the strategic interactions *interdependent*: firms' evaluation of profits in one market depends on profits realized in other markets. Then, the expected utility losses from simultaneous retaliations in more markets – a threat available only with multimarket contact – are larger than (the sum of) those from independent retaliations. Moreover, short-run profits from a simultaneous deviation in more markets – the optimal deviation with multimarket contact – are less valuable than (the sum of) short-run gains from independent deviations. These two effects always facilitate collusion. Of course, the pro-collusive effect of multimarket contact is strengthened when the circumstances identified by Bernheim and Whinston's are also present.

The wealth effects induced by a concave static objective function are shown to generate "scale economies" in cooperation. With multimarket contact collusion can be viable in a set of markets even when in the absence of multimarket contact it could not be supported in *any* of these markets. These wealth effects also ensure that conglomerate mergers facilitate collusion when the discount factor is relatively low, and *vice versa*.

The mechanism behind the results is quite general. We show that "multigame contact" facilitates cooperation in a large class of interdependent supergames other than oligopolies as

long as agents' static objective function is strictly *submodular* in the stage-games' material payoffs.

2. Why Concavity

By Fisher's Separation Theorem [13], when (a) firms are led by owners and (b) financial markets are perfect, the only interesting and relevant modelling assumption is the one of expected-profit-maximizing firms. In this case there is little more to say on multimarket contact and collusion apart from what has already been stated by Bernheim and Whinston. In reality, however, at least one of the two conditions is usually violated. Let us briefly argue why the assumption of a strictly concave objective function is the most interesting alternative.[1]

Empirical Evidence. Three decades of empirical findings on "income smoothing" revealed that managers are strongly averse to intertemporal substitution in profits, which (in the von Neuman-Morgenstern framework) implies that their static objective function is strictly concave.[2] Moreover, companies invest large amounts of resources to manage and hedge risks through various kinds of derivatives, which implies that they do not maximize expected profits (Géczy *et al.* [18]). Finally, there is evidence that investors themselves are averse to intertemporal substitution; they value assets with smooth returns more (Allen and Michaely [1]).[3]

Theoretical Reasons I: Managerial Objectives. When ownership is separated from control, firms tend to pursue objectives different from profit-maximization.[4] Financially constrained risk-averse managers smoothing their own income is one of the first explanations proposed for the evidence on income smoothing and hedging (Lambert [28], Dye [8], Smith and Stulz [34]). Managers' monetary incentive schemes also make firms' static objective function strictly concave,

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[1] See Spagnolo [35, 36] for more thorough discussions on this issue and further references.

[2] Recent evidence on income smoothing is provided by De Fond and Park [6], Degeorge *et al.* [7], Kasanen *et. al.* [27], Holthausen *et al.* [23], Gaver *et al.* [17], Merchant [29], Greenawalt and Sinkey [19], and Healy [22].

[3] One explanation proposed for income smoothing is that – by reducing the perceived volatility of cash flow – they increase firms' market valuation by risk-averse investors (Trueman and Titman [40], Ronen and Sadan [32]).

[4] Classical references include Simon [33], Baumol [2], Williamson [41], and Jensen and Meckling [25].

since they are usually capped.[5] Moreover, Fudenberg and Tirole [16] show that if managers earn rents, contracts are short-term, and performance measures are subject to "information decay," then the *optimal* incentive contract leads managers to smooth profits. Finally, when firms are leveraged, the risk of bankruptcy, with the consequent loss of job and reputation, leads managers to maximize a strictly concave objective function (Greenwald and Stiglitz [20, 21]).

Theoretical Reasons II: External Factors. It is often argued that firms *should* hedge to reduce their tax bill since corporate taxes are not perfectly linear.[6] The non-linearity of the corporate tax implies concavity of firms' objective function. Also, information asymmetries tend to make the cost of external finance strictly convex, which has similar implications with regard to firms' objectives.[7]

3. Bertrand Competition

Consider first Bernheim and Whinston's model of repeated Bertrand competition. Time is discrete, and in each duopolistic market k trade occurs simultaneously in each period t. Demand is always a decreasing and continuous function $Q_k(p_k)$ of price p_k. Firms are active in two markets and in each period simultaneously announce their current prices. When rival firms announce identical prices, they share the market equally; otherwise, all consumers buy from the firm with the lowest price. Let c_{ik} denote the constant marginal cost of production for firm i in market k (when firms and markets are identical we write $Q_k = Q$ and $c_{ik} = c$).

Under the standard assumptions of expected-profit-maximizing firms using trigger strategies, Bernheim and Whinston derive the irrelevance result: *"When identical firms with identical constant returns to scale technologies meet in identical markets, multimarket contact does not*

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[5] Healy [22] attributes income smoothing to managers' capped monetary bonus schemes, and provides evidence for his view. Joskow and Rose [26] find further evidence that boards tend to make managers' compensation capped. Indeed, shareholders may choose to purposely delegate control to managers averse to intertemporal substitution (e.g., under capped bonuses) as this facilitates collusion independently of multimarket contact (Spagnolo [36]).

[6] See for example Stulz [37], Smith and Stulz [34], Froot *et al.* [15], and The Economist [9].

[7] See Myers and Majluf [31], Fazzari *et al.* [12], Hubbard *et al.* [24], Bernanke and Gertler [3], or Greenwald and Stiglitz [20, 21].

aid in sustaining collusive outcomes" (1990, p.5).

Suppose, instead, that the static objective function of any firm i active in markets A and B is $U_i = \ln(1 + \pi_{iA} + \pi_{iB})$. As in [4], with multimarket contact, i.e. when the same two firms meet in both the markets in which they are active, the optimal punishment of reverting to the static equilibrium in all markets after any deviation is used, and firms find it optimal to deviate in all markets simultaneously. Then, firms can support the monopoly price in both markets if

$$\frac{1}{1-\delta} \ln \left(1 + 2[(p^m - c)\frac{Q(p^m)}{2}] \right) - \ln \left(1 + 2(p^m - c)Q(p^m) \right) \geq 0,$$

or, equivalently, if

$$\delta \geq \delta^* = \frac{\ln \left(1 + 2(p^m - c)Q(p^m) \right) - \ln \left(1 + (p^m - c)Q(p^m) \right)}{\ln \left(1 + 2(p^m - c)Q(p^m) \right)}. \tag{3.1}$$

Without multimarket contact, in each market only the threat of punishment in *that* same market can be used, and firms may consider deviating in one market only. Then, for collusion to be supportable in both markets the following pooled condition for independent deviations in the two markets must also be satisfied

$$\frac{1}{1-\delta} \ln \left(1 + 2[(p^m - c)\frac{Q(p^m)}{2}] \right) - \ln \left(1 + \frac{3}{2}(p^m - c)Q(p^m) \right) +$$

$$-\frac{\delta}{1-\delta} \ln \left(1 + \frac{1}{2}(p^m - c)Q(p^m) \right) \geq 0,$$

or, equivalently,

$$\delta \geq \delta^{**} = \frac{\ln \left(1 + \frac{3}{2}(p^m - c)Q(p^m) \right) - \ln \left(1 + (p^m - c)Q(p^m) \right)}{\ln \left(1 + \frac{3}{2}(p^m - c)Q(p^m) \right) - \ln \left(1 + \frac{1}{2}(p^m - c)Q(p^m) \right)}. \tag{3.2}$$

Both δ^* and δ^{**} are positive and less than one, but $\delta^{**} > \delta^*$ for any p^m, c, and $Q()$. Therefore when $\delta^{**} > \delta \geq \delta^*$ collusion is supportable with multimarket contact only.

4. A More General Result

Consider any finite set of oligopolistic markets $\Omega = \{A, B, C, ...\}$ and any finite set of firms $I = \{1, 2, 3, ..., N\}$ interacting repeatedly in several markets with a common intertemporal discount factor $\delta < 1$. Let S_{ik} denote firm $i's$ set of pure strategies in the static interaction in

market k, $s_{ik} \in S_{ik}$ denote one pure strategy, and $\widehat{s}_{ik}(s_{-ik})$ firm i's static best response to its opponents' strategy profile $s_{-ik} \in S_{-ik}$, where $S_{-ik} = \underset{j \neq i}{\times} S_{jk}$. Let $\pi_{ik}(.)$ denote firm i's profit function in market k, so that $\pi_{ik}^* = \pi_{ik}(s_{ik}^*, s_{-ik}^*)$ indicates firm i's market k profits from the strategy profile $s_k^* = (s_{ik}^*, s_{-ik}^*)$ and $\widehat{\pi}_{ik}^* = \pi_{ik}(\widehat{s}_{ik}(s_{-ik}^*), s_{-ik}^*)$ its market k profits from the static best response strategy $\widehat{s}_{ik}(s_{-ik}^*)$. Let $U = U(\sum_{k \in \Omega} \pi_{ik}(s_{ik}, s_{-ik}))$ be firms' continuous and monotonically increasing static objective function, and $\underline{\pi}_{ik}$ denote firm i's monetary payoff in one period of the punishment phase.[8] We can now state the following.

Proposition 1. *Suppose firms' static objective function U is strictly concave in profits. Then multimarket contact (always) relaxes the necessary and sufficient conditions for any set of profit streams to be supportable in subgame-perfect equilibrium by stationary punishments in any set of infinitely repeated oligopoly games.*

Proof: We prove the case of two markets, A and B. The generalization to N markets is straightforward. The following lemma will be useful.

Lemma 1. *Let $U{:}R{\rightarrow}R$ be a strictly concave function. Then for every x, y in R_{++} and z in R_+, $U(z) + U(x + y + z) < U(x + z) + U(y + z)$.*

Proof of Lemma 1: Define $s = (x+y)$, $\mu_x = \frac{x}{s}$ and $\mu_y = \frac{y}{s}$, so that $0 < \mu_i < 1$, $i = x, y$ and $g = (x + y + z)$. By the definition of strict concavity $U[\mu_i g + (1 - \mu_i)z] > \mu_i U(g) + (1 - \mu_i)U(z)$. Solving inside the squared brackets and summing over i we obtain the expression above. **Q.E.D.**

We proceed by contradiction. Without multimarket contact a firm i which is active in markets A and B respects a collusive agreement in market A delivering per-period profits π_{iA}^* if

$$\frac{1}{1 - \delta}U(\pi_{iA}^* + \pi_{iB}) - U(\widehat{\pi}_{iA}^* + \pi_{iB}) - \frac{\delta}{1 - \delta}U(\underline{\pi}_{iA} + \pi_{iB}) \geq 0,$$

and a symmetric condition holds for market B. The sum of these two conditions, that is,

$$\frac{2}{1-\delta}U(\pi_{iA}^* + \pi_{iB}^*) - U(\hat{\pi}_{iA}^* + \pi_{iB}^*)+$$

$$-U(\pi_{iA}^* + \hat{\pi}_{iB}^*) - \frac{\delta}{1-\delta}[U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB})] \geq 0, \qquad (4.1)$$

is a necessary condition for collusion being simultaneously supportable in both markets without multimarket contact. With multimarket contact, instead, the condition becomes

$$\frac{1}{1-\delta}U(\pi_{iA}^* + \pi_{iB}^*) - U(\hat{\pi}_{iA}^* + \hat{\pi}_{iB}^*) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) \geq 0. \qquad (4.2)$$

Suppose (4.2) can be equally or more stringent than (4.1). Then the difference between the LHS of (4.2) and the LHS of (4.1) can be weakly negative. Rearranging this difference, it could be

$$U(\hat{\pi}_{iA}^* + \hat{\pi}_{iB}^*) - U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) + U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB}) - U(\hat{\pi}_{iA}^* + \pi_{iB}^*) - U(\pi_{iA}^* + \hat{\pi}_{iB}^*) \leq$$

$$\leq \frac{1}{\delta}\left[U(\pi_{iA}^* + \pi_{iB}^*) + U(\hat{\pi}_{iA}^* + \hat{\pi}_{iB}^*) - U(\hat{\pi}_{iA}^* + \pi_{iB}^*) - U(\pi_{iA}^* + \hat{\pi}_{iB}^*)\right]. \qquad (4.3)$$

The RHS of (4.3) is negative by Lemma 1; the sign of its LHS is ambiguous. If the LHS is positive (4.3) is violated. If it is negative, multiply everything by -1 and rearrange into

$$\delta \geq \frac{-U(\pi_{iA}^* + \pi_{iB}^*) - K}{-U(\underline{\pi}_{iA} + \pi_{iB}^*) - U(\pi_{iA}^* + \underline{\pi}_{iB}) + U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) - K}, \qquad (4.4)$$

where $K = [U(\hat{\pi}_{iA}^* + \hat{\pi}_{iB}^*) - U(\hat{\pi}_{iA}^* + \pi_{iB}^*) - U(\pi_{iA}^* + \hat{\pi}_{iB}^*)]$. By Lemma 1, $U(\pi_{iA}^* + \pi_{iB}^*) < U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB}) - U(\underline{\pi}_{iA} + \underline{\pi}_{iB})$. Therefore the numerator is strictly larger than the denominator and because $\delta \leq 1$ (4.4) is never satisfied. Then (4.1) must be more stringent than (4.2). **Q.E.D.**

Conditions (4.1) and (4.2) are linear in the discount factor, collusive profits π_{ik}^*, and market structure ($\hat{\pi}_{ik}^*$ and $\underline{\pi}_{ik}$). Therefore, rearranging the proof one could alternatively state:

i) "Suppose (...). Then multimarket contact reduces the minimum discount factor at which any given set of profit streams can be supported (...) in any given set of oligopolistic supergames."

ii) "Suppose (...). Then, given the discount factor, the set of stationary profit streams supportable (...) with multimarket contact in any given set of repeated oligopolies is no smaller (in

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the sense of inclusion) than the set supportable without multimarket contact. Moreover, there exist $\underline{\delta}$ and $\overline{\delta}$, with $0 < \underline{\delta} < \overline{\delta} < 1$, such that for $\underline{\delta} < \delta < \overline{\delta}$ multimarket contact strictly enlarges the set of supportable profit streams."

iii) "Suppose (...). Then, given the discount factor, multimarket contact strictly enlarges (in the sense of inclusion) the set of oligopolistic supergames in which any given set of collusive profit streams can be supported (...)."

When each firm faces different opponents playing the market games independently, the threat available to enforce collusion in each market is retaliation in that market only. Multimarket contact allows firms to use the threat of a *simultaneous* punishment in more markets, which is stronger than the sum of the independent punishments because a firm being punished in one market has a higher marginal valuation of profits, therefore it values more the losses from punishments in other markets. Moreover, the threat of a simultaneous punishment ensures that with multimarket contact firms' optimal deviation is a simultaneous one in all markets. Because the marginal utility of profits is decreasing, the simultaneity of the deviation makes short-run gains from deviating less valuable. Both these effects always facilitate collusion.

5. Extensions

5.1. "Increasing Returns" in Cooperation

When firms' objective function is concave, multimarket contact may allow firms to sustain collusion in *all* markets even when without multimarket contact collusion could not be sustained in *any*. To see this, consider the modified Bernheim and Whinston model of Section 3. Collusion in only one of the existing markets is sustainable if

$$\delta \geq \delta' = \frac{\ln\left(1 + (p^m - c)Q(p^m)\right) - \ln\left(1 + \frac{1}{2}(p^m - c)Q(p^m)\right)}{\ln\left(1 + (p^m - c)Q(p^m)\right)}. \tag{5.1}$$

It easy to check that $\delta^{**} > \delta' > \delta^*$, for any p^m, c, and $Q()$. It follows that when $\delta' > \delta \geq \delta^*$ with multimarket contact the collusive price can still be sustained in both markets, while without

multimarket contact it cannot be sustained in *any* of them.[9]

5.2. Conglomeration, Mergers, and Collusion

One can argue that it is *conglomeration* that, by leading firms to operate in several segregated markets, insures them against too harsh punishments reducing their ability to sustain collusion. Multimarket contact then facilitates collusion by restoring such an ability at the pre-conglomeration level.[10] This interpretation applies as long as the term conglomeration refers to a *situation*. When the term refers to the *process* that leads firms to became conglomerates, the effect of conglomeration on firms' ability to collude is ambiguous. To see this, suppose a firm initially active in only one market acquires another firm active only in a different market. The merger guarantees the acquiring firm an independent stream of profits which makes it less sensitive to punishments, but also less interested in deviating in its original market. For the marginal merger we can write what follows.

Corollary 1. *Suppose firms' static objective function is strictly concave in profits, and let* $\alpha = \frac{U(\pi_{iA}^*) - U(\underline{\pi}_{iA})}{U(\hat{\pi}_{iA}^*) - U(\underline{\pi}_{iA})}$. *Then a merger with a marginally profitable firm – absent multimarket contact – reduces (increases) the minimum discount factor at which a firm* i *can sustain collusion in its original market A when* $\alpha U'(\hat{\pi}_{iA}^*) + (1 - \alpha)U'(\underline{\pi}_{iA}) < (>)U'(\pi_{iA}^*)$.

   Proof: Before the acquisition the condition for firm i to sustain collusion in market A is

$$\frac{1}{1-\delta}U(\pi_{iA}^*) - U(\hat{\pi}_{iA}^*) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA}) \geq 0 \Leftrightarrow \delta \geq \underline{\delta}(0) = \frac{U(\hat{\pi}_{iA}^*) - U(\pi_{iA}^*)}{U(\hat{\pi}_{iA}^*) - U(\underline{\pi}_{iA})}.$$

After the acquisition the condition becomes

$$\delta \geq \underline{\delta}(\pi_{iB}) = \frac{U(\hat{\pi}_{iA}^* + \pi_{iB}) - U(\pi_{iA}^* + \pi_{iB})}{U(\hat{\pi}_{iA}^* + \pi_{iB}) - U(\underline{\pi}_{iA} + \pi_{iB})}. \tag{5.2}$$

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[9]This effect depends both on the shape of the objective function and on the structure of monetary payoffs in the different markets; therefore it is difficult to generalize. However, for similar enough games/markets the scale effect is present with all most commonly used utility functions (quadratic, logarithmic, and other hyperbolic CRRA functions with elasticity of substitution lower than one).

[10]I thank Douglas Bernheim who suggested this alternative interpretation.

The marginal acquisition facilitates collusion when $Sign\left\{\left.\frac{\partial \underline{\delta}(\pi_{iB})}{\partial \pi}\right|_{\pi_{iB}=0}\right\} < 0$, that is, when

$$[U'(\hat{\pi}_{iA}^*) - U'(\pi_{iA}^*)]\,[U(\hat{\pi}_{iA}^*) - U(\underline{\pi}_{iA})] - [U'(\hat{\pi}_{iA}^*) - U'(\underline{\pi}_{iA})]\,[U(\hat{\pi}_{iA}^*) - U(\pi_{iA}^*)] < 0,$$

which after a few algebraic manipulations leads to the condition in the statement. **Q.E.D.**

Because mergers affect firms' evaluation of profits for many periods, firms' discount factor plays an important role with regards to their effects on firms' ability to collude.[11]

Corollary 2. *Suppose firms' static objective function is strictly concave in profits. Then a merger with a profitable firm – absent multimarket contact – increases (diminishes) the ability of a firm i to sustain collusive agreements in its original market when the discount factor is lower (higher) than a well defined intermediate level $0 < \widetilde{\delta} < 1$.*

Proof: A merger with a firm active in market B facilitates collusion in market A if

$$\frac{1}{1-\delta}U(\pi_{iA}^*) - U(\hat{\pi}_{iA}^*) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA}) <$$

$$\frac{1}{1-\delta}U(\pi_{iA}^* + \pi_{iB}) - U(\hat{\pi}_{iA}^* + \pi_{iB}) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA} + \pi_{iB}). \tag{5.3}$$

With a few algebraic manipulations the inequality reduces to

$$\delta < \widetilde{\delta} = \frac{[U(\pi_{iA}^* + \pi_{iB}) - U(\pi_{iA}^*)] - [U(\hat{\pi}_{iA}^* + \pi_{iB}) - U(\hat{\pi}_{iA}^*)]}{[U(\underline{\pi}_{iA} + \pi_{iB}) - U(\underline{\pi}_{iA})] - [U(\hat{\pi}_{iA}^* + \pi_{iB}) - U(\hat{\pi}_{iA}^*)]},$$

and $0 < \widetilde{\delta} < 1$ because $U' > 0$ and $U'' < 0$ imply

$$U(\underline{\pi}_{iA} + \pi_{iB}) - U(\underline{\pi}_{iA}) > U(\pi_{iA}^* + \pi_{iB}) - U(\pi_{iA}^*) > U(\hat{\pi}_{iA}^* + \pi_{iB}) - U(\hat{\pi}_{iA}^*).$$

Q.E.D.

6. Other Interdependent Supergames

Proposition 1 can be generalized to repeated games other than oligopolies. When players face simultaneously several repeated strategic interactions, the material payoffs from (or the outcomes

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[11] I am grateful to an anonymous referee whose comments convinced me to formalize this implication.

of) some of these may affect agents' evaluation of material payoffs from others even though they are of a completely different nature. Let $\mu_i = (\mu_{i1}, ..., \mu_{in})$ represent the vector of material payoffs from the n stage-games an agent i plays simultaneously each time period, and consider the following definitions.

Definition 1. *An agent i's static objective function U is strictly submodular in the stage-games' material payoffs if, for any two material payoff vectors $\mu_i' = (\mu_{i1}', ..., \mu_{in}')$ and $\mu_i'' = (\mu_{i1}'', ..., \mu_{in}'')$ such that neither $\mu_i' \geq \mu_i''$ nor $\mu_i' \leq \mu_i''$, it holds $U(\mu_i') + U(\mu_i'') > U\left[\min(\mu_{i1}', \mu_{i1}''), ..., \min(\mu_{in}', \mu_{in}'')\right]$ $+ \left[U\max(\mu_{i1}', \mu_{i1}''), ..., \max(\mu_{in}', \mu_{in}'')\right].$*

Submodularity of a function is a generalization of the concept of substitutability of its arguments (e.g. Topkis [39]). An alternative – but here equivalent – generalization of substitutability is that of decreasing differences.

Definition 2. *An agent i's static objective function U has strictly decreasing differences in (μ_{ik}, μ_{ih}) if for all (μ_{ik}', μ_{ik}'') and (μ_{ih}', μ_{ih}'') such that $\mu_{ik}' > \mu_{ik}''$ and $\mu_{ih}' > \mu_{ih}''$, it holds $U(\mu_{ik}', \mu_{ih}', \mu_{i-k-h}) - U(\mu_{ik}'', \mu_{ih}', \mu_{i-k-h}) < U(\mu_{ik}', \mu_{ih}'', \mu_{i-k-h}) - U(\mu_{ik}'', \mu_{ih}'', \mu_{i-k-h}).$*

We can now state our most general result.

Proposition 2. *Suppose agents interact simultaneously in a set of supergames and their static objective functions are strictly submodular – or have strictly decreasing differences – in material payoffs. Then "multigame contact" relaxes the necessary conditions for any cooperative outcome to be supported in subgame-perfect equilibrium by stationary punishment strategies.*

Proof: The proof follows closely that of Proposition 1 (see [35] for a more detailed proof). Let μ_{ik}^* denote player i's per-period material payoffs from cooperating in supergame k, $\widehat{\mu}_{ik}^*$ his material payoffs from deviating unilaterally, and $\underline{\mu}_{ik}$ his per-period payoffs in the punishment phase. Suppose the statement were false. Then the difference between net gains from simultaneously cooperating in the two supergames with and without multigame contact could be weakly negative, in which case it would hold

$$U(\widehat{\mu}_{iA}^*, \widehat{\mu}_{iB}^*) - U(\underline{\mu}_{iA}, \underline{\mu}_{iB}) + U(\underline{\mu}_{iA}, \mu_{iB}^*) + U(\mu_{iA}^*, \underline{\mu}_{iB}) - U(\widehat{\mu}_{iA}^*, \mu_{iB}^*) - U(\mu_{iA}^*, \widehat{\mu}_{iB}^*) \leq$$

$$\leq \frac{1}{\delta} \left[U(\mu_{iA}^*, \mu_{iB}^*) + U(\widehat{\mu}_{iA}^*, \widehat{\mu}_{iB}^*) - U(\widehat{\mu}_{iA}^*, \mu_{iB}^*) - U(\mu_{iA}^*, \widehat{\mu}_{iB}^*) \right]. \tag{6.1}$$

By Definition 1 or 2 the RHS of (5.4) is negative, while the LHS has ambiguous sign. If the LHS is positive (5.4) is violated. If it is negative multiply everything by -1 and rearrange as

$$\delta \geq \frac{-U(\mu_{iA}^*, \mu_{iB}^*) - K'}{-U(\underline{\mu}_{iA}, \mu_{iB}^*) - U(\mu_{iA}^*, \underline{\mu}_{iB}) + U(\underline{\mu}_{iA}, \underline{\mu}_{iB}) - K'}, \tag{6.2}$$

where $K' = [U(\widehat{\mu}_{iA}^*, \widehat{\mu}_{iB}^*) - U(\widehat{\mu}_{iA}^*, \mu_{iB}^*) - U(\mu_{iA}^*, \widehat{\mu}_{iB}^*)]$. By Definition 1 or 2 $U(\mu_{iA}^*, \mu_{iB}^*) < U(\underline{\mu}_{iA}, \mu_{iB}^*) + U(\mu_{iA}^*, \underline{\mu}_{iB}) - U(\underline{\mu}_{iA}, \underline{\mu}_{iB})$, therefore the numerator is strictly larger than the denominator and, because $\delta \leq 1$, (5.5) is also violated. **Q.E.D.**

Strict submodularity of – and strictly decreasing differences in – the objective function imply that material payoffs from different games are "kind of" substitutes. This is sufficient to replicate the effects of concavity behind Proposition 1.

References

[1] F. Allen and R. Michaely, Dividend policy, *in* "Finance," (R. Jarrow, V. Maksimovic, and W.T. Ziemba, Eds.) Handbooks in Operations Research and Management Science, Vol. 9., North-Holland, Amsterdam, 1995.

[2] W.J. Baumol, On the theory of oligopoly, *Economica* **25** (1958), 187-198.

[3] B.S. Bernanke and M. Gertler, Inside the black box: the credit channel of monetary policy transmission, *J. Econ. Perspect.* **9** (1995), 27-48.

[4] D.B. Bernheim and M.D. Whinston, Multimarket contact and collusive behavior, *RAND J. Econ.* **21** (1990), 1-26.

[5] A. Blume, Intraplay communication in repeated games," *Games Econ. Behav.* **6** (1994), 181-211.

[6] M.L. De Fond and C.W. Park, Smoothing income in anticipation of future earnings," *J. Acc. Econ.* **23** (1997), 115-139.

[7] F. Degeorge, P. Jayendu, and R. Zeckhauser, Earnings manipulation to exceed thresholds, mimeo, Kennedy School of Government, 1997.

[8] R.A. Dye, Earnings management in an overlapping generations model, *J. Acc. Res.* **26** (1988), 195-235.

[9] The Economist, Survey on corporate risk management, February 10th-16th 1996, 60.

[10] C. Edwards, "Conglomerate Bigness as a Source of Power" *NBER Conference Report on Business Concentration and Price Policy,* Princeton University Press, Princeton, 1955.

[11] J. Farrell and E. Maskin, Renegotiation in repeated games, *Games Econ. Behav.* **1** (1989), 327-60.

[12] S. Fazzari, R.G. Hubbard, and B. Petersen, Finance constraints and corporate investment, *Brookings Papers on Economic Activities* **1** (1988), 141-195.

[13] I. Fisher, "The Theory of Interest," Mcmillan, New York, 1930.

[14] J. Friedman, A noncooperative equilibrium for supergames, *Rev. Econ. Stud.* **38** (1971), 1-12.

[15] K.A. Froot, D.S. Sharfstein, and J.C. Stein, Risk management: coordinating corporate investment and financing policies, *J. Finance* **48** (1993), 1629-1658.

[16] D. Fudenberg and J. Tirole, A theory of income and dividend smoothing based on incumbency rents, *J. Polit. Economy* **103** (1995), 75-93.

[17] J.J. Gaver, K.M. Gaver, and J.R. Austin, Additional evidence on bonus plans and income management, *J. Acc. Econ.* **19** (1995), 3-28.

[18] C. Geczy, B.A. Minton, and C. Schrand, Why firms use currency derivatives, *J. Finance* **52** (1997), 1323-54.

[19] M.B. Greenawalt and J.F. Sinkey, Bank loan-loss provision and the income-smoothing hypothesis: an empirical analysis 1976-1984, *J. Finan. Services Res.* **1** (1988), 301-18.

[20] B.C. Greenwald and J.E. Stiglitz, Financial market imperfections and business cycles, *Quart. J. Econ.* **108** (1993), 77-114.

[21] B.C. Greenwald and J.E. Stiglitz, Asymmetric information and the new theory of the firm: financial constraints and risk behavior, *Amer. Econ. Rev.* **80** (1990), 160-65.

[22] P.M. Healy, The effect of bonus schemes on accounting decisions, *J. Acc. Econ.* **7** (1985), 85-107.

[23] R. Holthausen, D. Larker, and R. Sloan, Annual bonus schemes and the manipulation of earnings, *J. Acc. Econ.* **19** (1995), 29-74.

[24] R.G. Hubbard, A.K. Kashyap, and T.M. Whited, Internal finance and firm investment, N.B.E.R Working Paper No. 4392, 1993.

[25] M.C. Jensen and W. Meckling, Theory of the firm: managerial behavior, agency costs and ownership structure, *J. Finan. Econ.* **3** (1976), 305-360.

[26] P.L. Joskow and N. L. Rose, CEO pay and firm performance: dynamics, asymmetries and alternative performance measures, N.B.E.R. Working Paper No. 4976, 1994.

[27] E. Kasanen, J. Kinnunen, and J. Niskanen, Dividend-based earning management: Empirical evidence from Finland, *J. Acc. Econ.* **22** (1996), 283-312.

[28] R.A. Lambert, Income smoothing as rational equilibrium behavior, *Acc. Rev.* **59** (1984), 604-618.

[29] K.A. Merchant, "Rewarding Results: Motivating Profit Center Managers," Harvard Business School, Boston, 1989.

[30] B. McCutcheon, Do meetings in smoke-filled rooms facilitate collusion? *J. Polit. Economy* **105** (1997), 330-350.

[31] S.C. Myers and N.S. Majluf, Corporate financing and investment decisions when firms have information that investors do not have, *J. Finan. Econ.* **13** (1984), 187-221.

[32] J. Ronen and S. Sadan, "Smoothing Income Numbers: Objectives, Means and Implications," Addison-Wesley, Reading (Mass.), 1981.

[33] H. Simon, "Administrative Behavior," (2nd ed.) Macmillan, New York, 1957.

[34] C. Smith and R. Stulz, The determinants of firms' hedging policies, *J. Finan. Quant. Anal.* **28** (1985), 3-28.

[35] G. Spagnolo, Multimarket contact, concavity and collusion: on extremal equilibria of interdependent supergames, Working Paper No.104, Stockholm School of Economics, 1996.

[36] G. Spagnolo, Ownership, control, and collusion, Working Paper No.139, Stockholm School of Economics, 1996.

[37] R. Stulz, Optimal hedging policies, *J. Finan. Quant. Anal.* **19** (1984), 127-140.

[38] R. Stulz, Managerial discretion and optimal financing policy, *J. Finan. Econ.* **26** (1990), 3-27.

[39] D. Topkis, Equilibrium points in nonzero-sum n-person submodular games, *J. Contr. Optimiz.* **17** (1979), 773-787.

[40] B. Trueman and S.D. Titman, An explanation for accounting income smoothing, *J. Acc. Res.* **26** (1988), 127-139.

[41] O.E. Williamson, "Managerial Discretion and Business Behavior," Prentice-Hall, Engelwood Cliffs (N.J.), 1964.