

RESEARCH

Open Access

# On $\lambda$ -statistical convergence and strongly $\lambda$ -summable functions of order $\alpha$

Mikail Et<sup>1</sup>, Syed Abdul Mohiuddine<sup>2\*</sup> and Abdullah Alotaibi<sup>2</sup>

\*Correspondence:  
mohiuddine@gmail.com  
<sup>2</sup>Department of Mathematics,  
Faculty of Science, King Abdulaziz  
University, P.O. Box 80203, Jeddah,  
21589, Saudi Arabia  
Full list of author information is  
available at the end of the article

## Abstract

In this study, using the notion of  $(V, \lambda)$ -summability and  $\lambda$ -statistical convergence, we introduce the concepts of strong  $(V, \lambda, p)$ -summability and  $\lambda$ -statistical convergence of order  $\alpha$  of real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . Also some relations between  $\lambda$ -statistical convergence of order  $\alpha$  and strong  $(V, \lambda, p)$ -summability of order  $\beta$  are given.

**MSC:** 40A05; 40C05; 46A45

**Keywords:** statistical convergence; measurable function;  $(V, \lambda)$ -summability

## 1 Introduction

The idea of statistical convergence was given by Zygmund [1] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [2] and Fast [3] and then reintroduced by Schoenberg [4] independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory, measure theory, trigonometric series, turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Alotaibi and Alroqi [5], Belen and Mohiuddine [6], Connor [7], Dutta *et al.* [8–11], Et *et al.* [12–14], Fridy [15], Güngör *et al.* [16, 17], Kolk [18], Mohiuddine *et al.* [19–28], Mursaleen *et al.* [29–32], Nuray [33], Rath and Tripathy [34], Salat [35], Savaş *et al.* [36, 37], Tripathy [38] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

## 2 Definition and preliminaries

The definitions of statistical convergence and strong  $p$ -Cesàro convergence of a sequence of real numbers were introduced in the literature independently of one another and followed different lines of development since their first appearance. It turns out, however, that the two definitions can be simply related to one another in general and are equivalent for bounded sequences. The idea of statistical convergence depends on the density of

subsets of the set  $\mathbb{N}$  of natural numbers. The density of a subset  $E$  of  $\mathbb{N}$  is defined by

$$\delta(E) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \chi_E(k) \text{ provided the limit exists,}$$

where  $\chi_E$  is the characteristic function of  $E$ . It is clear that any finite subset of  $\mathbb{N}$  has zero natural density and  $\delta(E^c) = 1 - \delta(E)$ .

A sequence  $x = (x_k)$  is said to be statistically convergent to  $L$  if for every  $\varepsilon > 0$ ,  $\delta(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}) = 0$ . In this case we write  $x_k \xrightarrow{\text{stat}} L$  or  $S\text{-}\lim x_k = L$ . The set of all statistically convergent sequences will be denoted by  $S$ .

The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan in [39] and after then statistical convergence of order  $\alpha$  and strong  $p$ -Cesàro summability of order  $\alpha$  were studied by Çolak [40, 41] and generalized by Çolak and Bektaş [42].

Let  $\lambda = (\lambda_n)$  be a non-decreasing sequence of positive real numbers tending to  $\infty$  such that  $\lambda_{n+1} \leq \lambda_n + 1$ ,  $\lambda_1 = 1$ . The generalized de la Vallée-Poussin mean is defined by  $t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k$ , where  $I_n = [n - \lambda_n + 1, n]$  for  $n = 1, 2, \dots$ . A sequence  $x = (x_k)$  is said to be  $(V, \lambda)$ -summable to a number  $L$  if  $t_n(x) \rightarrow L$  as  $n \rightarrow \infty$  [43]. If  $\lambda_n = n$ , then  $(V, \lambda)$ -summability is reduced to Cesàro summability. By  $\Lambda$  we denote the class of all non-decreasing sequence of positive real numbers tending to  $\infty$  such that  $\lambda_{n+1} \leq \lambda_n + 1$ ,  $\lambda_1 = 1$ .

In [44] Borwein introduced and studied strongly summable functions. A real-valued function  $x(t)$ , measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , is said to be strongly summable to  $L = L_x$  if

$$\lim_{n \rightarrow \infty} \frac{1}{n} \int_1^n |x(t) - L|^p dt = 0, \quad 1 \leq p < \infty.$$

$[W_p]$  will denote the space of real-valued function  $x$ , measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . The space  $[W_p]$  is a normed space with the norm

$$\|x\| = \sup_{n \geq 1} \left( \frac{1}{n} \int_1^n |x(t)|^p dt \right)^{\frac{1}{p}}.$$

In this paper, using the notion of  $(V, \lambda)$ -summability and  $\lambda$ -statistical convergence, we introduce and study the concepts of strong  $(V, \lambda, p)$ -summability and  $\lambda$ -statistical convergence of order  $\alpha$  of real-valued functions  $x(t)$ , measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ .

Throughout the paper, unless stated otherwise, by ‘for all  $n \in \mathbb{N}_{n_0}$ ’ we mean ‘for all  $n \in \mathbb{N}$  except finite numbers of positive integers’ where  $\mathbb{N}_{n_0} = \{n_0, n_0 + 1, n_0 + 2, \dots\}$  for some  $n_0 \in \mathbb{N} = \{1, 2, 3, \dots\}$ .

### 3 Main results

In this section we give the main results of this paper. In Theorem 3.4 we give the inclusion relations between  $\lambda$ -statistically convergent functions of order  $\alpha$  for different  $\alpha$ 's of real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . In Theorem 3.8 we give the relationship between the strong  $(V, \lambda, p)$ -summability of order  $\alpha$  for different  $\alpha$ 's of real-valued functions which are measurable (in the Lebesgue sense) in

the interval  $(1, \infty)$ . In Theorem 3.10 we give the relationship between the strong  $[W_{\lambda,p}^\beta]$ -summability and  $[S_\lambda^\alpha]$ -statistical convergence of real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ .

**Definition 3.1** Let the sequence  $\lambda = (\lambda_n)$  be as above,  $\alpha \in (0, 1]$  and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . A real-valued function  $x(t)$  is said to be strongly  $(V, \lambda, p)$ -summable of order  $\alpha$  (or  $[W_{\lambda,p}^\alpha]$ -summable) if there is a number  $L = L_x$  such that

$$\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\alpha} \int_{n-\lambda_n+1}^n |x(t) - L|^p dt = 0, \quad 1 \leq p < \infty,$$

where  $I_n = [n - \lambda_n + 1, n]$  and  $\lambda_n^\alpha$  denote the  $\alpha$ th power  $(\lambda_n)^\alpha$  of  $\lambda_n$ , that is,  $\lambda^\alpha = (\lambda_n^\alpha) = (\lambda_1^\alpha, \lambda_2^\alpha, \dots, \lambda_n^\alpha, \dots)$ . In this case we write  $[W_{\lambda,p}^\alpha]$ - $\lim x(t) = L$ . The set of all strongly  $(V, \lambda, p)$ -summable functions of order  $\alpha$  will be denoted by  $[W_{\lambda,p}^\alpha]$ . For  $\lambda_n = n$  for all  $n \in \mathbb{N}$ , we shall write  $[W_p^\alpha]$  instead of  $[W_{\lambda,p}^\alpha]$  and in the special case  $\alpha = 1$  we shall write  $[W_{\lambda,p}]$  instead of  $[W_{\lambda,p}^\alpha]$ , and also in the special case  $\alpha = 1$  and  $\lambda_n = n$  for all  $n \in \mathbb{N}$  we shall write  $[W_p]$  instead of  $[W_{\lambda,p}^\alpha]$ .

**Definition 3.2** Let the sequence  $\lambda = (\lambda_n)$  be as above,  $\alpha \in (0, 1]$  and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . A real-valued function  $x(t)$  is said to be  $\lambda$ -statistically convergent of order  $\alpha$  (or  $[S_\lambda^\alpha]$ -statistical convergence) to a number  $L = L_x$  for every  $\varepsilon > 0$ ,

$$\lim_n \frac{1}{\lambda_n^\alpha} |\{t \in I_n : |x(t) - L| \geq \varepsilon\}| = 0.$$

The set of all  $\lambda$ -statistically convergent functions of order  $\alpha$  will be denoted by  $[S_\lambda^\alpha]$ . In this case we write  $[S_\lambda^\alpha]$ - $\lim x(t) = L$ . For  $\lambda_n = n$ , for all  $n \in \mathbb{N}$ , we shall write  $[S^\alpha]$  instead of  $[S_\lambda^\alpha]$  and in the special case  $\alpha = 1$ , we shall write  $[S_\lambda]$  instead of  $[S_\lambda^\alpha]$ .

The  $\lambda$ -statistical convergence of order  $\alpha$  is well defined for  $0 < \alpha \leq 1$ , but it is not well defined for  $\alpha > 1$  in general. For this let  $\lambda_n = n$  for all  $n \in \mathbb{N}$  and  $x(t)$  be defined as follows:

$$x(t) = \begin{cases} 1 & \text{if } t = 3n, \\ 0 & \text{if } t \neq 3n, \end{cases} \quad n = 1, 2, 3, \dots,$$

then both

$$\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\alpha} |\{t \in I_n : |x(t) - 1| \geq \varepsilon\}| \leq \lim_{n \rightarrow \infty} \frac{[\lambda_n] + 1}{3\lambda_n^\alpha} = 0$$

and

$$\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\alpha} |\{t \in I_n : |x(t) - 0| \geq \varepsilon\}| \leq \lim_{n \rightarrow \infty} \frac{2([\lambda_n] + 1)}{3\lambda_n^\alpha} = 0$$

for  $\alpha > 1$ , and so  $\lambda$ -statistically converges of order  $\alpha$  both to 1 and 0, i.e.,  $[S_\lambda^\alpha]$ - $\lim x(t) = 1$  and  $[S_\lambda^\alpha]$ - $\lim x(t) = 0$ . But this is impossible.

The proof of the following two results is easy, so we state without proof.

**Theorem 3.3** Let  $0 < \alpha \leq 1$  and  $x(t)$  and  $y(t)$  be real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then

- (i) If  $[S_\lambda^\alpha]$ - $\lim x(t) = L$  and  $c \in \mathbb{R}$ , then  $[S_\lambda^\alpha]$ - $\lim cx(t) = cL$ ;
- (ii) If  $[S_\lambda^\alpha]$ - $\lim x(t) = L_1$  and  $[S_\lambda^\alpha]$ - $\lim y(t) = L_2$ , then  $[S_\lambda^\alpha]$ - $\lim(x(t) + y(t)) = L_1 + L_2$ .

**Theorem 3.4** Let  $\lambda = (\lambda_n) \in \Lambda$  ( $0 < \alpha \leq \beta \leq 1$ ) and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then  $[S_\lambda^\alpha] \subseteq [S_\lambda^\beta]$ .

From Theorem 3.4 we have the following.

**Corollary 3.5** If  $x(t)$  is  $\lambda$ -statistically convergent of order  $\alpha$  to  $L$ , then it is statistically convergent to  $L$  for each  $\alpha \in (0, 1]$ .

**Theorem 3.6** Let  $\lambda = (\lambda_n) \in \Lambda$  ( $0 < \alpha \leq 1$ ) and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then  $[S^\alpha] \subseteq [S_\lambda^\alpha]$  if

$$\liminf_{n \rightarrow \infty} \frac{\lambda_n^\alpha}{n^\alpha} > 0. \tag{1}$$

*Proof* For given  $\varepsilon > 0$ , we have

$$\{t \leq n : |x(t) - L| \geq \varepsilon\} \supset \{t \in I_n : |x(t) - L| \geq \varepsilon\},$$

and so

$$\frac{1}{n^\alpha} |\{t \leq n : |x(t) - L| \geq \varepsilon\}| \geq \frac{\lambda_n^\alpha}{n^\alpha} \frac{1}{\lambda_n^\alpha} |\{t \in I_n : |x(t) - L| \geq \varepsilon\}|.$$

Taking the limit as  $n \rightarrow \infty$  and using (1), we get  $[S^\alpha]$ - $\lim x_k = L \implies [S_\lambda^\alpha]$ - $\lim x_k = L$ . □

From Theorem 3.6 we have the following result.

**Corollary 3.7** Let  $\lambda = (\lambda_n) \in \Lambda$  ( $0 < \alpha \leq 1$ ) and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then  $[S] \subseteq [S_\lambda^\alpha]$  if

$$\liminf_{n \rightarrow \infty} \frac{\lambda_n^\alpha}{n} > 0.$$

**Theorem 3.8** Let  $\lambda = (\lambda_n) \in \Lambda$ ,  $0 < \alpha \leq \beta \leq 1$ ,  $p$  be a positive real number and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then  $[W_{\lambda,p}^\alpha] \subseteq [W_{\lambda,p}^\beta]$ .

*Proof* Omitted. □

From Theorem 3.8 we have the following result.

**Corollary 3.9** Let  $\lambda = (\lambda_n) \in \Lambda$  and  $p$  be a positive real number and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ , then  $[W_{\lambda,p}^\alpha] \subseteq [W_{\lambda,p}]$ .

**Theorem 3.10** Let  $\lambda = (\lambda_n) \in \Lambda$ ,  $0 < \alpha \leq \beta \leq 1$ ,  $p$  be a positive real number and  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If a function  $x(t)$  is  $[W_{\lambda, p}^\alpha]$ -summable, then it is  $\lambda$ -statistically convergent of order  $\beta$ .

*Proof* For any sequence  $x(t) \in [W_{\lambda, p}^\alpha]$  and  $\varepsilon > 0$ , we have

$$\begin{aligned} \int_{n-\lambda_{n+1}}^n |x(t) - L|^p dt &= \int_{\substack{n-\lambda_{n+1} \\ |x(t)-L| \geq \varepsilon}}^n |x(t) - L|^p dt + \int_{\substack{n-\lambda_{n+1} \\ |x(t)-L| < \varepsilon}}^n |x(t) - L|^p dt \\ &\geq \int_{\substack{n-\lambda_{n+1} \\ |x(t)-L| \geq \varepsilon}}^n |x(t) - L|^p dt \\ &\geq |\{t \in I_n : |x(t) - L| \geq \varepsilon\}| \cdot \varepsilon^p \end{aligned}$$

and so that

$$\frac{1}{\lambda_n^\alpha} \int_{n-\lambda_{n+1}}^n |x(t) - L|^p dt \geq \frac{1}{\lambda_n^\beta} |\{t \in I_n : |x(t) - L| \geq \varepsilon\}| \cdot \varepsilon^p.$$

From this it follows that if  $x(t)$  is  $[W_{\lambda, p}^\alpha]$ -summable, then it is  $\lambda$ -statistically convergent of order  $\beta$ . □

From Theorem 3.10 we have the following results.

**Corollary 3.11** Let  $\alpha$  be a fixed real number such that  $0 < \alpha \leq 1$  and  $0 < p < \infty$ . The following statements hold:

- (i) If  $x(t)$  is strongly  $[W_{\lambda, p}^\alpha]$ -summable of order  $\alpha$ , then it is  $\lambda$ -statistically convergent of order  $\alpha$ .
- (ii) If  $x(t)$  is strongly  $[W_{\lambda, p}^\alpha]$ -summable of order  $\alpha$ , then it is  $\lambda$ -statistically convergent.

**Theorem 3.12** Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$  ( $0 < \alpha \leq \beta \leq 1$ ), and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If

$$\liminf_{n \rightarrow \infty} \frac{\lambda_n^\alpha}{\mu_n^\beta} > 0, \tag{2}$$

then  $[S_\mu^\beta] \subseteq [S_\lambda^\alpha]$ .

*Proof* Suppose that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$  and let (2) be satisfied. Then  $I_n \subset J_n$  and so that  $\varepsilon > 0$  we may write

$$\{t \in J_n : |x(t) - L| \geq \varepsilon\} \supset \{t \in I_n : |x(t) - L| \geq \varepsilon\}$$

and so

$$\frac{1}{\mu_n^\beta} |\{t \in J_n : |x(t) - L| \geq \varepsilon\}| \geq \frac{\lambda_n^\alpha}{\mu_n^\beta} \frac{1}{\lambda_n^\alpha} |\{t \in I_n : |x(t) - L| \geq \varepsilon\}|$$

for all  $n \in \mathbb{N}_{n_0}$ , where  $J_n = [n - \mu_n + 1, n]$ . Now, taking the limit as  $n \rightarrow \infty$  in the last inequality and using (2), we get  $[S_\mu^\beta] \subseteq [S_\lambda^\alpha]$ .  $\square$

From Theorem 3.12 we have the following results.

**Corollary 3.13** *Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$ , and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If (2) holds, then*

- (i)  $[S_\mu^\alpha] \subseteq [S_\lambda^\alpha]$  for each  $\alpha \in (0, 1]$ ,
- (ii)  $[S_\mu] \subseteq [S_\lambda]$  for each  $\alpha \in (0, 1]$ ,
- (iii)  $[S_\mu] \subseteq [S_\lambda]$ .

**Theorem 3.14** *Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$  ( $0 < \alpha \leq \beta \leq 1$ ), and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If (2) holds, then  $[W_{\mu p}^\beta] \subseteq [W_{\lambda p}^\alpha]$ .*

*Proof* Omitted.  $\square$

**Corollary 3.15** *Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$ , and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If (2) holds, then*

- (i)  $[W_{\mu p}^\alpha] \subseteq [W_{\lambda p}^\alpha]$  for each  $\alpha \in (0, 1]$ ,
- (ii)  $[W_{\mu p}] \subseteq [W_{\lambda p}]$  for each  $\alpha \in (0, 1]$ ,
- (iii)  $[W_{\mu p}] \subseteq [W_{\lambda p}]$ .

**Theorem 3.16** *Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$  ( $0 < \alpha \leq \beta \leq 1$ ), and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$  and (2) holds. If a real-valued function  $x(t)$  is strongly  $(V, \mu, p)$ -summable of order  $\beta$  to  $L$ , then it is  $\lambda$ -statistically convergent of order  $\alpha$  to  $L$ .*

*Proof* Let  $x(t)$  be a real-valued function such that  $x(t)$  is strongly  $(V, \mu, p)$ -summable of order  $\beta$  to  $L$  and  $\varepsilon > 0$ . Then we have

$$\begin{aligned} \int_{t \in J_n} |x(t) - L|^p dt &= \int_{\substack{t \in J_n \\ |x(t) - L| \geq \varepsilon}} |x(t) - L|^p dt + \int_{\substack{t \in J_n \\ |x(t) - L| < \varepsilon}} |x(t) - L|^p dt \\ &\geq \int_{\substack{t \in J_n \\ |x(t) - L| \geq \varepsilon}} |x(t) - L|^p dt \\ &\geq |\{t \in I_n : |x(t) - L| \geq \varepsilon\}| \cdot \varepsilon^p \end{aligned}$$

and so that

$$\frac{1}{\mu_n^\beta} \int_{t \in J_n} |x(t) - L|^p dt \geq \frac{\lambda_n^\alpha}{\mu_n^\beta \lambda_n^\alpha} |\{t \in I_n : |x(t) - L| \geq \varepsilon\}| \cdot \varepsilon^p.$$

Since (2) holds, it follows that if  $x(t)$  is strongly  $(V, \mu, p)$ -summable of order  $\beta$  to  $L$ , then it is  $\lambda$ -statistically convergent of order  $\alpha$  to  $L$ .  $\square$

**Corollary 3.17** *Let  $\lambda = (\lambda_n)$  and  $\mu = (\mu_n)$  be two sequences in  $\Lambda$  such that  $\lambda_n \leq \mu_n$  for all  $n \in \mathbb{N}_{n_0}$ , and let  $x(t)$  be a real-valued function which is measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . If (2) holds, then*

- (i) *A real-valued function  $x(t)$  is strongly  $(V, \mu, p)$ -summable of order  $\alpha$  to  $L$ , then it is  $\lambda$ -statistically convergent of order  $\alpha$  to  $L$ ;*
- (ii) *A real-valued function  $x(t)$  is strongly  $(V, \mu, p)$ -summable to  $L$ , then it is  $\lambda$ -statistically convergent of order  $\alpha$  to  $L$ ;*
- (iii) *A real-valued function  $x(t)$  is strongly  $(V, \mu, p)$ -summable to  $L$ , then it is  $\lambda$ -statistically convergent to  $L$ .*

#### Competing interests

The authors declare that they have no competing interests.

#### Authors' contributions

The authors contributed equally and significantly in writing this paper. All the authors read and approved the final manuscript.

#### Author details

<sup>1</sup>Department of Mathematics, Firat University, Elazig, 23119, Turkey. <sup>2</sup>Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah, 21589, Saudi Arabia.

#### Acknowledgements

The authors gratefully acknowledge the financial support from King Abdulaziz University, Jeddah, Saudi Arabia.

Received: 22 March 2013 Accepted: 16 October 2013 Published: 07 Nov 2013

#### References

1. Zygmund, A: Trigonometric Series. Cambridge University Press, Cambridge (1979)
2. Steinhaus, H: Sur la convergence ordinaire et la convergence asymptotique. *Colloq. Math.* **2**, 73-74 (1951)
3. Fast, H: Sur la convergence statistique. *Colloq. Math.* **2**, 241-244 (1951)
4. Schoenberg, IJ: The integrability of certain functions and related summability methods. *Am. Math. Mon.* **66**, 361-375 (1959)
5. Alotaibi, A, Alroqi, AM: Statistical convergence in a paranormed space. *J. Inequal. Appl.* **2012**, Article ID 39 (2012)
6. Belen, C, Mohiuddine, SA: Generalized weighted statistical convergence and application. *Appl. Math. Comput.* **219**, 9821-9826 (2013)
7. Connor, JS: The statistical and strong  $p$ -Cesàro convergence of sequences. *Analysis* **8**, 47-63 (1988)
8. Dutta, H, Başar, F: A generalization of Orlicz sequence spaces by Cesàro mean of order one. *Acta Math. Univ. Comen.* **80**(2), 185-200 (2011)
9. Dutta, H, Bilgin, T: Strongly  $(V^\lambda, A, \Delta_{m,r}^0, p)$ -summable sequence spaces defined by an Orlicz function. *Appl. Math. Lett.* **24**(7), 1057-1062 (2011)
10. Karakaya, V, Dutta, H: On some vector valued generalized difference modular sequence spaces. *Filomat* **25**(3), 15-27 (2011)
11. Tripathy, BC, Dutta, H: On some lacunary difference sequence spaces defined by a sequence of Orlicz functions and  $q$ -lacunary  $\Delta_m^n$  statistical convergence. *An. Univ. "Ovidius" Constanța, Ser. Mat.* **20**(1), 417-430 (2012)
12. Et, M: Strongly almost summable difference sequences of order  $m$  defined by a modulus. *Studia Sci. Math. Hung.* **40**(4), 463-476 (2003)
13. Et, M: Spaces of Cesàro difference sequences of order  $r$  defined by a modulus function in a locally convex space. *Taiwan. J. Math.* **10**(4), 865-879 (2006)
14. Et, M, Altin, Y, Choudhary, B, Tripathy, BC: On some classes of sequences defined by sequences of Orlicz functions. *Math. Inequal. Appl.* **9**(2), 335-342 (2006)
15. Fridy, J: On statistical convergence. *Analysis* **5**, 301-313 (1985)
16. Güngör, M, Et, M, Altin, Y: Strongly  $(V_\sigma, \lambda, q)$ -summable sequences defined by Orlicz functions. *Appl. Math. Comput.* **157**(2), 561-571 (2004)
17. Güngör, M, Et, M:  $\Delta^m$ -strongly almost summable sequences defined by Orlicz functions. *Indian J. Pure Appl. Math.* **34**(8), 1141-1151 (2003)
18. Kolk, E: The statistical convergence in Banach spaces. *Acta Comment. Univ. Tartu Math.* **928**, 41-52 (1991)
19. Mohiuddine, SA, Aiyub, M: Lacunary statistical convergence in random 2-normed spaces. *Appl. Math. Inform. Sci.* **6**(3), 581-585 (2012)
20. Mohiuddine, SA, Alghamdi, MA: Statistical summability through a lacunary sequence in locally solid Riesz spaces. *J. Inequal. Appl.* **2012**, Article ID 225 (2012)
21. Mohiuddine, SA, Alotaibi, A, Alsulami, SM: Ideal convergence of double sequences in random 2-normed spaces. *Adv. Differ. Equ.* **2012**, Article ID 149 (2012)
22. Mohiuddine, SA, Alotaibi, A, Mursaleen, M: Statistical convergence of double sequences in locally solid Riesz spaces. *Abstr. Appl. Anal.* **2012**, Article ID 719729 (2012)

23. Mohiuddine, SA, Alotaibi, A, Mursaleen, M: A new variant of statistical convergence. *J. Inequal. Appl.* **2013**, Article ID 309 (2013)
24. Mohiuddine, SA, Alotaibi, A, Mursaleen, M: Statistical convergence through de la Vallée-Poussin mean in locally solid Riesz spaces. *Adv. Differ. Equ.* **2013**, Article ID 66 (2013)
25. Mohiuddine, SA, Hazarika, B, Alotaibi, A: Double lacunary density and some inclusion results in locally solid Riesz spaces. *Abstr. Appl. Anal.* **2013**, Article ID 507962 (2013)
26. Mohiuddine, SA, Lohani, QMD: On generalized statistical convergence in intuitionistic fuzzy normed space. *Chaos Solitons Fractals* **42**, 1731-1737 (2009)
27. Mohiuddine, SA, Şevli, H, Cancan, M: Statistical convergence of double sequences in fuzzy normed spaces. *Filomat* **26**(4), 673-681 (2012)
28. Mohiuddine, SA, Şevli, H, Cancan, M: Statistical convergence in fuzzy 2-normed space. *J. Comput. Anal. Appl.* **12**(4), 787-798 (2010)
29. Mursaleen, M:  $\lambda$ -statistical convergence. *Math. Slovaca* **50**(1), 111-115 (2000)
30. Mursaleen, M, Alotaibi, A: Statistical summability and approximation by de la Vallée-Poussin mean. *Appl. Math. Lett.* **24**(3), 320-324 (2011)
31. Mursaleen, M, Mohiuddine, SA: Statistical convergence of double sequences in intuitionistic fuzzy normed spaces. *Chaos Solitons Fractals* **41**, 2414-2421 (2009)
32. Mursaleen, M, Mohiuddine, SA: On lacunary statistical convergence with respect to the intuitionistic fuzzy normed space. *J. Comput. Appl. Math.* **233**, 142-149 (2009)
33. Nuray, F:  $\lambda$ -strongly summable and  $\lambda$ -statistically convergent functions. *Iran. J. Sci. Technol., Trans. A, Sci.* **34**(4), 335-338 (2010)
34. Rath, D, Tripathy, BC: On statistically convergent and statistically Cauchy sequences. *Indian J. Pure Appl. Math.* **25**(4), 381-386 (1994)
35. Šalát, T: On statistically convergent sequences of real numbers. *Math. Slovaca* **30**, 139-150 (1980)
36. Savaş, E: Strong almost convergence and almost  $\lambda$ -statistical convergence. *Hokkaido Math. J.* **29**(3), 531-536 (2000)
37. Savaş, E, Mohiuddine, SA:  $\bar{\lambda}$ -statistically convergent double sequences in probabilistic normed spaces. *Math. Slovaca* **62**(1), 99-108 (2012)
38. Tripathy, BC: On generalized difference paranormed statistically convergent sequences. *Indian J. Pure Appl. Math.* **35**(5), 655-663 (2004)
39. Gadjev, AD, Orhan, C: Some approximation theorems via statistical convergence. *Rocky Mt. J. Math.* **32**(1), 129-138 (2002)
40. Çolak, R: Statistical Convergence of Order  $\alpha$ . *Modern Methods in Analysis and Its Applications*, pp. 121-129. Anamaya Pub., New Delhi (2010)
41. Çolak, R: On  $\lambda$ -statistical convergence. In: *Conference on Summability and Applications*, May 12-13, 2011, Istanbul, Turkey (2011)
42. Çolak, R, Bektaş, ÇA:  $\lambda$ -statistical convergence of order  $\alpha$ . *Acta Math. Sci.* **31**(3), 953-959 (2011)
43. Leindler, L: Über die la Vallee-Pousinsche Summierbarkeit Allgemeiner Orthogonalreihen. *Acta Math. Acad. Sci. Hung.* **16**, 375-387 (1965)
44. Borwein, D: Linear functionals connected with strong Cesàro summability. *J. Lond. Math. Soc.* **40**, 628-634 (1965)

10.1186/1029-242X-2013-469

**Cite this article as:** Et et al.: On  $\lambda$ -statistical convergence and strongly  $\lambda$ -summable functions of order  $\alpha$ . *Journal of Inequalities and Applications* 2013, **2013**:469

Submit your manuscript to a SpringerOpen<sup>®</sup> journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

---

Submit your next manuscript at ► [springeropen.com](http://springeropen.com)

---