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ON LAMB'S PLANE PROBLEM IN MICROPOLAR VISCOELASTIC HALF-SPACE WITH STRETCH

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ABSTRACT. A study is made of Lamb's plane problem in micropolar viscoelastic half-space with stretch. The viscoelasticity is characterized by the rate-dependent theories of micro-viscoelasticity generalizing the classical Kelvin-Voigt theory. The displacement components, force stress, couple stress and vector first moment are obtained for a half-space subjected to an arbitrary normal load. Two particular cases of a horizontal force and a torque which are time harmonic have been considered. Several limiting cases are obtained as special cases of the present analysis.

KEYWORDS AND PHRASES. Lamb's problem, Micropolar Viscoelastic Medium and Fourier Transforms.
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1. INTRODUCTION.

Lamb's problem [1] has received considerable attention by several researchers in different elastic media with various kinds of loading. In particular, Nowacki and Nowacki [2] have studied the Lamb problem in micropolar elastic media. Chadha [3] has investigated the Lamb problem in micropolar elastic media and discussed the propagation of waves in a semi-infinite micropolar elastic solid due to loading at the plane boundary of a semi-half space. Acharya and Sengupta [4] have recently studied the same problem in a thermo-elastic medium under the influence of temperture. They have examined the longitudinal and transverse thermo-elastic waves in a micropolar

semi-infinite space bounded by a plane in which a normal loading is applied. In a recent paper by Ray and Sengupta [5], a study is made of a two-dimensional waves in a micropolar thermo-viscoelastic medium. More recently, Kumar and Chadha [6] and Chadha, Kumar and Debnath [7] have made an investigation of the Lamb problem in a microelastic half-space with stretch, and in a thermoelastic micropolar medium with stretch.

Based upon the linear theory of micropolar viscoelastic waves due to Eringen [8-9], a study is made of Lamb's plane problem in micropolar viscoelastic half-space with the effect of stretch. The displacement components, force stress, couple stress, and vector first moment are determined for a half-space subjected to an arbitrary normal load. Two particular cases of a horizontal force and a torque which are time harmonic are cited as examples of the general theory.

2. BASIC CONSTITUTIVE AND FIELD EQUATIONS.

Making reference to Eringen [8-10] and Nowacki [11], the constitutive and field equations for micropolar viscoelastic solids with stretch in the absence of body forces, body moments and stretch forces are

$$t_{ji} = (\lambda + \lambda_1 \frac{\partial}{\partial t})u_{k,k}\delta_{ji} + \{(\mu-\alpha) + (\mu_1 - \alpha_1) \frac{\partial}{\partial t}\} (u_{l,j} + u_{j,i}) + 2 (\alpha + \alpha_1 \frac{\partial}{\partial t}) (u_{i,j} - \epsilon_{kji}\omega_k), \tag{2.1}$$

$$(m_{ji} = \beta_o \epsilon_{kji}\Phi_{,k} + (\beta + \beta_1 \frac{\partial}{\partial t}) \omega_{k,k}\delta_{ji} + \{(\gamma - \epsilon) + (\gamma - \epsilon) \frac{\partial}{\partial t}\} \omega_{j,i} + [(\gamma + \epsilon) + (\gamma + \epsilon) \frac{\partial}{\partial t}] \omega_{j,i}, \tag{2.2}$$

$$\lambda_j = \alpha_o \Phi_{,j} + \frac{1}{3} \beta_o \epsilon_{kji} \omega_{k,i}, \tag{2.3}$$

$$[(\mu + \alpha) + (\mu_1 + \alpha_1) \frac{\partial}{\partial t}] \nabla^2 \underline{u} + [(\lambda + \mu - \alpha) + (\lambda_1 + \mu_1 - \alpha_1) \frac{\partial}{\partial t}] \text{ grad div } \underline{u} + 2(\alpha + \alpha_1 \frac{\partial}{\partial t}) \text{ rot } \underline{\omega} = \rho \frac{\partial^2 \underline{u}}{\partial t^2}, \tag{2.4}$$

$$[(\gamma+ \epsilon) + (\gamma_1 + \epsilon_1) \frac{\partial}{\partial t}] \nabla^2 \underline{\omega} +[(\gamma+ \beta - \epsilon) + (\gamma_1 + \beta_1 - \epsilon) \frac{\partial}{\partial t}] \text{ grad div } \underline{\omega} -4(\alpha + \alpha_1 \frac{\partial}{\partial t}) \underline{\omega} + 2 (\alpha + \alpha_1 \frac{\partial}{\partial t}) \text{ rot } \underline{u} = J \frac{\partial^2 \underline{\omega}}{\partial t^2} \tag{2.5}$$

$$\alpha_o \nabla^2 \Phi - \eta \Phi_o = \frac{J}{2} \frac{\partial \Phi^2}{\partial t^2}, \tag{2.6}$$

where t_{ji}, m_{ji} and λ_j are the components of force stress, couple stress and vector first moment, ρ is the density, J is the rotational inertia, $\underline{u}$ is the displacement vector, $\underline{\omega}$ is the microrotation vector and Φ is the scalar microstretch, and λ, μ, α, β, γ, ϵ, α_o, β_o, η_o are material constants.

3. BASIC EQUATIONS AND THE BOUNDARY CONDITIONS

We consider a semi-infinite homogeneous, isotropic, micropolar viscoelastic solid with stretch. We take the rectangular Cartesian coordinates x, y, and z with the origin at the plane boundary of the half-space, $z \geq o$ and the z-axis is normal to the medium. We assume that there is a uniform stretch in the x-direction and a given loading function g(x,t) normal to the free surface $z = o$.

We consider the two-dimensional problem so that the displacement and microrotation are independent of the y coordinate. Hence, we may write $\underline{u} = (u_1, o, u_3)$ and $\underline{\omega} = (o, \omega_2, o)$. We introduce displacement potentials $\phi(x,z,t)$ and $\psi(x,z,t)$ defined by $u_1 = \phi_x + \psi_z$ and $u_3 = \phi_z - \psi_x$. Consequently, equations (2.4) - (2.6) can be written in terms of ϕ and ψ as

$$[(c_1^2 + c_1'^2 \frac{\partial}{\partial t}) \nabla^2 - \frac{\partial^2}{\partial t^2}] \phi = 0, \tag{3.1}$$

$$[(c_2^2 + c_2'^2) \frac{\partial}{\partial t} \nabla^2 - \frac{\partial^2}{\partial t^2}] \psi - (p_1 + p_1' \frac{\partial}{\partial t}) \omega_2 = 0, \tag{3.2}$$

$$[(c_3^2 + c_3'^2 \frac{\partial}{\partial t}) \nabla^2 - (\gamma_2 + \gamma_2' \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2})] \omega_2 + \frac{1}{2} (\gamma_2 + \gamma_2' \frac{\partial}{\partial t}) \nabla^2 \psi = 0, \tag{3.3}$$

$$[\nabla^2 - \gamma_3 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2}] \phi = 0, \tag{3.4}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad c_1^2 = (\lambda + 2\mu)/\rho, \quad c_1'^2 = (\lambda_1 + 2\mu_1)/\rho, \quad c_2^2 = (\mu+\alpha)/\rho$$

$$c_2'^2 = (\mu_1 + \alpha_1)/\rho, \quad c_3^2 = (\gamma + \varepsilon)/J, \quad c_3'^2 = (\gamma_1 + \varepsilon_1)/J,$$

$$c_4^2 = 2\alpha_o/J, \quad p_1 = 2\alpha/\rho, \quad p_1' = 2\alpha_1/\rho, \quad \gamma_2 = 4\alpha/J, \quad \gamma_2' = 4\alpha_1/J, \quad \gamma_3 = \eta_o/\alpha_o.$$

Eliminating ψ or ω_2 from (3.2) and (3.3), we obtain

$$[\{(c_2^2 + c_2'^2 \frac{\partial}{\partial t}) \nabla^2 - \frac{\partial^2}{\partial t^2}\} \{(c_3^2 + c_3'^2 \frac{\partial}{\partial t}) \nabla^2 - (\gamma_2 + \gamma_2' \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2})\}$$
$$+ \frac{1}{2} (p_1 + p_1' \frac{\partial}{\partial t}) (\gamma_2 + \gamma_2' \frac{\partial}{\partial t}) \nabla^2] (\psi, \omega_2) = 0 \tag{3.5ab}$$

The boundary conditions of the problem are

$$t_{33} = - g(x,t), \quad t_{31} = m_{32} = \lambda_3 = 0 \qquad \text{at } z = 0 \tag{3.6abcd}$$

where

$$t_{33} = (\lambda + \lambda_1 \frac{\partial}{\partial t}) \nabla^2 \phi + 2(\mu + \mu_1 \frac{\partial}{\partial t}) (\frac{\partial^2 \phi}{\partial z^2} - \frac{\partial^2 \psi}{\partial x \partial z}), \tag{3.7}$$

$$t_{31} = (\mu + \mu_1 \frac{\partial}{\partial t}) (2 \frac{\partial^2 \phi}{\partial z \partial z} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}) + (\alpha + \alpha_1 \frac{\partial}{\partial t}) (\nabla^2 \psi - 2\omega_2), \tag{3.8}$$

$$m_{32} = [(\gamma + \varepsilon) + (\gamma_1 + \varepsilon_1)\frac{\partial}{\partial t}]\frac{\partial \omega_2}{\partial z} - \beta_o \frac{\partial \Phi}{\partial x}$$

$$\lambda_3 = \alpha_o \frac{\partial \Phi}{\partial z} + \frac{\beta_o}{3}\frac{\partial \omega_2}{\partial x} \tag{3.9}$$

In addition, we assume that ϕ, ψ, ω_2 and Φ tend to zero as $z \to \infty$.

4. SOLUTION OF THE PROBLEM.

We apply the double Fourier transform $\tilde{\tilde{f}}(k,z,s)$ of $f(x,z,t)$ defined by (Myint-U and Debnath, [13]) as

$$\tilde{\tilde{f}}(k,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int \exp\{i(kx + st)\}\, f(x,z,t)\, dxdt \tag{4.1}$$

and its inverse is given by

$$f(x,z,t) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} \exp[-i(kx+st)]\, \tilde{\tilde{f}}(k,z,s)dkds$$

Applications of this transforms to equations (3.1) - (3.4) gives

$$[\frac{d^2}{dz^2} - s_1^2]\, \tilde{\tilde{\phi}} = 0, \tag{4.2}$$

$$[(\frac{d^2}{dz^2} - s_2^2)\,(\frac{d^2}{dz^2} - s_3^2)]\,(\tilde{\tilde{\psi}}, \tilde{\tilde{\omega}}_2) = 0, \tag{4.3}$$

$$(\frac{d^2}{dz^2} - s_4^2)\,\tilde{\tilde{\Phi}} = 0, \tag{4.4}$$

where

$$s_1^2 = k^2 - \frac{s^2}{c_1^2 - isc_1'^2}, \tag{4.5}$$

$$s_2^2 + s_3^2 - 2k^2 = -\frac{s^2}{c_2^2 - isc_2'^2} + \frac{\gamma_2 - is\gamma_2' - s^2}{c_3^2 - is\, c_3'^2} - \frac{1}{2}\frac{(p_1 - is\, p_1')(\gamma_2 - is\gamma_2')}{(c_2^2 - is\, c_2'^2)(c_3^2 - is\, c_3'^2)} \tag{4.6}$$

$$(s_2^2 - k^2)\,(s_3^2 - k^2) = \frac{s^2[s^2 - (\gamma_2 - is\gamma_2')]}{(c_2^2 - is\, c_2'^2)\,(c_3^2 - is\, c_3'^2)}, \tag{4.7}$$

$$s_4^2 = k^2 + \gamma_3 - \frac{s^2}{c_4^2}. \tag{4.8}$$

The solutions of (4.2) - (4.4) with the boundary conditions at infinity are

$$\tilde{\tilde{\phi}} = A\, e^{-s_1 z}, \quad \tilde{\tilde{\psi}} = B\, e^{-s_2 z} + C\, e^{-s_3 z} \tag{4.9ab}$$

$$\tilde{\tilde{\omega}}_2 = B_1\, e^{-s_2 z} + C_1 e^{-s_3 z}, \quad \tilde{\tilde{\Phi}} = D\, e^{-s_4 z} \tag{4.10ab}$$

where it is assumed that $Re(s_j) > 0$ for $j = 1, 2, 3, 4$ and

$$B_1 = a_2 B, \; C_1 = a_3 C, \tag{4.11ab}$$

$$a_j = \frac{1}{p_1 - i s p_1'} [(c_2^2 - i s c_2'^2)(s_j^2 - k^2) + s^2], \; j = 2,3 \tag{4.12ab}$$

Applying the Fourier transforms (3.1) to the boundary conditions (3.6abcd) - (2.10) and combined with (4.9ab) - (4.10ab), we obtain

$$(A, B, C, D) = \{\frac{\Delta_1}{\Delta}, \frac{\Delta_2}{\Delta}, \frac{\Delta_3}{\Delta}, \frac{\Delta_4}{\Delta}\} \tilde{\tilde{g}}(k,s) \tag{4.13abcd}$$

where

$$\Delta = (r_3 t_4 - r_4 t_3)(q_1 m_2 - q_2 m_1) + (r_2 t_4 - r_4 t_2)(q_3 m_1 - q_1 m_3),$$

$$\Delta_1 = m_3(r_2 t_4 - r_4 t_2) - m_2(r_3 t_4 - r_4 t_3), \tag{4.14abcd}$$

$$\Delta_2 = m_1(r_3 t_4 - r_4 t_3), \; \Delta_3 = m_1 (r_4 t_2 - r_2 t_4),$$

$$\Delta_4 = m_1(r_2 t_3 - r_3 t_2).$$

$$q_j = \begin{cases} (\lambda^* + 2\mu^*) s_j^2 - \lambda k^{*2}, & j=1 \\ -2i\mu^* k s_j, & j=2,3 \end{cases}$$

$$m_j = \begin{cases} 2i\mu^* k s_j, & j=1 \\ \mu^*(k^2 + s_j^2) + \alpha^*(s_j^2 - k^2 - 2a_j) & j=2,3 \\ (\gamma^* + \varepsilon^*) s_j a_j, & j=2,3 \end{cases}$$

$$r_j = \begin{cases} i\beta_o k, & j=4 \\ i\beta_o k a_j, & j=2,3 \end{cases}$$

$$t_j = \begin{cases} 3\alpha_o s_j & j=4 \end{cases}$$

$$\lambda^* = \lambda - is\lambda_1, \; \mu^* = \mu - is\mu_1, \; \alpha^* = \alpha - is\alpha_1, \; \gamma^* = \gamma - is\gamma_1, \; \varepsilon^* = \varepsilon - is\varepsilon_1$$

Application of the inverse Fourier transformation to (4.9ab) - (4.10ab) gives

$$\phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A e^{-s_1 z - i(kx+st)} dk ds, \tag{4.19}$$

$$\psi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (B e^{-s_2 z} + C e^{-s_3 z}) e^{-i(kx+st)} dk ds, \tag{4.20}$$

$$\omega_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (a_2 B e^{-s_2 z} + a_3 C e^{-s_3 z})\, e^{-i(kx+st)} dkds, \tag{4.21}$$

$$\Phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D\, e^{-s_4 z - i(kx+st)} dkds. \tag{4.22}$$

Using (4.19) - (4.22), we obtain the formal solutions for the displacement components, microrotations, force stress, couple stress and vector first moment in the form

$$u_1 = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} F_1(z,k,s)\, \tilde{\tilde{g}}(k,s)\, e^{-i(kx + st)} dkds, \tag{4.23}$$

$$u_3 = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} F_2(z,k,s)\, \tilde{\tilde{g}}(k,s)\, e^{-i(kx + st)} dkds, \tag{4.24}$$

$$\omega_2 = \frac{1}{2\pi} \iint_{-\infty}^{\infty} F_3(z,k,s)\, \tilde{\tilde{g}}(k,s) e^{-i(kx + st)} dkds \tag{4.25}$$

$$t_{33} = \frac{1}{2\pi} \iint_{-\infty}^{\infty} F_4(z,k,s)\, \tilde{\tilde{g}}(k,s) e^{-i(ks + st)} dkds, \tag{4.26}$$

$$t_{31} = \frac{1}{2\pi} \iint_{-\infty}^{\infty} F_5(z,k,s)\, \tilde{\tilde{g}}(k,s) e^{-i(ks + st)} dkds, \tag{4.27}$$

$$m_{32} = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} F_6(z,k,s)\, \tilde{\tilde{g}}(k,s) e^{-i(kx + st)} dkds, \tag{4.28}$$

$$\lambda_3 = -\frac{1}{6\pi} \iint_{-\infty}^{\infty} F_7(z,k,s)\, \tilde{\tilde{g}}(k,,s) e^{-i(kx + st)} dkds, \tag{4.29}$$

where

$$F_1(z,k,s) = \frac{1}{\Delta} [ik\Delta_1 e^{-s_1 z} + s_2\Delta_2 e^{-s_2 z} + s_3\Delta_3 e^{-s_3 z}], \tag{4.30}$$

$$F_2(z,k,s) = \frac{1}{\Delta} [s_1\Delta_1 e^{-s_1 z} - ik(\Delta_2 e^{-s_2 z} + \Delta_3 e^{-s_3 z})] \tag{4.31}$$

$$F_3(z,k,s) = \frac{1}{\Delta} (a_2 \Delta_2 e^{-s_2 z} + a_3 \Delta_3 e^{-s_3 z}), \tag{4.32}$$

$$F_4(z,k,s) = \frac{1}{\Delta} [\{(\lambda^* + 2\mu^*)s_1^2 - \lambda^* k^2\} \Delta_1 e^{-s_1 z} - 2i\mu^* k(s_2\Delta_2 e^{-s_2 z} + s_3\Delta_3 e^{-s_3 z})] \tag{4.33}$$

$$F_5(z,k,s) = \frac{1}{\Delta} [2i\mu^* k\, s_1\Delta_1 e^{-s_1 z} + \{(\mu^* + \alpha^*)s_2^2 + (\mu^* - \alpha^*) k^2 - 2\alpha^* \alpha_2\}\Delta_2 e^{-s_2 z}$$
$$+\{(\mu^* + \alpha^*)s_3^2 + (\mu^* - \alpha^*)k^2 - 2\alpha^* a_3\} \Delta_3 e^{-s_3 z}], \tag{4.34}$$

$$F_6(z,k,s) = \frac{1}{\Delta} [(\gamma^* + \varepsilon^*) (a_2 s_2 \Delta_2 e^{-s_2 z} + a_3 s_3 \Delta_3 e^{-s_3 z}) - i\beta_o k\Delta_4 e^{-s_4 z}], \tag{4.35}$$

$$F_7(z,k,s) = \frac{1}{\Delta} [i\beta_o k(a_2\Delta_2 e^{-s_2 z} + a_3\Delta_3 e^{-s_3 z}) + \alpha_o s_4 \Delta_4 e^{-s_4 z}] \tag{4.36}$$

5. PARTICULAR SOLUTIONS OF INTEREST.

We first consider a time harmonic concentrated normal load in the form

$$g(x,t) = P\,\delta(x)\,e^{i\omega t} \tag{5.1}$$

where P is a constant and $\delta(x)$ is the Dirac delta function and ω is the frequency. The double Fourier transform of $g(x,t)$ is

$$\tilde{\tilde{g}}(k,s) = P\,\delta(s-\omega)$$

Consequently the solutions assume the form

$$u_1 = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[F_1(z,k,s)]_{s=\omega}\exp[i(kx+\omega t)]dk \tag{5.2}$$

$$u_3 = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[F_2(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.3}$$

$$\omega_2 = \frac{P}{2\pi}\int_{-\infty}^{\infty}[F_2(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.4}$$

$$t_{33} = \frac{P}{2\pi}\int_{-\infty}^{\infty}[F_4(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.5}$$

$$t_{31} = \frac{P}{2\pi}\int_{-\infty}^{\infty}[F_5(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.6}$$

$$m_{32} = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[F_6(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.7}$$

$$\lambda_3 = -\frac{P}{6\pi}\int_{-\infty}^{\infty}[F_7(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.8}$$

Neglecting the viscous effects, we obtain the corresponding results for the micropolar elastic half space with stretch

$$u_1 = -P\int_{-\infty}^{\infty}[G_1(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk, \tag{5.9}$$

$$u_3 = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[G_2(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.10}$$

$$\omega_2 = \frac{P}{2\pi}\int_{-\infty}^{\infty}[G_3(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk, \tag{5.11}$$

$$t_{33} = \frac{P}{2\pi}\int_{-\infty}^{\infty}[G_4(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk, \tag{5.12}$$

$$t_{31} = \frac{P}{2\pi}\int_{-\infty}^{\infty}[G_5(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk, \tag{5.13}$$

$$m_{32} = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[G_6(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk, \tag{5.14}$$

$$\lambda_3 = -\frac{P}{2\pi}\int_{-\infty}^{\infty}[G_7(z,k,s)]_{s=\omega}\exp[-i(kx+\omega t)]dk \tag{5.15}$$

where

$$G_1(z,k,s) = \frac{1}{\Delta_o}\,[ik\Delta_{11}\, e^{-s_{11}z} + s_{21}\Delta_{21}e^{-s_{21}z} + s_{31}\Delta_{31}e^{-s_{31}z}], \tag{5.16}$$

$$G_2(z,k;s) = \frac{1}{\Delta_o}\,[s_{11}\Delta_{11}e^{-s_{11}z} - ik(\Delta_{21}e^{-s_{21}z} + \Delta_{31}e^{-s_{31}z})], \tag{5.17}$$

$$G_3(z,k,s) = \frac{1}{\Delta_o}\,(a_{21}\Delta_{21}e^{-s_{21}z} + a_{31}\Delta_{31}e^{-s_{31}z}), \tag{5.18}$$

$$G_4(z,k,s) = \frac{1}{\Delta_o}\,[\{(\lambda + 2\mu)s_{11}^2 - \lambda k^2\}\,\Delta_{11}e^{-s_{11}z} - 2i\mu k(s_{21}\Delta_{21}e^{-s_{21}z} + s_{31}\Delta_{31}e^{-s_{31}z})], \tag{5.19}$$

$$G_5(z,k,s) = \frac{1}{\Delta_o}\,[2i\mu k s_{11}\Delta_{11}e^{-s_{11}z} + \{(\mu+\alpha)s_{21}^2 + (\mu-\alpha)k^2 - 2\alpha a_{21}\}\Delta_{21}e^{-s_{21}z} + \{(\mu+\alpha)s_{31}^2 + (\mu-\alpha)k^2 - 2\alpha a_{31}\}\Delta_{31}e^{-s_{31}z}], \tag{5.20}$$

$$G_6(z,k,s) = \frac{1}{\Delta_o}\,[(\gamma + \varepsilon)(a_{21}s_{21}\Delta_{21}e^{-s_{21}z} + a_{31}s_{31}e^{-s_{31}z}) - i\beta_o k\Delta_{41}e^{-s_4 z})], \tag{5.21}$$

$$G_7(z,k,s) = \frac{1}{\Delta_o}\,[i\beta_o k(a_{21}\Delta_{21}e^{-s_{21}z} + a_{31}\Delta_{31}e^{-s_{31}z}) + 3\alpha_o s_4\Delta_{41}e^{-s_4 z}], \tag{5.22}$$

$$s_{11}^2 = k^2 - \frac{s^2}{c_1^2}, \tag{5.23}$$

$$s_{21}^2 + s_{31}^2 - 2k^2 = -\left[s^2\left(\frac{1}{c_2^2} + \frac{1}{c_3^2}\right) - \frac{\gamma_2}{c_3^2}\left(1 - \frac{p_1}{2c_2^2}\right)\right], \tag{5.24}$$

$$s_{21}^2 - k^2)\,(s_{31}^2 - k^2) = \frac{s^2(s^2-\gamma_2)}{c_2^2\, c_3^2}, \tag{5.25}$$

$$a_{j1} = \frac{1}{p_1}\left(s_{j1}^2 + \frac{s^2}{c_2^2} - k^2\right), \qquad j = 2,3 \tag{5.26}$$

$$\Delta_o = (r_{31}t_4 - r_4 t_{31})(q_{11}m_{21} - q_{21}m_{11}) + (r_{21}t_4 - r_4 t_{21})(q_{31}m_1 - q_{11}m_{31}), \tag{5.27abcd}$$

$$\Delta_{11} = m_{31}(r_{21}t_4 - r_4 t_{21}) - m_{21}(r_{31}t_4 - r_4 t_{31})$$

$$\Delta_{21} = m_{11}(r_{31}t_4 - r_4 t_{31}),\quad \Delta_{31} = m_{11}(r_4 t_{21} - r_{21}t_4),$$

$$\Delta_{41} = m_{11})r_{21}t_{31} - r_{31}t_{21}),$$

$$q_{ji} = \begin{cases} (\lambda + 2\mu)s_{j1}^2 - \lambda k^2, & j=1 \\ -2i\mu k s_{j1}, & j=2,3 \end{cases} \tag{5.28ab}$$

$$m_{ji} = \begin{cases} 2i\mu k s_{j1}, & j=1 \\ (\mu+\alpha)s_{j1}^2 + (\mu-\alpha)k^2 - 2\alpha a_{j1}, & j=2,3 \end{cases} \tag{5.29ab}$$

$$r_{j1} = (\gamma + \varepsilon)a_{j1}\, s_{j1}, \qquad j=2,3 \tag{5.30}$$

$$t_{j1} = i\beta_o k a_{j1} \qquad j=2,3. \tag{5.31}$$

If we neglect both the viscous and stretch effects, we obtain the corresponding displacements, microrotation and stresses for the micorpolar elastic half space. For the sake of brevity, we avoid writing down these results.

We next consider the effects of the harmonic torque with its axis parallel to the z-axis in the form

$$g(x,t) = M[\delta(x-a) - \delta(x+a)]e^{-i\omega t} \tag{5.32}$$

where M is the magnitude of the torque. The double Fourier transform of (5.32) gives

$$\tilde{\bar{g}}(k,s) = 2iM \sin ka\ \delta(s-\omega) \tag{5.33}$$

Using this expression in the equations (4.23) - (4.29), we obtain

$$u_1 = -Q \int_{-\infty}^{\infty} [F_1(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.34}$$

$$u_3 = Q \int_{-\infty}^{\infty} [F_2(z,k,s)]_{s=\omega} \sin(ka)\ e^{-ikx} dk, \tag{5.35}$$

$$\omega_2 = Q \int_{-\infty}^{\infty} [F_3(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.36}$$

$$t_{33} = Q \int_{-\infty}^{\infty} [F_4(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.37}$$

$$t_{31} = Q \int_{-\infty}^{\infty} [F_5(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.38}$$

$$m_{32} = -Q \int_{-\infty}^{\infty} [F_6(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.39}$$

$$\lambda_3 = -\frac{Q}{3} \int_{-\infty}^{\infty} (F_7(z,k,s)]_{s=\omega} \sin(ka) e^{-ikx} dk, \tag{5.40}$$

where $$Q = \frac{iM}{\pi} e^{-i\omega t}. \tag{5.41}$$

These results are in excellent agreement with those for the cases without viscous and/or stretch effects which have been discussed by several authors including McCarthy and Eringen [9], Kumar and Chadha [6] Nowacki and Nowaki [2], and Acharya and Sengupta [4].

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