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On Linkage of a Flow Shop Scheduling Model Including Job Block Criteria with a Parallel Biserial Queue Network

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Abstract: This paper is an attempt to establish a linkage between a flowshop scheduling model having job block criteria with a parallel biserial queue network linked with a common channel in series. The arrival and service pattern both follows Poisson law in queue network. The generating function technique, law of calculus and statistical tools have been used to find out the various characteristics of queue. Further the completion time of jobs in a queue system form the set up time for the first machine in the scheduling model. A heuristic approach to find an optimal sequence of jobs with a job block criteria with minimum total flow time when the jobs are processed in a combined system with a queue network is discussed. The proposed method is easy to understand and also provide an important tool for the decision makers when the production is done in batches. A computer programme followed by a numerical illustration is given to justify the algorithm.

Keywords: Queue Network, Mean Queue length, Waiting time, Processing time, Job-block, Makespan, Biserial Channel

1. Introduction

In flowshop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machines will optimize some well defined criteria. Every job will go on these machines in a fixed order of machines. The research into flow shop problems has drawn a great attention in the last decades with the aim to increase the effectiveness of industrial production. In queuing theory the objective is to reduce the waiting time, completion time (Waiting Time + Service Time) and mean queue length of the customers / jobs which when processed through a queue network. A lot of research work has already been done in the fields of scheduling and queuing theory separately. The heuristic algorithms for two and three stage production schedule for minimizing the make span have been developed by Johnson (1954), Ignall & Scharge (1965) have also developed a branch & bound technique for minimizing the total flow time of jobs in n x 2 flow shop. Research is also directed towards the development of heuristic and near exact procedures. Some of the noteworthy heuristics approaches are due to Campbell (1970), Maggu and Das (1985),, Nawaz et al.(1983), Rajendran (1992), Singh.T.P.(1985) etc. Jackson (1954) studied the time dependent behavior of queuing system with phase system. O'brien (1954) analyzed the transient queue model comprised of two queues in series where the service parameter depends upon their queue length. Maggu (1970) studied a network of two queues in biseries and find the total waiting time of jobs / customers. Very few efforts have been made so far to combine the study of a production scheduling model with a queue network. Singh & Kumar (2007, 2008, 2009) made an attempt to combine a scheduling system with a queue-network.

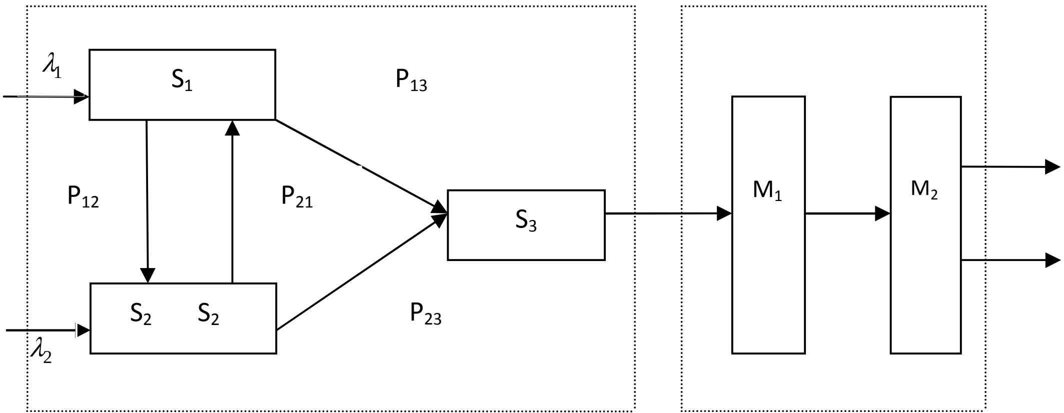
Kumar (2010) studied linkage of queue network with parallel biserial linked with a common channel to a flow shop scheduling model. This paper is an attempt to extend the study made by Kumar (2010) by introducing the idea of Job-Block criteria in scheduling model linked with a queue network having parallel biserial queues connected with a common channel. The basic concept of equivalent job for a job – block has been investigated by Maggu & Das (1977) and established an equivalent job-block theorem. The idea of job-block has practical significance to create a balance between a cost of providing priority in service to the customer and cost of giving service with non-priority. The decision maker may compare the total production cost in both cases and may decide how much to charge extra from the priority customers. The objective of the paper is to optimize two phases. In Phase one, the total Completion Time (Waiting time + Service time), Mean Queue length of the customers / jobs are optimized. In second phase by considering the completion time as setup time for the first machine, an optimal sequence of the customers / jobs in order to minimize the makespan including job block is obtained. Thus the problem discussed here is wider and practically more applicable and has significant results in the process industry.

2. Practical Situation

Many practical situation of the model arise in industries, administrative setup, banking system, computer networks, office management, Hospitals and Super market etc. For example, in a meal department of mall shop consisting two parallel biserial sections, one is for drinks and other is for food items and third section in series common to both is for billing. The customers taking drinks may also take some food items and vice-versa, then proceed for the billing. After billing the next two machines are for packing of the items and for the checking the bill and various items purchased. Here the concept of the job block is significant to give priority of one or more items with respect to others. It is because of urgency or demand of its relative importance.

Similarly in manufacturing industry, we can consider two parallel biserial channels one for cutting and other for turning. Some jobs after cutting may go to turning and vice-versa. Both these channels are commonly connected to the channel for chroming / polishing. After that the jobs has to pass thought two machine taken as inspection of quality of goods produced and second machine for the final packing. Here the concept of job block is significant in the sense to create a balance between the cost of providing priority in service to the customers / jobs and cost of giving services with non-priority customers / jobs. The decision maker may decide how much to charge extra from priority customers / jobs.

3. Mathematical Model



Phase I

Phase II

Figure 1: Linkage Model

Considered a queue network comprised of three service channels S_1, S_2 and S_3, where the channels S_1, S_2 are parallel biserial channels connected in series with a common channel S_3 and which is further linked with a flowshop scheduling system of n-jobs and 2-machines M_1 and M_2. The customers / jobs demanding service arrive in Poisson streams at the channels S_1 and S_2 with mean arrival rate λ_1, λ_2 and mean service rate μ_1 and μ_2 respectively. Let μ_3 be the mean service rate for the server S_3. Queues Q_1, Q_2 and Q_3 are said to be formed in front of the channels S_1, S_2 and S_3 respectively, if they are busy. Customers / Jobs coming at the rate λ_1 either go to the network of channels $S_1 \to S_2 \to S_3$ or $S_1 \to S_3$ with probabilities p_{12}, p_{13} such that $p_{12} + p_{13} = 1$ and those coming at rate λ_2 either goes to the network of the channels $S_2 \to S_1 \to S_3$ or $S_2 \to S_3$ with probabilities p_{21}, p_{23} such that $p_{21} + p_{23} = 1$.Further the completion time(waiting time + service time) of customers / jobs through Q_1, Q_2 & Q_3 form the setup times for machine M_1.

After coming out from the server S_3 .i.e. Phase I, customers / jobs go to the machines M_1 and M_2(in Phase II) for processing with processing time A_{i1} and A_{i2} in second Phase service. Our objective is to develop a heuristic algorithm to find an optimal sequence of the jobs / customers with minimum makespan in this Queue-Scheduling linkage system.

4. Assumptions

1. We assume that the arrival rate in the queue network follows Poisson distribution.
2. Each job / customer is processed on all the machines M_1, M_2, ------- in the same order and pre-emission is not allowed, .i.e. once a job is started on a machine, the process on that machine can not be stopped unless job is completed.
3. It is given to a sequence of k jobs i_1, i_2,--------, i_k as a block or group job in the order (i_1, i_2,-------, i_k) showing the priority of job i_1 over i_2.
4. For the existence of the steady state behaviour the following conditions hold good:

(i) $\rho_1 = \dfrac{(\lambda_1 + \lambda_2 p_{21})}{\mu_1(1 - p_{12}p_{21})} < 1$

(ii) $\rho_2 = \dfrac{(\lambda_2 + \lambda_1 p_{12})}{\mu_2(1 - p_{12}p_{21})} < 1$

(iii) $\rho_3 = \dfrac{p_{13}(\lambda_1 + \lambda_2 p_{21}) + p_{23}(\lambda_2 + \lambda_1 p_{12})}{\mu_2(1 - p_{12}p_{21})} < 1$.

5. Algorithm

The following algorithm provides the procedure to determine the optimal sequence of the jobs to minimize the flow time of machines M_1 and M_2 when the completion time (waiting time + service time) of the jobs coming out of Phase I is the setup times for the machine M_1.

Step 1: Find the mean queue length on the lines of Singh & Kumar (2005, 2006) using the formula

$$L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3}$$

Here $\rho_1 = \dfrac{(\lambda_1 + \lambda_2 p_{21})}{\mu_1(1 - p_{12}p_{21})}$, $\rho_2 = \dfrac{(\lambda_2 + \lambda_1 p_{12})}{\mu_2(1 - p_{12}p_{21})}$, $\rho_3 = \dfrac{p_{13}(\lambda_1 + \lambda_2 p_{21}) + p_{23}(\lambda_2 + \lambda_1 p_{12})}{\mu_2(1 - p_{12}p_{21})}$, λ_i is the mean arrival rates, μ_i is the mean service rates, p_{ij} are the probabilities.

Step 2: Find the average waiting time of the customers on the line of Little's (1961) using relation $E(w) = \dfrac{L}{\lambda}$, where $\lambda = \lambda_1 + \lambda_2$.

Step 3: Find the completion time(C) of jobs / customers coming out of Phase I, .i.e. when processed thought the network of queues Q_1, Q_2 and Q_3 by using the formula

$$C = E(W) + \frac{1}{\mu_1 p_{12} + \mu_1 p_{13} + \mu_2 p_{21} + \mu_2 p_{23} + \mu_3}.$$

Step 4: The completion time C of the customers / jobs through the network of queues Q_1, Q_2 and Q_3 will be the setup time for machine M_1. Define the two machines M_1, M_2 with processing time $A_{i1}^{'} = A_{i1} + C$ and A_{i2}.

Step 5: Find the processing time of job block $\beta = (k, m)$ on two machines M_1 and M_2 using equivalent job block theorem given by Maggu & Das (1977). Find $A_{\beta 1}$ and $A_{\beta 2}$ using

$$A_{\beta 1} = A'_{k1} + A'_{m1} - Min(A'_{m1}, A_{k2})$$
$$A_{\beta 2} = A_{k2} + A_{m2} - Min(A'_{m1}, A_{k2})$$

Step 6: Define a new reduced problem for machines M_1 and M_2 with processing time $A_{i1}^{'}$ and A_{i2} and replacing the job block (k, m) by a single equivalent job β with processing time $A_{\beta 1}$ and $A_{\beta 2}$.

Step 7: Apply Johnson's (1954) procedure to find the optimal sequence(s) with minimum elapsed time.

Step 8: Prepare In-Out tables for the optimal sequence(s) obtained in step 7. The sequence S_k having minimum total elapsed time will be the optimal sequence for the given problem.

6. Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
#include<math.h>

int n1[2],u[3],L[2];
int j[16],n;
float macha[16],machb[16],machc[16],maxv1[16];
float p[4];float r[3];
float g[16],h[16],a[16],b[16];float a1,b1,a2,b2,a3,b3,c1,c2,c3,P,Q,V,W,M;float q1,q2,q3,z,f,c;
void main()
{
        clrscr();
        int group[16];//variables to store two job blocks
        float minval, minv, maxv; float gbeta=0.0,hbeta=0.0;
        cout<<"Enter the number of customers and  Mean Arrival Rate for Channel S1:";
        cin>>n1[1]>>L[1];
        cout<<"\nEnter the number of customers and  Mean Arrival Rate for Channel S2:";
        cin>>n1[2]>>L[2];  n=n1[1]+n1[2];
        if(n<1 || n>15)
        {
                cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";
                getch();
                exit(0);
        }
```

```
for(int d=1;d<=3;d++)
	{
	cout<<"\nEnter the Mean Service Rate for the Channel S"<<d<<":"; cin>>u[d];
	}
for(int k=1;k<=4;k++)
	{
	cout<<"\nEnter the value of probability p"<<k<<":";cin>>p[k];
	}
for(int i=1;i<=n;i++)
	{
	j[i]=i;
	cout<<"\nEnter the processing time  of "<<i<<" job for machine A : ";cin>>a[i];
	cout<<"\nEnter the processing time  of "<<i<<" job for machine B : ";cin>>b[i];
	}
	a1=L[1]+L[2]*p[3];b1=(1-p[1]*p[3])*u[1];
	r[1]=a1/b1;a2=L[2]+L[1]*p[1];b2=(1-p[1]*p[3])*u[2];
	r[2]=a2/b2;b3=(L[1]+(L[2]*p[3]))*p[2]+(L[2]+(L[1]*p[1]))*p[4];c2=u[3]*(1-(p[1]*p[3]));
	c3=b3/c2;r[3]=c1+c3;M=L[1]+L[2];
	cout<<"r[1]\t\t"<<r[1]<<"\n";cout<<"r[2]\t\t"<<r[2]<<"\n";cout<<"r[3]\t\t"<<r[3]<<"\n";
	if(r[1],r[2],r[3]>1)
	{
		cout<<"Steady state condition does not holds good...\nExitting";
		getch();exit(0);
	}
	Q=(r[1]/(1-r[1]))+(r[2]/(1-r[2]))+(r[3]/(1-r[3]));
	cout<<"\nThe mean queue length is :"<<Q<<"\n";
	W=Q/M;
	cout<<"\nAverage waiting time for the customer is:"<<W<<"\n";
	z=u[1]*p[1]+u[1]*p[2]+u[2]*p[3]+u[2]*p[4]+u[3];
	f=1/z;c= W+f;
	cout<<"\n\nTotal completetion time of Jobs / Customers through Queue Network in Phase 1 :"<<c;
for(i=1;i<=n;i++)
	{
		g[i]=a[i]+c;h[i]=b[i];
	}
for(i=1;i<=n;i++)
	{
	cout<<"\n\n"<<j[i]<<"\t"<<g[i]<<"\t"<<h[i];cout<<endl;
	}
```

```
            cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):";
            cin>>group[0]>>group[1];
 //calculate G_Beta and H_Beta
if(g[group[1]]<h[group[0]])
            {
            minv=g[group[1]];
            }
else
            {
            minv=h[group[0]];
            }
            gbeta=g[group[0]]+g[group[1]]-minv;hbeta=h[group[0]]+h[group[1]]-minv;
            cout<<endl<<endl<<"G_Beta="<<gbeta;cout<<endl<<"H_Beta="<<hbeta;
int j1[16];int f=1;float g1[16],h1[16];
for(i=1;i<=n;i++)
{
if(j[i]==group[0]||j[i]==group[1])
{
f--;
}
else
{
j1[f]=j[i];
}
f++;
}
j1[n-1]=17;
for(i=1;i<=n-2;i++)
{
g1[i]=g[j1[i]];
h1[i]=h[j1[i]];
}
g1[n-1]=gbeta;
h1[n-1]=hbeta;
cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
{
cout<<j1[i]<<"\t"<<g1[i]<<"\t"<<h1[i]<<endl;
}
            float mingh[16];char ch[16];
```

```
  for(i=1;i<=n-1;i++)
  {
          if(g1[i]<h1[i])
                  {
                          mingh[i]=g1[i];ch[i]='g';
                  }
          else
                  {
                          mingh[i]=h1[i];ch[i]='h';
                  }

  }
          for(i=1;i<=n-1;i++)
          {
                  for(int j=1;j<=n-1;j++)
                  if(mingh[i]<mingh[j])
                  {
                  float temp=mingh[i]; int temp1=j1[i];
char d=ch[i];
                  mingh[i]=mingh[j]; j1[i]=j1[j]; ch[i]=ch[j];
                  mingh[j]=temp; j1[j]=temp1; ch[j]=d;
                  }
            }
           // calculate beta scheduling
float sbeta[16];
int t=1,s=0;
for(i=1;i<=n-1;i++)
          {
          if(ch[i]=='h')
          {
           sbeta[(n-s-1)]=j1[i];
           s++;
          }
else      if(ch[i]=='g')
          {
          sbeta[t]=j1[i];
          t++;
          }
          }
int arr1[16], m=1;
```

```
	cout<<endl<<endl<<"Job Scheduling:"<<"\t";
for(i=1;i<=n-1;i++)
	{
	if(sbeta[i]==17)
	{
	arr1[m]=group[0];
	arr1[m+1]=group[1];
	cout<<group[0]<<" "<<group[1]<<" ";
	m=m+2;
	continue;
	}
else
	{
	cout<<sbeta[i]<<" ";
	arr1[m]=sbeta[i];
	m++;
	}
	}
//calculating total computation sequence
	float time=0.0;
	macha[1]=time+g[arr1[1]];
for(i=2;i<=n;i++)
	{
	macha[i]=macha[i-1]+g[arr1[i]];
	}
	machb[1]=macha[1]+h[arr1[1]];
for(i=2;i<=n;i++)
	{
	if((machb[i-1])>(macha[i]))
	maxv1[i]=machb[i-1];
else
	maxv1[i]=macha[i];
machb[i]=maxv1[i]+h[arr1[i]];
	}

//displaying solution
cout<<"\n\n\n\n\n\t\t\t    #####THE SOLUTION##### ";
cout<<"\n\n\t*******************************************************************";
cout<<"\n\n\n\t    Optimal Sequence is : ";
for(i=1;i<=n;i++)
```



```
{
cout<<" "<<arr1[i];
}
cout<<endl<<endl<<"In-Out Table is:"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"\t"<<"Machine M2" <<"\t"<<endl;
cout<<arr1[1]<<"\t"<<time<<"--"<<macha[1]<<" \t"<<"\t"<<macha[1]<<"--"<<machb[1]<<" \t"<<endl;
for(i=2;i<=n;i++)
{
cout<<arr1[i]<<"\t"<<macha[i-1]<<"--"<<macha[i]<<""<<"\t"<<maxv1[i]<<"--"<<machb[i]<<"
"<<"\t"<<endl;
}
cout<<"\n\n\nTotal Elapsed Time (T) = "<<machb[n];
cout<<"\n\n\t*******************************************************************";
getch();
}
```

7. Numerical Illustration

Consider eight customers / jobs are processed through the network of three queues Q_1, Q_2 and Q_3 with the channels S_1, S_2 and S_3, where S_3 is commonly linked in series with each of the two parallel biserial channels S_1 and S_2. Let the number of the customers, mean arrival rate, mean service rate and associated probabilities are given as follows:

S. No	No. of Customers	Mean Arrival Rate	Mean Service Rate	Probabilities
1	$n_1 = 5$	$\lambda_1 = 5$	$\mu_1 = 12$	$p_{12} = 0.4$
2	$n_2 = 3$	$\lambda_2 = 4$	$\mu_2 = 9$	$p_{13} = 0.6$
			$\mu_3 = 10$	$p_{21} = 0.5$
				$p_{23} = 0.5$

Table 1

After completing the service at Phase I jobs / customers go to the machines M_1 and M_2 with processing time A_{i1} and A_{i2} respectively given as follows:

Jobs	1	2	3	4	5	6	7	8
Machine M_1 (A_{i1})	10	12	8	11	9	8	10	11
Machine M_2 (A_{i2})	13	12	10	9	8	12	7	8

Table 2

Further jobs 2 and 5 are processed as job block $\beta = (2, 5)$. The objective is to find an optimal sequence of the jobs / customers to minimize the makespan in this Queue-Scheduling linkage system by considering the first phase service into account.

Solution: We have

$$\rho_1 = \frac{\lambda_1 + \lambda_2 p_{21}}{(1 - p_{12} p_{21})\mu_1} = \frac{5 + 4 \times 0.5}{(1 - 0.4 \times 0.5)12} = \frac{7}{9.6}$$

$$\rho_2 = \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{12} p_{21})\mu_2} = \frac{4 + 5 \times 0.4}{(1 - 0.4 \times 0.5)9} = \frac{6}{7.2}$$

$$\rho_3 = \left[\frac{(\lambda_1+\lambda_2 p_{21})p_{13}+(\lambda_2+\lambda_1 p_{12})p_{23}}{\mu_3(1-p_{12}p_{21})}\right] = \frac{(5+4\times0.5)0.6+(4+5\times0.4)0.5}{11(1-0.4\times0.5)} = \frac{7.2}{8.8}.$$

Mean Queue Length = Average number of Jobs / Customers = $L = \frac{\rho_1}{1-\rho_1}+\frac{\rho_2}{1-\rho_2}+\frac{\rho_3}{1-\rho_3}$ = 12.2692

Average waiting time of the jobs / customers = $E(w) = \frac{L}{\lambda} = \frac{12.2692}{9} = 1.3632$.

The total completion time of Jobs / Customers when processed through queue network in Phase I

$$= C = E(W) + \frac{1}{\mu_1 p_{12}+\mu_1 p_{13}+\mu_2 p_{21}+\mu_2 p_{23}+\mu_3}.$$

$$= 1.3632 + \frac{1}{4.8+7.2+4.5+4.5+11} = 1.39445 \approx 1.39.$$

On taking the completion time C = 1.39 as the setup time, when jobs / customers came for processing with machine M_1. The new reduced two machine problem with processing times $A'_{i1} = A_{i1} + C$ and A_{i2} on machine M_1 and M_2 is as shown in table 3.

The processing times of equivalent job block β = (2, 5) by using Maggu & Das(1977) criteria are given by

$$A_{\beta1} = A'_{k1} + A'_{m1} - Min(A'_{m1}, A_{k2}) = 13.39$$

$$A_{\beta2} = A_{k2} + A_{m2} - Min(A'_{m1}, A_{k2}) = 9.61$$

The reduced two machine problem with processing times A'_{i1} and A_{i2} on machine M_1 and M_2 with jobs (2, 5) as a job-block β is as shown in table 3.

Using Johnson's (1954) algorithm, we get the optimal sequence(s)

S_1 = 3 – 6 – 1 - β - 4 – 8 – 7 = 3 – 6 – 1 – 2 – 5 - 4 – 8 – 7

S_2 = 6 – 3 – 1 - β - 4 – 8 – 7 = 6 – 3 – 1 – 2 – 5 - 4 – 8 – 7.

The In-Out flow table for the sequence S_1= 3 – 6 – 1 – 2 – 5 - 4 – 8 – 7 is as shown in table 4.

Therefore, the total elapsed time for sequence S_1 = 97.12 units.

The In-Out flow table for the sequence S_2= 6 – 3 – 1 – 2 – 5 - 4 – 8 – 7 is as shown in table 5.

Therefore the total elapsed time for sequence S_2= 97.12 = the total elapsed time for sequence S_1.

Therefore the optimal sequence(s) is S_1= 3 – 6 – 1 – 2 – 5 - 4 – 8 – 7 or S_2= 6 – 3 – 1 – 2 – 5 - 4 – 8 – 7.

8. Conclusion

The present paper is an attempt to study the queue network model combined with a two stage flowshop scheduling including job-block criteria with a common objective of minimizing the total elapsed time. A heuristic algorithm by considering the completion time of queuing network in Phase I as setup time for the first machine in Phase II. The concept of job-block is introduced to create a balance between the cost of providing priority in service to the customers / jobs and cost of giving services with non-priority customers / jobs. The study may further be extended by introducing various types of queuing models and different parameters for two and three stage flowshop scheduling models under different constraints.

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Tables

Table 3: The processing times A'_{i1} and A_{i2} on machine M_1 and M_2 is

Jobs	1	2	3	4	5	6	7	8
Machine M_1 (A'_{i1})	11.39	13.39	9.39	12.39	10.39	9.39	11.39	12.39
Machine M_2 (A_{i2})	13	12	10	9	8	12	7	8

Table 4: The In-Out flow table for the sequence S_1= 3 – 6 – 1 – 2 – 5 - 4 – 8 – 7 is

Jobs	Machine M_1	Machine M_2
3	0.0 – 9.39	9.39 – 19.39
6	9.39 – 18.78	19.39 – 31.39
1	18.78 – 30.17	31.39 – 44.39
2	30.17 – 43.56	44.39 – 56.39
5	43.56 – 53.95	56.39 – 64.39
4	53.95 – 66.34	66.34 – 75.34
8	66.34 – 78.73	78.73 – 86.73
7	78.73 – 90.12	90.12 – 97.12

Table 5: The In-Out flow table for the sequence S_2= 6 – 3 – 1 – 2 – 5 - 4 – 8 – 7 is

Jobs	Machine M_1	Machine M_2
6	0.0 – 9.39	9.39 – 21.39
3	9.39 – 18.78	21.39 – 31.39
1	18.78 – 30.17	31.39 – 44.39
2	30.17 – 43.56	44.39 – 56.39
5	43.56 – 53.95	56.39 – 64.39
4	53.95 – 66.34	66.34 – 75.34
8	66.34 – 78.73	78.73 – 86.73
7	78.73 – 90.12	90.12 – 97.12