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Czechoslovak Mathematical Journal, Vol. 43 (1993), No. 4, 743–751

Persistent URL: `http://dml.cz/dmlcz/128433`

Czechoslovak Mathematical Journal, 43 (118) 1993, Praha

ON LOCAL JOINT CAPACITIES OF OPERATORS

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(Received April 13, 1993)

Let $T = (T_1, \ldots, T_n)$ be an n-tuple of commuting operators in a Banach space X. Then the set of all $x \in X$ for which the local (Halmos-Stirling) capacity $\operatorname{cap}(T, x)$ is equal to the capacity $\operatorname{cap} T$ is dense in X. This generalizes the corresponding result for one operator [5].

Denote by $B(X)$ the algebra of all bounded operators in a Banach space X. Let $S \in B(X)$ and $x \in X$. The problem of describing the behaviour of all powers $S^n x$ (or all polynomials $p(S)x$) appears naturally in many questions of operator theory (e.g. local spectral theory or invariant subspace problem, cf. [1]).

The present paper was originally inspired by the paper of Halmos [2] and his notions of capacity in Banach algebras and quasialgebraic operators. He asked also whether every locally quasialgebraic operator is (globally) quasialgebraic, i.e. if there is a version of Kaplansky's theorem for quasialgebraic operators. An affirmative answer to this question was given in [4] and the result was improved in [5]. The present paper continues this study and generalizes the results for n-tuples of commuting operators.

Let $T = (T_1, \ldots, T_n)$ be an n-tuple of mutually commuting operators in a Banach space X.

We denote by $\sigma(T) \subset \mathbf{C}^n$ the Harte spectrum [3] of T, i.e. $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbf{C}^n$ does not belong to $\sigma(T)$ if and only if there exist operators $L_1, \ldots, L_n, R_1, \ldots, R_n \in B(X)$ such that

$$\sum_{i=1}^{n} L_i(T_i - \lambda_i) = I = \sum_{i=1}^{n} (T_i - \lambda_i) R_i.$$

Denote by $\sigma_e(T)$ the essential spectrum of T, i.e. the Harte spectrum of the commuting n-tuple $\pi(T) = (\pi(T_1), \ldots, \pi(T_n))$ in the Calkin algebra $B(X)|K(X)$, where

* The research was supported by the Alexander von Humboldt Foundation, Germany

$K(X)$ is the ideal of compact operators and $\pi \colon B(X) \to B(X)|K(X)$ is the canonical projection. We define formally $\sigma_e(T) = \emptyset$ for a commuting n-tuple T of operators in a finite-dimensional Banach space.

For an operator $S_1 \in B(X)$ denote by $r_e(S_1)$ the essential spectral radius of S_1, i.e. $r_e(S_1) = \max\{|\mu| \colon \mu \in \sigma_e(S_1)\}$.

Denote further by $\sigma_{\pi e}(T)$ the essential approximate point spectrum of T, i.e. $\lambda \in \sigma_{\pi e}(T)$ if and only if

$$\inf\Big\{\sum_{i=1}^{n} \|(T_i - \lambda_i)x\| \colon x \in M, \|x\| = 1\Big\} = 0$$

for every subspace $M \subset X$ of finite codimension.

We denote by $\mathscr{P}_r(n)$ the set of all polynomials in n variables with degree $\deg p \leqslant r$. Every $p \in \mathscr{P}_r(n)$ can be written in the form

$$p(z) = \sum_{|\alpha| \leqslant r} c_\alpha(p) z^\alpha$$

where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is an n-tuple of non-negative integers, $|\alpha| = \alpha_1 + \ldots + \alpha_n$, the coefficients $c_\alpha(p)$ are complex, $z = (z_1, \ldots, z_n) \in \mathbf{C}^n$ and $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$. If p is a polynomial in n variables and $K \subset \mathbf{C}^n$ a compact set then $\|p\|_K = \max\{|p(z)| \colon z \in K\}$. We say that a set $K \subset \mathbf{C}^n$ is algebraic if $p(K) \subset \{0\}$ for some non-zero polynomial p.

The first lemma uses the idea of extremal points of Fekete-Leja, see [8]. The authors are indebted to Professor J. Siciak for supplying the proof of it. Our proof is slightly modified.

Lemma 1. *Let n, r be positive integers and $K \subset \mathbf{C}^n$ a compact set. Then there exists a finite subset $K' \subset K$ with* card $K' = m \leqslant \binom{n+r}{n}$ *such that*

$$\|p\|_K \leqslant m \cdot \|p\|_{K'} \qquad (p \in \mathscr{P}_r(n)).$$

P r o o f. Denote by $L = \{p \in \mathscr{P}_r(n) \colon \|p\|_K = 0\}$ and let M be a complementary space of L in $\mathscr{P}_r(n)$, i.e. $M \cap L = \{0\}$ and $M + L = \mathscr{P}_r(n)$. Let $m = \dim M \leqslant \dim \mathscr{P}_r(n) = \binom{n+r}{n}$ and let $q_1, \ldots, q_m \in M$ be a basis of M. For $x_1, \ldots, x_m \in K$ denote by $V(x_1, \ldots, x_m) = \det(q_i(x_j))_{i,j=1}^m$. The polynomials $q_1, \ldots, q_m$ are linearly independent on K, so that there exist points $x_1, \ldots, x_m \in K$ such that the matrix $(q_i(x_j))_{i,j=1}^m$ is regular, i.e. $V(x_1, \ldots, x_m) \neq 0$. Let $k_1, \ldots, k_m \in K$ satisfy

$$|V(k_1, \ldots, k_m)| = \max\{|V(y_1, \ldots, y_m)| \colon y_1, \ldots, y_m \in K\}.$$

Then $V(k_1,\ldots,k_m)\neq 0$. For $j=1,\ldots,m$ define polynomials $L^{(j)}\in\mathscr{P}_r(n)$ by

$$L^{(j)}(z)=V\big(k_1,\ldots,k_{j-1},z,k_{j+1},\ldots,k_m\big)/V(k_1,\ldots,k_m).$$

Clearly $|L^{(j)}(z)|\leqslant 1$ for every $z\in K$. The polynomials $L^{(j)}$ are linear combinations of polynomials $q_1,\ldots,q_m$, so that $L^{(j)}\in M$ $(j=1,\ldots,m)$. Further $L^{(j)}(k_i)=\delta_{ij}$ (the Kronecker symbol), so that the polynomials $L^{(1)},\ldots,L^{(m)}$ are linearly independent and every polynomial $p\in M$ is a linear combination of them. Obviously

$$p(z)=\sum_{j=1}^{m}p(k_j)L^{(j)}(z)\qquad (p\in M, z\in K).$$

Set $K'=\{k_1,\ldots,k_m\}$. Every polynomial $p\in\mathscr{P}_r(n)$ can be written in the form $p=p_1+p_2$ for some $p_1\in L$ and $p_2\in M$, and $p_2=\sum\limits_{j=1}^{m}p_2(k_j)L^{(j)}$. Hence

$$\|p\|_K=\|p_2\|_K=\max\Big\{\Big|\sum_{j=1}^{m}p_2(k_j)L^{(j)}(z)\Big|: z\in K\Big\}\leqslant\sum_{j=1}^{m}|p_2(k_j)|\leqslant m\cdot\|p\|_{K'}.$$

□

Lemma 2. *Let E be a finite-dimensional subspace of an infinite dimensional Banach space X, let $\mathscr{M}$ be a finite-dimensional subspace of $B(X)$ and let $\varepsilon>0$. Then there exists a subspace $Z\subset X$ with* $\operatorname{codim} Z<\infty$ *such that*

$$\|T(e+z)\|\geqslant(1-\varepsilon)\max\{\|Te\|,\tfrac12\|Tz\|\}$$

for every $e\in E$, $z\in Z$ and $T\in\mathscr{M}$.

Proof. Let $T_1,\ldots,T_r$ be a basis in $\mathscr{M}$. Set $F=\bigvee\limits_{i=1}^{r}T_iE=\{Te\colon T\in\mathscr{M}, e\in E\}$. Clearly F is a finite-dimensional subspace of X. By [5], Lemma 1 there exists a subspace $Y\subset X$ with $\operatorname{codim}Y<\infty$ such that

$$\|f+y\|\geqslant(1-\varepsilon)\max\{\|f\|,\tfrac12\|y\|\}\qquad (f\in F, y\in Y).$$

Set $Z=\bigcap\limits_{i=1}^{r}T_i^{-1}Y$. As $\operatorname{codim}S^{-1}Y<\infty$ for every $S\in B(X)$, we have $\operatorname{codim}Z<\infty$. Let $e\in E$, $z\in Z$ and $T\in\mathscr{M}$. Then $Te\in F$ and $T_iz\in Y$ $(i=1,\ldots,r)$ so that $Tz\in Y$. Hence

$$\|T(e+z)\|\geqslant(1-\varepsilon)\max\{\|Te\|,\tfrac12\|Tz\|\}.$$

□

Lemma 3. *Let n, r be positive integers, let $T = (T_1, \dots, T_n)$ be an n-tuple of mutually commuting operators on a Banach space X such that $\sigma_e(T)$ is not algebraic. Let Y be a subspace of X with* $\operatorname{codim} Y < \infty$ *and let $\varepsilon > 0$. Then there exists $x \in Y$ such that $\|x\| = 1$ and*

$$\|p(T)x\| \geqslant \frac{1-\varepsilon}{2}\binom{n+r}{n}^{-2} r_e(p(T)) \qquad (p \in \mathscr{P}_r(n)).$$

Proof. Clearly X is infinite dimensional since $\sigma_e(T)$ is not algebraic.

Denote by $K = \sigma_{\pi e}(T)$. As the polynomially convex hulls of $\sigma_{\pi e}(T)$ and of $\sigma_e(T)$ coincide [7] and by the spectral mapping property for σ_e, we have, for every $p \in \mathscr{P}_r(n)$,

$$\|p\|_K = \max\{|p(z)| : z \in \sigma_{\pi e}(T)\} = \max\{|p(z)| : z \in \sigma_e(T)\} = r_e(p(T)).$$

Further $\|p\|_K \neq 0$ for $p \neq 0$ as the set $\sigma_e(T)$ is not algebraic. For a polynomial $p \in \mathscr{P}_r(n)$, $p = \sum\limits_{|\alpha| \leqslant r} c_\alpha(p) z^\alpha$ define a new norm by $|p| = \sum\limits_{|\alpha| \leqslant r} |c_\alpha(p)|$. The norms $|\cdot|$ and $\|\cdot\|_K$ are equivalent on $\mathscr{P}_r(n)$ so that there exists a positive constant c such that

$$|p| \leqslant c\|p\|_K \qquad (p \in \mathscr{P}_r(n)). \tag{1}$$

By Lemma 1 there exist elements $\lambda_1, \dots, \lambda_m \in K$, $m \leqslant \binom{n+r}{n}$ such that

$$\|p\|_K \leqslant m \cdot \max\{|p(\lambda_i)| : i = 1, \dots, m\} \qquad (p \in \mathscr{P}_r(n)). \tag{2}$$

We construct inductively points $x_1, \dots, x_m \in Y$. Suppose $x_1, \dots, x_k$ $(0 \leqslant k \leqslant m-1)$ are already found. Let E_k be the subspace generated by the vectors $x_1, \dots, x_k$ and let $\mathscr{M} = \{p(T) : p \in \mathscr{P}_r(n)\}$. By Lemma 2 there exists a subspace $Z_k \subset X$, $\operatorname{codim} Z_k < \infty$ such that

$$\|p(T)(e+z)\| \geqslant (1-\varepsilon')\max\{\|p(T)e\|, \tfrac{1}{2}\|p(T)z\|\} \quad (e \in E_k, z \in Z_k, p \in \mathscr{P}_r(n)) \tag{3}$$

where ε' is a positive number satisfying $\varepsilon' < 1$ and $(1-\varepsilon')^2(1 - m\varepsilon') \geqslant 1 - \varepsilon$.

Write $\lambda_{k+1} = (\lambda_{k+1,1}, \dots, \lambda_{k+1,n})$ and consider the subspace $W_k = Y \cap \bigcap\limits_{i=0}^{k} Z_i$. Clearly $\operatorname{codim} W_k < \infty$. By the definition of $\sigma_{\pi e}(T)$ we have

$$\inf\Big\{\sum_{i=1}^{n} \|(T_i - \lambda_{k+1,i})w\| : w \in W_k, \|w\| = 1\Big\} = 0$$

so that there exists $x_{k+1} \in W_k$, $\|x_{k+1}\| = 1$ such that

$$\|(T^\alpha - \lambda_{k+1}^\alpha)x_{k+1}\| \leqslant c^{-1}\varepsilon'$$

for every multiindex α, $|\alpha| \leqslant r$. Let $p = \sum_{|\alpha|\leqslant r} c_\alpha(p) z^\alpha \in \mathscr{P}_r(n)$. Then by (1),

$$\begin{aligned} \|(p(T) - p(\lambda_{k+1}))x_{k+1}\| &= \Big\| \sum_{|\alpha|\leqslant r} c_\alpha(p)(T^\alpha - \lambda_{k+1}^\alpha)x_{k+1} \Big\| \\ &\leqslant \sum_{|\alpha|\leqslant r} |c_\alpha(p)| \max\{\|(T^\alpha - \lambda_{k+1}^\alpha)x_{k+1}\| : |\alpha| \leqslant r\} \leqslant |p| \cdot c^{-1}\varepsilon' \leqslant \varepsilon' \|p\|_K. \end{aligned} \tag{4}$$

Suppose that we have found elements $x_1, \dots, x_m$ in this way. Set $x = a^{-1} \sum_{i=1}^m x_i$, where $a = \big\| \sum_{i=1}^m x_i \big\|$. Then

$$a \leqslant \sum_{i=1}^m \|x_i\| = m \qquad \text{and} \qquad a \geqslant (1 - \varepsilon')\|x_1\| = 1 - \varepsilon'$$

as $x_1 \in E_1$ and $x_2, \dots, x_m \in Z_1$. Clearly $x \in Y$ and $\|x\| = 1$. Let $p \in \mathscr{P}_r(n)$. Then, for $k = 1, \dots, m$, we have

$$\begin{aligned} \|p(T)x\| &= \Big\| a^{-1} \sum_{i=1}^m p(T)x_i \Big\| \geqslant (1 - \varepsilon')a^{-1} \Big\| \sum_{i=1}^k p(T)x_i \Big\| \geqslant \frac{1}{2}(1 - \varepsilon')^2 a^{-1} \|p(T)x_k\| \\ &\geqslant \frac{(1 - \varepsilon')^2}{2m} \Big(\|p(\lambda_k)x_k\| - \|(p(T) - p(\lambda_k))x_k\| \Big) \\ &\geqslant \frac{(1 - \varepsilon')^2}{2m} \Big(|p(\lambda_k)| - \varepsilon' \|p\|_K \Big) \end{aligned}$$

so that

$$\begin{aligned} \|p(T)x\| &\geqslant \frac{(1 - \varepsilon')^2}{2m} \Big(\max\{|p(\lambda_k)| : k = 1, \dots, m\} - \varepsilon' \|p\|_K \Big) \\ &\geqslant \frac{(1 - \varepsilon')^2}{2m} \|p\|_K (m^{-1} - \varepsilon') \\ &\geqslant \frac{1 - \varepsilon}{2m^2} \|p\|_K = \frac{1 - \varepsilon}{2m^2} r_e(p(T)). \end{aligned}$$

□

Theorem 4. *Let $T = (T_1, \dots, T_n)$ be an n-tuple of mutually commuting operators in a Banach space X such that $\sigma_e(T)$ is not algebraic, let $x \in X$ and $\varepsilon > 0$. Then there exists $y \in X$ and a constant $C = C(\varepsilon)$ such that $\|y - x\| < \varepsilon$ and*

$$\|p(T)y\| \geqslant C(1 + \deg p)^{-(2n+\varepsilon)} r_e(p(T))$$

for every polynomial p.

P r o o f. Find $k_0 \geqslant 1$ such that $\sum\limits_{i=k_0}^{\infty} \frac{1}{i^2} < \varepsilon$, $2^{k_0} \geqslant n$ and $k^2 \leqslant 2^{\varepsilon(k-1)}$ $(k \geqslant k_0)$. Denote by $C = \frac{1}{8k_0^2}(n+2^{k_0})^{-2n}$. Choose positive numbers ε_i $(i \geqslant k_0)$ such that $\varepsilon_i < 1$ and $\prod\limits_{i=k_0}^{\infty}(1-\varepsilon_i) \geqslant \frac{1}{2}$. We construct inductively points $y_{k_0}, y_{k_0+1}, \dots \in X$, $\|y_i\| = 1$. Suppose that $y_{k_0}, \dots, y_{k-1}$ are already given. Set $E_k = \bigvee\{x, y_{k_0}, \dots, y_{k-1}\}$. By Lemma 2 for $\mathcal{M} = \{p(T) \colon p \in \mathcal{P}_{2^k}(n)\}$ there exists a subspace $Z \subset X$ with $\operatorname{codim} Z < \infty$ such that

$$\|p(T)(e+z)\| \geqslant \left(1 - \frac{\varepsilon_k}{2}\right)\max\left\{\|p(T)e\|, \tfrac{1}{2}\|p(T)z\|\right\}$$

for every $e \in E_k$, $z \in Z$ and $p \in \mathcal{P}_{2^k}(n)$. By Lemma 3 there exists $y_k \in Z$ such that $\|y_k\| = 1$ and

$$\|p(T)y_k\| \geqslant \frac{1}{2}\left(1 - \frac{\varepsilon_k}{2}\right)\binom{n+2^k}{n}^{-2} r_e(p(T)) \qquad (p \in \mathcal{P}_{2^k}(n)).$$

Thus

$$\begin{aligned}\|p(T)(e+y_k)\| &\geqslant \left(1-\frac{\varepsilon_k}{2}\right)\max\left\{\|p(T)e\|, \frac{1}{4}\left(1-\frac{\varepsilon_k}{2}\right)\binom{n+2^k}{n}^{-2} r_e(p(T))\right\} \\ &\geqslant (1-\varepsilon_k)\max\left\{\|p(T)e\|, \frac{1}{4}\binom{n+2^k}{n}^{-2} r_e(p(T))\right\}\end{aligned} \tag{5}$$

for every $e \in E_k$ and $p \in \mathcal{P}_{2^k}(n)$).

Set $y = x + \sum\limits_{i=k_0}^{\infty} \frac{y_i}{i^2}$. Clearly $\|y - x\| \leqslant \sum\limits_{i=k_0}^{\infty} \frac{1}{i^2} < \varepsilon$. Let p be a polynomial of degree r. We distinguish two cases:

1) Let $r \leqslant 2^{k_0}$. Then, by (5), we have for $N \geqslant k_0$

$$\begin{aligned}&\left\|p(T)x + \sum_{i=k_0}^{N}\frac{1}{i^2}p(T)y_i\right\| \geqslant (1-\varepsilon_N)\left\|p(T)x + \sum_{i=k_0}^{N-1}\frac{1}{i^2}p(T)y_i\right\| \geqslant \dots \\ &\geqslant \prod_{i=k_0+1}^{N}(1-\varepsilon_i)\cdot\left\|p(T)x + \frac{1}{k_0^2}p(T)y_{k_0}\right\| \geqslant \prod_{i=k_0}^{N}(1-\varepsilon_i)\cdot\frac{1}{4k_0^2}\binom{n+2^{k_0}}{n}^{-2} r_e(p(T)) \\ &\geqslant \frac{1}{8k_0^2}(n+2^{k_0})^{-2n} r_e(p(T)) \geqslant C \cdot r_e(p(T)).\end{aligned}$$

2) Let $2^{k-1} < r \leqslant 2^k$ for some $k > k_0$. Then for $N \geqslant k$ we have

$$\begin{aligned}
\left\|p(T)x + \sum_{i=k_0}^{N} \frac{1}{i^2} p(T)y_i\right\| &\geqslant \prod_{i=k+1}^{N} (1-\varepsilon_i)\cdot \left\|p(T)x + \sum_{i=k_0}^{k} \frac{1}{i^2} p(T)y_i\right\| \\
&\geqslant \prod_{i=k}^{N}(1-\varepsilon_i)\cdot \frac{1}{4k^2}\binom{n+2^k}{n}^{-2} r_e(p(T)) \\
&\geqslant \frac{1}{8} 2^{-\varepsilon(k-1)}(n+2^k)^{-2n} r_e(p(T)) \\
&\geqslant \frac{1}{8} r^{-\varepsilon}(3r)^{-2n} r_e(p(T)) \\
&\geqslant Cr^{-(2n+\varepsilon)} r_e(p(T)).
\end{aligned}$$

So for every polynomial p we have

$$\|p(T)y\| = \lim_{N\to\infty}\left\|p(T)x + \sum_{i=k_0}^{N} \frac{1}{i^2} p(T)y_i\right\| \geqslant C(1+\deg p)^{-(2n+\varepsilon)} r_e(p(T)).$$

□

The notion of capacity for elements of a Banach algebra was introduced by Halmos [2] and extended to commuting n-tuples by Stirling [9].

Denote by $\mathscr{P}_k^1(n)$ the set of all polynomials $p(z) = \sum_{|\mu|\leqslant k} a_\mu(p) z^\mu \in \mathscr{P}_k(n)$ with $\sum_{|\mu|=k} |a_\mu(p)| = 1$. These polynomials were called monic in [9].

Let $T = (T_1, \ldots, T_n)$ be an n-tuple of mutually commuting operators in a Banach space X. The joint capacity of T was defined in [9] by

$$\operatorname{cap}(T) = \liminf_{k\to\infty} \operatorname{cap}_k(T)^{1/k}$$

where

$$\operatorname{cap}_k(T) = \inf\{\|p(T)\| : p \in \mathscr{P}_k^1(n)\}$$

(note that the liminf in the definition of cap T can be replaced by limit by [6]). For a compact subset $K \subset \mathbf{C}^n$ define the corresponding capacity by

$$\operatorname{cap} K = \liminf_{k\to\infty} (\operatorname{cap}_k K)^{1/k}$$

where

$$\operatorname{cap}_k K = \inf\{\|p\|_K : p \in \mathscr{P}_k^1(n)\}.$$

This capacity was studied in [10] and called the *homogeneous Tshebyshev constant* of a compact set K.

By [6] $\operatorname{cap} T = \operatorname{cap} \sigma(T) = \operatorname{cap} \sigma_e(T)$.

Let $T = (T_1, \dots, T_n) \in B(X)^n$ be a commuting n-tuple and let $x \in X$. We define the local capacity $\operatorname{cap}(T, x)$ by

$$\operatorname{cap}(T, x) = \liminf_{k \to \infty} \operatorname{cap}_k(T, x)^{1/k}$$

where

$$\operatorname{cap}_k(T, x) = \inf\{\|p(T)x\| : p \in \mathscr{P}_k^1(n)\}.$$

Clearly $\operatorname{cap}(T, x) \leqslant \operatorname{cap} T$ for every $x \in X$.

Theorem 5. *Let $T = (T_1, \dots, T_n)$ be an n-tuple of mutually commuting operators in a Banach space X. Then the set of all $y \in X$ with $\operatorname{cap}(T, y) = \operatorname{cap} T$ is dense in X.*

Proof. If $\sigma_e(T)$ is an algebraic set then $\operatorname{cap} \sigma_e(T) = 0$ so that $\operatorname{cap} T = 0$ and the assertion of Theorem 5 is satisfied trivially for every $y \in X$.

Suppose $\sigma_e(T)$ is not algebraic. Let $x \in X$ and $\varepsilon > 0$. Then there exists $y \in X$ with $\|y - x\| < \varepsilon$ and

$$\|p(T)y\| \geqslant C(1 + \deg p)^{-(2n+\varepsilon)} r_e(p(T))$$

for every polynomial p. Thus

$$\operatorname{cap}_k(T, y) = \inf\{\|p(T)y\| : p \in \mathscr{P}_k^1(n)\} \geqslant C(1 + k)^{-(2n+\varepsilon)} \inf\{r_e(p(T)) : p \in \mathscr{P}_k^1(n)\}$$

where

$$r_e(p(T)) = \sup\{|p(z)| : z \in \sigma_e(T)\}$$

so that

$$\operatorname{cap}_k(T, y) \geqslant C(1 + k)^{-(2n+\varepsilon)} \operatorname{cap}_k(\sigma_e(T)).$$

Hence

$$\operatorname{cap}(T, y) = \liminf_{k \to \infty} \operatorname{cap}_k(T, y))^{1/k} = \operatorname{cap}(\sigma_e(T)) = \operatorname{cap} T.$$

$\square$

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